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Emergency Service Vehicle Location Problem with Batch Arrival of Demands

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ABSTRACT

In this paper an emergency service vehicle (ESV) location problem has been considered in which it is assumed that each emergency call may require more than one ESV. In ESV location problem two factors should be known; the location of stations and the number of ESVs at each station. Hence, a nonlinear mixed integer programming model is presented in order to maximize the total response rate to the emergency calls. Moreover, a solution method based on genetic algorithm is provided and efficiency of the algorithm is evaluated with regard to the results from an exhaustive enumeration method. The model is applied to the real case study based on the data from Mashhad city to find the emergency gas stations and the required ESVs. Finally, a sensitivity analysis on the main parameters of the model is conducted and the managerial insights were reported. The results indicate that considering the fact that each call may require more than one ESV is very influential on the response rate and the assumption of each call requires just one ESV makes the results unrealistic.

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1. INTRODUCTION[1]

Every year a large number of people lose their lives due to accidents such as explosions caused by gas leaks, fires, etc. Although accidents are unavoidable, we can minimize their disastrous damages by taking preventive actions and proper planning such as locating fire brigades, emergency service stations, ambulances and police stations. Accordingly, ESV location problems are one of the most important decisions which influence the efficiency of relief operations. In relief operations, response time (time elapsed between an emergency call and its assistance) is a critical factor when deciding about emergency service system configurations. In particular, if the system cannot provide service immediately, injured people's lives will be endangered. Ideally, public expects immediately response in emergency situations; but this expectation is hard to be achieved regarding limited available resources such as vehicle and budget. Therefore, finding an optimal configuration for ESVs becomes an important issue [1].

Given demand zones and potential emergency service stations, two major decisions should be made in ESV location problem: 1) which potential stations should be opened as stations for ESVs and 2) how many ESVs should be placed in each station. ESV location model determines how to optimally coordinate and locate ESVs in order to reduce damages caused by accidents.

Generally, responding to an emergency accident is done through a call center. Call center receives emergency calls and gets information about the location and severity of corresponding accident. Then the required number of ESVs are dispatched from the nearest station. The response time to an emergency call from a specific demand zone is highly dependent to availability of the number of required ESVs which cover that zone. A demand zone can be covered if its distance/time to a station is equal or less that a predefined value. This value is called coverage distance or coverage radius [2]. Moreover, total number of ESVs, number of incoming calls and the required ESVs per call influence the availability of ESVs for responding to a call. So in this paper we aim to provide a model which determines the location of stations and

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the number of ESVs in each of them, simultaneously. The goal is to maximize emergency call coverage in all demand zones. Such model has been categorized in the maximum expected covering location problem (MEXCLP) model, which has been widely used to conveniently locate servers for general public services such as emergency medical services [3]. The common assumption in MEXCLP models is that each call requires just one ESV. However in reality, emergency calls may have different specifications and priorities that require different types and/or numbers of emergency services/vehicles [1]. Firefighting and gas emergency relief operations can be the examples in which more than one ESV would be required for relief operations. In this paper we develop a MEXCLP model that can assign more than one ESV to each call.

The remainder of this paper is organized as follows. Section 2 presents a brief review of the related published models and highlights the contributions of this research that positions our study in the existing literature. In section 3, problem definition is provided. In section 4, queuing system with batch arrival is applied to calculate the available probability of the ESVs in each station. Furthermore a non-linear mixed integer programming model for maximizing the expected covering is provided. In section 5, a genetic algorithm is presented and evaluated for solving the provided nonlinear model. The applicability of the proposed model is didcussed in section 6. The model is applied to gas emergency relief stations as a case study which arrives at important practical implications and managerial insights.. Section 7 concludes the paper with summary of the study and the results and guidance for future research.

2. LITERATURE REVIEW

The ESV location problem is an extension on Location Set Covering Problem (LSCP), which the first was proposed by Toregas et al. [4]. The LSCP is a compulsory coverage model and aims to find the minimum number of facilities to cover all demand zones. However, due to limited resources, full coverage is actually hard. In order to eliminate this defect, the Maximal Covering Location Problem (MCLP) was proposed by Church and ReVelle [5]. With a limited number of facilities, this model aims to maximize demand coverage. Chung et al. [6] proposed capacitated versions of the MCLP. They are considered only one fixed capacity level for all of the facilities sites. Yin and Mu [7] proposed modular capacitated maximal covering location problem (MCMCLP). In this model emergency facility sites could have different possible capacity levels due to varied numbers of stationed emergency vehicles. The LSCP and MCLP have a common

shortcoming; once a facility is busy, demand zones under its coverage are not covered. In the literature, there are two research areas to overcome this defect. One of them provides multiple coverage, such as the Double Standard Model (DSM), proposed by Gendreau et al. [8]. The DSM intends to allocate facilities among potential sites to provide a full coverage over a longer standard distance, while providing maximum coverage in a shorter distance. The objective function of the proposed model maximizes the demands that are met at least twice within shorter standard distance. Thus, demand areas are covered by two facilities as much as possible, so if their facilities are busy, they can provide service with the second facility. The second research area considered the busy probabilities and reliabilities of facilities which resulted in the Maximum Expected Covering Location Problem (MEXCLP) and the Maximum Availability Location Problem (MALP), proposed by Daskin [3] and ReVelle and Hogan [9], respectively. In the MEXCLP all facilities are assumed to have the same busy rate and operate independently. The objective function of this model is to maximize the expected coverage by a limited number of facilities. In MALPall facilities operate independently. The aim is to maximize the covered demands at a specified confidence level. Due to given confidence level and probability of engagement of facilities, the minimum number of facilities for servicing a demand zone can be determined. In the other words, the proposed model is going to maximize the demands that are satisfied with at least equal or greater facilities than the computed lower level for number of facilities. MALP models assume facilities are independent and the numbers of those are limited.

In all of these models, facilities are assumed to be independent and therefore busy probabilities are independent, but in reality, many of the demand zones are covered by several facilities. These facilities have preferences, so they cannot be assumed independent with the same priority. To eliminate the assumption of independence of facilities, Q-MALPand AMEXCLP are proposed by Marianov and ReVelle [10] and Batta et al. (1989) [11], respectively. Q-MALP is an extension of MALP. In this model lower level for the number of facilities is computed, using queuing theory. AMEXCLP embedded the hypercube queuing theory into the MEXCLP and studied the dependent busy probability using the correction factor derived in Larson [12]. Two assumptions of the basic hypercube model of Larson [12] are: (i) any server can travel (respond) to any call location and (ii) only one server is dispatched to a call defined by Iannoni et al. [13]. MEXCLP2 proposed by McLay [14] is the other model that uses hypercube. MEXCLP2 considered two types of facilities. Other extensions of hypercube models include Galvao et al. [15] and Toro-Díaz et al. [16]. A review

related to the use of hypercube queuing was defined by Galvao and Morabito [17].

Recently, Zhang et al. [18] employed uncertainty programming to cover uncertainty involved in LSCP and MCLP model. Unluyurt and Tuncer [19] used discrete event simulation to estimate the performance of emergency medical service location. Akdogan et al. [20] considered the location of ESVs in which multiple ESVs can be located in a single location with different service rates. They applied genetic algorithm to minimize mean response time of ESVs based on approximate queuing model. Belanger et al. [21] reviewed models and trends in location, relocation, and dispatching of emergency medical vehicles. Based on their reviews the main assumption in all papers is that each call requires just one vehicle. Souza et al. [22] and Rodrigues et al. [23] extended the hypercube model to analyze emergency medical services with different priorities in queues.

In this paper we introduced a mathematical model as an extension to MEXCLP with the following contributions:

1. The proposed model uses two radiuses to cover demand zones; minimum and maximum radiuses. The minimum radius equals to the standard time to meet urgent calls and the maximum radius equals to the standard time to respond ordinary calls. All demand zones should be covered within maximum radius and the number of them that are covered within minimum radius should be maximized as much as possible

2. More than one ESVs may be required for responding a call. The proposed model uses the queue system with batch arrival to dedicate more than one ESV to calls.

3. We consider both of complete and partial response. Partial response occurred when the number of available ESVs are less than the number of required.

4. The proposed model simultaneously determines the optimal location of the stations and the number of ESVs in each of them.

3. PROBLEM DEFINITION

There are a number of points which are potential locations for emergency service stations as well. In order to respond to calls, stations should be placed in such a way that all demand zones are covered in a specific time. Each demand zone has a stochastic number of emergency calls which should be responded by available ESVs in the stations. Each call includes a number of demands and one ESV is assigned to each demand. Two types of response are considered: emergency and ordinary. Accordingly two types of covering distances are defined, namely minimum and maximum covering distances. Minimum covering distance is based on the vital time required for servicing

an emergency call and the maximum covering distance is determined based on maximum acceptable time for servicing an ordinary call. Covering all demand zones within minimum covering distance is so costly that may cost more than the available budget, in addition emergency conditions often occurs occasionally. So, it is assumed that all demand zones should be covered in maximum covering distance. The objective is to maximize the expected immediately response to the emergency calls of demand zones which covered in minimum covering distance.

Each call may require more than one ESV. Accordingly, based on the number of available ESVs which cover specific demand zones, we may respond to a call completely or partially. Complete response occurs when the number of available ESVs is equal or greater than the number of required and partial response arises when the number of available ESVs is less than the number of required. In partial response all available ESVs are dispatched and it means that corresponding call is partially serviced. Due to importance of time in emergency response, each demand zone is allocated to its nearest emergency station. If there is no idle ESV, the call remains in queue until an ESV will be available. The calls in queue will be serviced in first in first out order. Other assumptions are as follows:

- The arrival rate of calls for each demand zone is known and arrives according to Poisson process.
- Call arrival rate of each station is equal to the summation of the arrival rate of calls for all demand zones in which this station is the nearest one for them.
- All ESVs are identical and the number of ESVs which assigned to a station is restricted due to space limitation.
- The service time of each ESV has exponential distribution.
- The number of ESVs required for a call is a random variable.

4. MATHEMATICAL MODELLING

This problem is in accordance with queuing system with batch arrival. A call, a demand and an ESV are batch, customer and server of a batch arrival queuing system, respectively. The following notations are required for calculating available probability of ESVs:

λ arrival rate of call

μ expected service time of an ESV

t number of ESVs required for a call, $t = 1, 2, \ldots, m$

p_t probability that a call requires t number of ESVs

E_m mean batch size, $E_m = \sum_{t=1}^{m} t p_t$

v number of ESVs which cover a demand zone

$\pi_{c,v}$ probability that c EVSs of v EVSs are busy

P   Occupation rate of the system, $\rho = \frac{E_m \lambda}{v\mu}$

In a queuing system with batch arrival $\pi_{c,v}$ is calculated according to following equations [1]:

$$\pi_{c,v} = q_c \pi_{0,v} \tag{1}$$

$$q_c = \frac{\alpha}{min(c,v)} \sum_{i=0}^{c-1} q_i \sum_{t=c-i}^{\infty} p_t \; ; \; c \geq 1 \tag{2}$$

where, $q_0 = 1$ and $\alpha = \frac{\lambda}{\mu}$. According to Equation (1), $\pi_{c,v}$ depends on $\pi_{0,v}$. By using the average number of available ESVs, $\pi_{0,v}$ can be obtained as follows:

$$v(1-\rho) = \sum_{c=0}^{v-1}(v-c)\pi_{c,v} \tag{3}$$

According to Equations (1) and **Error! Reference source not found.** we have:

$$v(1-\rho) = \sum_{c=0}^{v-1}(v-c)q_c\pi_{0,v} \tag{4}$$

Now $\pi_{0,v}$ can be calculated as follows:

$$\pi_{0,v} = \frac{v(1-\rho)}{\sum_{c=0}^{v-1}(v-c)q_c} \tag{5}$$

Now, expected immediate response to a call can be calculated based on available probability of ESVs. Assume that a call from a demand zones requires t ESVs while v ESVs placed in its nearest station. Expected immediate response can be calculated as follows:

$$Exresponse_{t,v} = \left[(100\%)\sum_{c=0}^{v-t}\pi_{c,v}\right] + \left[\sum_{c=1}^{t-1}(c/t) \times \pi_{v-c,v}\right] \tag{6}$$

In Equation (6), the first term calculates complete response. If there are at least t idle ESVs, all of required ESVs are dispatched and demands are responded completely. The probability that there are at least t idle ESVs is determined by $\sum_{c=0}^{v-t}\pi_{c,v}$. This probability is multiplied by 100% which means that 100% of demand of t ESVs is satisfied. The second term in Equation (6) stands for partial response. Partial response occurred when there are c idle ESVs in which $c<t$. In this case just c/t of the demand is satisfied. Multiplying this fraction into probability that exactly c ESVs are idle gives us expected immediate partial response.

    Now we can present a mathematical model to determine the location of stations and the number of ESVs in each station, simultaneously. The objective function of the model is to maximize the expected number of calls that are responded immediately within minimum covering distance. The following list summarizes the parameters used in the model:

I Set of demand zones,

J Set of potential locations for locating emergency stations,

i Index for demand zones

j Index for potential locations

R_u Maximum covering distance

R_l Minimum covering distance,

J_i Subset of locations which can cover demand zone i within minimum covering distance, $J_i = \{j \,|\, d_{ij} \leq R_l\}$

J'_i Subset of locations which can cover demandzone i within maximum covering distance, $J'_i = \{j \,|\, d_{ij} \leq R_u\}$

$T_{ij}^{R_u}$ Subset of locations which can cover demand zone i within maximum covering distance and located at least as close as j to i, $T_{ij}^{R_u} = \{k \,|\, k \in J_i, \, d_{ik} \leq d_{ij}\}$

N Total number of ESVs,

V_{max} Maximum number of ESVs which can be placed at a station,

m Maximum number of ESVs which are needed for responding a call,

p_t Probability that a call requires t ESVs , $t = \{1,2,\dots,m\}$

λ_i Arrival rate of calls from demand zone i,

λ_{it} Arrival rate of calls from demand zone i which need exactly t ESVs, $\lambda_{it} = p_t\lambda_i$

The variables in the model are as follows:

X_j Number of ESVs which is placed at location j

Y_{iv}^{Rl} 1, if exactly v ESVs cover demand zone i within minimum covering distance and 0 otherwise

F_j 1, if an emergency station places at location j and 0 otherwise

L_{ij}^{Rl} 1, if demand zone i is assigned to station j within minimum covering distance and 0 otherwise

L_{ij}^{Ru} 1, if demand zone i is assigned to station j within maximum covering distance and 0 otherwise,

$\pi_{c,v,j}$ Probability that c ESVs of v ESVs are busy at station j

λ'_j Arrival rate of calls to station j

Based on above parameter and variables non-linear mixed integer programming model for maximizing expected immediate response to calls with batch arrival of calls is presented as follows:

$$Max z = \sum_{i \in I} \sum_{j \in j'_i} \sum_{v=1}^{V_{max}} \sum_{t=1}^{m} L_{ij}^{Rl} Y_{iv}^{Rl} \lambda_{it} \left[\left(\sum_{c=0}^{v-t}\pi_{c,v,j}\right) + \left(\sum_{r=1}^{t-1}(r/t)\pi_{v-r,v,j}\right)\right] \tag{7}$$

s.t.

$$\sum_{j \in J} X_j \leq N \tag{8}$$

$$X_j \leq V_{max} \forall j \in J \tag{9}$$

$$\sum_{j \in J_i} L_{ij}^{Ru} = 1 \qquad \forall i \in I \tag{10}$$

$$L_{ij}^{R_u} \leq F_j \forall i \in I \,, j \in J \tag{11}$$

$$L_{ij}^{R_l} \leq L_{ij}^{R_u} \forall i \in I \,, j \in J'_i \tag{12}$$

$$\sum_{k \in T_{ij}^{R_u}} L_{ik}^{R_u} \geq F_j \ \forall i \in I \,, j \in J_i \tag{13}$$

$$\sum_{v=0}^{V_{max}} Y_{iv}^{R_l} = 1 \qquad\qquad \forall i \in I \tag{14}$$

$$\sum_{v=1}^{V_{max}} v Y_{iv}^{R_l} = \sum_{j \in J'_i} X_j L_{ij}^{R_l} \ \forall i \in I \tag{15}$$

$$\sum_{i \in I} \lambda_i L_{ij}^{R_u} = \lambda'_j \ \forall j \in J \tag{16}$$

$$E_m \lambda'_j < X_j \mu \forall j \in J \tag{17}$$

$$X_j \geq 0 \,, integer \forall j \in J \tag{18}$$

$$\lambda'_j \geq 0 \qquad\qquad \forall j \in J \tag{19}$$

$$F_j \in \{0,1\} \forall j \in J \tag{20}$$

$$Y_{iv}^{R_l} \in \{0,1\} \forall i \in I \,, v = 0,1,\dots,V_{max} \tag{21}$$

$$L_{ij}^{R_u} \in \{0,1\} \forall i \in I \,, j \in J_i \tag{22}$$

$$L_{ij}^{R_l} \in \{0,1\} \forall i \in I \,, j \in J'_i \tag{23}$$

The objective function definrd by constraint (7) maximizes expected immediate response to calls within minimum covering distance. Constraint (8) determines the total number of ESVs to be located. Constraint (9) limits the total number of ESVs located at each station. Constraint (10) ensures that each demand zone must be assigned to one station within maximum covering distance. Constraint (11) states the logical relationship between the location decisions and assignment of demand zones. Constraint (12) enforces that if a demand zone is assigned to a station within minimum covering distance, it should be assigned to the same station in maximum covering distance. Constraint (13) stands for assignment of demand zone to its nearest available station. Constraints (14) and (15) determine the exact number of ESVs that cover a demand zone. Constraint (16) calculates arrival rates for each station. Constraint (17) guarantees that queuing system stability condition is satisfied. Constraints (18)-(23) enforce restrictions on decision variables.

5. SOLUTION METHOD

The objective function of the presented model cannot be converted to the linear mode due to complexity of $\pi_{c,v,j}$. With known locations of stations, allocation of the demand zones to the stations based of nearest station is known and $\pi_{c,v,j}$ can be calculated. Then we have a linear model to determine the number of ESVs in each station. In this paper, genetic algorithm (GA) is used to locate stations. In GA, for each solution, corresponding

linear model has been solved using CPLEX to find the number of ESVs in each station.

5. 1. Solution Representation   Binary chromosome is used for solution representation. Gene with the value of "1" means that there is stations in the potential location, and "0" otherwise. Furthermore, the number of potential locations determines the number of genes in a chromosome. For example, Figure 1 shows a solution representation in which the length of the chromosome demonstrates that there are 10 candidate locations and binary values shows that we have four stations in locations 1, 4, 6 and 7, respectively.

5. 2. Mutation Operator   The mutation operator used in this paper randomly chooses a gene of a selected chromosome and change its value. To select chromosomes for mutation, a random number between 0 and 1 are assigned to all of chromosomes in solution pool. The selected chromosome is mutated if its assigned value is greater than mutation rate.

5. 3. Crossover Operation   The two point crossover is used in the proposed genetic algorithm where two distinct random integer numbers are generated. These two numbers will divide the selected chromosomes (parents) into three segments. Then the new chromosome is created by exchanging the first and third segment of the parents. Roulette wheel selection is used to select chromosomes for crossover. Figure 2 represents an example for two point crossover in which parents 1 and 2 are two parents which generate offspring1 and offspring 2.

5. 4. Population Initialization   To reduce solving time, the following lemmas are presented. These two lemmas define the upper and lower bounds for the number of stations, respectively.

Lemma 1: The minimum number of stations required to cover all of demand zones is equal to the number of stations obtained from solving the LSCP using the maximum coverage radius.

Proof: The objective function of the LSCP is to find the minimum number of facilities to cover all demand zones. In this research all of the demand zones should be covered within maximum coverage radius.
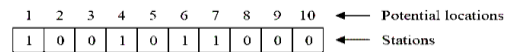

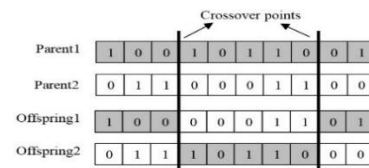**Figure 1.** Chromosome representation

Figure. 2. Two point crossover

Thus, the number of required stations cannot be less than the number of stations obtained from solving the LSCP using the maximum coverage radius.

Lemma 2: The maximum number of stations required to maximize the objective function is equal to the number of stations obtained from solving the LSCP using the minimum coverage radius.

Proof: The objective function of the model maximizes the average of calls that are responded immediately within a minimum coverage distance. The upper bound for the number of stations is equal to the number of stations obtained from solving the LSCP using the minimum coverage radius in which at least one ESV should be placed in each station. In case that the total number of ESVs obtained from solving the LSCP is less than the number of stations, the upper bound for the number of stations is equal to the total number of ESVs.

5. 5. Tuning the GA Parameters   A tuning procedure is carried out to find adequate values for parameters of the GA. There are three parameters that need to be set up: mutation rate, crossover rate and population size. We consider three levels for each parameter. Accordingly, 20, 60 and 100 are considered for population size, 0.6, 0.7 and 0.8 for crossover rate, and 0.01, 0.05 and 0.08 for mutation rate. 10 instances with different size have been established to set the parameters. Table 1 shows the rules for establishing instances regarding real world cases. Each instance is solved with all 27 combinations of GA parameters. Based on the gap percent between the best solution and the obtained solution the best values for crossover rate, mutation rate and population size are 0.7 , 0.08 and 20, respectively.

5. 6. Determine the Optimum Number of ESVs in Each Station   Each GA solution corresponds to a number of selected stations. For a solution of the GA, the following model determines the optimum number of vehicles in each station. The following list summarizes the notations.

TABLE 1. Rules of established instances

Parameters	Values
$|I|, |J|$	{10, 15, 20, 25, 30, 35, 40, 45, 50, 55}
x, y	U(0,2|I|)
λ_i	U(0,2)
λ_{i1}	$0.7 \lambda_i$
λ_{i2}	$0.2 \lambda_i$
λ_{i3}	$0.1 \lambda_i$
N	1.5|J|

J^{inst} Subset of selected stations in the GA solution,

J'^{inst}_i Subset of J^{inst} that can respond to the demand zone i within minimum coverage distance, i=1, 2, …, I;

$\pi_{c,v,i}$ Probability that c out of v ESVs in station i are busy when demand zone i is allocated, i = 1, 2,…, I.

k_i Index of nearest stations to demand zone i that cover i within minimum coverage distance, i=1, 2, …, I;

$$\begin{cases} k_i \in J'^{inst}_i , d_{ik_i} \le d_{ij} ; \forall j \in J'^{inst}_i , J'^{inst}_i \ne \emptyset \\ k_i \in (J \backslash J^{inst}) , d_{ik_i} \le d_{ij} ; \forall j \in (J \backslash J^{inst}) , J'^{inst}_i \ne \emptyset \end{cases}$$

The Linear Mixed Integer Programming (MIP) model to determine the optimal number of vehicles in a station is as follows (MODEL(II)):

$$Max z =$$

$$\sum_{i \in I} \sum_{v=1}^{V_{max}} \sum_{t=1}^{m} Y_{iv}^{R_i} \lambda_{it} \left[\left(\sum_{c=0}^{v-t} \pi_{c,v,i} \right) \right.$$
$$\left. + \left(\sum_{r=1}^{t-1} \left(r/t \right) \pi_{v-r,v,i} \right) \right] \tag{24}$$

s.t.

$$\sum_{j \in J^{inst}} X_j \le N \tag{25}$$

$$X_j \le V_{max} \qquad \forall j \in J^{inst} \tag{26}$$

$$\sum_{v=0}^{V_{max}} Y_{iv}^{R_i} = 1 \quad \forall i \in I \tag{27}$$

$$\sum_{v=1}^{V_{max}} v Y_{iv}^{R_i} = x_{k_i} \qquad \forall i \in I \tag{28}$$

$$X_j \ge 1 , integer \quad \forall j \in J^{inst} \tag{29}$$

$$X_j = 0 \quad \forall j \in \left(J \backslash J^{inst} \right) \tag{30}$$

$$Y_{iv}^{R_i} \in \{0,1\} \; \forall i \in I ,$$
$$v = 0,1,2,...,V_{max} \tag{31}$$

The objective function maximizes the average of calls that are responded immediately within a minimum coverage distance. Constraints (25-28) are similar to the Model (I). Constraints (29-31) ensure that the variables either binary or integer values.

5. 7. Computational Results   To evaluate the quality of the solutions obtained from the proposed GA, exhaustive enumeration method and the simplification method have been used. All of the instances are solved using ILOG CPLEX 12.3 on a computer with an Intel core i7 2.93 GHz processor with 3.49 GB of RAM.

5. 7. 1. Exhaustive Enumeration Method   In order to evaluate the GA performance, 10 instances have been generated based on Table 1. We run the GA five times in each instances and report the best objective. Results of GA have been compared with the one obtained from exhaustive enumeration in 2 hours (7200 seconds). Table 2 shows the results. In this table the column %GAP shows the gap percent between the objective functions obtained from GA and enumeration method. All of the gaps are less than or equal to zero. Negative values of the gap indicate that the GA obtained a better solution in shorter time.

5. 7. 2. Simplification Method   In this section, we have simplified the model by assuming that the maximum number of ESVs in each potential station is one and all of emergency calls only require one ESV. So each station is a M/M/1 queuing system. Then we have $\pi_{0,1,j} = 1 - \rho = 1 - \frac{\lambda_j'}{\mu}$ and the decision variables X_j and $Y_{iv}^{R_l}$ are eliminated the objective function is simplified as:

$$Max z = \sum_{i \in I} \sum_{j \in J_i'} L_{ij}^{R_l} \lambda_i \left(1 - \frac{\lambda_j^c}{\mu} \right) \tag{33}$$

In the simplified model, constraints (9), (14) and (15) are redundant constraints, constraint (8) convert to (34) and other constraints are the same as in Model (I).

$$\sum_{j \in J} F_j \leq N \tag{34}$$

The objective function of the simplified model is non-linear. $\alpha_{ij} = L_{ij}^{R_l} \lambda_j^c$ is proposed for linearization the objective function. Equation (35) with additional constraint (36-38) is the linear objective function.

$$\text{Max Z}$$

$$= \sum_{i \in I} \sum_{j \in J_i'} \lambda_i \left(L_{ij}^{R_l} - \frac{\alpha_{ij}}{\mu} \right) \tag{35}$$

$$\alpha_{ij} \geq \lambda_j' - M \left(1 - L_{ij}^{R_l} \right), \forall i \in I , j \in J_i' \tag{36}$$

$$\alpha_{ij} \leq ML_{ij}^{R_l} \quad \forall i \in I , j \in J_i' \tag{37}$$

$$\alpha_{ij} \geq 0 \quad \forall i \in I , j \in J' \tag{38}$$

The results of the GA and simplified model for different instances are shown in Table 3. CPLEX is used for obtaining the exact solution of the simplified model. In all instances GA provides a gap close to zero which means GA can provide exact or near exact solution for the problem.

6. ILLUSTRATIVE EXAMPLE

To evaluate the applicability of the proposed model, in this paper, locating the gas emergency stations in Mashhad is considered as a case study. There are 13 districts in Mashhad. Additionally each district is divided into several zones which constitutes 45 zones. The number of calls from each demand zone are directly related to the number of households. Historical data shows that the number of monthly calls at each zone are approximately equal to 2.5% of the number of its households. Each call may require one, two or three ESVs with probability of 0.7, 0.2 and 0.1, respectively. Minimum and maximum coverage radius are equal to 5 and 15 minutes, respectively. Service rate equals to 5 services per hour for each ESV. Total number of ESV is 15. According to *Lemma 1*, the minimum number of stations is equal to 8. Moreover, according to *Lemma 2*, the maximum number of stations is equal to 37.

TABLE 2. Comparison between complete enumeration method and genetic algorithm

Instance No.	|J|	Exhaustive Enumeration		GA		%GAP
		Best Objective	Time(s)	Best Objective	Time(s)	
1	10	5635.75	17	5635.75	463	0
2	15	8721.89	512	8721.89	475	0
3	20	13164.57	7200	13406.61	568	-1.84
4	25	14009.21	7200	14411	620	-2.87
5	30	17774.14	7200	18053.31	704	-1.57
6	35	19794.4	7200	20559.61	791	-3.87
7	40	22541.74	7200	23713.23	854	-5.20
8	45	24716.27	7200	25066.27	918	-1.42
9	50	29402.61	7200	30699.14	1013	-4.41
10	55	32914.67	7200	33556.57	1037	-1.95

Figure 3 illustrates the result of GA. 2 ESVs are located in station 7, and other stations have one ESV. The number of ESVs at each station is very low. In this situation 28.89% of calls are immediately responded within minimum covering distance. This value (28.89%) is called response rate and obtained bydividing the objective function value to total calls. This situation may be made because of the low number of available ESVs. Results of simplified model which is illustrated in Figure 4.

Within simplified model,43.72% of calls is immediately responded within shorter distance and configuration of stations are different from the previous situation. 14.83% difference in the level of immediate response between two situations is very important. Thus it can be concluded that the simplifying assumption caused a dramatic deviation from the real world situation.

TABLE 3. Comparison between exact solution and genetic algorithm

Instance No.	|J|	Exact	GA	%GAP
1	10	3205.01	3205.01	0
2	15	4697.35	4697.35	0
3	20	6879.82	6879.82	0
4	25	8513.21	8513.21	0
5	30	10034.65	10034.65	0
6	35	13367.51	13367.51	0
7	40	12955.27	12938.3	0.13
8	45	15625.49	15625.49	0
9	50	16976.09	16931.21	0.26
10	55	19854.83	19804.85	0.25
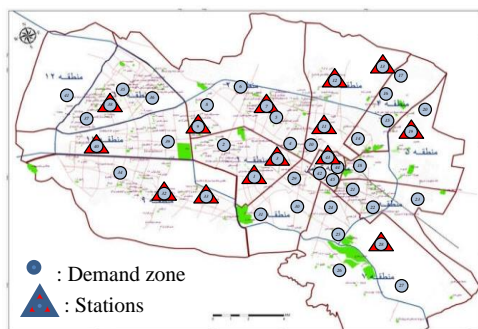
Figure 3. Result of GA
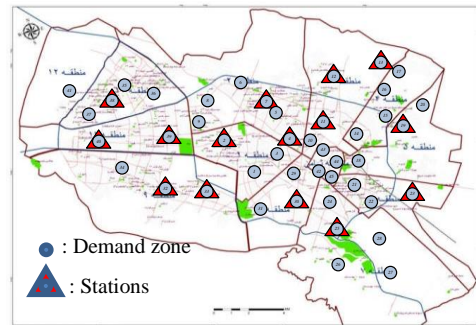
Figure 4. Result of GA when each call required one ESV

6. 1. Sensitivity Analysis of Effective Parameters
In this section we evaluate the impact of the number of ESVs, the arrival rate, and the service rate on the objective function value.

6. 1. 1. Impact of the Number of ESVs
Figure 5 shows the effect of the number of ESVs on response rate. Increasing the number of ESVs, improves the response rate. Regarding Figure 5, appropriate number of ESVs can be obtained according to the desired response rate. Figure 5 can be divided into three regions. In region (I), a gradual increase in the number of ESVs up to 50, can significantly improve the response rate up to 80%. Regarding region (II) there will be only 18% improve by increasing the number of ESVs from 50 to 100. One managerial insight is that increasing the number of ESVs in region (II) would be appropriate in case of existing extra budgets. Finally, as we can see in region (III), with 115 ESVs the response rate closes to 100%.

6. 1. 2. Impact of Service Rate
High quality equipment, better manpower utilization and other technical issues can reduce the service time and increase the response rate. Figure 6 shows the response rate by changing the service rate from 2 to 20 services per hour. By increasing the service rate from 5 to 10 services per hour, response rate increases from 29 to 40%. Moreover, response rate is %44.5 if service rate equal to 20 services per hour. It is clear that improving the service rate incurs more costs. So efforts to increase the service rate up to the 10 services per hour will be very effective.

6. 1. 3. Impact of Arrival Rate
This section evaluates the impact of various arrival rates on the response rate by changing the fraction of the number of households from 0.5 to %4. Figure 7 demonstrates the results.
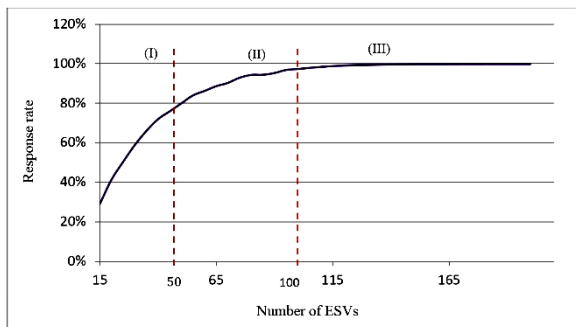
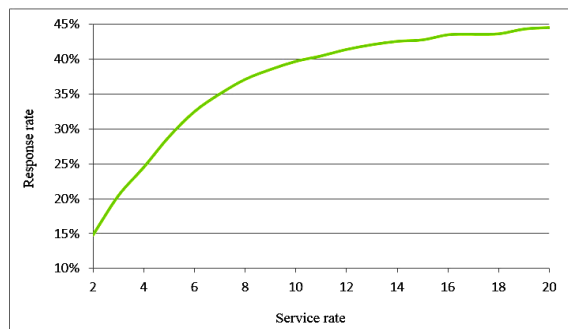**Figure 5**. Response rate as a function of number of ESVs

Figure 6. Response rate as a function of service rate

Obviously, response rate has a declining pattern by increasing the arrival rate. Figure 7 shows 20% increases in response rate by decreasing the fraction of the number of household from 2.5 to 0.5%. Moreover we have 6% decrease in response rate by increasing the fraction from 2.5 to 4%. The managerial insight is that decreasing emergency calls via increasing the community's awareness to follow safety requirement in using urban gas could increase service levels.
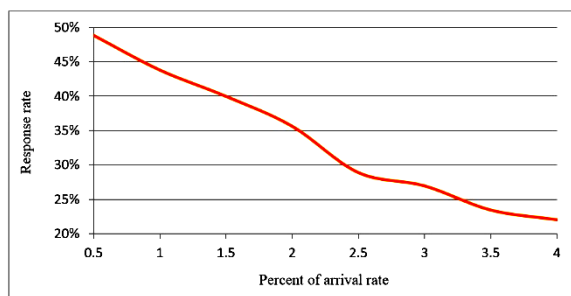

Figure 7. Response rate as a function of percent of arrival rate

7. CONCLUSION

In this paper an emergency vehicle locations problem has been considered. The underlying assumption in the literature is that each emergency call requires just one

ESV. However, this assumption contradicts what is happening in reality. For this reason, in this paper it is considered that in case of necessity it is possible to use more than one ESV for any call. In addition, in the proposed model, two radius of coverage are considered in which the minimum coverage distance related to the best response rate, and the larger one is related to the minimum service provision. The objective function is to maximize the response rate in the minimum coverage radius.

An exhaustive enumeration method has been used to solve the model. In this method, all possible combinations for station locations are considered, and then the proposed model for determining the number of ESVs of each station is optimally solved using CPLEX. Finally, the locations with the best objective function have been selected. However, despite the proposed lemmas to reduce the number of possible combination of stations location, it is not possible to get the optimal solutions for large instances at the satisfactory time. Therefore, a GA has been developed to solve the model. Computational experiments showed the efficiency of the provided GA algorithm. Finally in this paper, sensitivity analysis on the important parameters of the model have been conducted in Mashhad city as a case study. Results show that spending cost on providing more available ESVs, improving the service rates and the growing safety awareness of the society could have major effect on the level of response rate. This work can be extended by considering different types of ESVs and variable service rate.

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Emergency Service Vehicle Location Problem with Batch Arrival of Demands

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چکیده

در این مقاله جایابی خودروهای خدمات اضطراری در نظر گرفته شده است که در آن هر تماس اضطراری می‌تواند بیشتر از یک خودرو نیاز داشته باشد. در این مساله دو فاکتور باید مشخص شوند: مکان ایستگاه‌ها و تعداد خودرو در هر ایستگاه. به این منظور یک مدل برنامه‌ریزی عدد صحیح مختلط غیرخطی به منظور ماکزیمم کردن نرخ پاسخ به تماس‌های اضطراری ارائه می‌شود. علاوه بر این یک روش حل مبتنی بر الگوریتم ژنتیک ارائه شده و کارایی الگوریتم با مقایسه با یک الگوریتم شمارش کامل مقایسه ارزیابی خواهد شد. مدل برای یک مسئله واقعی مبتنی بر اطلاعات شهر مشهد پیاده شده که هدف آن پیدا کردن ایستگاه‌ها امداد گاز شهری و تعداد خودروی مورد نیاز هر ایستگاه است. در انتها تحلیل حساسیت روی پارامترهای اصلی مدل انجام شده و بینش‌های مدیریتی گزارش می‌شوند. نتایج نشان می‌دهد که در نظر گرفتن این فرض که هر تماس می‌تواند نیاز به بیش از یک خودرو داشته باشد روی نرخ پاسخ به تماس‌ها بسیار تاثیرگذار بوده و در نظر نگرفتن این فرض نتایج را غیرواقعی خواهد کرد.

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