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Finite Plate with Circular and Square Hole under Partial Loading

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ABSTRACT

In this paper a general analytical solution is obtained to find stress distribution in a finite elastic plate with a circular or square hole subjected to arbitrary biaxial partial loading using modified boundary condition by assuming plane stress conditions. The method employed is based on solution of circular hole in finite rectangular plate. This plate is mapped to circular ones and the partial loading is transformed to new boundaries as form of triangular functions. The Airy stress functions are selected according to these triangular functions and the unknown factors of Airy stress functions are derived by applying boundary conditions. The stresses in this plate with circular hole are mapped to plate with square hole using Muskhelishvili's complex variable method. The results of this method are compared with theoretical solution of infinite plate and finite elemnt method solution of finite plate. The results showed the dimensions of plate and square hole and length of biaxial partial loading affected on Von Mises stress around the square hole. Von Mises stress increases around the square hole by decreasing length of the plate or increasing hole's area.

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1. INTRODUCTION

Thin plates are used in marine, aerospace and building structures. Different types of holes are cut out of these plates to decrease the mass of structure or make the path of electrical cable and tube. These holes affect on stress distribution in the plate and stresses increase around the hole.

Many researchers have paid more attention to the problem of stress distribution around holes in an infinite elastic plate, and one of the most powerful methods for the problem is the complex variable method. Muskhelishvili [1] introduced stresses in the complex variable plane such as $\sigma_x - \sigma_y + 2i\sigma_{xy} = -4\,\partial^2/\partial\bar{z}^2$, $\sigma_x - \sigma_y = 4\partial^2/\partial z\partial\bar{z}$ so the stress in one plane can be mapped to another plane using $\emptyset$ as stress function and z=ω as mapping function. Gao [2] obtained the solution of an infinite isotropic plate with attraction-free elliptical hole subject to arbitrary biaxial loading by using complex potential method. Although, Savin [3] had studied the stress concentration around hole in an infinite medium under tension. Ukadgaonker and Rao [4] adapted the formulation given by Savin for stresses around holes in an isotropic plates under inplane loading and showed stress distribution around a given shape of hole depends on the combined effect of hole geometry, type of loading and laminate geometry. Vasil'ev and Fedorov [5] considered the plane problem of stress concentration near a circular hole in a thin unbounded plate loaded by normal and tangential stresses by relating the medium stress state to the geometry of the Riemannian space generated by the stresses. Kubair and Bhanu-Chandar [6] numerically investigated stress concentration factor due to a circular hole in functionally graded panels under uniaxial tension using the multiple is oparametric finite element formulation. Nageswara Rao et al. [7] found the stress distribution around holes in symmetric laminates Using Savin's basic solution for anisotropic plates. They studied square and rectangular holes in symmetric laminates of Graphite/epoxy and Glass/epoxy. It is noted that the type of loading mainly influences the maximum stress and its location. Mohammadi et al. [8] studied stress concentration around a hole in a radially inhomogeneous plate. The plate was infinite and subjected to uniform biaxial tension and pure shear. Najimi et al. [9] studied the preformatted plates made of

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functionally graded material (FGM) with and without a crack employing the finite element method (FEM). Louhghalam et al. [10] analysed the stress concentrations in plates with rectangular openings by a combined conformal mapping and Finite element approach. They proposed how coupling conformal mapping and Finite element method (FEM). Omidi and Suki [11] modelled the square AA5083-H116 aluminum plates as a finite plate with dimensions 300×300 having thickness of 3 mm and 5 mm under low velocity impact. Sharma [12] studied stress distribution around polygonal holes in an infinite plate using Muskhelishvili's complex variable method. The stress functions were obtained by evaluating Cauchy's integral for the given boundary conditions. Shahabian and Kazemi [13] experimentally investigated steel-plate girder web panels to partial loading indicates significant interaction between shear loading and patch loading. Khechai et al. [14] studied stress concentrations around circular hole in composite plate. Also compared different failure criteria. Grm and Batista [15] analysed stress around the polygonal hole shape in a finite plate by coupling of analytical and FEM solution. They correct the solution of infinite plate using FEM. Although expressing exact stress at the corner of polygon were impossible because of depending on number of meshes.

Pan et al. [16] studied stress distribution around a square hole in finite plate subjected uniaxial tension using Muskhelishvili's complex variable method. They validated their results with Savin's solution in an infinite plate and FEM in finite plate. They satisfied boundary condition using the boundary collocation method. Therefore, the accuracy of the results depend on place and count of collocation points on the boundaries. In addition, Jafari [17] used method like Pan's studying to analyzed stress around triangular hole in finite plate.

Above analytical methods are limited to study the stress analysis of an infinite plate with a hole and the last two reports satisfied boundary condition using collocation method that is based on count and collocation point on boundaries.

In this paper, the stresses around circular hole in finite plate are obtained using modified boundary condition. These stresses are mapped to finite plate with square hole.

2. PROBLEM

The boundary value problem in this paper is analyzing stress around square hole in finite plate as shown in 0 Two edges at $x = \pm a$ are subjected to σ_1 with length of 2L and another edges at $y = \pm b$ are subjected to σ_2 with length of 2M. the size of pale is $2a \times 2b$.

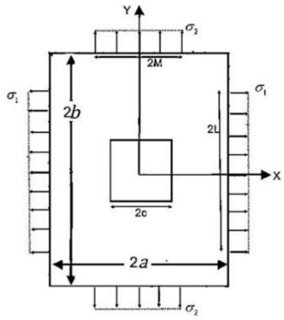


Figure 1. Finite plate with square hole

Three assumption are made in this analysis, i.e. (1) the 2D plane stress problem in the absence of body forces is studied, (2) the stress analysis is limited to the elastic range, and (3) the material is assumed linear elastic and isotropic.

3. SOLUTION PROCEDURE

Solution procedure has two main parts as follow:

(1) Analyzing stress in plate with unit circle cut out in the other words analyzing in the ζ plane

(2) Analyzing stress in plate with unit square cut out in the other words analyzing in the z plane.

The analytical solution is combined of Airy stress function [17] and Muskhelishvili's complex variable and mapping function methods. At first step z plane is mapped to ζ plane as 0. This mapping function is given in a general form [18] by:

$$z = \omega(\zeta) = R\left(\zeta + \sum_{k=1}^{N} \frac{m_k}{\zeta^k}\right) \quad (1)$$

where ω is mapping function from z plane to ζ plane. R is a constant that depend on hole's length and m_k are constants of mapping function that are equal to:

$$m_k = 0\left(k = 1,2,4,5,6,8,9,10\right)$$
$$m_3 = -\tfrac{1}{6}, m_7 = \tfrac{1}{56}, m_{11} = -\tfrac{1}{176} \quad (2)$$

In this paper, R=1.1786 for square hole is selected. This is an arbitrary constant.

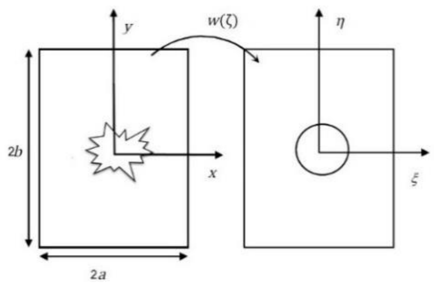


Figure 2. Mapping z plane to ζ plane

According to 0 if R=1.1786, the unit circle is mapped to square approximately with 2×2 dimensions and this is not affected on results. The different holes using various values of R are presented in 0.

Variable z $(z = x + iy)$ can be obtained using mapping function (1) and knowing that $\zeta = \xi + i\eta$ as follow:

$$x = Real\left[R\left(\zeta + \sum_{k=1}^{N}\frac{m_k}{\zeta^k}\right)\right]$$

$$y = \mathrm{Im}\left[R\left(\zeta + \sum_{k=1}^{N}\frac{m_k}{\zeta^k}\right)\right] \tag{3}$$

3. 1. Analyzing Stress in ζ Plane The main problem of 0 is mapped to plane $2a \times 2b$ with unit circle cut out as in 0.

3. 2. Replacing Rectangular Outer Boundary to Circle One The outer boundary is changed to a circle because the method of this paper is based on Airy stress function in polar coordinate. For example suppose a plate with circular hole subjected to prismatic compression loading σ_0 with length 2L on $x = \pm a$ as in 0.

A circular edge is drawn around outer boundary as 0.

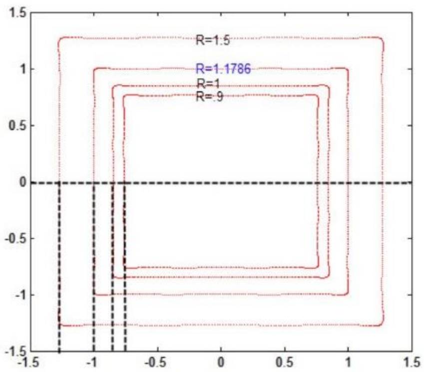


Figure 3. Results of mapping unit circle to square with varies R

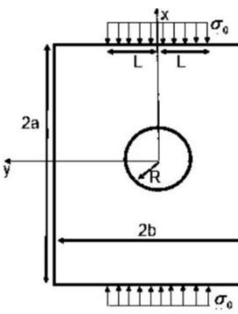


Figure 4. Unit circular hole in rectangular plate

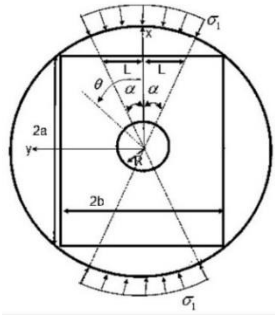


Figure 5. Replacing outer rectangular boundary to circle one

Resolving forces in the y-direction, we find:
On rectangular edge:

$$F_y = \sigma_0(2L) \tag{4-a}$$

On circular edge

$$F_y = \int_{-\alpha}^{\alpha}\sigma_1 R_1 \cos\theta d\theta = 2\sigma_1 R_1 \sin\alpha \tag{4-b}$$

where:

$$\alpha = \sin^{-1}\left(\frac{L}{\sqrt{L^2 + a^2}}\right), R_1 = \sqrt{a^2 + b^2} \tag{5}$$

By considering the equilibrium of the plate, the Equations (4-a) and (4-b) are equal. Therefore, σ_1 (the traction on circular edge) equals to:

$$\sigma_1 = \sqrt{\frac{L^2 + a^2}{a^2 + b^2}}\sigma_0 \tag{6-a}$$

In addition, in special condition where a=b=L equals to:

$$\sigma_1 = \sigma_0 \tag{6-b}$$

3. 3. Transform Tractions Into Appropriate Fourier Series The tractions on circular outer boundary is drawn in 0.
Tractions is transformed into appropriate Fourier series as follow

$$\sigma = \frac{2\alpha}{\pi}\sigma_1 + \sum_{n=1}^{\infty}\sigma_n \cos(n\theta) \tag{7}$$

where

$$\sigma_n = \begin{cases} 0 & \text{n odd} \\ \frac{4\sigma_1}{n\pi}\sin(n\alpha) & \text{n even} \end{cases} \tag{8}$$

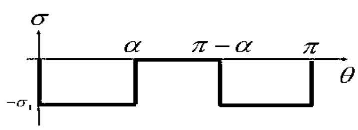


Figure 6. tractions on circular outer boundary

3. 4. Choosing Airy stress function By expanding $\sigma(\theta)$ as a Fourier series in polar coordinate, the Airy stress function can be selected by trial and error method to solve problem in 0 as follow:

$$\phi = A_0 r^2 + C_0 \ln r + D_0\theta + \left(A_n r^{n+2} + B_n r^{-n+2} + C_n r^n + D_n r^{-n}\right)\cos(n\theta) \tag{9}$$

where ϕ is Airy stress function and (r,θ) is polar coordinate in ζ plane and $A_0, C_0, D_0, A_n, B_n, C_n, D_n$ are unknown constant.

3. 5. Determining Boundary Condition The stress components are expressed in terms of orthogonal curvilinear coordinate system, i.e. σ_r, σ_θ and $\sigma_{r\theta}$. They are respectively radial, hoop and tangential stress components. In the Figure 5, the boundary conditions are given bellow:

$$\begin{aligned} &\sigma_r = \sigma_{r\theta} = 0 && r = R \text{ at} \\ &\sigma_r = \sigma_n \;,\; \sigma_{r\theta} = 0 && r = R_1 \text{ at} \end{aligned} \tag{10}$$

The stress component are derived from Equation (9) as given by:

$$\begin{aligned} \sigma_r = {} & 2A_0 + \frac{C_0}{r^2} \\ & + \left(-A_n(n+1)(n-2)\,r^n - B_n(n+2)(n-1)\,r^{-n}\right. \\ & \left. - C_n n(n-1)\,r^{(n-2)} - D_n n(n+1)\,r^{-(n+2)}\right)\cos(n\theta) \end{aligned} \tag{11}$$

$$\begin{aligned} \sigma_{r\theta} = {} & \frac{D_0}{r^2} \\ & + \left(A_n\, n(n+1)\,r^n - B_n n(n-1)\,r^{-n}\right. \\ & \left. + C_n n(n-1)\,r^{(n-2)} - D_n n(n+1)\,r^{-(n+2)}\right)\sin(n\theta) \end{aligned} \tag{12}$$

3. 6. Validation The hoop stresses around circular hole is derived from the present analytical solution and it is validated by finite element method using commercial software package ANSYS 13.0.

The circular plate with unit circle as in 0 is modeled in ANSYS by assuming a=b=L=1.78R and $\sigma_0 = 100$. The distribution of hoop stresses are shown in 0.

Unknown constant are derived using boundary condition at part (3.5.) and the hoop stresses are derived from Equation (9) as follow:

$$\begin{aligned} \sigma_\theta = {} & 2A_0 - \frac{C_0}{r^2} \\ & + \left(A_n(n+1)(n+2)\,r^n + B_n(n-2)(n-1)\,r^{-n}\right. \\ & \left. + C_n n(n-1)\,r^{(n-2)} + D_n n(n+1)\,r^{-(n+2)}\right)\cos(n\theta) \end{aligned} \tag{13}$$

This hoop stress is compared with FEM in Figure 8. The maximum error is 2.4%.

3. 7. Analyzing Stress in *z* Plane The plate with circular hole in ζ plane is mapped by mapping function ω to plate with square hole in z plane as in 0.

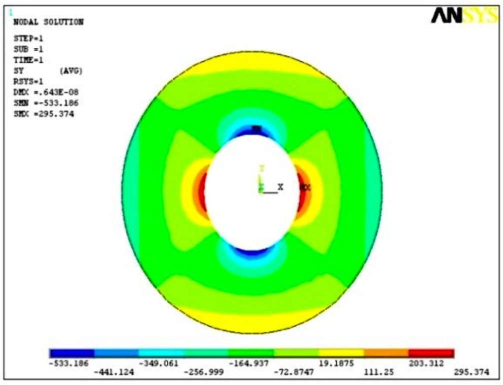


Figure 7. The hoop stresses in commercial software package ANSYS 13.0

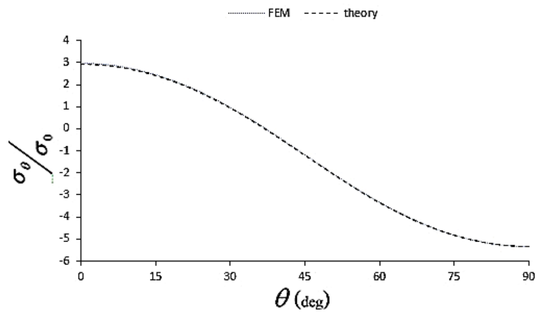
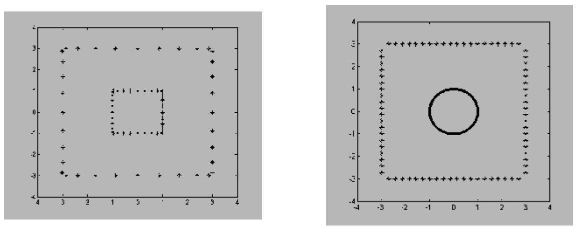


Figure 8. Comparing the hoop stresses are obtained by the present method and FEM



(b) (a)

$$\omega(\zeta) = \zeta + \frac{-1/6}{\zeta^3} + \frac{1/56}{\zeta^7} + \frac{-1/176}{\zeta^{11}}$$

Figure 9. Mapping ζ plane (a) to z plane (b)

In addition, the stresses in ζ plane can be mapped to z plane by mapping function.

3. 8. Mapping Stress In ζ Plane To z Plane The stresses can be define as function of airy stress function in complex variable in the both ζ and z plane as follow:

$$\begin{aligned} &\sigma_\xi - \sigma_\eta + 2i\,\sigma_{\xi\eta} = -4\frac{\partial^2\phi(\zeta)}{\partial\bar{\zeta}^2} \\ &\sigma_\xi + \sigma_\eta = 4\frac{\partial^2\phi(\zeta)}{\partial\zeta\,\partial\bar{\zeta}} \end{aligned} \tag{14}$$

$$\sigma_x - \sigma_y + 2i\,\sigma_{xy} = -4\frac{\partial^2 \phi}{\partial \bar{z}^2}$$
$$\sigma_x + \sigma_y = 4\frac{\partial^2 \phi}{\partial z\,\partial \bar{z}} \qquad (15)$$

where Equations (14) and (15) are expressing stress and $\phi(\zeta)$ and $\phi(z)$ are Airy stress functions in ζ and z plane respectively.

The Equation (15) can be related to Equation (14) Using the mapping function as follow:

$$\sigma_x - \sigma_y + 2i\,\sigma_{xy} = \frac{1}{\bar{\omega}'^2}\left(\sigma_\xi - \sigma_\eta + 2i\,\sigma_{\xi\eta}\right)$$
$$\sigma_x + \sigma_y = \frac{1}{|\omega'|^2}\left(\sigma_\xi + \sigma_\eta\right) \qquad (16)$$

while

$$\frac{\partial^2 \phi}{\partial \bar{z}^2} = \frac{\partial^2 \phi}{\partial \bar{\omega}^2} = \frac{1}{\bar{\omega}'^2}\frac{\partial^2 \phi}{\partial \bar{\zeta}^2} \qquad (17)$$

$$\frac{\partial^2 \phi}{\partial z\,\partial \bar{z}} = \frac{\partial^2 \phi}{\partial \omega \partial \bar{\omega}} = \frac{1}{|\omega'|^2}\frac{\partial^2 \phi}{\partial \zeta \partial \bar{\zeta}} \qquad (18)$$

The Equation (16) is changed to Equation (19) if the stresses in ζ plane are expressed in polar coordinate (r, θ) as follow:

$$\sigma_x - \sigma_y + 2i\,\sigma_{xy} = \frac{\zeta^2}{r^2\bar{\omega}'^2}\left(\sigma_r - \sigma_\theta + 2i\,\sigma_{r\theta}\right)$$
$$\sigma_x + \sigma_y = \frac{1}{|\omega'|^2}\left(\sigma_r + \sigma_\theta\right) \qquad (19)$$

where $\zeta = re^{i\theta}$. So the stresses are mapped to z plane using Equation (20).

$$\sigma_x = \frac{1}{2}\left[B + \mathrm{Re}(A)\right]$$
$$\sigma_y = \frac{1}{2}\left[B - \mathrm{Re}(A)\right] \qquad (20)$$
$$\sigma_{xy} = \mathrm{Im}(A)$$

where

$$A = \frac{\zeta^2}{\rho^2\bar{\omega}'^2}\left(\sigma_\rho - \sigma_t + 2i\,\sigma_{\rho t}\right)$$
$$B = \frac{1}{|\omega'|^2}\left(\sigma_\rho + \sigma_t\right) \qquad (21)$$

3. 9. Validation The hoop stresses around square hole is derived from the present analytical solution and is validated by finite element method using commercial software package ANSYS 13.0 and the result that are published by Savin [3]. In this part the model describe in 0 is solved using analytical solution that is presented by assumptions that a=b=L=2.5, R=1 and $\sigma_0 = 100$.

The geometrical parameters and boundary traction are mapped to z plane to modeling right hand side of 0 using FEM. The parameters are a=b=L=2.94, c=1 and $\sigma_1 = 70$. The rectangular edges are replaced by circular edges. hoop stresses are obtained as in 0 using FEM by dividing square hole edge to 629 part.

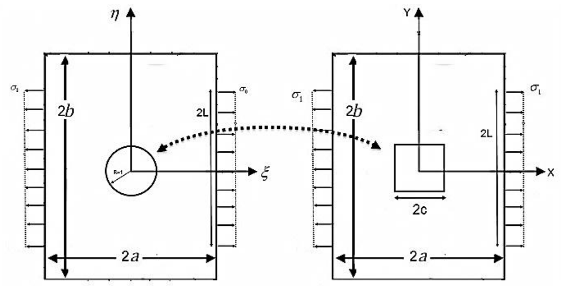


Figure 10. Validation mapping stress in $\boldsymbol{\zeta}$ plane to z plane

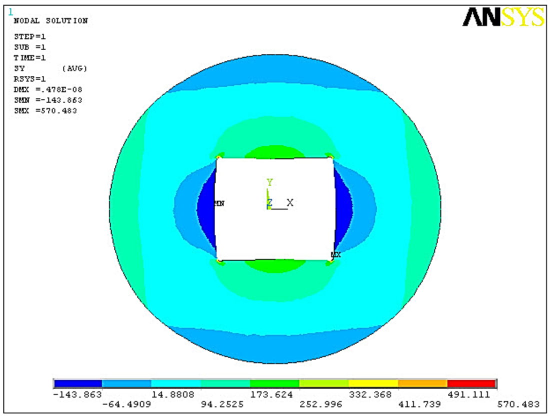


Figure 11. FEM modeling of right hand side of 0 10

The result of analytical solution that is presented in this part, FEM and Savin's results of plate in 0 are compared in Figure 12.

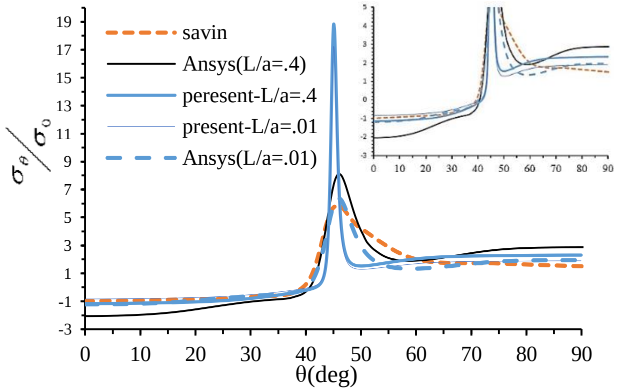


Figure 12. Normalise stress ($\boldsymbol{\sigma_\theta/\sigma_0}$) around square hole (finite and infinite plate) is subjected to uniaxial tension

4. RESULTS

4. 1. Results of Analysing Stress in ζ Plane The effect of hydrostatic pressure in circular hole on hoop stresses around circular hole is studied by assuming a=b=L=3R. The results are shown in 0. In this figure, "S_{in}" means normal stress at inner boundary of circular hole and parameter "S_{out}" means normal stress at outer boundary of plate.

As in Figure 13 Hoop stresses around circular hole increase as same as all angle position by increasing hydrostatic pressure. Therefore, it is interesting that by knowing the effect of hydrostatic pressure on one point at circular hole, it can be extended to another point on that.

4. 2. Results of Analysing Stress in z Plane In this section the effect of dimension of plate, square hole and partial loading in uniaxial and biaxial tension On Von Mises stresses around square hole are studied, because this stress is combination of all stresses at different direction.

The effect of length of partial loading on von Mises stress by assuming a=b=1.5c, $\sigma_1 = 1$ and $\sigma_2 = 0$ is studied as shown in 0Figure 14.

The Von Mises stress around square hole increase by increasing length of partial loading and this has a large effect on stress at corner point.

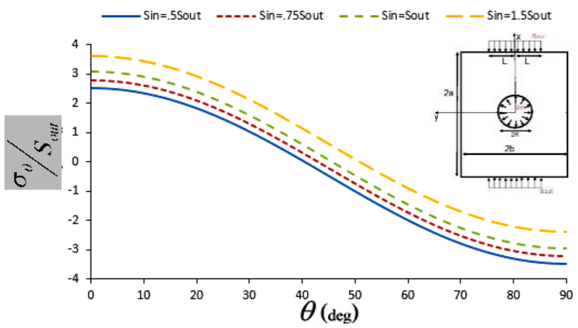


Figure 13. Effect of hydrostatic pressure in circular hole on hoop stress

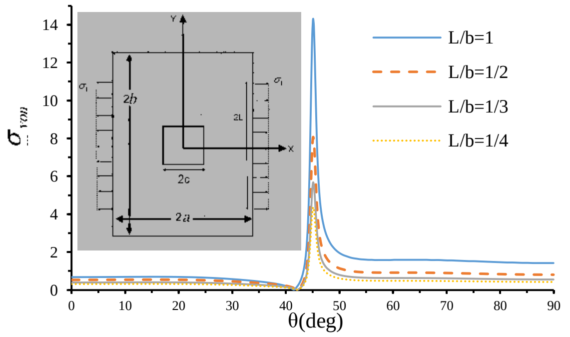


Figure 14. Effect of length of partial loading on von Mises stress around hole in uniaxial tension

Figure 15 Shows another attempt that is obtained by assuming a=b=1.5c and $\sigma_1 = \sigma_2 = 1$.

0Figure 15 shows the Von Mises stress at angle position greater than 45° increases by decreasing ratio of length of longitudinal tension to another one and this treatment changes at angle positions less than 45°. This means, the von Mises stress decreases by decreasing aspect ratio (M/L) but this is not correct when M/L=1/4.

The effect of the aspect ratio length of the edge to length of square hole (a/c in 0 Figure 1) On von Mises stress around square hole is studied as in Figure 16 by assuming a=b=L=M and $\sigma_1 = \sigma_2 = 1$.

The Von Mises stress decreases by increasing ratio a/c=1.5 to 5 and this stress do not vary when the ratio a/c is greater than five as infintine plate.

The effect of aspect ratio length of edges (b/a in 0Figure 1) On von Mises stress when the plate is subjected to biaxial loading is studied and the results are shown in Figure 170.

0Figure 17 shows the Von Mises stress increases by increasing the aspect ratio b/a at angle position between 0° to 45° but this stress decreases at angels greater than this range. This treatment changes at ratio b/a=3 as is shown in this figure.

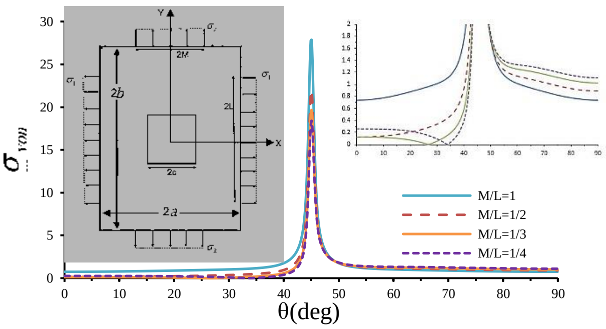


Figure 15. Effect of length of partial loading on von Mises stress around square hole in biaxial tension

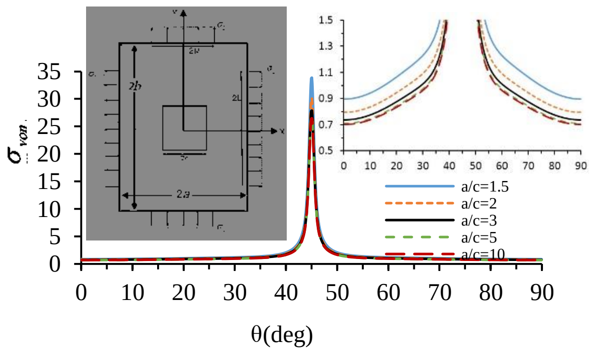


Figure 16. Effect aspect ratio (a/c) on von Mises stress around square hole in biaxial tension a=b=L=M

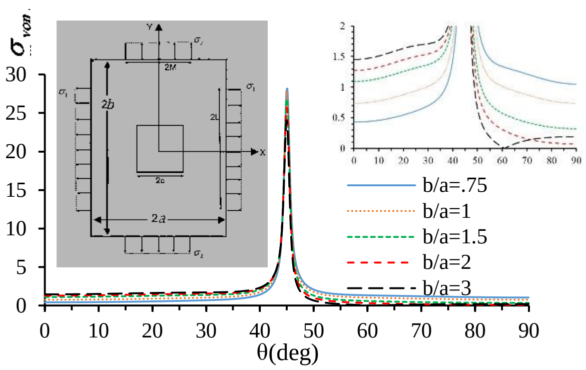


Figure 17. Effect aspect ratio (b/a) on von Mises stress around square hole in biaxial tension a=L, b=M, a/c=3

5. CONCLUSION

In this paper stress distribution around square hole in a finite plate that is subjected to biaxial tension is studied. The method is based on Airy stress function and modified boundary condition. The results are compared with FEM and Savin's results.

The results of plate with circular hole shows the length of plate effects on stress around hole and the hoop stress around circular hole increases by increasing hydrostatic pressure around the boundary of the hole.

Von Mises Stresses increase around square hole at angle position greater than 45° by decreasing the ratio M/L (length of traverse to longitudinal partial loading). Von Mises Stresses decrease around square hole in biaxial loading by increasing the ratio a/c (the length of plate's edge to length of square hole's edge) but this stress increases at angle position in the range 0° to 25° and 65° to 90° when the length of partial loading is half of the edges of plate. Von Mises Stresses increase around square hole at angle position less than 45° by the decreasing ratio b/a.

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Finite Plate with Circular and Square Hole under Partial Loading

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چکیده

در این مقاله تنش اطراف سوراخ مربع شکل در ورق با اندازه محدود تحت بار مکانیکی پاره‌ای در راستای طول و عرض این ورق با تغییر مرزهای ورق مستطیلی به منحنی شکل و فرض تنش صفحه ای به روش تحلیلی بررسی شده‌است. در روش به‌کاررفته، از حل صفحه محدود با سوراخ دایره‌ای در مرکز آن به عنوان مرجع استفاده‌شده است. مرزهای قائم ورق به منحنی تبدیل شده و بار مکانیکی وارده به بخشی یا تمام مرز قائم ورق به شکل توابع مثلثاتی تبدیل گردیده و تابع تنش ایری متناظر با آن انتخاب شده‌است. با اعمال شرایط مرزی، ضرایب مجهول تابع تنش ایری به دست آمده‌اند و تنش‌های اطراف سوراخ دایره‌ای در ورق محدود محاسبه شده است. به کمک روش توابع مختلط، توزیع تنش به دست آمده در این صفحه محدود با سوراخ دایره‌ای به صفحه محدود شامل گشودگی مربعی انتقال یافته است. نتایج حاصل از این روش با حل تئوری ورق با اندازه بینهایت و حل اجزا محدود ورق محدود مقایسه شده است. در قسمت نتایج تأثیر پارامترهای مؤثر بر تنش شامل طول اضلاع ورق، اندازه سوراخ بررسی گردیده و نشان داده شده‌است که با کاهش اندازه ابعاد ورق یا افزایش قطر سوراخ، مقدار ماکسیمم تنش ونمایسز اطراف مرز آن افزایش می‌یابد.

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