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Economic-statistical Design of NP Control Chart with Variable Sample Size and Sampling Interval

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ABSTRACT

The control charts are graphical tools and proven techniques to improve the performance of a process. Usually, the processes are not naturally controlled, so the use of control charts will help to reduce the variability and increase the stability of the process. In the traditional approach, control charts with fix sample size and constant sampling intervals were used to identify the changes in the process. While, control charts can be examined from different statistical and economical directions using different sampling schemes for achieving a better result. In past studies, it has been shown that *np* control charts show better results in detecting shifts by using variable sampling schemes but it is also important to consider how the cost of using variable sampling schemes will be, since the cost of a process depends on the parameters of the control chart. In this paper, the economic-statistical design of *np* control chart with variable sample size and sampling interval *(VSSI)* is formulated and then we compare and analyze the resulted obtained with other schemes. Results show significant improvement in terms of economic and statistical performance.

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1. INTRODUCTION

Shewhart traditional control charts for the first time was presented in 1924 and then in 1931, *X-bar* control chart was established to monitor the mean of process [1]. In order to design control charts, it is necessary to determine parameters such as sample size *(n)*, sampling interval *(h)* and control limits. In traditional control charts, equal sample size at a fixed sampling interval *(FSI)* have been used to monitor a process, but this scheme has low efficiency to detect small and medium shifts in the process mean.

Due to the weakness of Shewhart control charts in detecting small shifts and in order to improve the performance control charts, the idea of variable sample size (VSS), variable sampling interval (VSI) and variable sample size and sampling interval (VSSI) was introduced. For the first time, Reynolds et al. [2] in 1988 proposed the idea of variable sampling interval for X-bar charts. In VSI scheme, two different sampling intervals h1 and h2 ($h_1>h_2$) with a fixed sample size was implemented. When a sample point falls into the relax area, the long sampling interval h1 is used for the next sample and also if the sample point falls within the warning area, then the short sampling interval h2 is used for the next sample. After that, many studies were conducted to evaluate the effectiveness and power of this scheme in detecting the shifts in comparison with traditional scheme. For example, we can refer to Reynolds et al. [3], Vaughan [4], Liu et al. [5] and Faraz et al. [6].

Another approach is VSS that was presented at the first time by Prabhu et al. [7] in 1993. Unlike the VSI scheme, in VSS scheme, two different sample sizes n_1 and n_2 ($n_2>n_1$), with a fixed sampling interval were applied. When a sample point falls into the relax area, then the small sample size n_1 is used for the next sample and if the sample point falls into the warning area, then large sample size n_2 is applied for the next sample. Similarly, the different models of VSS were also studied in many papers like Zimmer et al. [8] and He et al. [9].

According to the good performance of the above schemes in detecting shifts, Prabhu et al. [10] combined

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both *VSI* and *VSS* schemes for *X-bar* chart. In this scheme, the performance is similar to previous schemes but variable sample size and variable sampling interval are used in designing control charts. For more information, we can refer to Mahadik et al. [11], Noorossana et al. [12], Zhou [13] and Sabahno and Amiri [14]. In all previous methods, the criteria such as average run length (ARL), average time to signal (ATS) and adjusted average time to signal (AATS) have been used for statistical comparison.

In all above studies, control charts are designed only based on the statistical criteria, while due to the wide applications of the control charts in the industry, considering the economic aspects of control charts is important. Duncan [15] in 1956 introduced the first model to design of X-bar control chart from the economic viewpoint. After that, Lorenzen et al. [16] provided a model with cost function for control charts. Other researchers including Montgomery [17], Fallahnezhad and Ahmadi [18], Katebi et al. [19], Fallahnezhad and Golbafian [20] provided a very good literature review on the economic design as well. Woodall [21] elaborated that economic schemes does not consider important statistical properties and also statistical schemes just focused on the statistical aspects that have higher cost in comparison with economic schemes. Saniga [22] in 1989 proposed a model for economic-statistical design of control charts in order to improve the performance of control charts. Economic-statistical design by Saniga [22] compared with his economic design leads to a slight increase in costs, but keeps the statistical power of the control chart in a desirable level. Subsequently, a variety of research was conducted in the field of economic-statistical design of various control charts. Amiri et al. [23] conducted the economic-statistical design of X-bar control chart which made a comparison between the VSSI scheme and other sampling schemes. Jahantigh and Safaeia [24] designed bi-objective of X-bar and S control chart, where the simultaneous reduction in out-of-control ATS and cost was reviewed. As other control charts, we can point out to the Moghaddam et al. [25] that by developing a multi-objective model using the evolutionary algorithm (NSGA-II), studied the economic-statistical design of cumulative count of conforming (CCC) control chart. Faraz et al. [26], Amiri and Jafarian-Namin [27], Lupo [28] and Fallahnezhad and Shamstabar [29] proposed optimization models for this problem.

The np control chart is widely used in industrial and service organizations, but few researches have been done in the field of attribute control charts. Luo et al. [30, 31] studied statistical performances of np control charts in the framework of VSS, VSI, VSSI and FSI schemes. Some economic studies are done on the attribute charts Montgomery et al. [32] and Chiu [33] proposed economic design of p and np chart. Chung [34] developed economic design of attribute control charts by considering assignable causes, Wang et al. [35] applied fuzzy optimization method to design np charts based on economic and statistical criteria. Kooli et al. [36, 37] studied economic design of np charts based on VSS and VSI schemes. Bashiri et al. [38] examined economic-statistical design of np chart by considering multi-objective methods.

One of the gaps and weaknesses in research on np control chart is related to the studies conducted in the field of economic-statistical design of most control charts with variable sampling schemes. No research is performed to evaluate the effect of using variable sampling schemes from the economic-statistical perspective for np control chart. In most economic-statistical models of variable sampling schemes have only been used as the average number of false alarms (ANF) as a constraint for designing the model. One of the main goals of these schemes is reducing ATS or AATS and to consider that applying this constraint can create a more realistic and practical model.

In this paper, economic-statistical design of np control chart has been investigated with the goal of reducing costs and applying appropriate statistical constraints based on variable sampling schemes. In Section 2, model parameters are introduced and it has been discussed to utilize Markov chain in design of control charts then a cost function has been formulated and a method is developed for solving optimization model. In Section 3, using the model of Lorenzen et al. [16], a cost functions is formulated for the np control chart based on economic-statistical design and the effects of various parameters have been studied. Finally, the conclusions are discussed in Section 4.

2. THE OPTIMIZATION MODEL AND COST FUNCTION

2. 1. Model Description and Markov Chain Properties

In this model, we use two different sample sizes (n_1, n_2) and two different sampling intervals (h_1, h_2) for VSSI-np chart. In VSSI scheme, we have two warning limits (WL_1, WL_2) and two control limits (UCL_1, UCL_2). The warning limits (WL_1, WL_2) are a guideline to change the sample size (n_1, n_2) and sampling intervals (h_1, h_2). Figure 1 depicts a general diagram for this scheme.

Some of the terminologies used in this article are introduced below:

p_0	The in-control defective proportion
p_+	The upward out-of-control defective proportion
n_i	The sample size in the state *i (i=1,2)*
h_i	The sampling interval in the state *i (i=1,2)*

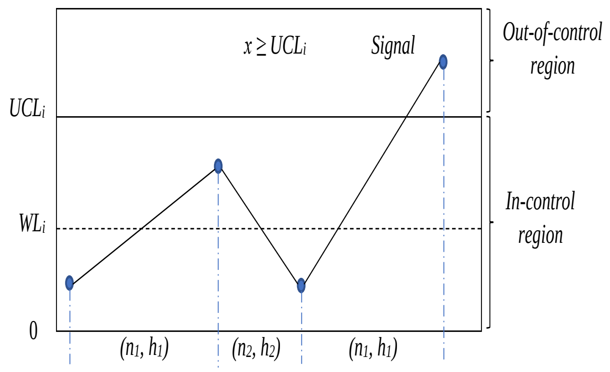

Figure 1. A demonstration of VSSI scheme

WL_i	The warning limit in the state *i (i=1,2)*
UCL_i	The upper control limit in the state *i (i=1,2)*
δ	Indicates the level of the shift in process
ANS	The average number of samples
ANI	The average number of inspected items

State i denotes the case that the sample size and sampling interval are ni and hi respectively. The upper warning and control limits are computed as follows:

$$UCL_i = n_i p_0 + 3\sqrt{n_i p_0 (1 - p_0)} \tag{1}$$

$$WL_i = n_i p_0 + 2\sqrt{n_i p_0 (1 - p_0)} \tag{2}$$

Then, from Bashiri et al. [38], the level of defective proportion, p_+ is defined as Equation (3):

$$p_+ = p_0 + \delta\sqrt{p_0 (1\text{-}p_0)} \tag{3}$$

According to Markov chain properties, in each stage of sampling and inspection, the following six transient states occur according to the status of the process, sample size and sampling interval.

State 1: 0≤np<WLi and the process is in-control; the next sample size and interval are n1, h1.

State 2: WLi≤np<UCLi and the process is in-control; the next sample size and interval are n_2, h_2.

State 3: np≥UCLi and the process is in-control; the next sample size and interval are n_2, h_2.

State 4: 0≤np<WLi and the process is out-of-control; the next sample size and interval are n_1, h_1.

State 5: WLi≤np<UCLi and the process is out-of-control; the next sample size and interval are n_2, h_2.

State 6: np≥UCLi and the process is out-of-control; the next sample size and interval are n_2, h_2. i= 1,2

State 6 is the absorbing state and if np≥UCLi then the control chart produces a signal because the number of defective units falls within the action region and if the current state is 3 (6), then the signal is a false alarm (true alarm).

The number of defective units, d, follows the binomial probability distribution function with parameters n, p as following:

$$binomial\,(n,p,d) = \frac{n\,!}{d\,!\left(n - d\right)!}\,p^d\,(1 - p)^{n-d} \tag{4}$$

and cumulative distribution function is denoted by F (n, k, p) and can be expressed as:

$$F(n,k,p) = \sum_{d=0}^{\lfloor k \rfloor} \frac{n\,!}{d\,!\left(n - d\right)!}\,p^d\,(1 - p)^{n-d} \tag{5}$$

Based on Markov chain, each p_{ij} element denotes the transition probability between the states i and j. Then, the transition probability matrix is obtained as following:

$$P = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} & p_{15} & p_{16} \\ p_{21} & p_{22} & p_{23} & p_{24} & p_{25} & p_{26} \\ p_{31} & p_{32} & p_{33} & p_{34} & p_{35} & p_{36} \\ 0 & 0 & 0 & p_{44} & p_{45} & p_{46} \\ 0 & 0 & 0 & p_{54} & p_{55} & p_{56} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Above matrix is derived from the properties of the Markov chain (Cinlar [39]) ,where p_{ij} can be calculated for VSSI-np chart as following:

$$p_{11} = \Pr\left(0 \le np < WL_1\right)e^{-\lambda h_1} = F\left(n_1, WL_1, p_0\right)e^{-\lambda h_1}$$

$$p_{12} = \Pr\left(WL_1 \le np < UCL_1\right)e^{-\lambda h_1} = F\left(n_1, UCL_1, p_0\right)e^{-\lambda h_1} - p_{11}$$

$$p_{13} = \Pr\left(np \ge UCL_1\right)e^{-\lambda h_1} = e^{-\lambda h_1} - p_{11} - p_{12}$$

$$p_{14} = \Pr\left(0 \le np < WL_1\right)\left(1 - e^{-\lambda h_1}\right) = F\left(n_1, WL_1, p_+\right)\left(1 - e^{-\lambda h_1}\right)$$

$$p_{15} = \Pr\left(WL_1 \le np < UCL_1\right)\left(1 - e^{-\lambda h_1}\right)$$

$$= F\left(n_1, UCL_1, p_+\right)\left(1 - e^{-\lambda h_1}\right) - p_{14}$$

$$p_{16} = \Pr\left(np \ge UCL_1\right)\left(1 - e^{-\lambda h_1}\right) = 1 - e^{-\lambda h_1} - p_{14} - p_{15}$$

$$p_{21} = p_{31} = \Pr\left(0 \le np < WL_2\right)e^{-\lambda h_2} = F\left(WL_2, n_2, p_0\right)e^{-\lambda h_2}$$

$$p_{22} = p_{32} = \Pr\left(WL_2 \le np < UCL_2\right)e^{-\lambda h_2}$$

$$= F\left(n_2, UCL_2, p_0\right)e^{-\lambda h_2} - p_{21}$$

$$p_{23} = p_{33} = \Pr\left(np \ge UCL_2\right)e^{-\lambda h_2} = e^{-\lambda h_2} - p_{21} - p_{22}$$

$$p_{24} = p_{34} = \Pr\left(0 \le np < WL_2\right)\left(1 - e^{-\lambda h_2}\right)$$
$$= F\left(n_2, WL_2, p_+\right)\left(1 - e^{-\lambda h_2}\right)$$
$$p_{25} = p_{35} = \Pr\left(WL_2 \le np < UCL_2\right)\left(1 - e^{-\lambda h_2}\right)$$
$$= F\left(n_2, UCL_2, p_+\right)\left(1 - e^{-\lambda h_2}\right) - p_{24}$$
$$p_{26} = p_{36} = \Pr\left(np \ge UCL_2\right)\left(1 - e^{-\lambda h_2}\right) = 1 - e^{-\lambda h_2} - p_{24} - p_{25}$$
$$p_{44} = \Pr\left(0 \le np < WL_1\right) = F\left(n_1, WL_1, p_+\right)$$
$$p_{45} = \Pr\left(WL_1 \le np < UCL_1\right) = F\left(n_1, UCL_1, p_+\right) - p_{44}$$
$$p_{46} = \Pr\left(np \ge UCL_1\right) = 1 - p_{44} - p_{45}$$
$$p_{54} = \Pr\left(0 \le np < WL_2\right) = F\left(n_2, WL_2, p_+\right)$$
$$p_{55} = \Pr\left(WL_2 \le np < UCL_2\right) = F\left(n_2, UCL_2, p_+\right) - p_{54}$$
$$p_{56} = \Pr\left(np \ge UCL_2\right) = 1 - p_{54} - p_{55}$$

An assumption used by researchers Faraz and Moghadam [40] and Jahantigh and Safaeia [24] is that the process remains in the in-control state using exponential distribution with parameter λ. In traditional control charts and *VSS* scheme, the speed that a control chart detects process mean shifts is evaluated by determining ARL. If the interval is variable, then we must evaluate the speed of control chart by calculating the ATS or AATS. Faraz et al. [6] express that the ATS index is used when the process starts in out-of-control state ($\delta>0$), while it is obvious this assumption cannot be so realistic. Therefore, taking into account the fact that the process starts in-control state ($\delta=0$), the adjusted ATS index, which is AATS, will be used for evaluating the performance of the schemes. AATS is the mean time from the process mean shift until the time that the control chart detects a signal. Based on the definition of the quality cycle, the mean time from the beginning of the production until the detection of first signal after the process shift is called the average time of the cycle (ATC). The ATC and AATS are calculated as follows [41]:

$$ATC = b'(I - Q)^{-1} h \tag{6}$$

$$AATS = ATC - 1/\lambda \tag{7}$$

where $h'=(h_1, h_2, h_2, h_1, h_2)$ is the sampling intervals vector, $b'=(p_1, p_2, p_3, 0, 0)$ is the initial probabilities vector for the state of the process, it is assumed that the vector b' is set to (0, 1, 0, 0, 0), for providing an extra protection and preventing problems that are encountered during start-up [41]. I is the identity matrix of order five and Q is the 5*5 transitional probability matrix among transient states.

Also, the values of ANS, ANI, ANF can be calculated as follow [41]:

$$ANS = b'(I - Q)^{-1} S \tag{8}$$

$$ANI = b'(I - Q)^{-1} N \tag{9}$$

$$ANF = b'(I - Q)^{-1} F \tag{10}$$

where, $S' = (1,1,1,1,1)$, $N' = (n_1, n_2, n_2, n_1, n_2)$ and $F' = (0,0,1,0,0)$.

2. 2. Economic-statistical Cost Function

The determination of an economic-statistical model requires certain assumptions; these assumptions are of particular importance in economic-statistical design because of the wide use of them, and make the framework model of a Markov chain to take over. Below are some of the assumptions used in this study.

- Only one assignable cause for the change in mean process is considered.
- Before the change in mean process, the process is in-control state.
- The occurrence of assignable cause is due to a Poisson distribution with a λ rate per unit time.
- In the *P* matrix, it is shown that the probability of returning from the out-of-control state to the in-control state is zero, that is, the process itself is not corrected, and only if the intervention of the manpower and the appropriate corrective actions are returned to the in-control state.
- The quality cycle starts from the in-control state for a process, until removing the assignable cause, repairing the process and finally, returning to the in-control state; in other words, the process is a renewal reward.

In 2014, Faraz et al. [26] developed the economic-statistical of the Hotelling's T^2 control chart and evaluated the performance of this control chart with variable sampling policies. Faraz et al. [26] proved that, the expected cycle time, *E(T)*, is given by:

$$\begin{aligned} E(T) &= 1/\lambda + (1-\gamma_1)T_0 * ANF + AATS + \bar{n}E + T_1 + T_2 \\ &= ATC + (1-\gamma_1)T_0 * ANF + \bar{n}E + T_1 + T_2 \end{aligned} \tag{11}$$

where $\gamma_1= 0$ or 1, T_0 is the expected time of investigating false alarms and if the process stops during investigating false alarms for each cycle then $\gamma_1=0$ and otherwise $\gamma_1=1$.

E is the expected time of inspecting and analyzing one item. Therefore, the total time to inspect and control a sample is equal to $\bar{n}E$ where $\bar{n}$ is the expected value of the sample size.

T_1 and T_2 are the expected time to detect the assignable cause and the expected time to repair the process, respectively.

In this study, using the cost model provided by Faraz et al. [26], the cost of producing a defective product in-control and out-of-control states, the costs of sampling and inspection, the cost of false alarms and the cost of detecting the deviation occurred in the mean process and eliminating them, and repairing the process, is considered.

Cost of defective products in-control and out-of-control states: These types of costs are expressed by Equation (12), generally include rework costs and product modifications, the cost of removing non-conforming products that are no longer capable of repair or other applications. Also, cover the costs of improper maintenance of raw materials and the cost of replacing and repairing guaranteed products.

$$C_0/\lambda + C_1\left(AATS + \bar{n}E + \gamma_1 T_1 + \gamma_2 T_2\right) \tag{12}$$

C_0 and C_1 are the expected cost of defective items when the process is in-control and out-of-control, respectively.

Cost of sampling and inspection: Includes costs of testing devices, inspection costs, and other related costs. Such costs are also shown in Equation (13), where a_1 and a_2 denote the fixed and variable cost of sampling and inspection, respectively.

$$\left(a_1 ANS + a_2 ANI\right) + \frac{\left(a_1 + a_2 n_2\right)\left(\bar{n}E + \gamma_1 T_1 + \gamma_2 T_2\right)}{h_2} \tag{13}$$

The cost of false alarms and removal and repair of the process: In this part, the cost of checking the false alarms with the cost of eliminating deflection and repairing the process are considered as two separate costs. Perhaps it is objected that these costs may vary depending on the type of deviation, but it should be taken into account that the correction of large deviations may cost more than small deviations, but the identification of small deviations is far more difficult and more complete [34]. Equation (14) represents these types of costs:

$$a_4 ANF + a_3 \tag{14}$$

a_4: The cost of a false alarm

a_3: The cost of repairing the process

Also, Faraz et al. [26] express the expected value of total cost per cycle *E(C)*, is given by:

$$E(C) = C_0/\lambda + C_1\left(AATS + \bar{n}E + \gamma_1 T_1 + \gamma_2 T_2\right) + a_4 ANF + a_3 + \left(a_1 ANS + a_2 ANI\right) + \frac{\left(a_1 + a_2 n_2\right)\left(\bar{n}E + \gamma_1 T_1 + \gamma_2 T_2\right)}{h_2} \tag{15}$$

In general, in economic-statistical designs, to evaluate the sampling policy, the Expected cost per hour *(EA)* is used and the *EA* is expressed by:

$$EA = E(C)/E(T) \tag{16}$$

Thus, the economic-statistical model can be formulated as follows:

$$\begin{aligned} &Min\ \ EA \\ &s.t.\quad 0 \le WL_i \le UCL_i \\ &\quad 1 \le n_1 \le n_2 \le 50 \\ &\quad 0.1 \le h_2 \le h_1 \le 8 \\ &\quad AATS \le 7 \\ &\quad ANF \le 0.5 \\ &\quad n_1, n_2 \in Z^+ \end{aligned} \tag{17}$$

In the above model, the maximum sampling interval is 8 hours, which is equivalent to one work shift and the maximum sample size is 50. In order to achieve the economic- statistical design, two statistical constraints are considered. The statistical constraint of AATS≤7 for quick detection of deviations and the constraint ANF≤0.5 is providing sufficient protection for process against false alarms. In most economic-statistical research studies, these constraints are less than 7 and 0.5.

2. 3. Optimal Method for Solving the VSSI-Np Control Chart

To determine optimal values of parameters n_1^*, n_2^*, h_1^* and h_2^* in economic-statistical *np* chart for VSSI scheme, following method is proposed:

1. A grid search procedure is employed for determining the sample size, n_1 from 1 to 50 and n_2 from n_1 to 50.
2. Based on the values of n_1 and n_2 obtained in step 1, the values of UCL_1, UCL_2, WL_1, WL_2 are obtained by Equations (1) and (2).
3. A grid search procedure is employed for determining the sampling interval, h_2 from 0.1 to 8 with step 0.1 and h1 from h_2 to 8 with step 0.1.
4. Compute the expected cost in each time unit (EA) by Equation (6).
5. If EA computed from step 4, has the minimum value and constraints are confirmed in Equation (17), then we select it as the optimal solution and also the values of parameters are selected as the optimal parameters.
6. If all values for h_1 and h_2 have been chosen, then go to next step otherwise go to step 3.
7. If all values of n_1 and n_2 have been chosen, then go to next step otherwise go to step 2.
8. The optimal values of parameters, sampling interval and sample size are obtained based on the minimum value of the cost objective function.

For other schemes like VSI, VSS and standard method, we apply the same solution algorithm but in VSI scheme, instead of applying n_1 and n_2, we consider a fixed value for n and then calculate UCL, WL in step 2. Also in VSS scheme, instead of using h_1 and h_2, we consider a fixed value of h. Also, we consider fixed values of n and h for the standard scheme and then the values of UCLi, WLi are obtained in step 2.

3. SENSITIVITY ANALYSIS

3. 1. Optimization and Comparison of Different Schemes

In the following part, the performance of sampling schemes will be studied in a numerical example. It also examines and compares the effectiveness of the proposed schemes to achieve the best scheme. The results of statistical and economic design will be analyzed in the cases of VSSI, VSS, VSI and standard control charts based on the cost objective function of Lorenzen et al. [17]. Input parameters are λ =0.05, γ_1= 1, γ_2 =0, E=T_0=T_1=0.0833, T_2 =0.75, C_0 =114.24, C_1= 949.2, a_1=5, a_2=4.22, a_3=a_4= 977.4 and p_0=0.0136. Table 1 shows the optimal design parameters n_1^*, n_2^*, h_1^* and h_2^* and values of *EA*, *AATS* and *ANF* based on the different values of shifts that are obtained using a search method given in Section 3-2 for *VSSI* scheme. Similarly, the optimal design parameters of other schemes are denoted in Tables 1–4. The column labeled Δ*EA(%)* measures the percentage reduction in cost by *VSSI* policy in comparison with other schemes.

The results denote that almost in all cases VSSI scheme has better performance in comparison with the other schemes. FSI scheme in all cases shows the weaker performance. Thus, the application of variable sampling schemes significantly affects the performance of control charts. For example, if the quality control department uses VSSI policy for detecting shifts with size of 0.9 in the process mean that the average cost of this scheme will be 297.15 per hour, while if we apply FSI scheme, then corresponding cost value is equal to 318.53 and the costs for schemes VSS and VSI will be equal to 307.84 and 308.89, respectively. Now, if the number of working days per year is 285 days with 8-hour shifts, the use of the VSSI scheme will reduce costs by 48746.40 per year compared to the FSI scheme. Also, the VSSI scheme compared to the VSS and VSI schemes shows a cost reduction of 24373.20 and 26767.20 per year, respectively.

TABLE 1. Optimal economic-statistical design and performance of the *VSSI* scheme

δ	n_1^*	n_2^*	h_1^*	h_2^*	*AATS*	*ANF*	*EA*
0.5	12	12	1	0.2	2.80	0.26	347.23
0.7	8	12	0.8	0.2	2.11	0.16	316.34
0.9	7	10	0.8	0.2	1.87	0.11	297.15
1.1	7	9	0.9	0.2	1.62	0.10	283.71
1.3	7	8	1	0.2	1.48	0.08	274.17
1.5	7	7	1	0.2	1.29	0.08	266.91

TABLE 2. Optimal economic-statistical design and performance of the *VSS* scheme

δ	n_1^*	n_2^*	h^*	*AATS*	*ANF*	*EA*	Δ*EA* $_{VSS\|VSSI}$*(%)*
0.5	14	14	0.9	2.95	0.33	359.11	-3.42
0.7	7	12	0.6	2.58	0.15	327.83	-3.63
0.9	7	12	0.7	2.07	0.13	307.84	-3.60
1.1	7	10	0.7	1.79	0.12	295.33	-4.10
1.3	7	9	0.7	1.51	0.11	286.18	-4.38
1.5	7	8	0.8	1.52	0.09	278.77	-4.44

TABLE 3. Optimal economic-statistical design and performance of the *VSI* scheme

δ	n^*	h_1^*	h_2^*	*AATS*	*ANF*	*EA*	Δ*EA* $_{VSI\|VSSI}$*(%)*
0.5	12	1	0.3	2.98	0.25	366.11	-5.44
0.7	12	1.2	0.4	2.20	0.21	330.91	-4.61
0.9	10	1.2	0.3	1.95	0.14	308.89	-3.95
1.1	9	1.1	0.3	1.60	0.12	293.02	-3.28
1.3	8	1.1	0.3	1.50	0.10	281.00	-2.49
1.5	7	1	0.2	1.29	0.08	271.23	-1.62

TABLE 4. Optimal economic-statistical design and performance of the *FSI* scheme

δ	n^*	h^*	*AATS*	*ANF*	*EA*	Δ*EA* $_{FSI\|VSSI}$*(%)*
0.5	17	1.2	2.87	0.35	370.99	-6.84
0.7	14	1.1	2.29	0.27	339.87	-7.44
0.9	12	1.1	2.06	0.20	318.53	-7.20
1.1	11	1.1	1.78	0.16	302.96	-6.79
1.3	10	1.1	1.62	0.14	290.92	-6.11
1.5	9	1	1.41	0.12	281.24	-5.37

It is also obvious that the inverse relationship between the sample size and shift size in the mean process is observed. Reducing the sample size with increasing shifts can be explained by the fact that large changes in the process average can easily be detected in comparison with small changes, which means reducing the need for large sample sizes. On the other hand, the sensitivity of the control charts to the sampling intervals is not significant. The reason for this is very clear; large sampling intervals leads to an increase in ATC and AATS, which in some ways neutralizes the effect of this factor on costs and does not have a significant impact on cost reductions. Therefore, suitable intervals help to increase the statistical power of the control scheme and limit the costs to the optimal level. In these conditions, increasing the amount of shifts and the stability of the sampling intervals, the AATS is improved and in each step decreases.

3. 2. Analysis Approach The response value of each parameter in all processes depends on several factors. For this reason, for a thorough and comprehensive examination of a process, the factors affecting the process should be determined. We conduct a Design of Experiments (DOE) study to build a model for exploring the sensitivity of the model parameters. We have used a fractional factorial design for studying the behavior of the response variables EA, AATS, ANF and optimal parameters of sampling intervals and sample sizes. Resolution of this model is IV with 32 experimental runs based on the generators F=ABC, G=ABD, H=ABE, J=ACD, K=ACE, L=ADE, M=BCD, N=BCE, O=BDE, P=CDE. The high and low levels of input parameters for experiment are presented in Table 5. These levels were chosen arbitrarily, a common approach in research papers dealing with the economic design of control charts.

Specifically, in Table 6, for each run we determine the optimal values of EA, AATS and ANF using a grid search procedure under the constraints $1 \leq n1 \leq n2 \leq 30$ and $0.25 \leq h2 \leq h1 \leq 8$ with 0.25 increments for VSSI-np scheme. The contribution of each factor to the total sum of squares is analyzed to determine the relative significance. In order to evaluate performance of control charts, we compare the EA of all schemes in Table 7. According to the results of the variable sampling schemes, the VSSI is better than VSS, VSI and FSI schemes based on data in the last column showing percentage difference indicating the superiority of this scheme in most cases.

Based on the analysis of the DOE, Table 8 shows the effective parameters that have the significant effects on the response variables of EA, AATS, ANF, n_1^*, n_2^*, h_1^* and h_2^*. We use the software Minitab 17 to analyze the results with a 5% significance level. The results of

TABLE 5. The ranges of the input parameters

Parameter	Low level (–)	High level (+)
γ_1	0	1
γ_2	0	1
T_0	0.05	3
T_1	0.05	3
T_2	0.3	10
C_0	75	300
C_1	500	1500
a_1	1	10
a_2	1	8
a_3	200	1500
a_4	100	800
E	0.06	1
λ	0.03	0.07
p_0	0.025	0.095
δ	0.7	1.8

normal probability tests are denoted in Table 8 where a positive (negative) sign indicates that increasing the value of the respective parameter leads to increase (decrease) in the response variable. Since most of the effects shown in Table 8 are intuitive and easy to explain, we concentrate on and discuss only the following findings, which are interesting and unexpected or at least not immediately obvious:

- The expected cost per time unit increases (*EA* increases) when the γ_2 and the expected costs of defective items in both cases of in-control and out-of-control C_0 and C_1 increases as expected. Also, if the parameter C_1 increases then the average number of false alarms in production process, *ANF* increases.
- The adjusted average time to signal *AATS* increased when the shift in process, δ and C_1 reduces. Also, if the variable cost of sampling and inspection a_2 increased then *AATS* and the detection power of the scheme reduced.
- The expected time of inspecting and analyzing E, and the parameters λ and δ affects significantly on the values of optimal sample size n_1^* and n_2^*.The optimal sampling frequency h_1^* and h_2^* increases when the expected time to repair, T_2 and a_2 increase. Also, the C_1 has a positive impact on h_2^*, and a_1 has negative effects on h_2^*.

TABLE 6. The results of the factorial experiments for *VSSI-np* scheme

Run	*EA*	*AATS*	*ANF*
1	121.44	1.76	0.42
2	520.72	2.50	0.44
3	600.98	1.40	0.42
4	309.50	4.21	0.32
5	394.28	2.07	0.47
6	349.66	2.08	0.46
7	219.39	1.69	0.48
8	322.37	1.15	0.09
9	215.73	1.56	0.40
10	380.02	3.12	0.20
11	321.28	2.38	0.04
12	492.17	0.48	0.22
13	416.34	1.06	0.49
14	256.96	3.09	0.47
15	184.82	3.72	0.29
16	619.38	3.58	0.27
17	135.78	0.95	0.05
18	260.96	0.73	0.47
19	699.17	0.85	0.25
20	225.58	4.85	0.46
21	307.39	5.37	0.30
22	212.90	2.22	0.45
23	789.17	4.33	0.49
24	461.68	5.31	0.43
25	343.06	4.48	0.49
26	269.27	2.66	0.20
27	394.37	6.42	0.26
28	559.40	1.63	0.49
29	260.40	0.63	0.43
30	160.15	2.58	0.07
31	253.75	1.04	0.25
32	1025.23	2.80	0.50
Average	377.60	2.58	0.34

For example, Figure 2 shows the normal probability plot of the effect estimates for the response variable (EA). This plot shows the magnitude of the impact on each factor through the difference between the average of responses at the upper and lower levels for that factor.

The small effects lie close to the line, and the significant effects deviate from the line. It is obvious that higher values of the costs (C_0 and C_1) and the binary parameter γ_2 lead to an increase in total cost. Also, other main factors or interaction effect do not have a significant impact on the value of EA.

TABLE 7. *EA* comparisons for adaptive *np* control charts

Run	***EA* comparisons**					
	FSI	***VSS***	***VSI***	Δ_{EA} **(%)**		
				VSSI/ FSI	***VSSI/ VSS***	***VSSI/ VSI***
1	132.01	123.68	125.57	8.01	1.81	3.29
2	529.32	527.06	529.32	1.62	1.20	1.62
3	697.88	600.98	697.88	13.88	0.00	13.88
4	327.81	309.50	327.81	5.59	0.00	5.59
5	396.73	394.63	396.55	0.62	0.09	0.57
6	382.90	349.66	372.90	8.68	0.00	6.23
7	216.71	219.65	216.55	-1.24	0.12	-1.31
8	323.86	324.30	322.29	0.46	0.60	-0.02
9	228.62	215.73	228.62	5.64	0.00	5.64
10	379.49	380.02	379.49	-0.14	0.00	-0.14
11	321.55	321.95	320.27	0.08	0.21	-0.32
12	494.13	492.17	494.13	0.40	0.00	0.40
13	433.50	416.34	426.45	3.96	0.00	2.37
14	258.29	256.96	258.29	0.51	0.00	0.51
15	214.85	188.21	214.11	13.98	1.80	13.68
16	699.30	622.54	696.59	11.43	0.51	11.08
17	136.09	138.83	134.64	0.23	2.20	-0.85
18	256.74	260.96	256.74	-1.64	0.00	-1.64
19	718.85	701.11	717.64	2.74	0.28	2.57
20	227.55	225.58	227.55	0.87	0.00	0.87
21	316.51	307.39	316.51	2.88	0.00	2.88
22	302.65	212.90	302.65	29.65	0.00	29.65
23	794.06	791.66	793.61	0.62	0.31	0.56
24	461.71	461.68	461.71	0.01	0.00	0.01
25	370.79	345.07	370.79	7.48	0.58	7.48
26	284.37	269.27	284.37	5.31	0.00	5.31
27	394.58	394.37	394.58	0.05	0.00	0.05
28	561.20	559.40	561.20	0.32	0.00	0.32
29	263.72	264.76	258.39	1.26	1.65	-0.78
30	263.72	161.89	159.90	39.27	1.07	-0.16
31	253.06	253.75	253.06	-0.27	0.00	-0.27
32	1016.78	1025.23	1016.78	-0.83	0.00	-0.83
Ave rage	395.60	378.66	391.15	4.55	0.28	3.46

TABLE 8. Summary results of significant parameter effects for the VSSI-np

Parameter	γ_2	T_2	C_0	C_1	a_1	a_2	E	λ	δ
EA	+		+	+					
AATS				-		+			-
ANF				+					
n_1^*							-	-	-
h_1^*		+		-	+	+			
n_2^*							-	-	-
h_2^*		+				+			

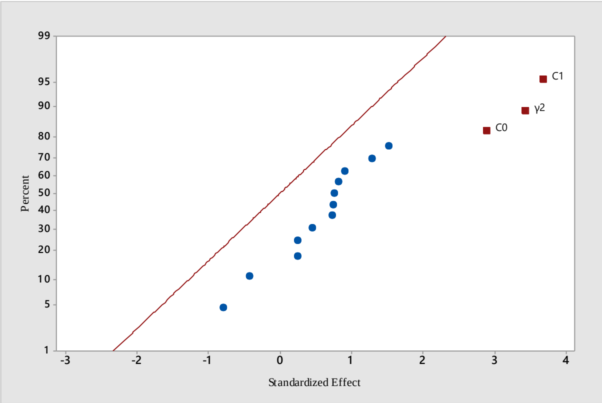

Figure 2. Normal plot of the standardized effects on the EA

4. CONCLUSION

The possibility to completely remove the changes in the production process is not feasible or, if possible, will increase the cost considerably. At the same time, the manufacturer expects the customer to receive an acceptable quality product. In this case, it is necessary to make a decision taking two opposing objectives into consideration. Therefore, efforts are made to reduce the process changes with the goal of reducing costs. Using control charts as an appropriate strategy for achieving the best conditions for the quality of products is considered. Traditional schemes used in designing control charts are both statistically and economically inappropriate. In this case, it is recommended to use variable sampling schemes to eliminate these two types of weaknesses. In this article, the economic-statistical design of *np* control chart based on the variable sample size and sampling interval is introduced. A grid search procedure was presented to solve the optimization model and it was shown that the *VSSI* scheme is more efficient than *VSS*, *VSI* and *FSI* schemes based on the economic and statistical performance. Statistical performances of optimal design parameters for each scheme have been evaluated. It has been found that the *VSSI* policy is able to reduce the average time to signal and the average number of false alarms compared to the other schemes.

Control charts are reaction tools against the changes, while using DOE techniques, one of the preventive strategies can be identified by identifying the factors affecting the process to produce quality products. We conducted a DOE with 32 experimental runs, by considering the results of the DOE model, we could determine effective parameters on EA (expected cost in hour), AATS (adjusted average time to signal), ANF (average size of false alarms), sampling intervals and sample sizes. Also, VSSI scheme in each run is compared with VSS, VSI and FSI charts. The results of *VSSI* scheme show significant improvement in performance of control chart compared to the other schemes.

For the future research, other sampling schemes with different capabilities, such as asynchronous change sample sizes or sampling intervals, can be used to examine the effect of shifts on cost and statistical criterions. Moreover, researchers can also consider the effect of other constraints on the proposed schemes.

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Economic-statistical Design of NP Control Chart with Variable Sample Size and Sampling Interval

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چکیده

نمودارهای کنترل ابزارهای ترسیمی و فنون اثبات شده‌ای برای بهبود کارایی یک فرآیند می‌باشند. معمولا فرآیندها به طور طبیعی در حالت تحت کنترل نیستند. بنابراین، استفاده از نمودارهای کنترل کمک شایانی به کاهش تغییرپذیری و افزایش ثبات در فرآیند خواهد کرد. در رویکرد سنتی از نمودارهای کنترل با اندازه نمونه و فاصله نمونه‌گیری ثابت برای شناسایی تغییرات در فرآیند استفاده می‌شد. در حالی که نمودارهای کنترل می‌توانند از جهات مختلف آماری و اقتصادی برای دست-یابی به نتیجه بهتر مورد ارزیابی قرار گیرند. در مطالعات قبلی که از طرح‌های نمونه‌گیری متغیر در طراحی نمودار کنترل np استفاده شده عملکرد بهتری در تشخیص انحرافات حاصل شده است ولی این موضوع مهم نیز مطرح است که هزینه استفاده از طرح‌های نمونه‌گیری متغیر به چه صورت خواهد بود، زیرا هزینه‌های یک فرآیند به پارامترهای نمودار کنترلی وابسته می‌باشد. در این مقاله طراحی آماری-اقتصادی نمودار کنترل np با اندازه نمونه و فاصله نمونه‌گیری متغیر (VSSI) صورت گرفته و در ادامه به مقایسه و تحلیل نتایج حاصل شده با طرح‌های دیگر پرداخته شده است. نتایج نشان دهنده‌ی بهبود قابل ملاحظه‌ای در عملکرد اقتصادی و آماری می‌باشند.

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