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Closed-loop Supply Chain Inventory-location Problem with Spare Parts in a Multi-Modal Repair Condition

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ABSTRACT

In this paper, a closed-loop location-inventory problem for spare parts is presented. The proposed supply chain network includes two echelons, namely (1) distribution centers (DCs) and repairing centers (RCs) and (2) operational bases. Multiple types of spare part are distributed among operational bases from distribution centers in the forward supply chain and failed spare parts from operational bases are sent back to the repairing centers to receive multi-modal repair services in the reverse supply chain. The RCs have no limitation to repair failed items and all of them should be repaired by minimizing total repairing cost. The main purpose of the proposed model is to minimize the total cost of inventory and location-allocation decisions in the proposed network in order to deal with the uncertainty nature of demand in both forward and reverse supply chains. Thus, a mixed-integer nonlinear programming model is formulated for the location-allocation problem that tries to choose which DCs and RCs to be opened and to determine the repair service mode of each failed items with objective function of minimizing the total cost. Furthermore, the validation of the model is tested by GAMS software for small-sized problem, and particle swarm optimization (PSO) is proposed to solve large-sized problems in a reasonable time. Finally, several sensitivity analyses are presented to evaluate the proposed model. Furthermore, according to computational results, the proposed heuristic algorithm is more efficient in both CPU-time and quality of solution for medium and large size problems.

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1. INTRODUCTION

Since the term of supply chain management (SCM) was introduced in 1982 [1], it has expanded in many aspects in the literature surveys and industries. The structure of a closed-loop supply chain has several activities for the forward flow of products and materials to the customer and also has some important ones in a reverse supply chain [2]. A classical form of the forward supply chain is a combination of processes to satisfy customers' demands and includes all probable entities, such as suppliers, manufacturers, transporters, warehouses, retailers, and customers [3]. According to the American Reverse Logistics Executive Council, reverse logistics is defined as a procedure including planning, implementation and controlling all products and information throughout two point of origin and destination in an effective cost [4]. The closed-loop supply chain is constructed when both the forward and reverse supply chains are considered.

In this paper, a closed-loop location inventory supply chain network design problem is presented to minimize the total cost of the supply chain network. The considered network, which is depicted in Figure 1, consists of two types of flows. The forward one is the demand satisfaction flow to distribute the spare parts from distribution centers among operational bases. The second one is a reverse flow to send back the failed spare parts from the operational bases (i.e., consumption points) to the repairing centers (RCs). The failed parts are repaired in one of slow or fast service modes in the RCs. The considered closed-loop supply chain inventory-location is very complicated in many different

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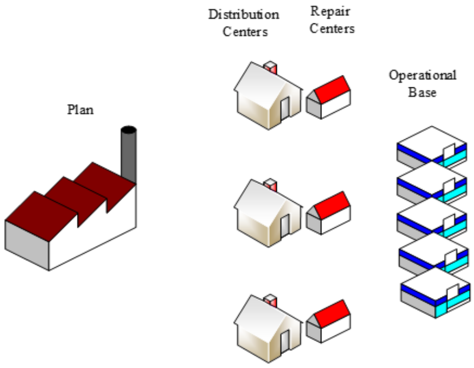

Figure 1. Proposed closed-loop supply chain network

aspects: first, the original closed-loop supply chain inventory-location is known to be NP-hard, and second, considering multiple types for spare parts and repairing services beside service level constraint are caused complexity in the mathematical model. In order to handle uncertainty in input data and dealing with complexity of new constraints, the required computation time is increased. In this regard, the second aim of this paper is to propose an efficient and powerful evolutionary algorithm to solve the proposed mathematical model in a reasonable amount of time.

The first aim of this paper is to develop a new mixed-integer non-linear mathematical model for a closed-loop supply chain inventory-location problem. The model aims to (1) minimize the supply chain total cost, (2) determine which DCs and RCs can be opened, (3) properly allocate operational bases to DCs and RCs and (4) consider inventory decisions in the proposed network. Also, two different repairing modes in slow and fast servicing are proposed to repair the failed items, the demand of multiple types of spare parts are satisfied in the proposed closed-loop supply chain network, and using the service level constraint to determine each failed items should be repair in one repairing service mode to reduce supply chain total cost.

The remainder of the paper is organized into seven sections. After the introduction, Section 2 presents the related literature on a closed-loop location-inventory spare part supply chain. In Section 3, the problem description and mathematical formulation are described to determine the location of distribution centers (DCs) and RCs by considering inventory decisions. The model validation and solution method is presented in Sections 4 and 5, respectively. Section 6 presents the computational experiments and sensitivity analyses. Finally, conclusions and future research directions are drawn in Section 7.

2. LITERATURE REVIEW

In this section, the recent studies on a closed-loop location-inventory supply chain for spare parts are reviewed. Shen et al. [5] developed a nonlinear integer mathematical model to formulate a facility location problem considering inventory costs and a proper portion of transport costs. Their proposed model is solved using a Lagrangian relaxation algorithm. Also, two examples with 88 and 150 nodes are used to evaluate the algorithm. The results demonstrate that the number of selected facilities increases when fixed order costs are remarkably decreased. Daskin et al. [6] introduced a nonlinear integer programming model to formulate a location–inventory problem considering risk pooling (LRMP). Diabet et al. [7] introduced a multi-echelon inventory–location model, in which both the location decisions and inventory policies are considered at warehouses and customers simultaneously.

Gzara et al. [8] studied two network integrated inventory-location decisions and introduced two mixed-integer nonlinear programming (MINLP) model related to two kind of target service levels and part-warehouse requirements. In results, the proposed approach was identified as the most efficient among other approaches in the surveys. Olsson [9] introduced a lateral transshipment mathematical model for a single-echelon inventory spare part network with two locations. In the studied spare part inventory system, a Poisson distribution is used to determine the demand of each location independently. Furthermore, the proposed model was solved by a doubly stochastic Poisson process technique. Also, the performance of the model is measured by a simulation approach. Tahirov et al. [10] presented a closed-loop supply chain model for products and spare parts and considered three cost structures related to the collection of used items in the reverse network. Firstly, the used items are accumulated from customer by a manufacturer. Secondly, accumulating procedure is determined by contracting between the manufacturer and customer or a third party. This model is performed for 12 numerical examples using Matlab software. Also, the results showed that the closed-loop supply chain and its inventory policy significantly are influenced by the costs of collection and transportation.

Diabat et al. [11] introduced a mathematical model for a closed-loop inventory location network with single echelon to fulfill the uncertain demand of a single product. They proposed a nonlinear model with joint location-allocation and spare part inventory decisions. Furthermore, the model is solved by a Lagrangian relaxation algorithm. The numerical results illustrated that the number of facilities opened in the supply chain decreases when the inventory cost increases while it increases as the increasing of the transportation cost Samouei et al. [12] studied a multi-echelon network for spare part inventory. Moreover, backorder and quantity discount are considered in this network. They developed the mathematical model to minimize the total cost of transportation, distribution, repairing, holding, purchasing, and backordering of the items over the planning horizon according to quantity discount that external supplier gives customers. Furthermore, the Fordyce and Webster algorithm is proposed to solve the model. Also, the efficiency of the proposed algorithm is evaluated by numerical examples. Ghomi-avili et al. [13] presented a closed-loop supply chain by considering completely disruption risk with several scenarios. They studied a resilience concept by two resilience factors (i.e., lateral transshipment and extra inventory) to formulate the problem as a mixed-integer programming. Finally, a sensitivity analysis is conducted to investigate the effects of resilience factors on the proposed network. Ahmadi Yazdi and Honarvar [14] proposed a closed-loop supply chain integrated pricing policy. They developed a mixed-integer linear model using a stochastic approach. The value of the stochastic solution (VSS) is used due to evaluate the precise of the proposed approach. Furthermore, the optimal values of sales price are computed based on forward and reverse logistics.

Zarrinpoor et al. [15] presented a reliable hierarchical location-allocation mathematical model related to the facility disruption risk. Also, a multi-level multi-flow hierarchy is considered with the relations among different levels. Furthermore, an exact solution based on the Benders decomposition method is used to solve the model. Maleki et al. [16] developed a bi-objective MILNLP model for remanufacturing facility considering decision variable for price of remanufactured products. There are different kinds of incoming nonconforming products with several workstations. These workstations are independent and have specified capacity. The *M/M/1/K* queening system is used in each remanufacturing workstation. The proposed mathematical model was solved using GAMS to obtain optimal solution with maximizing the total profit and minimizing the average length of queening at workstations. Panda et al. [17] presented a closed-loop supply chain considering the impact of corporate social responsibility (CSR) between manufacturer and customers through product recycling. Cárdenas-Barrón et al. [18] studied a supplier selection problem for multi-product multi-period inventory lot sizing. Furthermore, the efficient reduce and optimize approach (ROA) is applied to solve the model. 150 benchmark examples are presented for different sizes. Also, the result of examples demonstrate the efficiency of the proposed heuristic algorithm against the CPLEX MLP solver.

Namdar et al. [19] presented a capacitated distribution network design under disruption and proposed a scenario-based mixed-integer linear programing (MILP) model to cope with disruption risk and enhance reliability of the distribution network. These multiple strategies are: transshipment, facility location and fortification. The model was verified by executing several examples and eventually results showed that the transshipment strategy is most efficient strategy against fortification strategy to deal with the uncertainty nature of network. Yosefi et al. [20] studied two supply chain networks for the same product and opened a new distribution center to satisfy the additional demand, so they competed with each other to keep their market. Thus, a non-linear bi-level model is proposed to formulate these competition. Due to the NP-hard nature of the model, an ant colony algorithm is applied to obtain the best solution. According to another application of meta-heuristic algorithms in supply chain networks, Qi et al. [21] used modified binary-particle swarm optimization (MBPSO) algorithm to obtain the proper solution for a multi-level distribution network. This algorithm can obtain near-optimal solutions by parallel global search in high accuracy and fast convergence for the hybrid integer mathematical model with a weighted objective and multiple constraints. Kamali et al. [22] presented a closed-loop supply chain (CLSC) for some products with a continuous price decrease. In order to solve the mixed-integer linear programming model, several meta-heuristics are applied (i.e., artificial bee colony (ABS), particle swarm optimization (PSO), deferential evolution (DE) and genetic algorithm (GA)). In order to evaluate the proposed methods, the results of them are compared with the deterministic solution. Eventually, the artificial bee colony method reveals the high quality results for the proposed model with the lowest error value than other methods.

According to the above discussion, the literature review demonstrates that there is a gap in a closed-loop supply chain inventory-location. Furthermore, there is a few studies in the context of multiple types of repairing services. Moreover, many parameters in the real world situation are tainted with uncertainty, which can impose a high degree of uncertainty on the designed network.

In the real world, there are various types of spare parts which are used in different industries to conduct maintenance tasks (e.g., maintaining the production machines and aircrafts). It commonly happens that some items should be replenishment or can be repaired, depending on their nature.

Besides, solving complex the closed-loop supply chain inventory-location is one of the most challenging issues as they are known to be NP-hard. However, a few papers in the literature have developed evolutionary algorithms to deal with these models. To overcome these shortcomings and fulfill these gaps, we develop a mathematical model for an closed-loop supply chain inventory-location under uncertainty to investigate the integrated inventory and location decisions in a closed-loop supply chain, an efficient algorithm, called partial swarm optimization (PSO) algorithm is also proposed to solve the developed mathematical model and obtain near-optimal solutions in a reasonable amount of time.

The main contributions of this paper, which differentiate our effort from related studies, are as follows:

- Developing a new MINLP model for a closed-loop supply chain inventory-location problem.
- Considering two different repairing modes for repairing the failed items based on their importance or customer's preferences.
- As each machine or equipment consist of multiple spare parts, we considered multi types of spare parts in our proposed closed-loop supply chain network.
- We use a service level constraint to allocate each failed item to proper repairing mode while the minimum service level should be satisfied and also, all the failed items needed to repair should be allocate to only one of the repairing modes which are obtained the minimum total cost.

3. PROBLEM DESCRIPTION

This paper considers a closed-loop network, in which multi-type of spare parts are shipped from a single supplier through regional distribution centers (DCs) to several operational bases under uncertain demands. Furthermore, the failed spare parts are collected from operational bases and are sent to the repair centers (RCs). We suppose that the failed spare parts can be repaired in a multi-modal repair condition, namely slow and fast. The repair time at the slow mode is more than the fast mode. In addition, the cost of fast repair is higher than that of the slow mode. The repaired spare parts are as good as new ones and can be sent to satisfy the demand at the operational bases.

Q_{ipl} is defined as the demand at operational base i for spare part p on a day l. It is assumed that the daily demands of each kind of spare parts for particular day l at the bases are independent with a normal distribution (i.e., $Q_{ipl} \sim N(\mu_{ip}, \sigma_{ip}^2)$). The rate of failed spare parts of the forward supply chain at base i of spare part p on a day l is similarly defined by q_{ipl}, which are independent random variables with a normal distribution (i.e., $q_{ipl} \sim N(\lambda_{ip}, \delta_{ip}^2)$). Although, the expected number of daily failed spare parts is a fixed ratio of the daily demands of bases as $\mu_p = \gamma.\lambda_{ip}$ (where $\gamma \in [0,1]$), we assume that all the failed spare parts are repairable and each of failed item are assigned to one of two service repair modes (i.e., slow and fast repairs).

In this model, the inventory costs are integrated to the location mathematical model by considering an economic order quantity (EOQ) approximation for the spare parts at DCs and the failed spare parts awaiting repair at RCs. The safety stock cost is computed based on fixed lead time. Furthermore, the model is formulated to determine the location of DCs and RCs, assignment of bases to DCs in the forward supply chain, assignment of operational bases to RCs in the reverse supply chain and to minimize the inventory costs for the proposed closed-loop supply chain. The indices, parameters and variables are defined in Table 1.

TABLE 1. Indices, parameters and variables

Indices	
I	Set of operational bases, indexed by i
J	Set of potential regional distribution centers, indexed by j (where j and k are aliases)
K	Set of potential repair centers, indexed by k
P	Set of spare part items, indexed by p
Parameters	
f_j	Fixed annual cost of a distribution center at location j
l_k	Fixed annual cost of a repair center at location k
d_{ij}	Cost per unit shipped from distribution center j to base i
d'_{ij}	Cost per repaired unit shipped from distribution center j to base i
d_{ik}	Cost per unit shipped from base i to repair center k
χ	Number of days per year
β	Transportation cost weight
θ	Inventory cost weight
γ	Ratio of expected average repairable spare parts daily arrive in repair center
SL	Service level
z_α	Standard normal deviate with $p(z \leq z_\alpha)$
F_j	Fixed order cost from DC j to supplier
g_j	Fixed shipping cost from DC j to supplier
L_p	lead time of spare part p from external supplier to DC
L_r	Repairing time needed for repairing each failed item by the repairing mode r at the RCs
C_r	Unit repairing cost of each failed item by the repairing mode r
h_p	Holding cost of safety stock of spare part p at distribution centers
h'_p	Holding cost of safety stock of spare part p at repair centers
V	Priority of each kind of spare part
np	Number of multi type of spare parts
Decision Variable	
I_{pr}	A binary variable, equal to 1 if spare part p is repaired by repairing mode r
x_j	A binary variable, equal to 1 if location j is selected as a distribution center; otherwise, 0
w_k	A binary variable, equal to 1 if location k is selected as a Repair center; otherwise, 0
y_{ij}	A binary variable, equal to 1 if the base i served by distribution center located at site j; otherwise, 0
z_{ik}	A binary variable, equal to 1 if the repairable item at the base i are serviced by the repair center located at site k; otherwise, 0

Model Assumptions:

1) The distribution network is uncapacitated.
2) Demand is uncertain with a normal distribution.
3) Each operational bases has independent normal distribution demand.
4) All of the failed spare parts can be repaired in slow or fast repairing mode.

$$\text{Min} \sum_{j\in J} f_j x_j + \sum_{k\in K} l_k w_k$$

Forward supply chain:

$$+\beta\sum_{j\in J}\sum_{i\in I}\sum_{p\in P}\chi\ \mu_{ip}\left(d_{ij}+a_j\right)y_{ij}$$
$$+\beta\sum_{j\in J}\sum_{i\in I}\sum_{p\in P}\chi\ \gamma.\mu_{ip}.d_{ij}'.y_{ij}$$
$$+\sum_{j\in J}\sqrt{2\theta\left(F_j+\beta g_j\right)\sum_{i\in I}\sum_{p\in P}\chi.\mu_{ip}.y_{ij}.h_p}$$
$$+\theta.z_\alpha.\sum_{j\in J}\sqrt{\sum_{i\in I}\sum_{p\in P}L_p.\sigma_{ip}^2.h_p^{\ 2}.y_{ij}}$$

Reverse supply chain:

$$+\left(\beta\sum_{i\in I}\sum_{p\in P}\sum_{k\in K}\chi.\gamma.\mu_{ip}.\text{d}_{ik}.\text{z}_{ik}\right)$$
$$+\theta.z_\alpha.\sum_{k\in K}\sqrt{\sum_{i\in I}\sum_{p\in P}\sum_{r\in R}\left[{L_r.I_{pr}}\big/{np}\right]\gamma^2.\sigma_{ip}^2.h_{ip}'^{\,2}.z_{ik}} \qquad (1)$$
$$+\sum_{i\in I}\sum_{p\in P}\sum_{r\in R}C_{pr}.\text{I}_{pr}.\chi.\gamma.\mu_{ip}$$

The proposed mathematical model minimizes the total cost of the closed-loop spare parts inventory-location; the first two terms of objective function depicts the fixed location cost of DCs and RCs. The next fourth line shows the total cost of the forward supply chain, which is contained the delivery cost of spare parts to the operational bases from, in which DCs are assigned to them, the delivery cost of sending the failed items to the RCs from operational bases, total expected inventory costs that purpose *a*, *g* and *F* are related to the location of each DC and the last term of forward supply chain represent the total expected safety stock inventory cost based on risk pooling of the uncertainty of demand. Safety stock is commonly set to be proportional to standard deviation of the demand occurring throughout the lead time. Reducing in variability of demand can be achieved by reducing in safety stock. The aggregating demand from all bases is smaller than the sum of the variances demand of those bases. Therefore, the amount of safety stock which is needed for pooled demand is usually less than the sum of safety stock of each base's demand. As a result, using the risk pooling strategy reduces the variability in demand, and in consequence reducing supply chain cost, especially inventory costs [11, 23-25]. The last three lines represent the costs of reverse supply chain; the seventh term represents the total expected inventory cost at the RCs assigned to the bases. The eighth term shows the total expected safety stock inventory cost based on risk pooling related to uncertainty in failed items.

$$\sum_{j\in J} y_{ij} = 1 \qquad \forall i\in \text{I} \qquad (2)$$

$$y_{ij} - x_j \le 0 \qquad \forall i\in I,\ j\in J \qquad (3)$$

$$\sum_{k\in K} z_{ik} = 1 \qquad \forall i\in I \qquad (4)$$

$$z_{ik} - w_k \le 0 \qquad \forall i\in I, \forall k\in \text{K} \qquad (5)$$

$$w_k - \sum_{i\in I} y_{ik} \le 0 \qquad \forall k\in \text{K} \qquad (6)$$

$$\sum_{i\in I}\sum_{p\in P}\chi.\gamma.\mu_{ip}.I_{pr} \ \le\ \text{Cap}_r \quad \forall r\in R, k\in K \qquad (7)$$

$$SL \ \le\ \frac{\sum_{i\in I}\sum_{p\in P}\sum_{r\in R}\chi.\gamma.\mu_{ip}.I_{pr}.\text{v}_p.\left({1}\big/{L_r}\right)}{\sum_{i\in I}\sum_{p\in P}\chi.\gamma.\mu_{ip}.\text{v}_p} \qquad (8)$$

$$\sum_{r\in R} I_{pr} = 1 \qquad \forall p\in P \qquad (9)$$

$$y_{ij} \in \{0,1\} \qquad \forall i\in \text{I},\quad \forall j\in \text{J} \qquad (10)$$

$$z_{ik} \in \{0,1\} \qquad \forall i\in I, \forall k\in \text{K} \qquad (11)$$

$$x_i \in \{0,1\} \qquad \forall i\in \text{I} \qquad (12)$$

$$w_k \in \{0,1\} \qquad \forall k\in \text{K} \qquad (13)$$

$$I_{pr} \in \{0,1\} \qquad \forall p\in P, r\in R \qquad (14)$$

Constraint (1) represents that each operational base's demand is assigned to a DC j. Constraint (2) shows that assignment to DCs occur if a DC is opened at location *j*. Constraint (3) ensures that failed items from bases are assigned to RC k. Constraint (4) represents that the failed items can be repaired only in repair centers. Constraint (5) represents that each RC can be opened when the DC has been opened at location and the bases are assigned to this DC. Constraint (6) represents the number of assigned failed items to each repair mood is limited with capacity of repair mood. Constraint (7) represents that assigning failed item to repairing mood should be satisfied target service level. Constraint (8) ensures that each kind of failed spare part assigned to one of the repair mood. Finally, constraints (9) to (13) represent binary decision variables.

4. MODEL VALIDATION

In order to validate the presented mathematical model, a set of small and medium-scale test problems are solved based on the objective function using random generated data, which are shown in Table 2. The model is solved by General Algebraic Modeling System (GAMS) software – version 27.7.1. This software employs the language compiler several solvers within high performance to solve optimization problems.

TABLE 2. Parameter values

Parameters	Value
χ	360
β	0.01
θ	0.1
γ	0.2
z_α	1.96
SL	0.95
μ_{ip}	~ Uniform (200,500)
σ_{ip}	~ Uniform (0.5,1)
f_j	~ Uniform(6.5×106,1.5×107)
l_k	~ Uniform(6.5×106,1.5×107)
d_{ij}	~ Uniform(6.5×109,1.5×1010)
d_{ij}'	~ Uniform(6.5×109,1.5×1010)
d_{ik}	~ Uniform(6.5×109,1.5×1010)
F_j	~ Uniform(1×104,4.5×104)
g_j	~ Uniform(1×104,4.5×104)
a_j	~ Uniform (5×104,2×106)
h_p	~ Uniform(1000.2500)
h_p'	~ Uniform(50,150)
L_p	~ Uniform (100,150)
L_r	{5,10}
C repair	{1500,1000}
Cap r	~ Uniform (100,300)

As can been seen, Table 3 presents the optimal values of the objective functions (OFV) and CPU time for different test problems. The results reveal that the CPU time of the problem increases due to the complexity of the problem. Since the number of operational bases $|I|$ and the variety of spare part types $|P|$ increase, CPU time remarkably increase.

As it reveals, the comparison between dataset S_4 and S_5 demonstrates that changing the number of spare part types from 10 to 30 highly increase CPU time. Also, comparing the dataset S_7 and S_8, illustrates that the required CPU time is significantly increased up to 17022.17 seconds when the number of operational bases changed from 50 to 100. These numerical results show that the impact of these two parameters i.e., $|I|$ and $|P|$ on CPU time and OFV are more than the impact of the number of DCs and RCs.

Due to the nature of spare part inventory, the number of spare part types used in industries and warehouses is usually more than hundreds. Thus, we perform our model for large-sized problems; however, GAMS is unable to reach the feasible solution through 48 hours of the model runtime because of the complexity and NP-hard nature of the proposed model. Since, the meta-heuristics are able to overcome these complexity of problems, we propose the partial swarm optimization (PSO) algorithm to solve the defined nonlinear model under high complexity in reasonable runtime.

5. SOLUTION APPROACH

In this section, a brief survey on the proposed meta-heuristic algorithm is conducted.

TABLE 3. Numerical results of GAMS for small and medium-scale problems

	Problem Dimension	Output	
No.	$\|I\|\times\|J\|\times\|K\|\times\|P\|$	OFV (Rial)	CPU time (sec)
S_0	5×5×5×5	4.5909×10^{11}	0.05
S_1	10×8×8×5	9.1896×10^{11}	0.14
S_2	10×8×8×10	1.8436×10^{12}	0.33
S_3	20×12×12×10	3.5956×10^{12}	1.82
S_4	30×12×12×10	5.4015×10^{12}	4.76
S_5	30×12×12×30	1.6662×10^{13}	79.54
S_6	50×25×25×30	2.8350×10^{13}	625.53
S_7	50×25×25×60	5.9373×10^{13}	824.22
S_8	100×25×25×60	1.1929×10^{14}	17022.17

In the following sub-section, PSO is performed using Matlab R2015a for large-scale test problems of our model to obtain near-optimal solutions in reasonable runtime. Also, the same small and medium-scale test problems are solved using PSO and GAMS to compare their results and the gap between them. The computational results of this algorithm are shown in Sub-section 5.2.

5. 1. General Description of PSO Particle swarm optimization (PSO) is an evolutionary computational technique based on social behavior. It was introduced by Kennedy and Eberhart in 1995 [26]. Swarm in PSO is the similar to the population in the genetic algorithm (GA) that generates random solutions. The random solutions in PSO and GA are particles and chromosomes, respectively. Unlike the GA, the potential solutions (i.e., particles) are by their own randomized velocity in PSO. So, it flows in solution space.

It is initialized with the initial population defined as a set of particles. Each particle has a randomized velocity that is moved through the solution space. Based on the position of each particle in solution space, the fitness value is computed and the best one which is related to the best position is selected [27]. The general form of PSO is described as follows:

$$\vec{v}_{k+1} = \vec{a} \otimes \vec{v}_k + \vec{b}_1 \otimes \vec{r}_1 \otimes \left(\vec{p}_1 - \vec{x}_k\right) + \vec{b}_2 \otimes \vec{r}_2 \otimes \left(\vec{p}_2 - \vec{x}_k\right) \tag{15}$$

$$\vec{x}_{k+1} = \vec{c} \otimes \vec{x}_k + \vec{d} \otimes \vec{v}_{k+1} \tag{16}$$

where $\otimes$ is a symbol that denotes the vector multiplication in each iteration. The velocity and position vectors are updated based on their current values using momentum factor $(\vec{a}).(\vec{p}_1)$ is the own previous best position and $(\vec{p}_2)$ is the best position in the swarm. Also, $\vec{x}_k$ is a particle position that is updated within its current value and the velocity that computed from Equation (15). $\vec{c}$ and $\vec{d}$ are the coefficients. Also, $\vec{r}_1$ and $\vec{r}_2$ are the random numbers between [0, 1]. The original steps of executing the PSO algorithm are shown if Figure 2 is also described briefly as follows:

Step 1: The initial population of particles are randomly generated. Each particle has its own position and velocity in solution space of the problem.

Step 2: Each particle is evaluated using fitness function.

Step 3: Evaluation value of the position of each particle is compared with the best fitness value reached by the best position called $p\ best$. If the current value has a better value than $p\ best$, then the current value is set as the $p\ best$, and the best position is equal to the current position of article in problem search space.

Step 4: The fitness value of the particle's evaluation is compared with the population's overall best value which obtains by all particles called $g\ best$ (i.e., global best). If the current value has better value than $g\ best$ then the current value set as the $g\ best$.

$$v_{k+1} = a.v_k + b_1 r_1 \ (p_1 - x_k) + b_2 r_2 \ (p_2 - x_k) \tag{17}$$

$$x_{k+1} = c.x_k + d.v_{k+1} \tag{18}$$

Step 5: This loop to Step 2 is repeated to meet maximum number of iterations (or other stopping criteria, such as value of fitness, maximum desired run time) to obtain global solution with the best fitness value can be obtained in solution space.

PSO has been used in wide range of applications. It is most popular because of its few parameters and ability to find the global optimum solution with the high probability and also fast convergence rate [27-30].

5. 2. Computational Results A set of large-scale random datasets as shown in Table 4 are examined using a PSO algorithm. Because the GAMS runtime for our model takes more hours for large-scale problems, PSO is used to obtain near-optimal solutions in less runtime. The numerical results are demonstrated in Table 4. An important result obtained from this table is that the behavior of the OFV and CPU time of large-scale problems solved by PSO is the same as the results of GAMS.

Comparison of S_9 and S_{10} shows a dramatic increase in the OFV because of increasing in the number of operational $|I|$ bases and spare part types $|P|$. Also, the CPU time in order to obtain near-optimal solutions is increased when the number of $|I|$ and $|P|$ are increasing. As a variety of spare part types increases, in comparison of problems S_{11} and S_{12}, both the OFV and CPU time increase as well.

As expected, the results obtained by PSO for large-scale problems reveal that the impact of the number of $|I|$ indices is much more than $|P|$, $|K|$ and $|J|$.

TABLE 4. Numerical results of PSO for large-scale problems

	Problem Dimension	Output	
No.	$\lvert I\rvert \times \lvert J\rvert \times \lvert K\rvert \times \lvert P\rvert$	**OFV (Rial)**	**CPU time (Sec.)**
S_9	100×45×45×100	4.3915×10^{16}	1074.22
S_{10}	300×45×45×700	7.9549×10^{20}	2573.78
S_{11}	300×45×45×900	1.8945×10^{21}	2892.59
S_{12}	300×45×45×1100	9.2395×10^{21}	3204.51

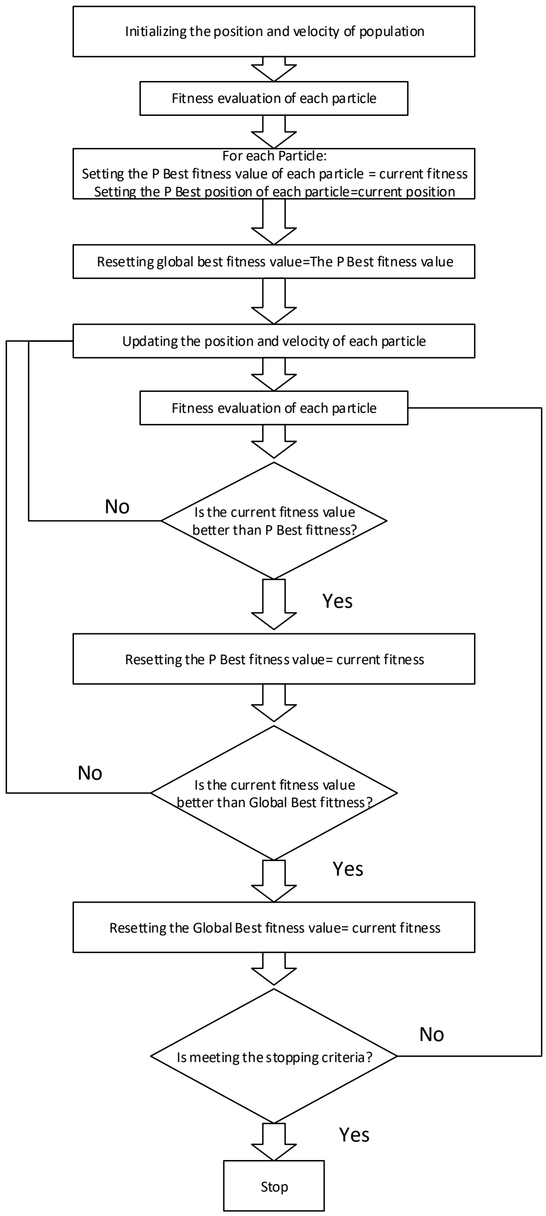

Figure 2. Flochart of PSO

The PSO algorithm is evaluated and the results are reported in Table 5. The results illustrate that the PSO algorithm can obtain good OFVs in reasonable CPU time. According to the results, there is no considerable gap between optimal OFVs which are achieved form GAMS and the function values which are obtained from PSO. Thus, this algorithm is reliable to perform for large-scale problems.

6. SENSITIVITY ANALYSIS

In this section, several sensitivity analyses to validate the proposed model are conducted based on the different parameter values of transportation cost weight (β) and inventory cost weight (θ). We obtained the optimal objective function value (OFV) and CPU time for different values of β and θ for dataset problem S_5 (i.e., ($|I|\times|J|\times|K|\times|P| = 30\times12\times12\times30$)). Table 6 presents the change of the OFV (i.e., total cost of supply chain) and CPU time when the weight of inventory and transportation is changed. As it reveals, when the value of θ and β increases, there is an increase in the OFV; however, the CPU time has no a constant behavior.

Figure 3 illustrates linear enhancement in the OFV Vs. β variation. Furthermore, in Figure 4, an increasing trend can be seen in the total cost of the whole supply chain (i.e., OFV) by increasing the inventory cost (θ).

As shown in Table 4 and Figure 4, the linear increment is demonstrated in the total supply chain cost by increasing θ and β. However, the model is more sensitive when the weight of the inventory cost increases.

TABLE 5. Comparison of GAMS and PSO results for small and medium-scale problems

	GAMS outputs			PSO output	
No	OFV (Rial)	CPU time (Sec)	GAP (%)	OFV (Rial)	CPU time (Sec)
S_0	4.5909×10^{11}	00.05	2.49	4.7083×10^{11}	3.27
S_1	9.1896×10^{11}	00.14	1.26	9.3073×10^{11}	5.18
S_2	1.8436×10^{12}	00.33	4.18	1.9241×10^{12}	10.07
S_3	3.5956×10^{12}	1.72	6.12	3.8301×10^{12}	28.77
S_4	5.4015×10^{12}	4.76	2.31	5.5291×10^{12}	35.05
S_5	1.6662×10^{13}	79.54	3.13	1.7201×10^{13}	21.35
S_6	2.8350×10^{13}	639.53	5.54	3.0012×10^{13}	58.92
S_7	5.9373×10^{13}	824.22	5.78	6.3012×10^{13}	69.52
S_8	1.1929×10^{14}	17022.1	6.26	1.2726×10^{14}	104.52
S_9	-	-	-	4.3915×10^{16}	1074.2

TABLE 6. Sensitivity analysis of θ and β

Value of θ and β	Sensitivity analysis for β		Sensitivity analysis for θ	
	OFV (Rial)	CPU time	OFV (Rial)	CPU time
0.01	1.6662×10^{13}	79.5	5.5762×10^{12}	56.6
0.5	2.2800×10^{13}	28.4	3.6703×10^{13}	63.5
1	2.6553×10^{13}	146.3	5.1719×10^{13}	79.7
1.5	3.0306×10^{13}	32.1	6.3242×10^{13}	64.4
2	3.4058×10^{13}	83.9	7.2956×10^{13}	34.4
2.5	3.7811×10^{13}	133.8	8.1515×10^{13}	43.0
3	4.1564×10^{13}	97.4	8.9252×10^{13}	35.6

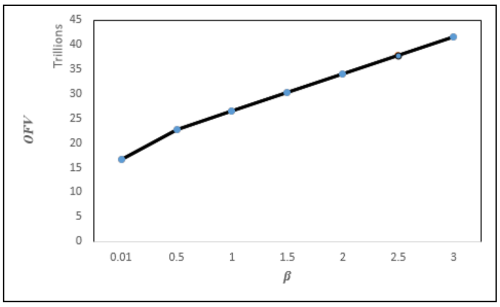

Figure 3. Variation of β vs. the OFV

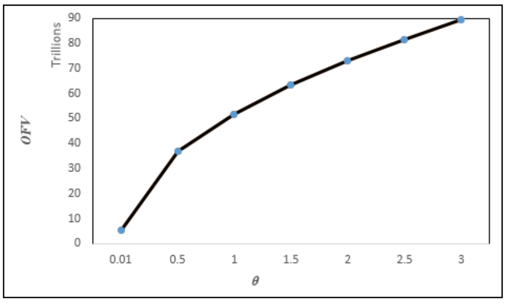

Figure 4. Variation of θ vs. the OFV

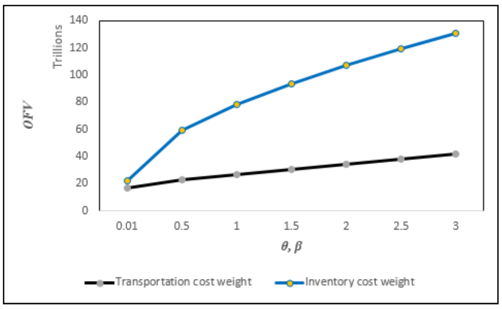

Figure 5. Variation of θ and β vs. the OFV

Furthermore, the ascending trend of the OFV in variation of θ is more than β. Thus, θ has more influence in increasing the total cost of the supply chain. For a large amount of θ and β, there is a monotonous increase in the OFV, and also a greater increase in the OFV can be seen for the value of 0.01 to 0.5 of both θ and β. It is more efficient for the decision maker in order to determine the appropriate value of θ and β.

7. CONCLUSION

In this paper, a closed-loop location inventory supply chain network design problem is studied within multi types of products to optimize the distribution and repair centers (RCs) location and to allocate each failed items to slow or fast repair service mode by minimizing the total supply chain cost and service level constraint.

The problem was formulated as a mixed-integer nonlinear location allocation model. In the forward supply chain, new products are distributed to operational bases through distribution centers (DCs) and the failed items are collected by using the reverse supply chain. Also, the failed items are repaired in repair centers (RCs), which are allocated in the location of DCs. The importance of the proposed formulation is allocation of each failed spare part to suitable repairing mode while service level is satisfied

The proposed model was solved using GAMS software for several small and medium-scale problems. Because of complexity and NP-hard nature of large-scale problems, GAMS software cannot solve large-scale ones within 24 hours. Therefore, particle swarm optimization (PSO) was proposed as an efficient meta-heuristic algorithm to solve large-scale problems in reasonable runtime. Moreover, several sensitivity analyses were conducted for different values of β and θ and illustrated the change of the objective function value (OFV) based on different values of these parameters. The results show that any changes in β and θ can influence the total cost. Also, these analyses demonstrate that the total cost is more sensitive to the weight of the inventory cost more than the transportation cost. Therefore, it is important to determine the accurate value of them for reducing supply chain total cost. Therefore, future research can consider the reliability of each repaired item. Also, the model can be developed by considering robust optimization to deal with uncertainty of repairing demand in a reverse supply chain to make model more flexible. Another extension is to consider some limitations on the depot space or capacity. Moreover, the development of meta-heuristics can be conducted to compare with each other and select the most efficient one.

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Closed-loop Supply Chain Inventory-location Problem with Spare Parts in a Multi-Modal Repair Condition

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چکیده

در این مقاله، مساله مکان یابی– موجودی برای انتقال قطعات یدکی در یک شبکه زنجیره تامین بسته دوسطحی شامل ایستگاه-های عملیاتی و مراکز تعمیر و توزیع مورد بررسی قرار می‌گیرد. در این زنجیره تامین، انواع مختلفی از قطعات یدکی از طریق مراکز توزیع به ایستگاه‌های عملیاتی، برای پاسخ گویی به نیاز این ایستگاه‌ها منتقل می‌شوند. همچنین قطعات معیوب از طریق شبکه زنجیره تامین معکوس جمع‌آوری شده و به مراکز تعمیر انتقال می‌یابند. این قطعات در مراکز تعمیر به دو روش خدمات تعمیراتی سریع و آهسته بررسی می‌شوند. به علاوه، در این مراکز هیچ گونه محدودیتی برای پذیرش قطعات یدکی معیوب به دو روش تعمیراتی وجود ندارد و تمامی قطعات معیوب با کمترین هزینه تعمیر خواهند شد. هدف مساله، انتخاب تعدادی از سایت‌های کاندید برای احداث مراکز توزیع و مراکز تعمیر به منظور برآورده ساختن تقاضای احتمالی قطعات یدکی و پاسخ گویی به تقاضای احتمالی تعمیر قطعات یدکی و همچنین انتخاب یکی از دو روش تعمیری (سریع یا آهسته) برای تعمیر قطعات یدکی معیوب می‌باشد. به منظور بررسی صحت مدل ارایه شده، چندین نمونه عددی با استفاده از نرم افزار بهینه سازی GAMS حل می‌شود. همچنین، به دلیل پیچیدگی مساله و نیاز به زمان اجرای بسیار طولانی برای رسیدن به جواب بهینه در مسایل با ابعاد بزرگ، الگوریتم بهینه‌سازی ازدحام ذرات (PSO) پیشنهاد می‌شود. در نهایت، چندین تحلیل حساسیت برای بررسی کارایی و رفتار مدل ارایه می‌گردد. همچنین، طبق نتایج محاسبات انجام شده الگوریتم فراابتکاری PSO از لحاظ کیفیت جواب و مدت زمان محاسبات برای مسایل با اندازه‌های متوسط و بزرگ بسیار کارا می‌باشد.

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