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Solving Critical Path Problem in Project Network by a New Enhanced Multi-objective Optimization of Simple Ratio Analysis Approach with Interval Type-2 Fuzzy Sets

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ABSTRACT

Decision making is an important issue in business and project management that assists finding the optimal alternative from a number of feasible alternatives. Decision making requires adequate consideration of uncertainty in projects. In this paper, in order to address uncertainty of project environments, interval type-2 fuzzy sets (IT2FSs) are used. In other words, the rating of each alternative and weight of each criterion are expressed by IT2FSs. Moreover, for obtaining weights of criteria, interval type-2 fuzzy analytic hierarchy process (AHP) method is employed. In addition, a new enhanced model of multi-objective optimization on the basis of simple ratio analysis (MOOSRA) method is developed with IT2F-relative preference relation. Finally, to illustrate applicability of the introduced approach, an existing application from literature is adopted and solved.

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1. INTRODUCTION

Multi-criteria decision making (MCDM) is one of the fastest growing areas. Since the acceptance of MCDM in areas of operation research and management science, several methodologies have been developed based on this discipline. In the conventional MCDM approaches, evaluation ratings and criteria weights are expressed by exact values. In other words, the problem is considered under certain environments. While in reality, the decision environment is not certain and has vagueness.

To address uncertainty, evaluation ratings and criteria weights in fuzzy MCDM (FMCDM) problems are expressed by imprecision and vagueness. As a result, ratings and weights are displayed by linguistic terms [1-4] which are then transferred into fuzzy numbers [5-7]. To solve FMCDM problems, defuzzification or fuzzy generalization was used to generalize classical MCDM methods under fuzzy environment. As a matter of fact, defuzzification causes loss of fuzzy messages.

One practical approach in FMCDM is using the fuzzy preference relation. It is a total ordering relation, which satisfies reciprocal and transitive laws on fuzzy numbers. Moreover, it satisfies a total ordering relation on fuzzy numbers. Given these facts, it can be concluded that applying preference relation is more reasonable in comparison with defuzzification in ranking fuzzy numbers. In other words, defuzzification cannot present preference degree between two fuzzy numbers and loses some messages of fuzziness [8].

Another new approach that has not been properly applied in project management is Multi-objective optimization on the basis of simple ratio analysis (MOOSRA). This relatively new multi-objective optimization method computes the simple ratio of beneficial and non-beneficial criteria in a decision making process [9, 10]. This method is categorized as one of the multi- objective optimization approaches. The MOOSRA method in comparison to the MOORA method does not reflect the negative performance scores

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and is less sensitive to large variation in the values of the criteria [11, 12]. In order to avoid negative outcomes, the MOOSRA method applies the simple ratio of the overall scores of the beneficial and non-beneficial criteria. Moreover, this process applies output and input evaluation to assist decision making process [10].

In relation to new research on fuzzy multi-criteria decision making that, Wei [13] has expressed interval valued hesitant fuzzy uncertain linguistic aggregation operators in multiple attribute decision making problems. Lu and Wei [14] have introduced the multiple attribute decision making (MADM) problem based on the arithmetic and geometric aggregation operators with dual hesitant fuzzy uncertain linguistic information. Moreover, interval valued dual hesitant fuzzy linguistic geometric aggregation operators in multiple attributes decision making problem have been presented by Wei et al. [15]. Furthermore, a cross entropy of picture fuzzy sets for multiple attribute decision making problems was introduced by Wei [16].

Apart from applying the right set of decision making methods, an important approach in FMCDM is using proper fuzzy sets. Given the high degree of uncertainty in project environment, applying classic fuzzy sets could cause some issues. Type-1 fuzzy sets (T1FSs) despite their novelty in expressing uncertainty in lack of historical data have several shortcomings. One is that in these sets decision maker (DM) has to use a precise value in a range [0, 1] to express membership degree [17]. One of the approaches used to overcome issues of T1FSs is using T2FS. The concept of type-2 fuzzy sets (T2FSs) initially introduced by Zadeh [18] have fuzzy membership function. In other words, each member of these sets has a fuzzy membership degree [19]. Type-2 fuzzy sets involve more uncertainties in comparison with type-1 fuzzy sets. They provide the DM with additional degrees of freedom to express the uncertainty and the fuzziness of the real world situations. Several scholars have recently applied T2FSs to address uncertain project environment [20, 21].

Critical path method (CPM) is the most commonly used project management technique for planning projects in real-world applications. This method assists project managers to obtain the completion time and critical activities of the project. The main aim is to assist decision makings concerning concentration of resources to reduce project completion time. In other word, critical path is the route in which the total float time equals zero. In past, the time was a determinative criterion to specify the critical path, but gradually cost, quality and safety added to time as effective criteria for determining critical path [22].

Regarding research in project scheduling area, Chanas and Zieliński [23] have expressed a solution for determine critical path by using fuzzy activity times. Moreover, a method for specifying critical path by means of fuzzy sets theory was presented by Liang and Han [24]. Chen and Hsueh [25] have introduced a simple fuzzy approach to determine critical path by using linear programing. Furthermore, a new method for project scheduling problems by trapezoidal fuzzy numbers was presented by Shanmugasundari and Ganesan [26]. Kaur and Kumar [27] have introduced a linear programing approach for solving critical path problems with fuzzy parameters. Also, a project scheduling method using triangular intuitionistic fuzzy numbers was expressed by Elizabeth and Sujatha [28]. Kazemi et al. [29] have presented an approach to specify critical path with random fuzzy activity times.

To conclude from the above, given the advantages of T2FSs, applying them in project environment is a practical approach. However, only a small number of studies have applied them to address project uncertainty. On the other hand, given the advantages of the relative preference relation and MOOSRA method for decision making, they have not been applied in project decision making problems. This paper, in order to overcome the existing gaps in the literature, presents a new approach that combines the extension of relative preference relation to interval type-2 fuzzy sets and the development of the MOOSRA method under IT2F uncertainty. Moreover, this paper offers a new ranking index based on the aforementioned methods. To illustrate the novelties of this paper, the following is presented:

- To properly address the existing uncertainty in the project environment, IT2FSs are applied.
- An enhanced MOOSRA under IT2FS uncertainty is introduced to address the multi-criteria decision making problem. This approach provides this decision-making problem with better ability in ranking the alternatives.
- An extension of the relative preference relation under IT2F-environment is developed for obtaining priority degree of each alternative by considering efficient criteria.
- A hybrid method under IT2FSs is developed to gain a final ranking.

The paper proceeds as follows. Section 2 expresses developed preference relation method under IT2FSs. Section 3 introduces proposed method. Section 4 presents the application of proposed method. Section 5 expresses sensitivity and comparative analysis. Finally, Section 6 concludes the paper.

2. INTRODUCING A NEW IT2F RELATIVE PREFERENCE RELATION METHOD

In this section, a new approach of relative preference relation is developed based on the concept of IT2FSs. The method is presented as follows:

If $\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$ are two trapezoidal IT2F numbers, then the fuzzy preference relation *P* is a fuzzy subset of *R*R* with membership function $\mu_p(\tilde{\tilde{A}},\tilde{\tilde{B}})$ representing preference degree of $\tilde{\tilde{A}}$ over $\tilde{\tilde{B}}$. We define:

$$\mu_P(\tilde{\tilde{A}},\tilde{\tilde{B}})=\frac{1}{2}\begin{bmatrix}\mu_P(\tilde{A}^U,\tilde{B}^U)+\\ \mu_P(\tilde{A}^L,\tilde{B}^L)\end{bmatrix}=$$
$$\frac{1}{2}\begin{bmatrix}\frac{1}{2}\left(\frac{\begin{matrix}(a^U_{l1}-b^U_{u4})+(a^U_{h2}-b^U_{g3})+\\(a^U_{g3}-b^U_{h2})+(a^U_{u4}-b^U_{l1})\end{matrix}}{2\|T^U\|}+1\right)+\\ \frac{1}{2}\left(\frac{\begin{matrix}(a^L_{l1}-b^L_{u4})+(a^L_{h2}-b^L_{g3})+\\(a^L_{g3}-b^L_{h2})+(a^L_{u4}-b^L_{l1})\end{matrix}}{2\|T^L\|}+1\right)\end{bmatrix} \tag{1}$$

where,

$$\|T^U\|=\begin{cases}\dfrac{(t^{+U}_{l1}-t^{-U}_{u4})+(t^{+U}_{h2}-t^{-U}_{g3})+(t^{+U}_{g3}-t^{-U}_{h2})+(t^{+U}_{u4}-t^{-U}_{l1})}{2} & if\ \ t^{+U}_{l1}\geq t^{-U}_{u4}\\ \dfrac{(t^{+U}_{l1}-t^{-U}_{u4})+(t^{+U}_{h2}-t^{-U}_{g3})+(t^{+U}_{g3}-t^{-U}_{h2})+(t^{+U}_{u4}-t^{-U}_{l1})}{2}+2(t^{-U}_{u}-t^{+U}_{l}) & if\ \ t^{+U}_{l1}\prec t^{-U}_{u4}\end{cases} \tag{2}$$

$$\|T^L\|=\begin{cases}\dfrac{(t^{+L}_{l1}-t^{-L}_{u4})+(t^{+L}_{h2}-t^{-L}_{g3})+(t^{+L}_{g3}-t^{-L}_{h2})+(t^{+L}_{u4}-t^{-L}_{l1})}{2} & if\ \ t^{+L}_{l1}\geq t^{-L}_{u4}\\ \dfrac{(t^{+L}_{l1}-t^{-L}_{u4})+(t^{+L}_{h2}-t^{-L}_{g3})+(t^{+L}_{g3}-t^{-L}_{h2})+(t^{+L}_{u4}-t^{-L}_{l1})}{2}+2(t^{-L}_{u4}-t^{+L}_{l1}) & if\ \ t^{+L}_{l1}\prec t^{-L}_{u4}\end{cases} \tag{3}$$

where,

$$\begin{aligned}
&t^{+L}_{l1}=\max\{a^L_{l1},b^L_{l1}\},t^{+L}_{h2}=\max\{a^L_{h2},b^L_{h2}\},\\
&t^{+L}_{g3}=\max\{a^L_{g3},b^L_{g3}\},t^{+L}_{u4}=\max\{a^L_{u4},b^L_{u4}\}\\
&t^{-L}_{l1}=\min\{a^L_{l1},b^L_{l1}\},t^{-L}_{h2}=\min\{a^L_{h2},b^L_{h2}\},\\
&t^{-L}_{g3}=\min\{a^L_{g3},b^L_{g3}\},t^{-L}_{u4}=\min\{a^L_{u4},b^L_{u4}\}\\
&t^{+U}_{l1}=\max\{a^U_{l1},b^U_{l1}\},t^{+U}_{h2}=\max\{a^U_{h2},b^U_{h2}\},\\
&t^{+U}_{g3}=\max\{a^U_{g3},b^U_{g3}\},t^{+U}_{u4}=\max\{a^U_{u4},b^U_{u4}\}\\
&t^{-U}_{l1}=\min\{a^U_{l1},b^U_{l1}\},t^{-U}_{h2}=\min\{a^U_{h2},b^U_{h2}\},\\
&t^{-U}_{g3}=\min\{a^U_{g3},b^U_{g3}\},t^{-U}_{u4}=\min\{a^U_{u4},b^U_{u4}\}
\end{aligned} \tag{4}$$

In addition, if $S=\{\tilde{\tilde{X}}_1,\tilde{\tilde{X}}_2,...,\tilde{\tilde{X}}_n\}$ indicate a set consisting of *n* trapezoidal fuzzy numbers in case: $\tilde{\tilde{X}}_j=((x^U_{jl1},x^U_{jh2},x^U_{jg3},x^U_{ju4};H_1(X^U_j),H_2(\tilde{X}^U_j)),$ where $(x^L_{jl1},x^L_{jh2},x^L_{jg3},x^L_{ju4};H_1(\tilde{X}^L_j),H_2(\tilde{X}^L_j)))\ \in S$ *j=1,2,..,n*. By extension principle, $\bar{\tilde{\tilde{X}}}=(1/n)\otimes(\tilde{\tilde{X}}_1,\tilde{\tilde{X}}_2,...,\tilde{\tilde{X}}_n)=((\bar{x}^U_{jl1},\bar{x}^U_{jh2},\bar{x}^U_{jg3},\bar{x}^U_{ju4};\min_j H_1(\bar{\tilde{X}}^U_j),\min_j H_2(\bar{\tilde{X}}^U_j)),(\bar{x}^L_{jl1},\bar{x}^L_{jh2},\bar{x}^L_{jg3},\bar{x}^L_{ju4};\min_j H_1(\bar{\tilde{X}}^L_j),\min_j H_2(\bar{\tilde{X}}^L_j)))$ which is assumed to be average of $\tilde{\tilde{X}}_1,\tilde{\tilde{X}}_2,...,\tilde{\tilde{X}}_n$. Then, the relative preference relation P^* with membership function $\mu_{p^*}(\tilde{\tilde{X}}_j,\bar{\tilde{\tilde{X}}})$ representing relative preference degree of $\tilde{\tilde{X}}_j$ over $\bar{\tilde{\tilde{X}}}$ in *S*, where $j=1,2,...,n$ is defined as:

$$\mu_{p^*}(\tilde{\tilde{X}}_j,\bar{\tilde{\tilde{X}}})=\frac{1}{2}\begin{bmatrix}\mu_{p^*}(\tilde{X}^U_j,\bar{\tilde{X}}^U)+\\ \mu_{p^*}(\tilde{X}^L_j,\bar{\tilde{X}}^L)\end{bmatrix}=$$
$$\frac{1}{2}\begin{bmatrix}\frac{1}{2}\left(\frac{\begin{matrix}(x^U_{jl1}-\bar{x}^U_{u4})+(x^U_{jh2}-\bar{x}^U_{g3})+\\(x^U_{jg3}-\bar{x}^U_{h2})+(x^U_{ju4}-\bar{x}^U_{l1})\end{matrix}}{2\|T^U_S\|}+1\right)+\\ \frac{1}{2}\left(\frac{\begin{matrix}(x^L_{jl1}-\bar{x}^L_{u4})+(x^L_{jh2}-\bar{x}^L_{g3})+\\(x^L_{jg3}-\bar{x}^L_{h2})+(x^L_{ju4}-\bar{x}^L_{l1})\end{matrix}}{2\|T^L_S\|}+1\right)\end{bmatrix} \tag{5}$$

where

$$\|T^U_S\|=\begin{cases}\dfrac{(t^{+U}_{sl1}-t^{-U}_{su4})+(t^{+U}_{sh2}-t^{-U}_{sg3})+(t^{+U}_{sg3}-t^{-U}_{sh2})+(t^{+U}_{su4}-t^{-U}_{sl1})}{2} & if\ \ t^{+U}_{sl1}\geq t^{-U}_{su4}\\ \dfrac{(t^{+U}_{sl1}-t^{-U}_{su4})+(t^{+U}_{sh2}-t^{-U}_{sg3})+(t^{+U}_{sg3}-t^{-U}_{sh2})+(t^{+U}_{su4}-t^{-U}_{sl1})}{2}+2(t^{-U}_{su4}-t^{+U}_{sl1}) & if\ \ t^{+U}_{sl1}\prec t^{-U}_{su4}\end{cases} \tag{6}$$

$$\|T^L_S\|=\begin{cases}\dfrac{(t^{+L}_{sl1}-t^{-L}_{su4})+(t^{+L}_{sh2}-t^{-L}_{sg3})+(t^{+L}_{sg3}-t^{-L}_{sh2})+(t^{+L}_{su4}-t^{-L}_{sl1})}{2} & if\ \ t^{+L}_{sl1}\geq t^{-L}_{su4}\\ \dfrac{(t^{+L}_{sl1}-t^{-L}_{su4})+(t^{+L}_{sh2}-t^{-L}_{sg3})+(t^{+L}_{sg3}-t^{-L}_{sh2})+(t^{+L}_{su4}-t^{-L}_{sl1})}{2}+2(t^{-L}_{su4}-t^{+L}_{sl1}) & if\ \ t^{+L}_{sl1}\prec t^{-L}_{su4}\end{cases} \tag{7}$$

$$
\begin{aligned}
&t_{Sl1}^{+L}=\max_j\left\{x_{jl1}^{L}\right\},t_{sh2}^{+L}=\max_j\left\{x_{jh2}^{L}\right\},\\
&t_{sg3}^{+L}=\max_j\left\{x_{jg3}^{L}\right\},t_{su4}^{+L}=\max_j\left\{x_{ju4}^{L}\right\}\\
&t_{sl1}^{-L}=\min_j\left\{x_{jl1}^{L}\right\},t_{sh2}^{-L}=\min_j\left\{x_{jh2}^{L}\right\},\\
&t_{sg3}^{-L}=\min_j\left\{x_{jg3}^{L}\right\},t_{su4}^{-L}=\min_j\left\{x_{ju4}^{L}\right\}\\
&t_{sl1}^{+U}=\max_j\left\{x_{jl1}^{U}\right\},t_{sh2}^{+U}=\max_j\left\{x_{jh2}^{U}\right\},\\
&t_{sg3}^{+U}=\max_j\left\{x_{jg3}^{U}\right\},t_{su4}^{+U}=\max_j\left\{x_{ju4}^{U}\right\}\\
&t_{sl1}^{-U}=\min_j\left\{x_{jl1}^{U}\right\},t_{sh2}^{-U}=\min_j\left\{x_{jh2}^{U}\right\},\\
&t_{sg3}^{-U}=\min_j\left\{x_{jg3}^{U}\right\},t_{su4}^{-U}=\min_j\left\{x_{ju4}^{U}\right\}
\end{aligned}
\tag{8}
$$

3. PROPOSED ENHANCED DECISION APPROACH

In this paper, IT2FSs are used to consider uncertainty. Also, the MOOSRA method is extended to IT2FSs. Moreover, the method presented by [30] based on relative preference relation is extended to IT2FSs for obtaining priority degree of each alternative and ranking the IT2F numbers. Finally, an enhanced model based on these two methods is introduced for ranking the alternatives by considering important criteria. IT2F sets are more powerful than type-1 fuzzy sets. Each member of IT2F set has a type-1 fuzzy membership degree. Type-2 fuzzy sets involve more uncertainties in comparison with type-1 fuzzy sets. They provide the DM with additional degrees of freedom to express the uncertainty and the fuzziness of the real-world situations. Also, MOOSRA method in comparison to the MOORA method does not reflect the negative performance scores and is less sensitive to large variation in the values of the criteria. In order to avoid negative outcomes, the MOOSRA method applies the simple ratio of the overall scores of the beneficial and non-beneficial criteria. Moreover, this procedure applies output and input evaluation to assist decision making process. In this paper, to use the advantages of IT2FSs and MOOSRA method, the enhanced MOOSRA method will be developed under an IT2F-environment.

Step 1. Form a team of experts who are responsible to determine the best alternative considering the evaluating criteria. Experts' judgments on qualitative criteria are converted to their equivalent IT2FNs presented in Table 1.

Step 2. Construct the decision matrix Y_p of the pth DM and construct the average decision matrixes $\overline{Y}$, respectively. The aggregated IT2F-information of the alternatives on each criterion are obtained via Equation (9).

TABLE 1. Linguistic terms and their corresponding interval type-2 fuzzy sets [31]

Linguistic terms	Interval type-2 fuzzy sets
Very Low (VL)	((0,0,0,0.1;1,1), (0,0,0,0.05;0.9,0.9))
Low (L)	((0,0.1,0.1,0.3;1,1), (0.05,0.1,0.1,0.2;0.9,0.9))
Medium Low (ML)	((0.1,0.3,0.3,0.5;1,1), (0.2,0.3,0.3,0.4;0.9,0.9))
Medium (M)	((0.3,0.5,0.5,0.7;1,1), (0.4,0.5,0.5,0.6;0.9,0.9))
Medium High (MH)	((0.5,0.7,0.7,0.9;1,1), (0.6,0.7,0.7,0.8;0.9,0.9))
High (H)	((0.7,0.9,0.9,1;1,1), (0.8,0.9,0.9,0.95;0.9,0.9))
Very High (VH)	((0.9,1,1,1;1,1), (0.95,1,1,1;0.9,0.9))

Integration method is as follows:

$$
Y_p=(\tilde{\tilde{f}}_{ij}^{\,p})_{m\times n}=
\begin{array}{c} \\ A_1 \\ \vdots \\ A_m \end{array}
\begin{array}{c}
\begin{array}{ccc} C_1 & \cdots & C_n \end{array}\\
\begin{pmatrix}
\tilde{\tilde{f}}_{11}^{\,p} & \cdots & \tilde{\tilde{f}}_{1n}^{\,p}\\
\vdots & \ddots & \vdots\\
\tilde{\tilde{f}}_{m1}^{\,p} & \cdots & \tilde{\tilde{f}}_{mn}^{\,p}
\end{pmatrix}
\end{array}
\tag{9}
$$

$\overline{Y}=(\tilde{\tilde{f}}_{ij})_{m\times n}$, where, $\tilde{\tilde{f}}_{ij}=(\dfrac{\tilde{\tilde{f}}_{ij}^{\,1}\oplus\tilde{\tilde{f}}_{ij}^{\,2}\oplus\ldots\oplus\tilde{\tilde{f}}_{ij}^{\,k}}{k})$. $\tilde{\tilde{f}}_{ij}$ is an IT2FS and $1\le i\le m, 1\le j\le n, 1\le p\le k$. Also, k denotes the number of DMs; also, $A_1,A_2,...,A_m$ are feasible alternatives and $C_1,C_2,...,C_n$ are evaluation criteria.

Step 3. Compute the normalized IT2F decision matrix. The data are normalized using the following:

$$
\tilde{\tilde{f}}_{ij}=\begin{bmatrix}
(\dfrac{a_{ij1}^{U}}{d_j},\dfrac{a_{ij2}^{U}}{d_j},\dfrac{a_{ij3}^{U}}{d_j},\dfrac{a_{ij4}^{U}}{d_j};H_1(\tilde{A}_i^{U}),H_2(\tilde{A}_i^{U}),\\
(\dfrac{a_{ij1}^{L}}{d_j},\dfrac{a_{ij2}^{L}}{d_j},\dfrac{a_{ij3}^{L}}{d_j},\dfrac{a_{ij4}^{L}}{d_j};H_1(\tilde{A}_i^{L}),H_2(\tilde{A}_i^{L})
\end{bmatrix}
\tag{10}
$$
$$j=1,2,\ldots,n,\ i=1,2,\ldots,m$$

where

$$
d_j=\sqrt{\sum_{i=1}^{m}\sum_{p=1}^{4}(a_{ijp}^{L})^2+\sum_{i=1}^{m}\sum_{p=1}^{4}(a_{ijp}^{U})^2},
\tag{11}
$$
$$p=\{1,2,3,4\}\ \ for\ \ \forall j=1,2,...,n$$

Step 4. Determine weight of each criterion by using AHP method.

In this step, AHP method and pairwise comparison matrix are used to obtain weight of each criterion. Linguistic variables are used to express judgments of experts. Table 2 presents the linguistic variables and their corresponding IT2FNs. Judgments of experts are aggregated by means of Equation (9).

TABLE 2. Definition and interval type-2 fuzzy scales of the linguistic variables [32]

Linguistic variables	Interval type-2 fuzzy scales
Absolutely strong (AS)	((7,8,9,9;1,1), (7.2,8.2,8.8,9;0.9,0.9))
Very strong (VS)	((5,6,8,9;1,1), (5.2,6.2,7.8,8.8;0.9,0.9))
Fairly strong (FS)	((3,4,6,7;1,1), (3.2,4.2,5.8,6.8;0.9,0.9))
Slightly strong (SS)	((1,2,4,5;1,1), (1.2,2.2,3.8,4.8;0.9,0.9))
Exactly equal (E)	((1,1,1,1;1,1), (1,1,1,1;1,1))
If factor *i* has one of the above linguistic variables assigned to it when compared with factor *j*, then *j* has the reciprocal value when compared with *i*	Reciprocals of the above

Step 4. 1. Construct the IT2F pairwise comparison matrix A_k of the *k*th DM and construct the average decision matrix $\bar{A}$. Each element of the pairwise comparison matrix is an IT2FS that denotes the comparative importance of two criteria. The pairwise comparison matrix is presented as follows:

$$A_K = (\tilde{\tilde{a}}_{ij}^k)_{n\times n} = \begin{pmatrix} 1 & \tilde{\tilde{a}}_{12}^k & \cdots & \tilde{\tilde{a}}_{1n}^k \\ \tilde{\tilde{a}}_{21}^k & 1 & \cdots & \tilde{\tilde{a}}_{2n}^k \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{\tilde{a}}_{n1}^k & \tilde{\tilde{a}}_{n2}^k & \cdots & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & \tilde{\tilde{a}}_{12}^k & \cdots & \tilde{\tilde{a}}_{1n}^k \\ 1/\tilde{\tilde{a}}_{12}^k & 1 & \cdots & \tilde{\tilde{a}}_{2n}^k \\ \vdots & \vdots & \ddots & \vdots \\ 1/\tilde{\tilde{a}}_{1n}^k & 1/\tilde{\tilde{a}}_{2n}^k & \cdots & 1 \end{pmatrix}, \quad \bar{A} = (\bar{\tilde{\tilde{a}}}_{ij})_{n\times n} \tag{12}$$

Step 4. 2. Examine the consistency of the fuzzy pairwise comparison matrices. Let $A = [a_{ij}]$ be a positive reciprocal matrix and $\tilde{\tilde{A}} = [\tilde{\tilde{a}}_{ij}]$ be a fuzzy positive reciprocal matrix. If the result of the comparisons of $A = [a_{ij}]$ is consistent, then it can be concluded that the outcome of the comparisons of $\tilde{\tilde{A}} = [\tilde{\tilde{a}}_{ij}]$ is also consistent [33].

Step 4. 3. Calculate the geometric mean of *k* type-2 fuzzy sets. Compute the fuzzy geometric mean for each criterion. The geometric mean of each row of $\tilde{\tilde{A}} = [\tilde{\tilde{a}}_{ij}]$ is calculated as:

$$\tilde{\tilde{r}}_i = \sqrt[n]{\tilde{\tilde{A}}_{i1} \otimes \tilde{\tilde{A}}_{i2} \otimes \cdots \otimes \tilde{\tilde{A}}_{in}} \tag{13}$$

Step 4. 4. Compute the weights of the criteria by normalization. The IT2F weight of the *i*th criterion is calculated as:

$$\tilde{\tilde{w}}_i = \tilde{\tilde{r}}_i \otimes \left[\tilde{\tilde{r}}_1 \oplus \tilde{\tilde{r}}_2 \oplus \cdots \oplus \tilde{\tilde{r}}_n\right]^{-1} \tag{14}$$

Step 5. Construct weighted normalized decision matrix by multiplying weight of each criterion to decision matrix.

$$\bar{Y}_W = (\tilde{\tilde{v}}_{ij})_{m\times n} = \bar{Y} \times \bar{w} = \begin{pmatrix} \tilde{\tilde{f}}_{11} \times \tilde{\tilde{w}}_1 & \cdots & \tilde{\tilde{f}}_{1n} \times \tilde{\tilde{w}}_n \\ \vdots & \ddots & \vdots \\ \tilde{\tilde{f}}_{m1} \times \tilde{\tilde{w}}_1 & \cdots & \tilde{\tilde{f}}_{mn} \times \tilde{\tilde{w}}_n \end{pmatrix} = \begin{pmatrix} \tilde{\tilde{v}}_{11} & \cdots & \tilde{\tilde{v}}_{1n} \\ \vdots & \ddots & \vdots \\ \tilde{\tilde{v}}_{m1} & \cdots & \tilde{\tilde{v}}_{mn} \end{pmatrix} \tag{15}$$

Step 6. Compute the overall performance score of each alternative (y_i) by using IT2F-MOOSRA. The extension of MOOSRA to IT2FSs is presented as follows:

$$\tilde{\tilde{y}}_i = \frac{\sum_{j=1}^{g} \tilde{\tilde{V}}_{ij}}{\sum_{j=g+1}^{n} \tilde{\tilde{V}}_{ij}} = \begin{bmatrix} ((y_{i1}^U, y_{i2}^U, y_{i3}^U, y_{i4}^U \\ ;H_1(y_i^U), H_2(y_i^U)), \\ ((y_{i1}^L, y_{i2}^L, y_{i3}^L, y_{i4}^L; \\ H_1(y_i^L), H_2(y_i^L)) \end{bmatrix} \tag{16}$$

Step 7. Compute the priority of each alternative based on criteria by using the following sub steps:

In this step, a model for obtaining priority of each alternative based on criteria by using the presented method of Wang [30] by using the concept of relative preference relation and IT2FSs is introduced.

At first, by using Equations (5)-(8), the relative preference degree of obtained weight in step 4 over average is calculated. Then, the normalized decision matrix presented in step 3 is changed as follows:

$$\begin{array}{c c} & \begin{array}{ccc} A_1 & \cdots & A_m \end{array} \\ \begin{array}{c} C_1 \\ \vdots \\ C_n \end{array} & \begin{pmatrix} \tilde{\tilde{f}}_{11}^p & \cdots & \tilde{\tilde{f}}_{1m}^p \\ \vdots & \ddots & \vdots \\ \tilde{\tilde{f}}_{n1}^p & \cdots & \tilde{\tilde{f}}_{nm}^p \end{pmatrix} \end{array} \tag{17}$$

Here, the priority degree (PD) of each alternative based on evaluation criteria is calculated as follows:

$$(PD_1, PD_2, ..., PD_m) = (1/n) \otimes \begin{bmatrix} \mu_{p*}(\tilde{\tilde{w}}_1, \bar{\tilde{\tilde{w}}}), \\ \mu_{p*}(\tilde{\tilde{w}}_2, \bar{\tilde{\tilde{w}}}), ..., \\ \mu_{p*}(\tilde{\tilde{w}}_n, \bar{\tilde{\tilde{w}}}) \end{bmatrix} \begin{pmatrix} \tilde{\tilde{f}}_{11}^p & \cdots & \tilde{\tilde{f}}_{1m}^p \\ \vdots & \ddots & \vdots \\ \tilde{\tilde{f}}_{n1}^p & \cdots & \tilde{\tilde{f}}_{nm}^p \end{pmatrix} \tag{18}$$

where, $PD_1, PD_2, ..., PD_m$ are the priority degrees of $A_1, A_2, ..., A_m$ alternatives based on evaluation criteria, also the PD_i is obtained as follows:

$$PD_i = \frac{1}{n} \otimes \begin{bmatrix} \mu_{p^*}(\tilde{\tilde{w}}_1, \bar{\tilde{\tilde{w}}}), \\ \mu_{p^*}(\tilde{\tilde{w}}_2, \bar{\tilde{\tilde{w}}}), ..., \\ \mu_{p^*}(\tilde{\tilde{w}}_n, \bar{\tilde{\tilde{w}}}) \end{bmatrix} \begin{bmatrix} \tilde{\tilde{f}}_{1i} \\ \tilde{\tilde{f}}_{2i} \\ \vdots \\ \tilde{\tilde{f}}_{ni} \end{bmatrix} = \frac{1}{n} \otimes \begin{bmatrix} (\mu_{p^*}(\tilde{\tilde{w}}_1, \bar{\tilde{\tilde{w}}}) \otimes \tilde{\tilde{f}}_{1i}) \\ \oplus (\mu_{p^*}(\tilde{\tilde{w}}_2, \bar{\tilde{\tilde{w}}}) \otimes \tilde{\tilde{f}}_{2i}) \\ \oplus ... \oplus (\mu_{p^*}(\tilde{\tilde{w}}_n, \bar{\tilde{\tilde{w}}}) \otimes \tilde{\tilde{f}}_{ni}) \end{bmatrix}, i = 1, 2, ..., m \tag{19}$$

Since $\mu_{p^*}(\tilde{\tilde{w}}_n, \bar{\tilde{\tilde{w}}})$ is a crisp value and $\tilde{\tilde{f}}_{ni}$ is a IT2F number, PD_i is a IT2F number. In fact, relation (37) demonstrates priority degree (*PD*) of each alternative which is obtained by multiply the weight of each criterion over average, computed by relative preference relation, in IT2F normalized decision matrix. In other words, by using relative preference relation and its advantages, multiplying IT2F weights in IT2F decision matrix is avoided and easier computations in the proposed method are resulted.

Step 8. Combine the outcome of MOOSRA method in step 6 with the results of step 7 to present a single ranking (*SR*) as follow:

$$S\tilde{\tilde{R}} = \beta \otimes \tilde{\tilde{y}}_i + (1-\beta) \otimes P\tilde{\tilde{D}}_i \tag{20}$$

The value of β is determined according to DM's opinion. $S\tilde{\tilde{R}}$ illustrates linear combination of MOOSRA method and priority degree of each alternative to obtain a better final ranking by utilization advantages of two methods. In fact, $S\tilde{\tilde{R}}$ uses the advantages of MOOSRA method and advantages of priority degree, simultaneously. Some of these advantages are expressed as follows: avoiding negative performance scores (in comparison with MOORA method), being less sensitive to large variation in the values of the criteria (in comparison with MOORA method), considering more uncertainty due to applying IT2FSs, avoiding multiplying two IT2F matrixes by using advantages of relative preference relation and using relative preference relation that does not ignore fuzzy message in comparison to defuzzification.
The value of β is determined by DMs and by means of β DMs can assign various importance to each method.

Step 9. This step's result is IT2F numbers. In order to rank these numbers, relative preference relation is used as follows:

$$S\tilde{\tilde{R}}_i = ((sr^U_{il1}, sr^U_{ih2}, sr^U_{ig3}, sr^U_{iu4}; H_1(S\tilde{R}^U_i), H_2(S\tilde{R}^U_i)), sr^L_{il1}, sr^L_{ih2}, sr^L_{ig3}, sr^L_{iu4}; H_1(S\tilde{R}^L_i), H_2(S\tilde{R}^L_i))) \tag{21}$$

Then,

$$\bar{S\tilde{\tilde{R}}} = \frac{1}{m}(S\tilde{\tilde{R}}_1 \oplus S\tilde{\tilde{R}}_2 \oplus ... \oplus S\tilde{\tilde{R}}_m) = ((\bar{sr}^U_{il1}, \bar{sr}^U_{ih2}, \bar{sr}^U_{ig3}, \bar{sr}^U_{iu4}; \min_i H_1(S\tilde{R}^U_i), \min_i H_2(S\tilde{R}^U_i)), (\bar{sr}^L_{il1}, \bar{sr}^L_{ih2}, \bar{sr}^L_{ig3}, \bar{sr}^L_{iu4}; \min_i H_1(S\tilde{R}^L_i), \min_i H_2(S\tilde{R}^L_i))) \tag{22}$$

Here, by using Equations (5)-(8) the final ranking is obtained via computing relative preference relation of $S\tilde{\tilde{R}}_i^*$ over $\bar{S\tilde{\tilde{R}}}^*$.

4. APPLICATION

To better demonstrate the applicability of the proposed approach, an existing application example from the literature [22] is adopted and solved in this section. The aim of this application example is to determine the critical path of a project. The main activities of this project are depicted in Figure 1. To find the critical path, four criteria will be considered: time in days, cost in Euros, and quality and safety which will be assessed using the fuzzy linguistic variables are shown in Table 1. The data addressing various criteria for the project, expressed as IT2FSs by three experts, are shown in Tables 3-5.

Ranking the IT2F final results based on relation preference over the average value is carried out. The outcome is illustrated in Table 6.

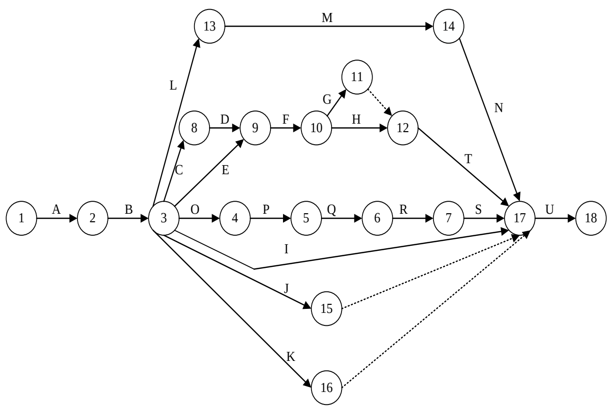

Figure 1. Proposed framework

TABLE 3. IT2F information of activities on quality and safety criteria

ACT	Quality			Safety		
	DM_1	DM_2	DM_3	DM_1	DM_2	DM_3
A	MH	MH	M	ML	M	ML
B	ML	MH	M	ML	M	ML
C	ML	ML	L	L	M	ML
D	H	MH	M	H	MH	MH
. . .						
R	MH	MH	M	M	M	ML
S	MH	M	ML	M	M	ML
T	MH	MH	M	M	ML	ML
U	MH	MH	M	ML	M	ML

TABLE 4. IT2F information of activities on time criterion (days)

ACT	Predecessors	$DM_1 \ldots DM_3$
A	-	((0.4,0.5,0.6,0.7;1,1), (0.45,0.55,0.6,0.65;0.9,0.9))
B	A	((0.15,0.3,0.45,0.55;1,1), (0.2,0.35,0.4,0.5;0.9,0.9))
C	B	((0.15,0.2,0.3,0.4;1,1), (0.17,0.25,0.3,0.35;0.9,0.9))
⋮	⋮	⋮
S	R	((0.75,1,1.5,2;1,1), (0.9,1.2,1.5,1.9;0.9,0.9))
T	G,H,N	((0.2,0.3,0.4,0.5;1,1), (0.25,0.35,0.4,0.45;0.9,0.9))
U	I,J,K,S,T	((0.2,0.3,0.4,0.5;1,1) (0.25,0.35,0.4,0.45;0.9,0.9))

TABLE 5. IT2F information of activities on cost criterion (100 euros)

ACT.	DMs
A	((7,7.5,7.75,8;1,1), (7.25,7.6,7.7,7.9;0.9,0.9))
B	((2.85,3,3.1,3.25;1,1), (2.95,3,3.1,3.15;0.9,0.9))
C	((4,5,6,7;1,1), (4.5,5.5,5.9,6.8;0.9,0.9))
D	((21,22,27,30;1,1), (21.5,22.5,25.5,28;0.9,0.9))
⋮	⋮
R	((4,5,6,7;1,1), (4.5,5,5.5,6.5;0.9,0.9))
S	((7,7.5,8.25,9;1,1), (7.25,7.6,8,8.5;0.9,0.9))
T	((4.5,5,5.75,6.5;1,1), (4.7,5.2,5.5,6;0.9,0.9))
U	((4.5,5,5.75,6.5;1,1), (4.7,5.2,5.5,6;0.9,0.9))

TABLE 6. Final ranking

Path	Critical Path	Preference degree	Ranking
1	A-B-L-M-N-T-U	0.5404	4
2	A-B-E-F-G-T-U	0.5559	3
3	A-B-C-D-F-H-T-U	0.5563	2
4	A-B-O-P-Q-R-S-U	0.5573	1
5	A-B-I-U	0.4190	7
6	A-B-J-U	0.4396	5
7	A-B-K-U	0.4315	6

5. SENSITIVITY AND COMPARATIVE ANALYSIS

In order to perform sensitivity analysis of the method, the value of β in Equation (20) is changed and the results are presented. In fact, the amount of β denotes the importance value of MOOSRA method and $1-\beta$ depicts the importance of the priority degree of each alternative based on each criterion. At first, the amount of β is set to 0.1; therefore, the amount of $1-\beta$ equals 0.9. The value of β is increased to 1 in intervals of 0.1. The model is solved by $\beta=0.1$ in step 1 and $\beta=0.2$ in step 2 and so on, except step 11 in which the amount of β is 0. Final results of model over 11 experiments have been shown in Figure 2. Path 4 in 64% of times is the best alternative.

To illustrate the advantages of proposed method, the problem of critical path selection is solved by proposed method under type-1 fuzzy sets which is depicted in Table 7. Moreover, the critical path was also selected by using fuzzy TOPSIS [31]. Using TOPSIS method to find the critical path provided the same results are the presented method. The results demonstrate reliability and capability of the proposed method. Also, the results of solving problem by type-1 fuzzy sets were compared to results of solving problem under IT2FSs which is illustrated in Tables 6 and 7. It can be observed that the results are the same.

The basic difference between the two kinds of fuzzy sets is that the memberships of a type-1 fuzzy set are crisp numbers whereas the memberships of a type-2 fuzzy set are type-1 fuzzy sets.

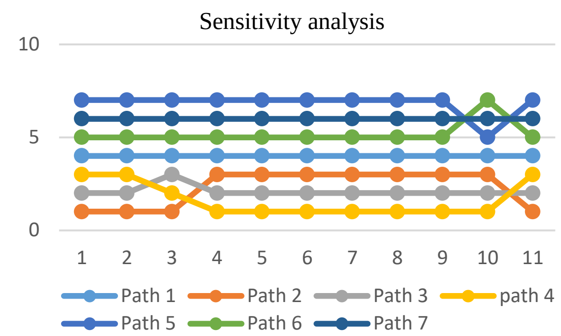

Figure 2. Sensitivity analysis

TABLE 7. Results of critical path selection under classic fuzzy environments

Path	Proposed method	Final ranking	Chen and Lee [31]	Final ranking
1	0.542439	4	0.53837	4
2	0.561703	3	0.550021	3
3	0.562406	2	0.550134	2
4	0.559808	1	0.554812	1
5	0.413025	7	0.425010	7
6	0.434514	5	0.44477	5
7	0.426105	6	0.436883	6

Nevertheless, type-2 fuzzy sets involve more uncertainties than type-1 fuzzy sets. In fact, IT2FSs are more flexible and capable than type-1 fuzzy sets and that is the reason why in this paper the proposed method is developed under IT2FS uncertainty.

6. CONCLUSION

Considering uncertainty in real-world situations is an important issue in decision making problems. Since interval type-2 fuzzy sets (IT2FSs) can express uncertainty better than type-1 fuzzy sets, in this paper the fuzzy sets were applied to address uncertainty. Moreover, a new enhanced model based on IT2F-MOOSRA method and IT2F relative preference relation has been presented for decision making and obtaining best alternative by considering efficient criteria. By extending a new method, priority degree of each alternative has been obtained; then, the results of this enhanced method have been combined with the results of MOOSRA method to achieve a ranking. In addition to obtain the final ranking, relative preference relation has been used to compare IT2F numbers. At the end, an example from the literature has been presented and solved by the presented process. Finally, sensitivity analysis and discussion of final results have been presented. The proposed method can be applied to group decision making problems in many different project management problems. Furthermore, the proposed method provided a useful way to handle fuzzy multiple attributes group decision-making problems in a more flexible and more intelligent manner due to the fact that it used IT2FSs to represent the evaluating values and the weights of attributes. Cross entropy can be added to this method to determine the weight of each criterion. In addition, this method can be developed in hesitant, dual hesitant, interval valued hesitant and interval valued dual hesitant fuzzy sets.

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Solving Critical Path Problem in Project Network by a New Enhanced Multi-objective Optimization of Simple Ratio Analysis Approach with Interval Type-2 Fuzzy Sets

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چکیده

تصمیم‌گیری یک مساله مهم در کسب و کار و مدیریت پروژه است که به پیدا کردن آلترناتیو بهینه از بین آلترناتیوهای شدنی کمک می‌کند. تصمیم‌گیری نیاز به توجه کافی به عدم قطعیت در پروژه ها دارد. در این مقاله، به منظور مواجهه با عدم قطعیت‌های موجود در پروژه، مجموعه‌های فازی نوع ۲ بازه‌ای استفاده می‌شود. به عبارت دیگر ارزیابی آلترناتیوها و وزن هر معیار به صورت اعداد فازی نوع ۲ بازه‌ای بیان می‌شود. علاوه بر این، برای به دست آوردن وزن معیارها از روش وزن دهی فازی نوع ۲ بازه‌ای *AHP* استفاده می‌شود. به علاوه، یک مدل ترکیبی از روش *MOOSRA* به همراه ارتباط اولویت نسبی فازی نوع ۲ بازه‌ای توسعه داده می‌شود. در انتها برای نشان دادن قابلیت روش معرفی شده یک مثال کاربردی موجود در مرور ادبیات حل می‌شود.

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