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Fitting the Three-parameter Weibull Distribution by using Greedy Randomized Adaptive Search Procedure

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P A P E R   I N F O

A B S T R A C T

The Weibull distribution is widely employed in several areas of engineering because it is an extremely flexible distribution with different shapes. Moreover, it can include characteristics of several other distributions. However, successful usage of Weibull distribution depends on estimation accuracy for three parameters of scale, shape and location. This issue shifts the attentions to the requirement for effective methods of Weibull parameters estimation. It is a known fact that the estimation procedure is inherently a very complicated procedure when the three-parameter Weibull distribution is of interest. Hence, this study suggests a computational approach, greedy randomized adaptive search procedures, with several neighborhood local searches to enhance the quality of estimations. Computational experiments are also implemented to assess the quality of estimations as opposed to benchmark grid search algorithm.

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1. INTRODUCTION

The probabilistic Weibull model is an extremely flexible distribution because of its different curve shapes. This property has made it capable in fitting of a wide range of experimental data very well and consequently has given rise to widespread real applications. For example, it is the most widely employed distribution for failure analysis in various types of systems in which decreasing and increasing hazard rates are taken into account [1]. Moreover, the Weibull distribution has been used in radar systems to model the dispersion of the received signal level caused by clutters. Furthermore, the distribution is especially useful for doing statistical analysis of satellite reliability and simulation of significant wave height [2]. Other application areas include wind energy potential [3], biomedical sciences [4], air quality determination [5], food products drying technology [6], model for growth/decline in product sales [7], and optimum adhesive thickness in structural adhesives joints [8].

The probability density function of the three-parameter Weibull distribution is as below [9]:

$$f_X(x;\ \alpha,\beta,\delta) = \frac{\beta}{\alpha}\left(\frac{x-\delta}{\alpha}\right)^{\beta-1} e^{\left(\frac{x-\delta}{\alpha}\right)^{\beta}};\quad \alpha,\beta > 0, x \geq \delta \qquad (1)$$

where, α, β and δ are known as the scale, shape and location parameters, respectively. Because of its importance, many techniques have been suggested to estimate the three-parameter Weibull distribution parameters. However, the most existing estimation approaches relaxes one of its parameters to estimate the other two because of a known fact that the estimation procedure of the three-parameter Weibull distribution family is intrinsically complicated. In real world situations, successful application of Weibull distribution depends on having appropriate statistical estimates of the three parameters.

The rest of this study is organized as follows: Section 2 reviewed related literature. Section 3 describes the MLE estimation of three-parameter Weibull distribution. In Section 4, Greedy Randomized Adaptive Search Procedures (GRASP) algorithm are

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designed. Section 5 discussed the computational results, and section 6 concludes the paper.

2. RELATED WORKS

Regarding the three-parameter Weibull distribution, Qiao and Tsokos [10] presented a stepwise algorithm including eight steps for fitting a Weibull distribution. Nosal and Nosal [11] employed array processing language and Monte Carlo simulation to investigate the performance of the gradient random search minimization procedure for estimating three parameters of Weibull distribution to a given data set. Teimouri et al. [12] classified the estimation techniques into five major categories: the approach of moments; the approach of maximum likelihood estimation; the approach of logarithmic moments; the percentile approach; and the approach of L-moments. They provided a comprehensive comparison of these estimation methods. Bartolucci et al. [13], Bartkute and Sakalauskas [14], Jukic etal. [15], Jukic and Markovich [16], and Markovich and Jukic [17] suggested moments-based approaches for estimation the Weibull distribution parameters. The most common way for estimation the parameters of the density function from an observed data set is the maximum likelihood estimation (MLE) technique [18-21]. Luus and Jammer [22] demonstrated that MLE gives the most reliable parameter estimation compared to the errors-in-variables and least-squares approaches. Ismail [23] employed a Newton–Raphson algorithm to maximize the MLE for hybrid censored data. He also through a Monte Carlo simulation calculated the mean square errors and biases of the MLE in order to investigate their performances.

Since the estimation procedure of the three-parameter Weibull distribution is a quite difficult problem, some researchers suggested intelligent-based heuristic approaches to discover satisfying solutions. Abbasi et al. [24] applied simulated annealing as one of the most popular meta-heuristics to maximize the likelihood function formed to estimate three-parameter Weibull distribution. Later on, in order to improve the performance of their method, Abbasi et al. [25] developed a hybrid variable neighborhood search and simulated annealing method. Furthermore, Abbasi et al. [26] suggested a neural network-based method which estimates the three parameters by using mean, standard deviation, median, skewness and kurtosis of the data. Wang [27] suggested bare bones particle swarm optimization algorithm to estimate the two-parameter Weibull distribution through maximizing MLE. Moeni et al. [28] developed a modified cross entropy method, as one of the modern simulation-based optimization methods, in the context of MLE of a three-parameter Weibull distribution. Yang and Yue [29] proposed a kernel density estimation-based method utilizing the

genetic algorithm and neural network to estimate the three-parameter Weibull distribution. Orkcu et al. [30] employed differential evolution algorithm to maximize the MLE function. Recently, Orkcu et al. [31] showed that the performance of particle swarm optimization considerably depends greatly on its control parameters such as acceleration coefficients and inertia weight in the parameter estimation problem of Weibull distribution.

For more details, we refer the interested readers to the surveys by Luus and Jammer [22] and Cousineau [32] which reviewed estimation methods for three-parameter Weibull distribution.

3. PARAMTER ESTIMATION USING MLE

Estimation theory plays an important role in statistical analysis and engineering designs. In the last decade, several techniques such as maximum likelihood estimation, moments method [33, 34], graphical procedure, and weighted least square method [35, 36] have been introduced to estimate parameters. Among the existing approaches to estimate the parameters of a given probability distribution based on a data set, the MLE method is the most popular estimation technique because of its applicability in complex estimation problems. Furthermore, it is widely known that MLE provides asymptotically unbiased estimators with the minimum variance.

This study suggests employing the MLE method to estimate three-parameters of Weibull distribution due to the desirable statistical properties of estimators obtained by this technique. Let $x_1, x_2, \ldots, x_n$ be a random sample of size n drawn from probability density function of the three-parameter Weibull distribution $f_{X_i}(x_i; \alpha, \beta, \delta)$. Since $x_i s$ are independent, their joint probability density function is the product of the individual probability density functions. Consequently, the likelihood function for Weibull distribution is equal to Equation (2).

$$L(x_1, x_2, \ldots, x_n; \alpha, \beta, \delta) =$$
$$\prod_{i=1}^{n} \frac{\beta}{\alpha} \left(\frac{x_i - \delta}{\alpha}\right)^{\beta-1} e^{\left(\frac{x_i - \delta}{\alpha}\right)^{\beta}} \; ; \quad \alpha, \beta > 0, x \geq \delta \tag{2}$$

The aim is to determine a vector value for α, β, and δ that maximizes the likelihood function. In practice, the maximization of the natural logarithm of the likelihood function, called the log-likelihood, is extremely more convenient. Hence, to maximize L, equivalently log-likelihood function is utilized, which for the three-Weibull distribution is as Equation (3).

$$Ln\big(L(x_1, x_2, \ldots, x_n; \alpha, \beta, \delta)\big) = nLn\left(\frac{\beta}{\alpha}\right) +$$
$$\sum_{i=1}^{n} \left[-\left(\frac{x_i - \delta}{\alpha}\right)^{\beta} + (\beta - 1)Ln\left(\frac{x_i - \delta}{\alpha}\right) \right] \tag{3}$$

Optimal estimate values can be obtained either by maximizing Equation (3) or through solving the

following system of differential equations. However, it is very hard to evaluate the gradient terms in this problem because of the large number of parameters and multi-modal nature the log-likelihood function for the three-Weibull distribution. Hence, solving system of Equations (4-6) and also applying classical gradient-based methods cannot be interesting ways.

$$\frac{\partial Ln(L(x_1, x_2, ..., x_n; \alpha, \beta, \delta))}{\partial \alpha} = 0 \tag{4}$$

$$\frac{\partial Ln(L(x_1, x_2, ..., x_n; \alpha, \beta, \delta))}{\partial \beta} = 0 \tag{5}$$

$$\frac{\partial Ln(L(x_1, x_2, ..., x_n; \alpha, \beta, \delta))}{\partial \delta} = 0 \tag{6}$$

To overcome these difficulties, this study develops a numerical search method based on greedy randomized adaptive search procedures to handle the parameter estimation of three-parameter Weibull distribution as an optimization problem with log-likelihood objective function.

4. GREEDY RANDOMIZED ADAPTIVE SEARCH PROCEDURE

The problem of optimization to select among a finite number of options appears in industry frequently. Considerable researches have been carried out over the last three decades to devise optimal seeking techniques so as to converge to optimal solution without requiring any explicit evaluation of each option. This work has been treated with an increasing rise to the context of combinatorial optimization, and gains invaluable capability to solve real world problems. However, some open problems remain regarding finding global optimum, local optimum trapping, premature convergence, etc. In addition, most real world problems existing in industry are computationally intractable, or very large scale which make the use of exact methods more costly. In such situations, the application of heuristic and metaheuristics are common strategies in finding appropriate near optimal solutions with less computation difficulty. The success of these techniques is highly related to their capability in preventing to trap at local area, and utilizing the structural property of problem. To this end, some common remedy strategies like restart mechanisms, randomization process, and preprocessing can be utilized. By exploiting these mechanisms, different heuristic and metaheuristic search methods have been established which improves our ability to achieve acceptable solutions for difficult real world problems.

Among various heuristic and metaheuristic approaches which are available to the operations research audiences, the GRASP is a relatively new one. It is an iterative randomized sampling search method which gives a solution of the problem at each iteration.

The best solution obtained during all GRASP iterations is introduced as the final result. There are two phases during GRASP iterations: (i) construction phase in which an initial solution is constructed via an adaptive randomized greedy heuristic; and (ii) local search phase in which a heuristic is applied to current solution in hope of achieving a better solution. In current paper, we devise the various components in GRASP and to fit the parameters of Weibull distributions described in previous section. A general pseudo-code of GRASP is shown by Figure 1. The problem input is taken in Line 1 of the pseudo-code. In lines 2–6, the GRASP iterations are executed. The iterations are terminated when stopping criterion like maximum number of iterations is met. The construction phase take places in Line 3 and the local search phase occurs in line 4. If an improvement happens, the update process runs in line 5. In subsequent sections, we describe these two phases with more details.

4. 1. GRASP Construction Phase   A feasible solution is iteratively generated in GRASP construction phase step by step by adding one element at a time. During one specific iteration of this phase, a greedy function is considered to order all of elements in a candidate solution list and recognize the next element to be added. The greedy function evaluates the appropriateness of each element to be selected. Since the appropriateness of each element is changed at each iteration of this phase, this method should be adaptive. This leads to reflecting the changes created by the last element selected in previous stage. The GRASP utilizes a list of top candidates which is entitled as restricted candidate list (RCL). To construct the RCL used in this phase, the incremental cost associated with the incorporation of element into the current partial solution is assessed by greedy function. At any GRASP iteration, the restricted candidate list RCL is made up of elements with the best incremental costs. This list can be limited by the number of elements in the list υ.

To randomly select one of the best candidates in the RCL, the GRASP uses a probabilistic mechanism. The selected element should not be necessarily the best one in the list in order to allow the GRASP for obtaining different diverse solutions at each iteration.

Procedure GRASP
1.   Input Instance;
2.   **for** Stopping criterion not satisfied
3.   Construct greedy randomized solution;
4.   Local search;
5.   Update solution;
6.   **end**
7.   Return best solution found
end GRASP

Figure 1. GRASP procedure

Figure 2 gives the pseudo-code for the GRASP construction phase. Line 1 initializes the solution to be generated in construction phase. The solution is constructed within loop in line 2 to 7, and the RCL is constructed in line 3 of pseudo-code. In lines 4 and 5 a candidate S is randomly chosen from RCL, and is used to form the solution. In line 6 of code, the greedy function is called and the effect of the selected element S on the appropriateness associated with every element is evaluated and measured.

4. 2. GRASP Local Search Phase
It is inevitable that the solutions found by construction phase be locally optimal solutions. As remedial technique, it is frequently common to design and employ a local search procedure. The local search procedures move from one solution to a different one in the feasible region by applying changes in solutions, until a criterion is satisfied. It is usually used on optimization problems to find a solution optimizing an objective measure among a list of candidate solutions. The local search gets the constructed solution as input and attempts to enhance it via an iterative process. It starts from the current solution and then iteratively moves to a neighbor region, which is feasible. At each iteration, the current solution is successively replaced with a new better solution founded in the neighborhood of the current solution. The neighborhood of the constructed solutions is explored by intensifying the search in vicinity of solution with the hope of improving the solution that we already have at hand. The local search terminates when there is no potential improvement in the neighborhood of current solution. Also, it has been proved that utilizing of two or more local search strategies helps the GRASP to prevent trapping in a local optimum. The important problem in designing a local search is the design of the neighborhood mechanisms. A neighborhood mechanism determines the way to gain a new solution by modifying the input solution. The local solution founded by a particular neighborhood mechanism is not necessarily same as the local solution established by another mechanism, and hence, the use of several neighborhood mechanism get us the flexibility to guide the search to more appropriate regions.

Procedure Construct greedy randomized solution
1.   Solution = {};
2.   **for** Solution construction not done
3.   Make RCL;
4. S = Select element at random (RCL);
5.   Solution=Solution ∪ S;
6.   Adapt Greedy Function (S)
7.   **end**
end Construct greedy randomized solution
Figure 2. Construction phase procedure

In local search which is utilized within GRASP algorithm, four neighborhood mechanisms are considered in current paper: (i) insertion, (ii) swap, (iii) twist, and (iv) random. Given a schedule l, the insertion neighborhood is related to all the solutions that can be gained by getting a Weibull parameter value from its place in l and re-inserting it into new position. Thus the insert mechanism removes the parameter in ith position from current solution and insert it into a new random position j. Given a solution l, the swap is related to all the neighbor solutions that can be attained by swapping the numbers of two parameters form three parameters in Weibull distribution. In other words, this neighborhood swaps the parameters at the ith position and the jth position in the current solution l. In twist neighborhood, a new candidate solution is created by taking a subset of the Weibull parameters in current solution and re-inserting the selected subset in reverse order into their position. For random neighborhood, a parameter of Weibull distribution is selected randomly and a random value is replaced with its current value.

5. NUMERICAL ILLUSTRATION

In this section, we aim at demonstrating the procedure of proposed estimation based GRASP algorithms and evaluate performance of estimations achieved via illustrative examples. To this end, four examples are considered and discussed in sequel. The parameters for examples $1-4$ are $(\alpha, \beta, \delta) = (2, 3, 8), (4, 5, 6), (6, 7, 4)$, and $(8, 9, 2)$ respectively. To generate the examples, a three parameter random number generator is implemented in MATLAB 2010a and the samples $X = (X_1, X_2, ..., X_n)$ are attained for each example separately. The sample size n is an important parameter in analyzing the GRASP algorithms which will be selected computationally. Moreover, the number of elements in the RCL list is another important parameter influencing the performance of algorithms. Tables 1-4 depict the experimental results for 4 examples in term of log-likelihood objective function. The first columns indicate the number of elements in RCL, while second columns show sample size. The next columns show the results of a benchmark grid search algorithm. The grid search is a kind of full enumeration search method which is used to solve an optimization problem with discretized domain. It recognizes a discrete feasible region with an equally divided grid and finds the best solutions in that region by comparing the fitness of new solution to that of best obtained solution so far. The updating process is implemented when the improvement occurs. Since computational efforts incurred by any grid search are outstandingly high, we limit run time to maximum run time of all GRSAP algorithms.

TABLE 1. Log-likelihood values for example 1 $(\alpha, \beta, \delta) = (2, 3, 8)$

Number of elements in the RCL list υ	Sample size	Algorithms				
		Grid Search	GRASP-Swap	GRASP-Insert	GRASP- Twist	GRASP- Rand
$\upsilon = 8$	$n = 20$	-24.4817	-23.346	-24.971	-23.380	-21.878
	$n = 50$	-45.9458	-61.761	-60.414	-80.761	-51.207
	$n = 100$	-100.2260	-118.051	-126.634	-89.394	-103.588
	$n = 200$	-196.6091	-286.542	-243.184	-228.283	-212.072
$\upsilon = 5$	$n = 20$	-24.4817	-19.412	-22.093	-18.595	-23.874
	$n = 50$	-45.9458	-50.671	-60.995	-57.665	-52.583
	$n = 100$	-100.2260	-102.441	-116.783	-121.629	-104.103
	$n = 200$	-196.6091	-241.226	-234.774	-220.718	-209.251
$\upsilon = 2$	$n = 20$	-24.4817	-25.722	-21.210	-23.291	-21.639
	$n = 50$	-45.9458	-52.590	-58.718	-57.523	-52.825
	$n = 100$	-100.2260	-110.003	-113.118	-118.133	-111.449
	$n = 200$	-196.6091	-223.596	-218.252	-219.872	-213.642
Average		-91.815	-109.613	-108.429	-104.937	-98.175

TABLE 2. Log-likelihood values for example 2 with $(\alpha, \beta, \delta) = (4, 5, 6)$

Number of elements in the RCL list υ	Sample size	Algorithms				
		Grid Search	GRASP-Swap	GRASP-Insert	GRASP- Twist	GRASP- Rand
$\upsilon = 8$	$n = 20$	-19.515	-27.877	-24.478	-26.326	-26.762
	$n = 50$	-57.284	-76.700	-64.963	-65.902	-68.376
	$n = 100$	-110.497	-118.043	-124.893	-132.393	-123.136
	$n = 200$	-248.308	-275.076	-257.397	-253.228	-245.109
$\upsilon = 5$	$n = 20$	-19.515	-24.635	-25.887	-26.021	-21.230
	$n = 50$	-57.284	-68.773	-65.738	-63.243	-63.844
	$n = 100$	-110.497	-136.545	-122.829	-120.752	-129.695
	$n = 200$	-248.308	-258.450	-263.615	-259.211	-250.670
$\upsilon = 2$	$n = 20$	-19.515	-22.148	-29.888	-22.570	-25.315
	$n = 50$	-57.284	-58.978	-64.681	-56.314	-62.261
	$n = 100$	-110.497	-119.242	-130.703	-131.008	-121.393
	$n = 200$	-248.308	-238.692	-255.194	-262.913	-254.489
Average		-108.901	-118.763	-119.189	-118.323	-116.023

TABLE 3. Log-likelihood values for example 3 with $(\alpha, \beta, \delta) = (6, 7, 4)$

Number of elements in the RCL list υ	Sample size	Algorithms				
		Grid Search	GRASP-Swap	GRASP-Insert	GRASP- Twist	GRASP- Rand
$\upsilon = 8$	n = 20	-31.270	-25.767	-36.762	-29.686	-25.224
	n = 50	-64.276	-60.012	-73.024	-71.128	-61.646
	n = 100	-129.817	-141.742	-131.879	-137.811	-135.302
	n = 200	-262.573	-294.119	-291.653	-269.693	-263.568
$\upsilon = 5$	n = 20	-31.270	-25.702	-31.347	-27.329	-22.396
	n = 50	-64.276	-67.923	-69.312	-64.481	-63.935
	n = 100	-129.817	-141.572	-132.707	-145.435	-139.725
	n = 200	-262.573	-272.282	-278.882	-254.274	-251.230
$\upsilon = 2$	n = 20	-31.270	-26.925	-26.521	-31.587	-26.521
	n = 50	-64.276	-72.337	-74.484	-65.392	-72.074
	n = 100	-129.817	-261.257	-142.149	-128.010	-124.805
	n = 200	-262.573	-126.637	-270.605	-256.578	-258.267
Average		-121.984	-126.356	-129.944	-123.45	-120.391

TABLE 4. Log-likelihood values for example 4 with $(\alpha, \beta, \delta) = (8, 9, 2)$

| Number of elements in the RCL list υ	Sample size	Algorithms				
		Grid Search	GRASP-Swap	GRASP-Insert	GRASP-Twist	GRASP-Rand
$\upsilon = 8$	$n = 20$	-27.627	-27.441	-37.592	-24.349	-27.232
	$n = 50$	-65.907	-74.460	-71.767	-68.480	-72.765
	$n = 100$	-134.917	-130.559	-127.852	-130.838	-132.656
	$n = 200$	-263.583	-275.454	-295.777	-267.403	-290.788
$\upsilon = 5$	$n = 20$	-27.627	-27.077	-30.059	-27.169	-28.063
	$n = 50$	-65.907	-67.559	-63.864	-68.825	-69.236
	$n = 100$	-134.917	-164.137	-138.502	-137.741	-142.329
	$n = 200$	-263.583	-276.920	-278.594	-271.512	-307.777
$\upsilon = 2$	$n = 20$	-27.627	-26.009	-24.681	-20.366	-27.488
	$n = 50$	-65.907	-71.034	-64.910	-70.832	-65.971
	$n = 100$	-134.917	-140.717	-132.444	-132.556	-130.897
	$n = 200$	-263.583	-259.344	-288.095	-278.741	282.883
Average		-123.009	-128.393	-129.511	-124.901	-84.3599

When run time violated, the best obtained solution is recorded and its log-likelihood function are reported. Figure 3 shows a simple pseudo-code of grid search algorithm.

The next remaining columns present the value of achieved log-likelihood objective functions by algorithms GRASP-Swap, GRASP-Insert, GRASP-Twist and GRASP-Rand respectively. As we expected, the objective function gets negative values which is due to nature of log function for input probabilities between 0 and 1. The aim is to find the value of parameters (α, β, δ) with highest value of log-likelihood function.

Since the proposed GRASP algorithms are random based search methods, the way they approaches their final statuses is of interest. In order to further assess the performance of GRASP algorithms, the quality of solutions obtained by algorithms are depicted in sequel. Figure 4 shows the log-likelihood objective functions for average values of log-likelihood objective functions. The grid search is a relatively full enumeration method which is expected to reach better solutions. After that, as can be seen from results, the rank of algorithms in terms of quality of solutions is GRASP- Rand, GRASP-Twist, GRASP-Swap and GRASP-Insert.

Procedure Grid search
1.   Construct grid network;
2.   **for** maximum parameter values (α, β, δ) are not violated
3.   evaluate the current solution point on network;
4.   updating process;
7.   **end**
end Grid search

Figure 3. Grid search procedure
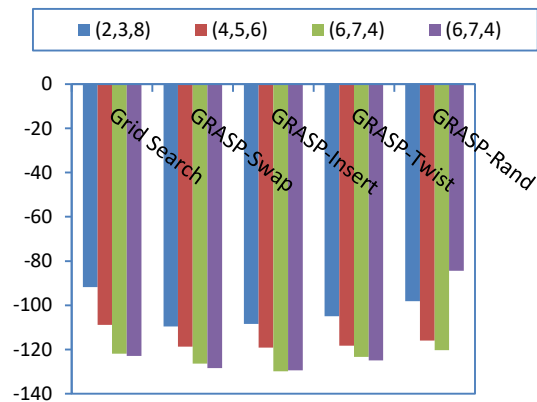
Figure 4. The result of comparisons

6. CONCLUSION

The Weibull distribution plays an important role in several real world applications such as reliability and lifetime studies. This issue has attracted many attentions to precise estimation of the Weibull parameters. The estimation of parameters of three-parameter Weibull distribution is intractable analytically. In this research, the GRASP algorithm with four different local search schemes was developed to maximize the log-likelihood function of a three-parameter Weibull distribution. The performance of the suggested algorithms was assessed and compared with the benchmark grid search method. The obtained results supported the appropriate performance of estimations attained in terms of both accuracy and efficiency. As a direction for future research, it is interesting to devise other numerical search methods like other recent metaheuristics and compare the results.

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Fitting the Three-parameter Weibull Distribution by using Greedy Randomized Adaptive Search Procedure

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P A P E R I N F O	چکیده

توزیع وایبول به طور گسترده‌ای در موضوعات مختلف مهندسی مورد استفاده قرار می‌گیرد، چراکه یک توزیع بسیار منعطف با اشکال متنوع است. به علاوه این توزیع ویژگی‌های تعداد دیگری از توابع توزیع را نیز در بر می‌گیرد. ولیکن استفاده‌ی موفقیت آمیز از این توزیع مستلزم تخمین دقیق سه پارامتر آن یعنی پارامتر مقیاس، پارامتر شکل و پارامتر مکان است. این مساله تلاش‌ها را به سمت توسعه روش‌هایی جهت برآورد دقیق پارامترهای این توزیع سوق داده است. سختی تخمین پارامترهای توزیع وایبول مخصوصاً زمانیکه هر سه مجهول هستند به عنوان یک فرآیند کاملاً پیچیده معروف است. از اینرو، این مطالعه یک رویکرد محاسباتی یعنی روش جستجوی انطباقی تصادفی حریصانه با تعدادی جستجوی محلی همسایگی به منظور ارتقای کیفیت برآوردها را پیشنهاد می‌نماید. همچنین تعدادی آزمایش محاسباتی جهت ارزیابی کیفیت برآوردها در مقایسه با الگوریتم جستجوی شبکه‌ای اجرا شدند.

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