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Determining the Sample Size for Estimation of the *CCC-R* Control Chart Parameters Based on Estimation Costs

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P A P E R I N F O	*A B S T R A C T*
	In today's highly competitive industrial environment due to fast technology development, quality practitioners will have to detect out-of-control situations and take actions whenever is necessary as soon as possible. Accordingly, new statistical procedures have been enhanced incessantly both to handle high yield processes along with looking for methods of minimizing all quality cost. *CCC-r* (Cumulative Count Conforming-rarely) chart, the extended method of *CCC* charts, is commonly applied when nonconforming items are rarely observed. Since the values of the parameters used for the design of the charts' control parameters are usually unknown in practice, the practitioners need to estimate them by using an in-control retrospective sample. It has been shown that parameter estimation affects the control charts' properties severely. This study develops a model based on estimation costs and Average Number of Inspected Items for *CCC-r* chart when nonconforming fraction is unknown. The unknown parameters estimated based on different values of sample sizes and sensitivity analysis was performed. *doi*: 10.5829/idosi.ije.2017.30.02b.12

1. INTRODUCTION

As one of the basic Statistical Process Control (SPC) tools, control chart has an important role to maintain process stability. Quality practitioners use control charts to draw conclusions about the state of the process and monitor performance to retain control situation in the production process.

The control chart conclusions depended on whether the applied monitoring approach is a phase I or phase II method. In phase I, historical data of the process is analyzed to understand the variation of the process over time, to evaluate the process stability, and to estimate the in-control parameters. In phase II, the process is monitored in real-time to quickly detect shifts from the baseline established in phase I [1]. As highlighted by Szarka and Woodall [2], the sampled items in high yield processes are often represented by a Bernoulli process; that is, the items are independently classified as conforming or nonconforming. In addition, Szarka and Woodall [2] stated that the in-control value of p, p_0, is typically assumed to be known. However, accurate parameter estimation for such processes is very difficult and may require a larger sample size than available in practice. Psarakis et al. [3] emphasized that a key factor on designing reliable control charts with estimated parameters is the accuracy of the process parameter estimators obtained during the phase I implementation of charts. There are several measures employed to evaluate the efficiency of the proposed estimators. Also, Psarakis et al. [3] proved that the most reliable control charts are based on the most efficient and robust parameter estimators, especially in cases where normality assumption is violated and diffused or localized disturbances are present. Moreover, since parameter estimation has an impact on the in-control and out-of-control performance of control charts, thus the performance of each estimated chart has to be evaluated. Many researchers have studied the effects of estimation on control chart performance. Jones et al. [4], Chakraborti [5], Testik [6], and Ryu and Wan [7]

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studied variable control charts with estimated parameters. Yang et al. [8] and Chakraborti and Human [9] reported the effects of parameter estimation on attribute control charts. Jensen et al. [10] analyzed the effect of parameter estimation on control charts and concluded that the effect of parameter estimation on control chart properties should not be ignored. Shepherd et al. [11] presented a comparison of the in-control ARL (average run length) properties of the attribute chart for Markov-dependent data with known and estimated parameters. Lee and Lee [12] evaluated the performance of Shewhart chart with estimated parameters in terms of the average of median run length (AMRL) and the standard deviation of MRL (SDMRL) metrics. Since the run length distribution is skewed when the process is either in-control or out-of-control, the median run length (MRL) can be used as alternative measure instead of the ARL. Saleh et al. [13] studied the effect of the phase I estimation error on the cumulative sum (CUSUM) chart. They designed the CUSUM chart by adjusting the control limits using a bootstrap-based design technique. To reduce the effect of estimation error on the chart's performance

However, accurate parameter estimation for high yield processes is very difficult and may require a larger sample size than that available in practice.

High-quality or high-yield processes refer to those production or processes which are capable of producing defective items rarely and have a very low nonconforming proportion. In these states, traditional statistical process control tools had been found inadequate to provide the obligatory decision support mechanisms to evaluate the process. Bersimis et al. [14] expressed that this is because of the rules used in these charts. So, if a small or moderate shift occurs in these processes, then the out-of-control fraction of non-conforming items will still be very small, and as a consequence, it is highly probable that no defective items will be observed in the inspected sample. Therefore, for small or moderate shifts in nonconforming fraction, standard p or np charts are incapable to diagnose a change in a high quality process because there are a small number of nonconforming items, and usually a high number of conforming items are detected before a nonconforming one.

Due to the properties of zero defect processes, the high yield control chart, also known as the Cumulative Conformance Count (CCC) chart or Geometric chart, has gained wide application in the industry. This chart first introduced by Calvin [15], counts the number of conforming items produced between successive nonconforming ones. It has been known for its ability to detect improvements in high yield production processes while overcoming the problem of possible false alarms experienced by the Shewhart chart when a defective item rarely occurs. Zhang et al. [16] suggested an improved technique for high-quality processes under

group inspection and concept of control charts for high-quality process under sampling inspection. Chen et al. [7] offered a CCC control chart with variable sampling intervals and control limits. Their numerical comparisons showed that the performance of the CCC control chart could be improved by some modifications.

In recent years, attention to rising sensitivity of control charts considerably increased because of technology improvements. An alternative strategy to enhance sensitivity is to monitor the cumulative count of items produced until a fixed number "r" of nonconforming items are observed. This chart is called CCC-r control chart and it has better performance in finding the shifts of the non-conforming proportion in high-quality processes. The idea of CCC-r control chart has been proposed by Xie et al. [17] and further discussed by Sun and Zhang [18]. The cumulative count of conforming items will follow the Negative Binomial distribution, which is a generalization of the Geometric distribution.

Yang et al. [8] proved that an accurate parameter estimation might require a large sample size in the high-quality processes. They studied the effect of sample size and run length distribution on the CCC chart with estimated control limits. Zhang et al. [1] presented geometric charts with estimated parameters and they used the standard deviation of the average run length and the standard deviation of the average number of inspected items to show that much larger sample sizes in phase I are needed in practice than the sample size implied by previous researches. Chiu and Tsai [19] explored the impact of estimated nonconforming fraction, p_0, on a one-sided CCC control chart. They have shown that values of conditional probability of a false alarm rate, conditional average run length, and standard deviation are close to the nominal values for a large sample. Zhang et al. [20] investigated the effect of performance of CCC chart with variable sampling intervals with estimated control limits. The best design parameters of CCC chart with variable sampling intervals with known process nonconforming rate p_0 are determined when such nonconforming fraction is estimated with Bayesian estimator. Yang et al. [8] recommended taking a historical sample of size m from the Bernoulli process and using the maximum likelihood estimator for p_0. Zhang et al. [1] and Lee et al. [21] proposed the Bayesian estimator. Tang and Cheong [22] proposed another sampling scheme based on observing a specified number of nonconforming items. As a result, the size of the phase I sample is a negative binomial random variable. Rakitzis and Costagliola [23] investigated the performance of Shewhart-type control charts for zero-inflated processes with estimated parameters and proposed practical guidelines for the statistical design of the examined charts, when the size of the preliminary sample is predetermined.

An efficient implementation of a control chart requires the optimal selection of its design parameters, the appropriate sample size, sampling frequency, and control limits. They can be selected according to an economic-statistical objective before applying the classical control methods so that minimum cost in production cycle is obtained.

Several methods are proposed for the economic design of control charts. Duncan [24] was the first to present a mathematical model for the design of control charts achieving a pure economic objective function based on the minimization of a total cost per hour. Yilmaz and Burnak [25] discussed about importance of cost consideration. Furthermore, they explained that any activities in production and quality control should be put into the context of cost saving. In most researches only one objective function named cost function is minimized subject to statistical constraints which is not an effective method for economic-statistical design of control charts. Moghaddam et al. [26] generalized a multi objective model for designing CCC control chart where optimal values of sampling interval and the lower control limit are obtained such that cost function as well as out of control average run length are minimized, while in-control average run length is considered as a statistical constraint.

Nowadays, in using control chart, economic factors must be taken into account to achieve optimum level for designing the charts. From the economic view, all quality costs such as those related to sampling, testing, investigating out of control signals, eliminating special cases, and rectifying the process are influenced by process parameters.

With respect to the literature review, it is important to consider control costs in determining chart parameters so that control chart could be operating in an optimum condition and production process costs is minimized specifically in high quality production.

Since reaching to a defective need to investigating many samples in high quality processes and it leads to additional time and cost, so it's been tried to estimate non-conforming fraction in a way such that inspection's cost would be in the minimum value in this research.

In addition, there is an error in estimating non-conforming fraction of the process, and this causes additional cost. The Average Number of Inspected items is used as performance measure in order to analyze the accuracy of estimation. Therefore, with considering formulation of CCC-r control chart, the purpose of this study is to estimate parameters by considering estimation related costs, which have not been addressed in previous studies. At first, the probability distribution of estimators is developed and then the Upper control limit and Lower control limit of the CCC-r chart are calculated based on estimation of non-conforming fraction. Then, a cost scheme is designed by considering investigation cost of samples and estimation error cost

based on Average Number of Inspected items. In this scheme, the needed sample size to estimate the non-conforming fraction is obtained when the objective function is in its optimum value.

This paper is organized as fallows. Firstly, the formulations of CCC-r control chart will be reviewed, and estimation of non-conforming fraction is discussed. Secondly, a cost scheme to obtain the optimum sample size is explained. Then, a numerical example and sensitivity analysis of the model will be given in section four. At last, the conclusion is presented.

2. REVIEW OF *CCC-r* CONTROL CHART

The idea of *CCC* chart can be extended for process monitoring by considering the number of items inspected until a fixed number of nonconforming items are observed. This control chart is called the cumulative count of conforming chart (*CCC-r* chart). In a *CCC-r* chart, the number of items until the detection of r_{th} nonconforming item (*x*) is plotted to monitor the nonconforming fraction of the process, p_0. If the value $p_0 \in (0.1)$ is known, *x* is a random variable of negative binomial distribution. The probability mass function (*pmf*) and the cumulative distribution function (*cdf*) are presented by Xie et al. [27] as follows:

$$f_{r.p_0}(x) = \binom{x-1}{r-1}(1-p_0)^{x-r}p_0{}^r \; ; for \; x = r.r+1. ... \quad (1)$$

$$F_{r.p_0}(n) = p(x \leq n) = \\ \sum_{i=r}^{n}\binom{i-1}{r-1}(1-p_0)^{i-r}p_0{}^r \; ; for \; n = r.r+1. ... \quad (2)$$

when *r* is equal to one (the case of *CCC* chart) then the distribution reduces to geometric probability distribution.

For an acceptable risk of false alarm, α, Xie et al. [28] explained that the upper control limit, UCL and the lower control limit, LCL of the *CCC*-r chart could be obtained as the solutions of the following equations:

$$F(UCL - 1.r.p_0) = \sum_{i=r}^{UCL-1}\binom{i-1}{r-1}p_0{}^r = 1 - \frac{\alpha}{2} \quad (3)$$

And:

$$F(LCL.r.p_0) = \sum_{i=r}^{LCL}\binom{i-1}{r-1}p_0{}^r(1-p_0)^{i-r} = \frac{\alpha}{2} \quad (4)$$

While the centerline, CL, is given as the solution of following equation

$$F(CL.r.p_0) = \sum_{i=r}^{CL}\binom{i-1}{r-1}p_0{}^r(1-p_0)^{i-r} = \frac{1}{2} \quad (5)$$

As is clear from the above equations, the control limits of the *CCC-r* chart increase when process fraction nonconforming, p_0, approaches zero. The control limits of the *CCC-r* charts with a large *r,* are much larger than that small *r* for the same value of p_0 level. The probability of not observing any signal in two-sided *CCC-r* chart is denoted by $\beta(p_0)$:

$$\beta(p_0) = p(LCL < x < UCL \, | p_0) =$$
$$\sum_{i=LCL+1}^{UCL-1} \binom{i-1}{r-1} p_0{}^r (1-p_0)^{i-r} \qquad (6)$$

Average run length (ARL), which represents the average number of samples taken until the occurrence of one signal can be calculated:

$$ARL = \frac{1}{1-\beta(p_0)} = \frac{1}{1-\sum_{i=LCL+1}^{UCL-1}\binom{i-1}{r-1}p_0{}^r(1-p_0)^{i-r}} \qquad (7)$$

It is obvious that ARL is the mean value of Geometric distribution with success probability of $1-\beta(p_0)$. In addition to that, the average number of items inspected (ANI) is a different performance measure of control charts. Moreover, we can emphasize that ANI is more informative than ARL in high quality processes because the ANI counts the number of items inspected whereas the ARL does not. ANI can be determined as:

$$ANI = \frac{r}{p_0} ARL = \frac{r}{p_0} * \frac{1}{1-\sum_{i=LCL+1}^{UCL-1}\binom{i-1}{r-1}p_0{}^r(1-p_0)^{i-r}} \qquad (8)$$

where $\frac{r}{p_0}$ is equal to the expected number of sample size in each stage that correspond to the mean value of negative binomial distribution.

2. 1. CCC-r Chart with Estimated Parameters

The control limits of the traditional CCC-r chart in Equation (3) and (4) are based on assumption that the in control non-conforming fraction, p_0 ,is known.

If the value of p_0 is unknown, then we must estimate it by using initial sampling in the Phase I of quality control plan. Assume that M items are inspected and L items are found to be non-conforming. If x is defined as the nonconformity proportion the posterior probability of random variable x, is obtained as following:

$$f(x|M.L) = \frac{g(M.L|x)f(x)}{g(M.L)} = \frac{g(M.L|x)f(x)}{\int_0^1 g(M.L|z)f(z)dz} =$$
$$\frac{\binom{M}{L}x^L(1-x)^{M-L}}{\binom{M}{L}z^L(1-z)^{M-L}} = \frac{\binom{M}{L}x^L(1-x)^{M-L}}{\binom{M}{L}\frac{\Gamma(L+1)\Gamma(M-L+1)}{\Gamma(M+2)}} = \qquad (9)$$
$$\frac{(M+1)!}{L!(M-L)!}x^L(1-x)^{M-L}$$

Thus, the value of nonconforming fraction can be estimated as following,

$$\widehat{p_0} = \int x.f(x|M.L)\,dx = \frac{L+1}{M+2} \qquad (10)$$

when the value of p_0 is unknown, then we apply $\widehat{p_0}$ as an estimator, where L is the number of nonconforming items among M items sampled and L is a binomial random variable with parameters M and p_0. Consequently, by applying an estimation for p_0, control limits of CCC-r chart will change. The estimated control limits for two-sided CCC-r chart are obtained as:

$$\sum_{i=r}^{\widehat{UCL}-1}\binom{i-1}{r-1}(\tfrac{L+1}{M+2})^r\left(1-(\tfrac{L+1}{M+2})\right)^{i-r} = 1-\frac{\alpha}{2} \qquad (11)$$

$$\sum_{i=r}^{\widehat{LCL}}\binom{i-1}{r-1}(\tfrac{L+1}{M+2})^r\left(1-(\tfrac{L+1}{M+2})\right)^{i-r} = \frac{\alpha}{2} \qquad (12)$$

$\widehat{UCL}$ is upper control limit when the value of nonconforming fraction is unknown and $\widehat{LCL}$ is lower control limit.

Since the estimated parameter is utilized to design economic CCC-r chart based on ANI which is used as performance measure, thus first probability of not observing any signal on the chart when non-conforming fraction has been estimated must be briefly explained. Since $\widehat{p_0}$ is estimated by $\frac{L+1}{M+2}$, thus $\beta(p_0)$ is a function of a total items sampled M and the number of non-conforming items L. The properties of CCC-r control chart were derived conditionally on the estimated value $\widehat{p_0}$. If x was defined as the number of inspected items until, the detection of r_{th} nonconforming item then $\beta(p_0)$ can be written as:

$$\beta(p_0.M|L) = p(\widehat{LCL} < x < \widehat{UCL}\,|L) =$$
$$\left(\sum_{i=\widehat{LCL}}^{\widehat{UCL}-1}\binom{i-1}{r-1}\widehat{p_0}{}^r(1-\widehat{p_0})^{i-r}\Big|L\right) \qquad (13)$$

Equation (13) is equal to the conditional probability of not observing any signal in the case of an unknown parameter p_0 which is conditioned on a value of the random variable L. The expected value of conditional probability in Equation (13) by considering binomial distribution of L can be stated as:

$$E(\beta(p_0.M|L)) =$$
$$\begin{cases} 0 & if \; L = 0 \; or \; L = M \\ \sum_{l=0}^M p(\widehat{LCL} < x < \widehat{UCL}\,|L=l) * p(L=l) \end{cases}$$
$$= \begin{cases} 0 & if \; L = 0 \; or \; L = M \\ \sum_{l=0}^M (\sum_{i=\widehat{LCL}+1}^{\widehat{UCL}-1}\binom{i-1}{r-1}\widehat{p_0}{}^r(1-\widehat{p_0})^{i-r} * \binom{M}{l}p_0{}^l.(1-p_0)^{M-l}) \end{cases} \qquad (14)$$

The expected value of probability of not observing any signal on the chart is denoted by $E(\beta(p_0.M|L))$ when non-conforming fraction is estimated. As expressed before and considering an unknown non-conforming fraction, it can be demonstrated that $\widehat{ANI}$ is equal to expected number of items inspected in each sampling stage $(E(\frac{r}{\widehat{p_0}}))$ multiplied by expected number of sampling stages or ARL ($\widehat{ARL} = \frac{1}{1 - E(\beta(p_0.M|L))}$). $E(\frac{r}{\widehat{p_0}})$ is obtained as following:

$$E\left(\frac{r}{\widehat{p_0}}\right) = \sum_{l=0}^M E\left(\frac{r}{\widehat{p_0}}\Big|L=l\right) * p(L=l) = \sum_{l=0}^M \frac{r}{\frac{l+1}{M+2}} *$$
$$\binom{M}{l}.p_0{}^l.(1-p_0)^{M-l} \qquad (15)$$

Thus:

$$\widehat{ANI} = E\left(\frac{r}{\widehat{p_0}}\right) * ARL = \sum_{l=0}^M \frac{r}{\frac{l+1}{M+2}}\binom{M}{l}.p_0{}^l.(1-p_0)^{M-l} * \frac{1}{1 - E(\beta(p_0.M|L))} \qquad (16)$$

With respect to previous discussions, there is an optimal choice of M, which minimizes the expected total cost and provides best estimator in control chart. The

formulations of a cost scheme for determining the optimal value of M is explained in the next section.

3. AN AOOROCH TO OBTAIN REQUIRED SAMPLE SIZE IN PARAMETER ESTIMATION

As mentioned in introduction section, a large sample size is needed in order to estimate unknown parameters in high quality related control charts. This large sample is leading to high estimation costs which has not been studied in previous studies, and consequently in this research a new cost function is introduced which could estimate parameters at the least level of cost.

The proposed model is made up two different parts. One part is for estimation error of non-conforming fraction and the other is for the size of investigated samples. The cost parameters of the model are as following:

1- The cost C_{est} is required to consider the estimation error: Since average number of inspected items will be used to examine chart performance, thus its estimation error should be considered with a particular cost. The cost of estimation error is multiplied with error term $(\widehat{ANI} - ANI_{p_0})$ where $\widehat{ANI}$ is the estimated value, which is used in proposed control chart. ANI_{p_0} is true value of ANI and is calculated based on $\beta(p_0)$ in Equation (8).

2- The cost C_{inv} is required to consider the cost of inspection in the investigated sample: It is needed to inspect M items to find L which is the number of nonconforming items for estimating the value of p_0. Therefore, the cost to inspect all M items should be considered.

Subsequently, total cost function based on the average number of inspected items as the performance measure of *CCC-r* control chart is as following:

$$TC = C_{est}(\widehat{ANI} - ANI_{p_0}) + MC_{inv} \qquad (17)$$

To illustrate the proposed cost function, some numerical examples are solved in next section.

4. PERFORMANCE STUDY

When the sample of size M will be in optimum value, then the total cost approaches its minimum value. With this presumption, the sample size M is assumed as decision variable in this paper to reach the minimum cost in the process. A grid search procedure written in Visual Basic is used to determine the optimal value of decision variable in proposed approach. The optimal value of M, which is experimented in a fixed range of 5 to 120 is obtained by considering a specified value for the acceptable risk of false alarm to determine control limits, C_{inv} and C_{est} as fixed costs. The fixed

parameters are stated in Table 1. For all values of M in each experiment, the following steps are repeated.

Firstly, the estimation of non-conforming fraction and chart control limits are calculated. Then, the values of $\widehat{ANI}$ and ANI_{p_0} are evaluated based on real values of control chart parameters and estimation parameters. Finally, the total cost function is obtained according to fixed costs. Finally, the minimum total cost is achieved by comparing all values of total cost functions in each iteration.

When the non-conforming fraction is estimated precisely, the estimation error statement would be minimum value and estimation of nonconforming fraction will be closer to its real value.

4. 1. Sensitivity Analysis   According to Table 1 and specified fixed range of M, we survey on optimal condition for different values of p_0 and r so that the parameters in each experiment would be changed separately. Table 2 can be used to analyze the sensitivity of the model to the variation of various parameters. In this table, six different values are considered for r. For each value of r, different values of nonconforming fractions are experimented when acceptable risk of false alarm is constant.

As shown in Table 2, when r is fixed by increasing the value of p_0, the value of M decreases; thus, a lower value of sample size is needed for larger value of p_0. Also, in this condition, the value of cost function decreases by increasing the value of p_0. By variations of r when the value of p_0 is fixed, increasing the value of r leads to the reduction of M and cost function value. Also in this case, the value of $\widehat{p_0}$ is closer to true nonconforming fraction when r is lower. Similarly, it can be seen that estimated values are more precise for larger values of nonconformity fraction.

5. CONCLUSION

In this paper, detailed description of *CCC-r* control chart, which is a suitable type of control chart for high yield processes, was studied. The properties of a two-sided control chart with unknown non-conforming fraction were investigated. The probability of non-conforming estimators was developed based on sample size and negative binomial sampling scheme. It was denoted that the estimated values affected the upper and lower control limits.

TABLE 1. Fixed parameters

Parameter		Value
Fixed costs	C_{est}	100
	C_{inv}	5
	α	0.0027

TABLE 2. Parameter estimation in different experiments

Exp. No.	p_0	Chart parameters			Decision variable	Estimated p_0	Total cost
		LCL	UCL	r	Optimum M		
1	0.006	1	1098	1	86	0.006772459	15391938055
2	0.008	1	823	1	69	0.008091835	5114445741
3	0.009	1	731	1	63	0.00870416	3243794317
4	0.01	1	658	1	59	0.009060385	2156752927
5	0.02	1	328	1	34	0.013976038	142219294.1
6	0.04	1	162	1	19	0.021924724	9001904.468
7	0.05	1	129	1	16	0.024451482	3679185.401
8	0.006	10	1479	2	63	0.010530012	8.39E+11
9	0.008	8	1110	2	50	0.012870051	3.72E+11
10	0.009	7	986	2	46	0.013745067	2.89E+11
11	0.01	6	886	2	42	0.014901346	2.45E+11
12	0.02	4	441	2	22	0.022350546	20122354427
13	0.04	2	219	2	15	0.031887434	2290033325
14	0.05	2	175	2	12	0.038597149	674130944
15	0.006	37	1806	3	67	0.009683641	2.50E+12
16	0.008	28	1355	3	53	0.001187835	1.31E+12
17	0.009	25	1204	3	46	0.01345067	9.90E+11
18	0.01	23	1083	3	42	0.014901364	7.26E+11
19	0.02	12	539	3	23	0.02513891	1.84E+11
20	0.04	7	267	3	13	0.039213424	992538416
21	0.05	6	213	3	11	0.042753853	3855330466
22	0.006	79	2107	4	72	0.008761	5.00E+12
23	0.008	60	1582	4	55	0.0112789	2.612E+12
24	0.009	53	1405	4	49	0.012590382	2.064E+12
25	0.01	48	1263	4	44	0.01396981	1.58E+12
26	0.02	25	630	4	24	0.023683859	2.74E+12
27	0.04	13	313	4	12	0.049213424	36694756578
28	0.05	111	249	4	10	0.043753853	143235268
29	0.006	134	2392	5	78	0.007817142	6.73E+12
30	0.008	101	1795	5	59	0.009712234	4.31E+12
31	0.009	90	1595	5	53	0.011260068	3.29E+12
32	0.01	81	1434	5	48	0.012345803	2.61E+11
33	0.02	42	715	5	26	0.022350546	4.53E+12
34	0.04	22	355	5	13	0.039213424	58824370158
35	0.05	18	283	5	11	0.043753853	26875557453
36	0.006	198	2664	6	100	0.009049871	1.12E+13
37	0.008	149	2001	6	63	0.009275172	6.40E+12
38	0.009	133	1777	6	56	0.010391941	4.83E+12
39	0.01	120	1598	6	51	0.011301057	3.79E+12
40	0.02	61	796	6	28	0.021121266	7.26E+12
41	0.04	32	396	6	14	0.039213424	9306250519
42	0.05	26	316	6	12	0.043753853	44110445325

Since the cost parameters is not taken into account in previous studies for parameter estimation and this could be an important issue in high quality processes because of low defective samples, thus a cost function is developed in this research. We tried to minimize the summation of cost of estimation error and cost of inspecting required samples to estimate unknown parameters. In order to evaluate performance of the estimator, Average Number of Inspected items is used as performance measure in proposed function. Due to the importance of sample size in high quality processes, it is considered as the decision variable in the cost function. The variations of the estimated non-conforming fraction and sample size are studied in various conditions of control chart parameters.

As expected, the results of sensitivity analysis show that cost function increases when nonconforming fraction decreases, so when non-conforming fraction is low, the cost of estimation would be very high. The increasing non-conforming fraction has an inverse effect on optimal value of the sample size as expected. Moreover, for fixed values of non-conforming fraction, the optimal value of sample size decreases as the value of r increases. Therefore, when the value of r increases then the value of optimal sample size decreases which leads to increase the sensitivity of CCC-r control chart.

Further research on this topic may include evaluating the performance of proposed model when different estimators are used. On the other hand, it could be beneficial to apply this proposed estimators and cost function on the performance of all types of control charts when nonconforming fraction is low.

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Determining the Sample Size for Estimation of the *CCC-R* Control Chart Parameters Based on Estimation Costs

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چکیده

در محیط‌های صنعتی بسیار رقابتی امروز که به واسطه پیشرفت سریع تکنولوژی ایجاد شده‌اند، متخصصین کیفیت موقعیت‌هایی را که خارج از کنترل بوده شناسایی و در صورت نیاز، در زمان مناسب اقدام به عمل می‌کنند. در نتیجه، روش‌های آماری جدید برای رسیدگی به فرایندهای باکیفیت، و همچنین برای یافتن روش‌هایی که هزینه‌های کیفیت را به حداقل می‌رسانند، به صورت پیوسته ارتقا یافته‌است. نمودار کنترلی *CCC-r* که حالت توسعه یافته‌ای از نمودار کنترلی *CCC* است، زمانی مورد استفاده قرار می‌گیرد که نمونه‌های معیوب بسیار کمی مشاهده می‌شود. به دلیل این که مقادیر پارامترهای مورد استفاده در طراحی پارامترهای نمودار کنترلی در واقعیت نامعلوم است، متخصصین کیفیت باید این مقادیر را بر اساس نمونه‌های تحت کنترل گذشته براورد نمایند. نشان داده شده‌است که براورد پارامتر تاثیر زیادی بر روی خصوصیات نمودار کنترل دارد. در این مطالعه مدلی براساس هزینه‌های براورد و میانگین اقلام بازرسی شده در زمان نامعلوم بودن نسبت اقلام معیوب در نمودارهای کنترلی *CCC-r* ارائه شده‌است. پارامترهای نامعلوم براساس مقادیر مختلف اندازه نمونه براورد شده و تحلیل حساسیت در این مورد انجام می‌شود.

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