International Journal of Engineering

Journal Homepage: www.ije.ir

Unreliable Server $M^x/G/1$ Queue with Loss-delay, Balking and Second Optional Service

P. Sharma*

Department of Mathematics, School of Science, Noida International University, Gautam Budh Nagar, Greater Noida, India

PAPER INFO

Paper history:
Received 20 July 2016
Received in revised form 26 January 2017
Accepted 30 January 2017

Keywords:
Bulk queue
Loss and delay
Essential service
Optional service
Supplementary variables
Queue size

ABSTRACT

This investigation deals with $M^X/G/1$ queueing model with setup, bulk- arrival, loss-delay and balking. The provision of second optional service apart from essential service by an unreliable server is taken into consideration. We assume that the delay customers join the queue when server is busy whereas loss customers depart from the system. After receiving the essential service, the customers may opt for the optional service with some probability or may leave the system. The server is unreliable and hence may breakdown in both essential and optional service cases and requires a setup time before being repaired. The service during essential service, setup times and repair times for both cases are general distributed while the service time during optional service is exponential. Using the supplementary variable technique, the equations governing the model are constructed. The steady state queue size distribution is then obtained using Laplace transform and probability generating functions. The numerical results for various performance indices have been obtained and illustrated graphically.

doi: *10.5829/idosi.ije.2017.30.02b.11*

1. INTRODUCTION

Queueing models are widely used to study the behavior of various types of congestion problems of day to day as well as industrial problems which are encountered in telecommunication, manufacturing systems, computer systems and many others. In such systems the jobs may arrive in batches and after getting the essential service may opt for optional service or leave the system. The server may be unreliable and requires a setup time before being repaired. Bulk arrivals queues have also find several applications in telecommunication and computer systems, transportation and distribution systems, manufacturing and production systems, etc. In recent past, the important contributions on queueing systems with bulk arrivals are due to Borthakur and Medhi [1], Chaudhry and Templeton [2], and many other researchers. Lee et al. [3] has studied M/G/1 batch arrival queue with N-policy and multiple vacations. Batch arrival retrial queues with multiple vacations and starting failures have been studied by Krishna Kumar and Madheswari [4]. $M^X/G/1$ queue with feedback and server vacations based on a single vacation policy was investigated by Madan and Al-Rawwash [5]. Ke and Lin [6] proposed maximum entropy approach for batch-arrival queue under N policy with an un-reliable server and single vacation. A batch arrival retrial queueing system with two phases of service and service interruption was analyzed by Choudhury et al. [7]. Jain and Chauahn [8] developed a working vacation queue with second optional service and unreliable server. Strategic behavior in a observable fluid queue with an alternating service process was analyzed by Economou and Manou [9].

In many real life situations, due to overloading or long run of operating time, the servers may break down. Queueing problems with service station subject to break down were studied by many researchers; to cite a few, we refer Avi-Itzhak and Naor [10] and also Neuts and Lucantoni [11]. Ke [12, 13] proposed server breakdowns for M/G/1 queueing system. Recently, Ke [14] analyzed a batch arrival queues under vacation policies with server breakdowns and startup/closedown times. An M/G/1 retrial queueing system with two phases of service subject to the server breakdown and

*Corresponding Author's Email: *sharma_ibspankaj@rediffmail.com* (P. Sharma)

repair was made by Choudhury and Deka [15]. Wang et al. [16] developed optimization of the T policy M/G/1 queue with server breakdowns and general startup times. A queue with working breakdowns was analyzed by Kalidas and Kasturi [17]. Yang and Wu [18] studied cost minimization analysis of a working vacation queue with N-policy and server break down.

Queues with balking have received considerable attention of researchers working in the area of queueing theory (cf. Abou-El-Ata and Shawky [19]; Abou-El-Ata and Hariri [20]. Thomo [21] proposed a multiple vacation $M^X/G/1$ model with balking. Jain and Sharma [22] studied controllable multi server queue with balking. Jain and Sharma [23] developed finite capacity queueing system with queue dependent servers and discouragement. Cochran and Broyles [24] developed nonlinear queuing regressions to increase emergency department patient safety: Approximating reneging with balking. Machine repair problem with spares, balking, reneging and N-policy for vacation was developed by Sharma [25]. Yu et al. [26] considered equilibrium strategies of the unobservable M/M/1 queue with balking and delayed repair.

Due to some constraints, often customers would not like to wait for service when all servers are busy and hence, are lost to the system. The loss customers are those which cannot wait in the system when all service positions are occupied whereas the delay customers can wait in the system in such a case. Single unreliable server interdependent loss and delay queueing model with controllable arrival rate under N-Policy was discussed by Sharma [27]. User optimal state dependent routing in parallel tandem queues with loss was made by Spicer and Ziedins [28]. Fan [29] developed a queueing model for mixed loss-delay systems with general inter arrival processes for wide-band calls. Kim et al. [30] considered erlang loss queueing system with batch arrivals operating in a random environment. Network queue and loss analysis using histogram-based traffic models was analyzed by Orallo and Carbo [31]. Transient analysis of loss and delay bulk service Markovian queue under N-policy was discussed by Sharma [32]. Gupta et al. [33] developed optimal revenue management in two class pre-emptive delay dependent Markovian queues.

In this paper, we consider a queue-dependent $M^X/G/1$ queueing system with loss-delay customers, balking, second optional service and setup time before repairing of unreliable server. The remaining paper is organized as follows. In section 2, we give the model description and notations to be used for mathematical formulation. The queue size distribution and performance indices are given in sections 3 and 4, respectively. In section 5, we deduce some special cases by setting appropriate parameters. To explore the effect of various parameters on performance indices, the sensitivity analysis is carried out in section 6. The noble features and future scope of the model proposed are outlined in the final section 7.

2. MODEL DESCRIPTION

Consider an $M^X/G/1$ queue with loss-delay customers. The system consists of an unreliable server who renders essential and optional services to the jobs. For modeling the queueing system the following assumptions are made:

The loss and delay customers arrive in batch of size X in Poisson fashion with arrival rate λ_l and λ_d, respectively. The customers may balk with balking probability ε when server is in busy state, and ε_1 *and* ε_2 (ε_1' *and* ε_2') are the joining probability when the server is in broken down (under set up) state for essential and optional service case, respectively. Since the server is unreliable it may breakdown during both essential and optional service period and is sent for repair; the repairman starts the repair after some setup time. After the repair the server starts serving the customer with the same efficiency as before breakdown. Only delay customers arrive when server is in broken down state and under the setup or repair state.

The time of essential service is assumed to be general distributed. The time of optional service is assumed to be exponentially distributed. After the completion of the essential service the customer may opt for the second service with probability θ; which is optional or may leave the system with probability (1- θ). The following notations are used for modeling purpose:

X random variable denoting the batch size

c_k probability distribution of X, i.e. c_k= Pr(X=k)

C(z) probability generating functions (PGF) of the batch size X

$\bar{X} = C'(1)$ mean batch size [E(X)].

$X_2 = C''(1)$ 2nd factorial moment of batch size

B(.), H_i(.), G_i(.) Cumulative distribution functions (c.d.f) of service time (B), setup time (H) and repair time (G) where index i=1 and 2 corresponding to essential and optional service.

b(x), h_i(x), g_i(x) probability density functions (p.d.f) of B, H_i, G_i(i=1,2) $b^*(.), h_i^*, g_i^*$ Laplace-Stieltje's transform (LST) of B, H_i, G_i where i=1,2 θ probability of opting the optional service $1/\mu_i$ mean times of essential and optionalservice.

α_i (i=1, 2) is failure rate of the server when he has broken down during essential and optional service, respectively.

Denote $\delta_i = 1 + \alpha_i(\nu_i + \gamma_i),\ i = 1,2$; $\Lambda_0 = \lambda_\ell + \lambda_d$ Now $\beta_1(x) = \frac{b(x)}{1-B(x)}, \nu_i(y) = \frac{h_i(y)}{1-H_i(y)}$ *and* $\gamma_i(y) = \frac{g_i(y)}{1-G_i(y)}, i = 1,2$, are the hazard rate for essential service time, setup time and repair time, respectively.

The second moment of essential service time, setup time and repair time are given by

$\beta_1^{(2)} = (-1)^2 b^{*2}(0),\ \nu_i^{(2)} = (-1)^2 h_i^{*(2)}(0)$

$and\ \gamma_i^{(2)} = (-1)^2 g_i^{*(2)}(0),\ i = 1,2$

respectively.

The transient probabilities for different states are defined as follows:

Q(t) probability that the server is idle at time t

$P_n^{(1)}(t,x)dx$ joint probability at time t, with n customers in the queue, the server is on and a customer is being provided essential service with elapsed service time lying between x and x + dx.

$P_n^{(2)}(t,x)$ probability at time t, with n customers In the queue, the server is on and a customer is being provided the optional service.

$S_n^{(1)}(t,x,y)dy$ joint probability at time t, with customers in the queue, the elapsed service time is equal to x, the server is broken-down when providing essential service and is waiting for repair with elapsed setup time lying between y and y + dy.

$S_n^{(2)}(t,y)\,dy$ joint probability at time t, with n customers in the queue, and the server is broken-down when providing optional service and is waiting for repair with elapsed setup time lying between y and y + dy.

$R_n^{(1)}(t,x,y)\,dy$ joint probability at time t, with n customers in the queue, the elapsed service time is equal to x, the server is broken-down when rendering essential service and the server is under repair with elapsed repair time lying between y and y+ dy.

$R_n^{(2)}(t,y)$ dy joint probability at time t, with n customers in the queue, the server is broken-down when rendering optional service and the server is under repair with elapsed repair time lying between y and y+dy.

3. QUEUE SIZE DISTRIBUTION

Using the above assumptions and the notations, we construct the governing equations of our model as follows:

3. 1. Governing Equations

$$\left(\frac{d}{dt} + \Lambda_0\right)Q(t) = (1-\theta)\int_0^\infty P_0^{(1)}(t,x)\mu_1(x)dx + \mu_2 P_0^{(2)}(t) \tag{1}$$

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial t}\right)P_n^{(1)}(t,x) = -(\Lambda_0\varepsilon + \mu_1(x) + \alpha_1)P_n^{(1)}(t,x) + \int_0^\infty \beta_1(y)R_n^{(1)}(t,x,y)dy + \Lambda_0\varepsilon\sum_{k=1}^{n} c_k P_{n-k}^{(1)}(x),\ \text{n} > 0 \tag{2}$$

$$\left(\frac{\partial}{\partial y} + \frac{\partial}{\partial t}\right)S_n^{(1)}(t,x,y) = -(\lambda_d\varepsilon_1 + \upsilon_1(y))S_n^{(1)}(t,x,y) + \lambda_d\varepsilon_1\sum_{k=1}^{n} c_k S_{n-k}^{(1)}(t,x,y),\quad n > 0 \tag{3}$$

$$\left(\frac{\partial}{\partial y} + \frac{\partial}{\partial t}\right)R_n^{(1)}(t,x,y) = -(\lambda_d\varepsilon_1' + \beta_1(y))R_n^{(1)}(t,x,y) + \lambda_d\varepsilon_1'\sum_{k=1}^{n} c_k R_{n-k}^{(1)}(t,x,y),\quad n > 0 \tag{4}$$

$$\frac{d}{dt}P_n^{(2)}(t) = -(\Lambda_0\varepsilon + \mu_2 + \alpha_2)P_n^{(2)}(t) + \theta\int_0^\infty \mu_1(x)P_n^{(1)}(t,x) + \int_0^\infty \beta_2(y)R_n^{(2)}(t,y)dy + \Lambda_0\varepsilon\sum_{k=1}^{n} c_k P_{n-k}^{(2)}(t), \tag{5}$$

$$\left(\frac{\partial}{\partial y} + \frac{\partial}{\partial t}\right)S_n^{(2)}(t,y) = -(\lambda_d\varepsilon_2 + \upsilon_2(y))S_n^{(2)}(t,y) + \lambda_d\varepsilon_2\sum_{k=1}^{n} c_k S_{n-k}^{(2)}(t,y),\quad n > 0 \tag{6}$$

$$\left(\frac{\partial}{\partial y} + \frac{\partial}{\partial t}\right)R_n^{(2)}(t,x,y) = -(\lambda_d\varepsilon_2' + \beta_2(y))R_n^{(2)}(t,y) + \lambda_d\varepsilon_2'\sum_{k=1}^{n} c_k R_{n-k}^{(2)}(t,y),\quad n > 0 \tag{7}$$

The above equations are to be solved subject to the following boundary conditions:

$$P_n^{(1)}(t,0) = (1-\theta)\int_0^\infty P_{n+1}^{(1)}(t,x)\mu_1(x)dx + \mu_2 P_{n+1}^{(2)}(t),\quad n > 0 \tag{8}$$

$$P_0^{(1)}(t,0) = (1-\theta)\int_0^\infty P_1^{(1)}(t,x)\mu_1(x)dx + \mu_2 P_1^{(2)}(t,x) + \Lambda_0\varepsilon\sum_{k=1}^{n} c_k Q_{n-k}(t),\quad n > 0 \tag{9}$$

$$S_n^{(1)}(t,x,0) = \alpha_1 P_n^{(1)}(t,x),\quad n > 0 \tag{10}$$

$$R_n^{(1)}(t,x,0) = \int_0^\infty S_n^{(1)}(t,y)\upsilon_1(y)dy,\quad n > 0 \tag{11}$$

$$S_n^{(2)}(t,0) = \alpha_2 P_n^{(1)}(t),\quad n > 0 \tag{12}$$

$$R_n^{(2)}(t,0) = \int_0^\infty S_n^{(2)}(t,y)\upsilon_2(y)dy,\quad n > 0 \tag{13}$$

Normalizing condition is given by

$$Q(t) + \sum_{n=0}^{\infty}\left\{P_n^{(2)}(t) + \int_0^\infty S_n^{(2)}(t,y)dy + \int_0^\infty R_n^{(2)}(t,y)dy + \int_0^\infty P_n^{(1)}(t,x)dx + \int_0^\infty\int_0^\infty S_n^{(1)}(t,x,y)dxdy + \int_0^\infty\int_0^\infty R_n^{(1)}(t,x,y)dxdy\right\} = 1 \tag{14}$$

Taking Laplace-Stieltjes transforms, above Equations (1)-(7) yield:

$$(s + \Lambda_0)Q^*(s) - 1 = (1-\theta)\int_0^\infty P_0^{*(1)}(s,x)\mu_1(x)dx + \mu_2 P_0^{*(2)}(s), n > 0 \tag{15}$$

$$\frac{\partial}{\partial x}P_n^{*(1)}(s,x) + (s + \Lambda_0\varepsilon + \mu_1(x) + \alpha_1)P_n^{*(1)}(s,x) = \Lambda_0\varepsilon\sum_{k=1}^{n} c_k P_{n-k}^{*(1)}(s,x) + \int_0^\infty \beta_1(y)R_n^{*(1)}(s,x,y)dy,\quad n > 0 \tag{16}$$

$$\frac{\partial}{\partial y}S_n^{*(1)}(s,x,y)+(s+\lambda_d\varepsilon_1+\upsilon_1(y))S_n^{*(1)}(s,x,y)=\lambda_d\varepsilon_1\sum_{k=1}^{n}c_kS_{n-k}^{*(1)}(s,x,y),\quad n>0 \tag{17}$$

$$\frac{\partial}{\partial y}R_n^{*(1)}(s,x,y)+(s+\lambda_d\varepsilon_1'+\beta_1(y))R_n^{*(1)}(s,x,y)=\lambda_d\varepsilon_1'\sum_{k=1}^{n}c_kR_{n-k}^{*(1)}(s,x,y),\quad n>0 \tag{18}$$

$$(s+\Lambda_0\varepsilon+\mu_2+\alpha_2)P_n^{*(2)}(s)=\theta\int_0^\infty\mu_1(x)P_n^{*(1)}(s,x)dx+\int_0^\infty\beta_2(y)R_n^{*(2)}(s,y)dy+\Lambda_0\varepsilon\sum_{k=1}^{n}c_kP_{n-k}^{*(2)}(s),\quad n>0 \tag{19}$$

$$\frac{\partial}{\partial y}S_n^{*(2)}(s,y)+(s+\lambda_d\varepsilon_2+\upsilon_2(y))S_n^{*(2)}(s,y)=\lambda_d\varepsilon_2\sum_{k=1}^{n}c_kS_{n-k}^{*(2)}(s,y),\quad n>0 \tag{20}$$

$$\frac{\partial}{\partial y}R_n^{*(2)}(s,y)+(s+\lambda_d\varepsilon_2'+\beta_2(y))R_n^{*(2)}(s,y)=\lambda_d\varepsilon_2'\sum_{k=1}^{n}c_kR_{n-k}^{*(2)}(s,y),\quad n>0 \tag{21}$$

Similarly, taking Laplace-Stieltjes transforms of boundary conditions (8)-(13), we get:

$$P_n^{*(1)}(s,0)=(1-\theta)\int_0^\infty P_{n+1}^{*(1)}(s,x)\mu_1(x)dx+\mu_2P_{n+1}^{*(2)}(s),\ n>0 \tag{22}$$

$$P_0^{*(1)}(s,0)=(1-\theta)\int_0^\infty P_1^{*(1)}(s,x)\mu_1(x)dx+\mu_2P_1^{*(2)}(s,x)+\Lambda_0\varepsilon\sum_{k=1}^{n}c_kQ_{n-k}^{*}(s),\quad n>0 \tag{23}$$

$$S_n^{*(1)}(s,x,0)=\alpha_1P_n^{*(1)}(s,x),\quad n>0 \tag{24}$$

$$R_n^{*(1)}(s,x,0)=\int_0^\infty\upsilon_1(y)S_n^{*(1)}(s,y)dy,\quad n>0 \tag{25}$$

$$S_n^{*(2)}(s,0)=\alpha_2P_n^{*(1)}(s),\quad n>0 \tag{26}$$

$$R_n^{*(2)}(s,0)=\int_0^\infty\upsilon_2(y)S_n^{*(2)}(s,y)dy,\quad n>0 \tag{27}$$

Laplace-Stieltjes transforms of normalizing condition yields:

$$Q^*(s)+\sum_{n=0}^{\infty}\left\{P_n^{*(2)}(s)+\int_0^\infty S_n^{*(2)}(s,y)dy+\int_0^\infty R_n^{*(2)}(s,y)dy+\int_0^\infty P_n^{*(1)}(s,x)dx+\int_0^\infty\int_0^\infty S_n^{*(1)}(s,x,y)dxdy+\int_0^\infty\int_0^\infty R_n^{*(1)}(s,x,y)dxdy\right\}=1 \tag{28}$$

3. 2. Generating Functions Define the following generating functions:

$$\begin{aligned}
&P^{*(1)}(s,x,z)=\sum_{n=0}^{\infty}P_n^{*(1)}(s,x)z^n\\
&P^{*(2)}(s,z)=\sum_{n=0}^{\infty}P_n^{*(2)}(s)z^n,\\
&R^{*(1)}(s,x,y,z)=\sum_{n=0}^{\infty}R_n^{*(1)}(s,x,y)z^n\\
&R^{*(2)}(s,y,z)=\sum_{n=0}^{\infty}R_n^{*(2)}(s,y)z^n,\\
&S^{*(1)}(s,x,y,z)=\sum_{n=0}^{\infty}R_n^{*(1)}(s,x,y)z^n\\
&S^{*(2)}(s,y,z)=\sum_{n=0}^{\infty}R_n^{*(2)}(s,y)z^n,\quad |z|\le 1
\end{aligned} \tag{29}$$

Multiplying Equations (16)-(21) by z^n and summing over n, we obtain:

$$\frac{\partial}{\partial x}P^{*(1)}(s,z,x)+[s+\mu_1(x)+\alpha_1+\Lambda_0\varepsilon-\Lambda_0\varepsilon C(z)]P^{*(1)}(s,x)=\int_0^\infty\beta_1(y)R^{*(1)}(s,z,x,y)dy \tag{30}$$

$$\frac{\partial}{\partial y}S^{*(1)}(s,z,x,y)+[s+\upsilon_1(y)+\lambda_d\varepsilon_1-\lambda_d\varepsilon_1C(z)]S^{*(1)}(s,z,x,y)=0 \tag{31}$$

$$\frac{\partial}{\partial y}R^{*(1)}(s,z,x,y)+[s+\beta_1(y)+\lambda_d\varepsilon_1'-\lambda_d\varepsilon_1'C(z)]R^{*(1)}(s,z,x,y)=0 \tag{32}$$

$$[s+\mu_2+\alpha_2+\Lambda_0\varepsilon-\Lambda_0\varepsilon C(z)]P^{*(2)}(s,z)=\theta\int_0^\infty\mu_1(x)P^{*(1)}(s,z,x)dx+\int_0^\infty\beta_2(y)R^{*(2)}(s,z,y)dy \tag{33}$$

$$\frac{\partial}{\partial y}S^{*(2)}(s,z,y)+[s+\upsilon_2(y)+\lambda_d\varepsilon_2-\lambda_d\varepsilon_2C(z)]S^{*(2)}(s,z,y)=0 \tag{34}$$

$$\frac{\partial}{\partial y}R^{*(2)}(s,z,y)+[s+\beta_2(y)+\lambda_d\varepsilon_2'-\lambda_d\varepsilon_2'C(z)]R^{*(2)}(s,z,y)=0 \tag{35}$$

Similarly, multiplying Equation (22) by z^n and Equation (23) by z^0 and summing over n and using Equation (15), we get:

$$zP^{*(1)}(s,0,z)=1-[s+\Lambda_0\varepsilon-\Lambda_0\varepsilon C(z)]Q^*(s)+(1-\theta)\int_0^\infty P^{*(1)}(s,x,z)\mu_1(x)dx+\mu_2P^{*(2)}(s,z) \tag{36}$$

Proceeding in similar manner with Equations (24)-(27), we have:

$$S^{*(1)}(s,z,x,0)=\alpha_1 P^{*(1)}(s,z,x) \tag{37}$$

$$R^{*(1)}(s,z,x,0)=\int_0^\infty \upsilon_1(y)S^{*(1)}(s,z,y)dy \tag{38}$$

$$S^{*(2)}(s,z,0)=\alpha_2 P^{*(2)}(s,z) \tag{39}$$

$$R^{*(2)}(s,z,0)=\int_0^\infty \upsilon_2(y)S^{*(2)}(s,z,y)dy \tag{40}$$

3. 3. Computation of Partial Generating Functions

Solving the differential Equations (31), (32), (34) and (35), and using Equations (37)-(40), after some algebraic calculations, we have:

$$S^{*(1)}(s,x,y,z)=\alpha_1 P^{*(1)}(s,x,z)e^{-[s+\lambda_d\varepsilon_1\{1-C(z)\}]y}\overline{H}_1(y) \tag{41}$$

$$S^{*(1)}(s,y,z)=\alpha_2 P^{*(2)}(s,z)e^{-[s+\lambda_d\varepsilon_2\{1-C(z)\}]y}\overline{H}_2(y) \tag{42}$$

$$R^{*(1)}(s,x,y,z)=\alpha_1 P^{*(1)}(s,x,z)h_1^*[s+\lambda_d\varepsilon_1\{1-C(z)\}] \times e^{-[s+\lambda_d\varepsilon_1'\{1-C(z)\}]y}\overline{G}_1(y) \tag{43}$$

$$R^{*(2)}(s,y,z)=\alpha_2 P^{*(2)}(s,z)h_2^*[s+\lambda_d\varepsilon_2\{1-C(z)\}] \times e^{-[s+\lambda_d\varepsilon_2'\{1-C(z)\}]}\overline{G}_2(y) \tag{44}$$

3. 3. 1. Busy States

Equations (41)-(42), give:

$$\int_0^\infty \beta_1(y)R^{*(1)}(s,z,x,y)dy=\alpha_1 P^{*(1)}(s,z,x)h_1^*[s+\lambda_d\varepsilon_1\{1-C(z)\}] \times g_1^*[s+\lambda_d\varepsilon_1'\{1-C(z)\}] \tag{45}$$

$$\int_0^\infty \beta_2(y)R^{*(2)}(s,z,y)dy=\alpha_2 P^{*(2)}(s,z)h_2^*[s+\lambda_d\varepsilon_2\{1-C(z)\}] \times g_1^*[s+\lambda_d\varepsilon_2'\{1-C(z)\}] \tag{46}$$

Using Equation (45) in Equation (30), we obtain after some algebra:

$$P^{*(1)}(s,x,z)=P^{*(1)}(s,0,z)\exp\{-f_1(s,z)\}\overline{B}(x) \tag{47}$$

$$\begin{aligned} \textit{where } f_1(s,z)&=s+\Lambda_0\varepsilon\{1-C(z)\}+\alpha_1 \\ &-\alpha_1 h_1^*[s+\lambda_d\varepsilon_1\{1-C(z)\}]g_1^*[s+\lambda_d\varepsilon_1'\{1-C(z)\}] \\ f_2(s,z)&=s+\Lambda_0\varepsilon\{1-C(z)\}+\alpha_2 \\ &-\alpha_2 h_2^*[s+\lambda_d\varepsilon_2\{1-C(z)\}]g_2^*[s+\lambda_d\varepsilon_2'\{1-C(z)\}] \end{aligned} \tag{48}$$

$$\textit{and } \int_0^\infty P^{*(1)}(s,x,z)=P^{*(1)}(s,0,z)b^*\{f_1(s,z)\} \tag{49}$$

Using Equations (33), (45) and (49), we get:

$$P^{*(2)}(s,z)=\frac{\theta b^*\{f_1(s,z)\}P^{*(1)}(s,z,0)}{f_2(s,z)+\mu_2} \tag{50}$$

Integrating Equation (47) w.r.t. 'x', we obtain:

$$P^{*(1)}(s,z)=[\frac{1-b^*\{f_1(s,z)\}}{\{f_1(s,z)\}}]P^{*(1)}(s,z,0) \tag{51}$$

3. 3. 2. Setup States

Performing similar procedure for Equations (41) and (42), we obtain:

$$S^{*(1)}(s,z)=\alpha_1\left[\frac{1-b^*\{f_1(s,z)\}}{f_1(s,z)}\right]\frac{h_1^*[s+\lambda_d\varepsilon_1\{1-C(z)\}]}{s+\lambda_d\varepsilon_1\{1-C(z)\}}P^{*(1)}(s,0,z) \tag{52}$$

$$S^{*(2)}(s,z)=\alpha_2\left[\frac{\theta b^*\{f_1(s,z)\}}{f_2(s,z)+\mu_2}\right]\frac{h_2^*[s+\lambda_d\varepsilon_2\{1-C(z)\}]}{s+\lambda_d\varepsilon_2\{1-C(z)\}}P^{*(1)}(s,0,z) \tag{53}$$

3. 3. 3. Repair States

Integrating Equation (43) w.r.t. 'y', we get:

$$R^{*(1)}(s,x,z)=\alpha_1 P^{*(1)}(s,0,z)h_1^*[s+\lambda_d\varepsilon_1\{1-C(z)\}] \times\frac{\overline{g}_1^*[s+\lambda_d\varepsilon_1'\{1-C(z)\}]}{[s+\lambda_d\varepsilon_1'\{1-C(z)\}]} \tag{54}$$

Again integrating Equation (54) w.r.t. 'x', we finally find:

$$R^{*(1)}(s,z)=\alpha_1\left[\frac{1-b^*\{f_1(s,z)\}}{\{f_1(s,z)\}}\right]\frac{\overline{g}_1^*[s+\lambda_d\varepsilon_1'\{1-C(z)\}]}{[s+\lambda_d\varepsilon_1'\{1-C(z)\}]} \times h_1^*[s+\lambda_d\varepsilon_1\{1-C(z)\}]P^{*(1)}(s,0,z) \tag{55}$$

Likewise, we obtain:

$$R^{*(2)}(s,z)=\alpha_2\left[\frac{\theta b^*\{f_1(s,z)\}}{f_2(s,z)+\mu_2}\right]\frac{\overline{g}_2^*[s+\lambda_d\varepsilon_2'\{1-C(z)\}]}{[s+\lambda_d\varepsilon_2'\{1-C(z)\}]} \times h_2^*[s+\lambda_d\varepsilon_2\{1-C(z)\}]P^{*(1)}(s,0,z) \tag{56}$$

From Equation (36), we get:

$$P^{*(1)}(s,z,0)=\frac{[f_2(s,z)+\mu_2][1-\{s+\Lambda_0-\Lambda_0 C(z)\}Q^*(s)]}{[z-b^*\{f_1(s,z)\}][f_2(s,z)+\mu_2]+\theta b^*\{f_1(s,z)\}\{f_2(s,z)\}} \tag{57}$$

3. 4. Evaluation of Generating Functions

Now, the probability generating function of the number of customers in the queue, irrespective of the type of service being provided is obtained as:

$$P^*(s,z)=\sum_{n=1}^{2}P^{*(n)}(s,z)+S^{*(n)}(s,z)+R^{*(n)}(s,z)$$

$$=P^{*(1)}(s,z,0)\left[\begin{aligned} &\frac{1-b^*\{f_1(s,z)\}}{\{f_1(s,z)\}} \\ &\times\{1+\alpha_1\frac{\overline{h}_1^*[s+\lambda_d\varepsilon_1\{1-C(z)\}]}{s+\lambda_d\varepsilon_1\{1-C(z)\}} \\ &+\alpha_1\frac{\overline{g}_1^*[s+\lambda_d\varepsilon_1'\{1-C(z)\}]}{s+\lambda_d\varepsilon_1'\{1-C(z)\}}h_1^*[s+\lambda_d\varepsilon_1\{1-C(z)\}]\} \\ &+\theta\frac{b^*\{f_1(s,z)\}}{f_2(s,z)+\mu_2} \\ &\times\{1+\alpha_2\frac{\overline{h}_2^*[s+\lambda_d\varepsilon_2\{1-C(z)\}]}{s+\lambda_d\varepsilon_2\{1-C(z)\}} \\ &+\alpha_2\frac{\overline{g}_2^*[s+\lambda_d\varepsilon_2'\{1-C(z)\}]}{s+\lambda_d\varepsilon_2'\{1-C(z)\}}h_{21}^*[s+\lambda_d\varepsilon_2\{1-C(z)\}]\} \end{aligned}\right] \tag{58}$$

Putting z=1 in Equation (58), we get:

$$Q^*(s)+P^*(s,1)=\frac{1}{s} \tag{59}$$

Applying the arguments of the Rouche's theorem in Equation (58), we obtain the unknown probability $Q^*(s)$.

4. PARTICULAR MODELS

We shall now discuss some special cases of the above model by specifying some parameters as follows:

Model I: *Model without setup time*.

The queue size distribution in this case is given by:

$$P^*(s,z)=\sum_{n=1}^{2}\left\{P^{*(n)}(s,z)+R^{*(n)}(s,z)\right\}$$
$$=P^{*(1)}(s,z,0)\left[\begin{array}{l}\dfrac{1-b^*\{f_1'(s,z)\}}{\{f_1'(s,z)\}}\left\{1+\alpha_1\dfrac{\bar{g}_1^*[s+\lambda_d\varepsilon_1'\{1-C(z)\}]}{s+\lambda_d\varepsilon_1'\{1-C(z)\}}\right\}\\ +\theta\dfrac{b^*\{f_1'(s,z)\}}{f_1'(s,z)+\mu_2}\left\{1+\alpha_2\dfrac{\bar{g}_2^*[s+\lambda_d\varepsilon_2'\{1-C(z)\}]}{s+\lambda_d\varepsilon_2'\{1-C(z)\}}\right\}\end{array}\right] \tag{60}$$

$$where\ f_1'(s,z)=s+\Lambda_0\varepsilon\{1-C(z)\}+\alpha_1 -\alpha_1 g_1^*[s+\lambda_d\varepsilon_1'\{1-C(z)\}]$$
$$f_2'(s,z)=s+\Lambda_0\varepsilon\{1-C(z)\}+\alpha_2 -\alpha_2 g_2^*[s+\lambda_d\varepsilon_2'\{1-C(z)\}]$$

Model II: In this case we consider the model without loss customer and without balking behavior. So, the new arrival rate is $\lambda=\Lambda_0=\Lambda_0\varepsilon=\lambda_d\varepsilon_i=\lambda_d\varepsilon_i'\ \ where\ i=1\ and\ 2$.

Now, our model corresponds to $M^X/G/1$ queue with second optional service, setup and unreliable server. In this model, the queue size distribution is obtained as:

$$P^*(s,z)=\sum_{n=1}^{2}P^{*(n)}(s,z)+S^{*(n)}(s,z)+R^{*(n)}(s,z)$$
$$=\frac{N(s,z)}{D(s,z)}\left[\frac{1}{s+\lambda-\lambda C(z)}-Q^*\right]$$
$$where\ \ N(s,z)=[1-b^*\{\psi_1(s,z)\}][\psi_2(s,z)+\mu_2] +\theta\, b^*\{\psi_1(s,z)\}\psi_2(s,z) \tag{61}$$

and

$$D(s,z)=[z-b^*\{\psi_1(s,z)\}][\psi_2(s,z)+\mu_2] +\theta b^*\{\psi_1(s,z)\}]\psi_2(s,z)$$
$$\psi_1(s,z)=s+\lambda\{1-C(z)\}+\alpha_1 -\alpha_1 h_1^*[s+\lambda\{1-C(z)\}]g_1^*[s+\lambda\{1-C(z)\}]$$
$$\psi_2(s,z)=s+\lambda\{1-C(z)\}+\alpha_2 -\alpha_2 h_2^*[s+\lambda\{1-C(z)\}]g_2^*[s+\lambda\{1-C(z)\}]$$

Model III: $M^X/G/1$ queue with second optional service and unreliable server.

In this case, we set arrival rate as $\lambda=\Lambda_0=\Lambda_0\varepsilon=\lambda_d\varepsilon_i=\lambda_d\varepsilon_i'\quad where\ i=1\ and\ 2$.

Now, the queue size distribution is given by:

$$P^*(s,z)=\sum_{n=1}^{2}P^{*(n)}(s,z)+R^{*(n)}(s,z)$$
$$=\frac{N'(s,z)}{D'(s,z)}\left[\frac{1}{s+\lambda-\lambda C(z)}-Q^*\right]$$
$$where\ N'(s,z)=[1-b^*\{\xi_1(s,z)\}][\xi_2(s,z)+\mu_2] +\theta\, b^*\{\xi_1(s,z)\}\xi_2(s,z) \tag{62}$$
$$D'(s,z)=[z-b^*\{\xi_1(s,z)\}][\xi_2(s,z)+\mu_2] +\theta b^*\{\xi_1(s,z)\}]\xi_2(s,z)$$

and

$$\xi_1(s,z)=s+\lambda\{1-C(z)\}+\alpha_1-\alpha_1 g_1^*[s+\lambda\{1-C(z)\}]$$
$$\xi_2(s,z)=s+\lambda\{1-C(z)\}+\alpha_2-\alpha_2 g_2^*[s+\lambda\{1-C(z)\}]$$

5. PERFORMANCE INDICES

(a) Long run probabilities of the server's states

Now, we evaluate the probabilities for different states of the server by applying the well-known Tauberian property, which is stated as:

$$P^*(z)=s\lim_{s\to 0}P^*(s,z)=\frac{N(0,z)}{D(0,z)}\left[0-Q\right]$$

Putting z=1 in the above equation and applying L'Hospital's rule, we get:

$$P(0,1)=\frac{\rho_1\delta_1+\theta\rho_2\delta_2}{\left[1-\rho_1\delta_1-\theta\rho_2\delta_2\right]}\left[-Q\right]\text{, } where\ \ \rho_i=\frac{\lambda\bar{X}}{\mu_i} \tag{63}$$

The normalizing condition is given by $Q+P(0,1)=1$.

The probability of the server being idle is:

$$Q=\begin{cases}1-\dfrac{\bar{X}\Phi_1}{\mu_1}-\theta\dfrac{\bar{X}\Phi_2}{\mu_2}; & for\ \text{mod}\,el\ I\\ 1-\rho_1\delta_1-\theta\rho_2\delta_2; & for\ \text{mod}\,el\ II\ and\ III\end{cases}$$

where

$$\Phi_1=(\Lambda_0\varepsilon+\lambda_d\varepsilon_1'\alpha_1\gamma_1)\ \ and\ \Phi_2=(\Lambda_0\varepsilon+\lambda_d\varepsilon_2'\alpha_2\gamma_2) \tag{64}$$

and

$$\zeta_i=1+\alpha_i\gamma_i,\quad i=1,2$$

The probability of the server being busy:

$$P_B=\lim_{z\to 1}\lim_{s\to 0}s\sum_{l=1}^{k}P^{*(l)}(s,z)$$
$$=\begin{cases}\dfrac{\Lambda_0\varepsilon\bar{X}}{\mu_1}+\theta\dfrac{\Lambda_0\varepsilon\bar{X}}{\mu_2}; & for\ \text{mod}\,el\ I\\ \rho_1+\theta\rho_2; & for\ \text{mod}\,el\ II\ and\ III\end{cases} \tag{65}$$

The probability of the server being under setup (only for case II):

$$P_S=\lim_{z\to 1}\lim_{s\to 0}s\sum_{n=1}^{2}S^{*(n)}(s,z)\ \ =\rho_1\alpha_1\nu_1+\theta\rho_2\alpha_2\nu_2 \tag{66}$$

The probability of the server being under repaired:

$$P_R = \lim_{z\to 1}\lim_{s\to 0} s\sum_{n=1}^{2} R^{*(n)}(s,z)$$
$$= \begin{cases} \dfrac{\Lambda_0 \varepsilon \lambda_d \varepsilon_1' \overline{X}}{\mu_1}\gamma_1 + \theta \dfrac{\Lambda_0 \varepsilon \lambda_d \varepsilon_2' \overline{X}}{\mu_2}\gamma_2; & \text{for model I} \\ \rho_1\gamma_1 + \theta\rho_2\gamma_2; & \text{for model II} \\ \rho_1\alpha_1\gamma_1 + \theta\rho_2\alpha_2\gamma_2; & \text{for model III} \end{cases} \tag{67}$$

(b) Average Queue Length

The average number of customers in the queue is:

$$E(Q) = \lim_{s\to 0} s\widetilde{P}(s,1)$$

$$= \begin{cases} \dfrac{1}{2\left[1-\overline{X}\Phi_1\beta_1 - \theta\dfrac{\overline{X}\Phi_2}{\mu_2}\right]}\left[(\overline{X}\Phi_1)^2\beta_1^{(2)} + \overline{X}X_2\left\{\Phi_1\beta_1 + \theta\dfrac{\Phi_2}{\mu_2}\right\} \right. \\ \quad + 2\theta\overline{X}^2\left(\dfrac{\beta_1}{\mu_2}\Phi_1\Phi_2 + \dfrac{1}{\mu_2^2}\Phi_2^2\right) \\ \quad \left. + \Lambda_0\varepsilon(\overline{X})^2\left(\lambda_d\varepsilon_1'\alpha_1\beta_1\gamma_1^{(2)} + \theta\dfrac{\lambda_d\varepsilon_2'\alpha_2\gamma_2^{(2)}}{\mu_2}\right)\right]; \text{for model I} \\ \dfrac{1}{2[1-\rho_1\delta_1-\theta\rho_2\delta_2]}\left[(\lambda\overline{X})^2\delta_1^2\beta_1^{(2)} + \lambda X_2(\delta_1\beta_1 + \theta\dfrac{\delta_2}{\mu_2}) \right. \\ \quad \left. + 2\theta\left(\rho_1\delta_1\rho_2\delta_2 + (\rho_2\delta_2)^2\right) + \lambda\overline{X}\left(\rho_1\alpha_1\Delta_1 + \theta\rho_2\alpha_2\Delta_2\right)\right]; \\ \qquad \text{for model II} \\ \dfrac{1}{2[1-\rho_1\zeta_1-\theta\rho_2\zeta_2]}\left[(\lambda\overline{X})^2\zeta_1^2\beta_1^{(2)} + \lambda X_2(\zeta_1\beta_1 + \theta\dfrac{\zeta_2}{\mu_2}) \right. \\ \quad \left. + 2\theta\left(\rho_1\zeta_1\rho_2\zeta_2 + (\rho_2\zeta_2)^2\right) + \lambda\overline{X}\left(\rho_1\alpha_1\gamma_1^{(2)} + \theta\rho_2\alpha_2\gamma_2^{(2)}\right)\right]; \\ \qquad \text{for model III} \end{cases} \tag{68}$$

where, $\Delta_i = \gamma_i^{(2)} + 2\nu_i\gamma_i + \nu_i^{(2)}, i = 1, 2$

6. SPECIAL CASES

Now, we deduce results for average queue length for some special cases as follows:

Case I: In case of single arrival model i.e. when X=1, our model tallies with that of Wang et al. [16], and we get:

$$E(Q) = \frac{1}{2[1-\rho_1\zeta_1-\theta\rho_2\zeta_2]}\left[\lambda^2\zeta_1^2\beta_1^{(2)} + \theta\frac{\zeta_2}{\mu_2} + 2\theta\left(\rho_1\zeta_1\rho_2\zeta_2 + (\rho_2\zeta_2)^2\right) + \lambda\left(\rho_1\alpha_1\gamma_1^{(2)} + \theta\rho_2\alpha_2\gamma_2^{(2)}\right)\right] \tag{69}$$

Case II: For M/G/1 queueing model with reliable server and second optional service we put X=1 and $\alpha_1=\alpha_2=0$ in Equation (68). This model is similar to the one studied by Madan and Al-Rawwash [5]. In this case, the average queue length reduces to:

$$E(Q) = \frac{1}{2\left[1-\lambda\beta_1 - \dfrac{\theta\lambda}{\mu_2}\right]}\left[\lambda^2\beta_1^{(2)} + \frac{\theta}{\mu_2} + 2\theta\left(\frac{\lambda^2\beta_1}{\mu_2} + \frac{\lambda^2}{\mu_2}\right)\right] \tag{70}$$

Case III: On substituting $\theta=0$, X=1 and $\alpha_1=\alpha_2=0$ in Equation (68), we get results for classical M/G/1 model (cf. Gross and Harris,) as:

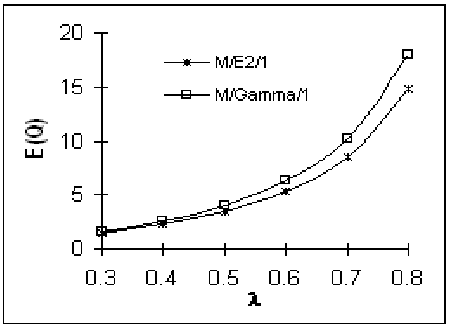
$$E(Q) = \frac{\lambda^2\beta_1^{(2)}}{2[1-\lambda\beta_1]} \tag{71}$$

7. SENSITIVITY ANALYSIS

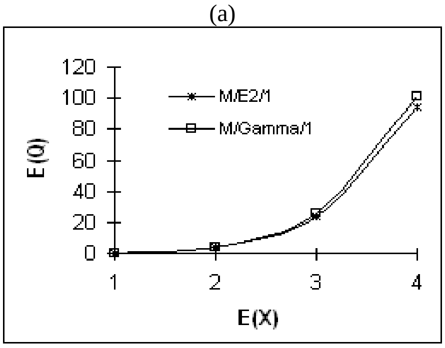
In this section, the analytical results obtained in the previous section are numerically computed by a program developed in MATLAB software. We explore the effect of different parameters on various performance measures. In MATLAB program, we make the following assumptions for computation purpose:

- Service time distribution for essential service is 2-Erlang
- Batch size distribution of the arrival is geometric with mean 2
- Repair times of the server when broken down during both essential and optional services are 2-Erlang
- The default parameters are $\lambda=0.3$, $\theta=0.5$, $\mu_1=7$, $\mu_2=3$, $\alpha_1=1$, $\alpha_2=2$, $\beta_1=1/2$, $\beta_2=1/3$

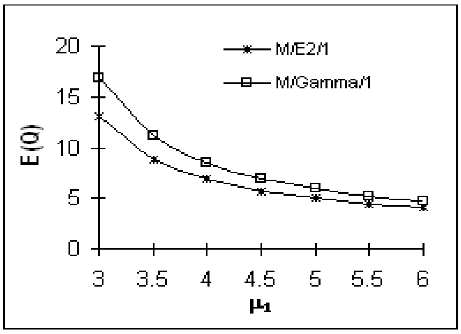
We study the effects of various parameters on average queue length E(Q) as depicted in Figures 1(a)-(f). In Figures 1(a)-(f), we display the trends of average queue length against arrival rate λ, mean batch size E(X) or $\overline{X}$, service rate μ_1, and μ_2, failure rate α and repair rates β for different distributions.



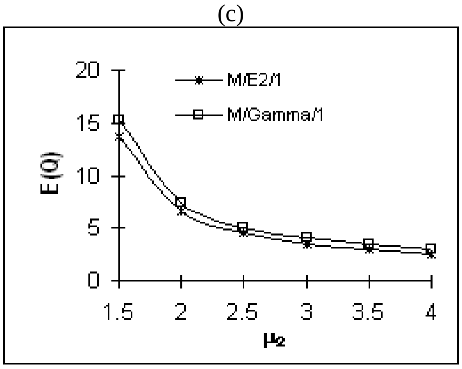
(a)



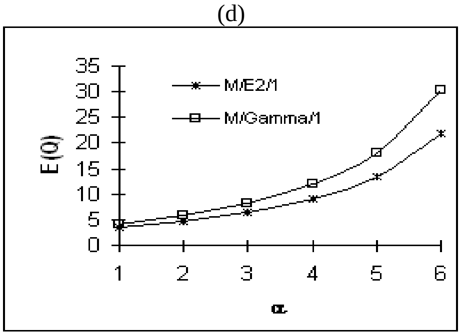
(b)



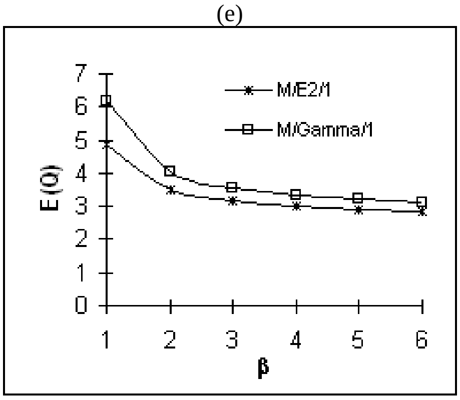
(c)



(d)



(e)



(f)

Figure 1. Average queue length for different distribution by varying (a) arrival rate λ, (b) batch size $\overline{X}$ (c) service rate μ_1 (d) service rate μ_2 (e) failure rate α and (f) repair rate β

In Figures 1(a)-(f), we notice that for Gamma distribution, the queue length is greater as compared to 2-Erlang distribution. We also observe that the E(Q) increases with the increase in the arrival rate as depicted in Figure 1(a). The queue length also increases as increase the mean batch size, as shown in Figure 1 (b). Figures 1(c) and (d) demonstrates the effect of service rate $\mu_1$and μ_2 on E(Q). We note that the queue length first decreases sharply up to $\mu_1=4$ ($\mu_2=2$) and there after decreases gradually. From Figures 1(e) and (f), we see that with the increase (decrease) in the failure (repair) rate, the queue length increases (decrease), which is what we expect from experience.

8. CONCLUSION

In this paper, we have discussed the loss-delay queue wherein customers arriving in batches. We have dealt with a model which incorporated the realistic situations of unreliable server and second optional service. Making use of Laplace transform and probability generating function, the queue size distribution is explicitly derived. The sensitivity analysis done provides an insight to the decision makers to design and control sensitive descriptors for the improvement of the model.

The other noble feature of our investigation is the inclusion of balking behavior which makes our model more realistic to the present day scenario in computer, telecommunication, transport, manufacturing and many other congestion situations. The concept of setup of the repairmen is also seems to be more closer to real time system as before starting repair, the repairman needs some time in traveling, in fixing its tools, etc.

9. REFERENCES

1. Borthakur, A. and Medhi, J., "A queueing system with arrival and service in batches of variable size", ***Cahiers du. CERO***, Vol. 16, (1974), 117-126.
2. Chaudhry, M. and Templeton, J.G., "First course in bulk queues", (1983).
3. Lee, S.S., Lee, H.W., Park, J.O. and Chae, K.C., "Analysis of the mx/g/1 queue with n-policy and multiple vacations", ***Journal of Applied Probability***, Vol. 31, (1994), 476-496.
4. Kumar, B.K. and Madheswari, S.P., "M^ x/g/1 retrial queue with multiple vacations and starting failures", ***Opsearch-New Delhi-***, Vol. 40, No. 2, (2003), 115-137.
5. Madan, K.C. and Al-Rawwash, M., "On the m x/g/1 queue with feedback and optional server vacations based on a single vacation policy", ***Applied Mathematics and Computation***, Vol. 160, No. 3, (2005), 909-919.
6. Ke, J.-C. and Lin, C.-H., "Maximum entropy approach for batch-arrival queue under n policy with an un-reliable server and single vacation", ***Journal of Computational and Applied Mathematics***, Vol. 221, No. 1, (2008), 1-15.
7. Choudhury, G., Tadj, L. and Deka, K., "A batch arrival retrial queueing system with two phases of service and service interruption", ***Computers & Mathematics with Applications***, Vol. 59, No. 1, (2010), 437-450.
8. Jain, M. and Chauhan, D., "Working vacation queue with second optional service and unreliable server", ***International Journal of Engineering***, Vol. 25, No. 3, (2012), 223-230.

9. Economou, A. and Manou, A., "Strategic behavior in an observable fluid queue with an alternating service process", ***European Journal of Operational Research***, Vol. 254, No. 1, (2016), 148-160.

10. Avi-Itzhak, B. and Naor, P., "Some queuing problems with the service station subject to breakdown", ***Operations Research***, Vol. 11, No. 3, (1963), 303-320.

11. Neuts, M.F. and Lucantoni, D.M., "A markovian queue with n servers subject to breakdowns and repairs", ***Management Science***, Vol. 25, No. 9, (1979), 849-861.

12. Ke, J.-C., "The optimal control of an m/g/1 queueing system with server vacations, startup and breakdowns☆", ***Computers & Industrial Engineering***, Vol. 44, No. 4, (2003), 567-579.

13. Ke, J.-C., "Optimal nt policies for m/g/1 system with a startup and unreliable server", ***Computers & Industrial Engineering***, Vol. 50, No. 3, (2006), 248-262.

14. Ke, J.-C., "Batch arrival queues under vacation policies with server breakdowns and startup/closedown times", ***Applied Mathematical Modelling***, Vol. 31, No. 7, (2007), 1282-1292.

15. Choudhury, G. and Deka, K., "An m/g/1 retrial queueing system with two phases of service subject to the server breakdown and repair", ***Performance Evaluation***, Vol. 65, No. 10, (2008), 714-724.

16. Wang, T.-Y., Wang, K.-H. and Pearn, W.L., "Optimization of the t policy m/g/1 queue with server breakdowns and general startup times", ***Journal of Computational and Applied Mathematics***, Vol. 228, No. 1, (2009), 270-278.

17. Kalidass, K. and Kasturi, R., "A queue with working breakdowns", ***Computers & Industrial Engineering***, Vol. 63, No. 4, (2012), 779-783.

18. Yang, D.-Y. and Wu, C.-H., "Cost-minimization analysis of a working vacation queue with n-policy and server breakdowns", ***Computers & Industrial Engineering***, Vol. 82, (2015), 151-158.

19. Abou-El-Ata, M. and Hariri, A., "The m/m/c/n queue with balking and reneging", ***Computers & operations research***, Vol. 19, No. 8, (1992), 713-716.

20. Abou-El-Ata, M. and Shawky, A., "The single-server markovian overflow queue with balking, reneging and an additional server for longer queues", ***Microelectronics Reliability***, Vol. 32, No. 10, (1992), 1389-1394.

21. Thomo, L., "A multiple vacation model mx¦ g¦ 1 with balking", ***Nonlinear Analysis: Theory, Methods & Applications***, Vol. 30, No. 4, (1997), 2025-2030.

22. Jain, M., Sharma, P. and Sharma, P., "Controllable multi-server queue with balking", ***International Journal of Engineering***, Vol. 18, No. 3, (2005), 263-271.

23. Jain, M., "Finite capacity m/m/r queueing system with queue-dependent servers", ***Computers & Mathematics with Applications***, Vol. 50, No. 1-2, (2005), 187-199.

24. Cochran, J.K. and Broyles, J.R., "Developing nonlinear queuing regressions to increase emergency department patient safety: Approximating reneging with balking", ***Computers & Industrial Engineering***, Vol. 59, No. 3, (2010), 378-386.

25. Sharma, P., "Machine repair problem with spares, balking, reneging and n-policy for vacation".

26. Yu, S., Liu, Z. and Wu, J., "Equilibrium strategies of the unobservable m/m/1 queue with balking and delayed repairs", ***Applied Mathematics and Computation***, Vol. 290, (2016), 56-65.

27. Sharma, P., "Single unreliable server interdependent loss and delay queueing model with controllable arrival rate under n-policy", ***International Journal of Scientific and Research Publications***, Vol., No. 215.

28. Spicer, S. and Ziedins, I., "User-optimal state-dependent routeing in parallel tandem queues with loss", ***Journal of Applied Probability***, Vol. 43, No. 01, (2006), 274-281.

29. Fan, M., "A queueing model for mixed loss-delay systems with general interarrival processes for wide-band calls", ***European Journal of Operational Research***, Vol. 180, No. 3, (2007), 1201-1220.

30. Kim, C.S., Dudin, A., Klimenok, V. and Khramova, V., "Erlang loss queueing system with batch arrivals operating in a random environment", ***Computers & Operations Research***, Vol. 36, No. 3, (2009), 674-697.

31. Hernández-Orallo, E. and Vila-Carbo, J., "Network queue and loss analysis using histogram-based traffic models", ***Computer Communications***, Vol. 33, No. 2, (2010), 190-201.

32. Sharma, P., "Transient analysis of loss and delay bulk service markovian queue under n-policy", ***Journal of Rajasthan Academy of Physical Sciences***, Vol. 13, No. 3, (2014), 221-234.

33. Gupta, M.K., Hemachandra, N. and Venkateswaran, J., "Optimal revenue management in two class pre-emptive delay dependent markovian queues", ***Applied Mathematical Modelling***, (2016).

Unreliable Server $M^X/G/1$ Queue with Loss-delay, Balking and Second Optional Service

P. Sharma

Department of Mathematics, School of Science, Noida International University, Gautam Budh Nagar, Greater Noida, India

PAPER INFO

Paper history:
Received 20 July 2016
Received in revised form 26 January 2017
Accepted 30 January 2017

Keywords:
Bulk queue
Loss and delay
Essential service
Optional service
Supplementary variables
Queue size

چکیده

این بررسی با مدل صف بندی MX /G /1 با راه اندازی، ورود بالک، از دست دادن تاخیر و رد کردن سرو کار دارد. ارائه خدمات اختیاری دوم جدا از خدمات ضروری توسط یک سرور غیر قابل اعتماد در نظر گرفته شده است. ما فرض می کنیم که مشتریان تاخیری زمانی به صف بندی می پیوندند که سرور مشغول است در حالی که مشتریان از دست داده، از سیستم جدا می مانند. پس از دریافت خدمات ضروری، مشتریان ممکن است احتمالا سرویس اختیاری را انتخاب کنند و یا ممکن است سیستم را ترک کنند. سرور غیر قابل اعتماد هست و از این رو ممکن است در هر دو مورد خدمات ضروری و اختیاری با شکست مواجه شود و قبل از تعمیر نیاز به زمان راه اندازی دارد. خدمات در طول خدمات ضروری، زمان راه اندازی و زمان تعمیر برای هر دو مورد، به صورت کلی توزیع شده است در حالی که زمان سرویس در طول خدمت اختیاری نمایی است. با استفاده از روش متغیر تکمیلی، معادلات حاکم بر مدل ساخته می شوند. سپس، توزیع اندازه صف بندی حالت یکنواخت با استفاده از تبدیل لاپلاس و احتمالا تابع مولد به دست آمده است. نتایج عددی برای شاخص عملکرد های مختلف به دست آمده و به صورت گرافیکی نشان داده شده است.

***doi**: 10.5829/idosi.ije.2017.30.02b.11*