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A Self-starting Control Chart for Simultaneous Monitoring of Mean and Variance of Simple Linear Profiles

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ABSTRACT

In many processes in real practice at the start-up stages the process parameters are not known a priori and there are no initial samples or data for executing Phase I monitoring and estimating the process parameters. In addition, the practitioners are interested in using one control chart instead of two or more for monitoring location and variability of processes. In this paper, we consider a simple linear profile in which the relationship between a response variable and one explanatory characterizes the quality of a process. We proposed a self-starting Max-CUSUM control chart based on recursive residuals to monitor mean vector (including intercept and slope) and variability (variance of error term) of a simple linear profile simultaneously from the start-up stages of the process. We developed Max-CUSUM control chart to monitor simple linear profile in Phase II. Then, we compared our proposed control charts with the best one in the literature through simulation studies. The simulation results showed that our proposed control charts have better performance compared to competitive control charts under moderate and large shifts in terms of out-of-control (OC) ARLs. Finally, the application of the proposed self-starting control chart is illustrated through a real case in the leather industry.

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1. INTRODUCTION

Statistical process control (SPC) has been widely used to monitor industrial processes in which control charts are the most important tools. In ordinary processes, there are always one or more quality characteristics which should be monitored over time. However, in some cases the quality of a process is characterized by a relationship between a response variable and one or more explanatory variables referred to as profile in the literature. There are many researches on monitoring profiles, especially simple linear profiles that we will mention here. Kang and Albin [1] developed two control charts to monitor simple linear profiles in Phase II. Kim et al. [2] proposed three exponentially weighted moving average (EWMA) control charts which monitor intercept, slope and standard deviation in simple linear profiles. Zhang et al. [3] proposed a control chart based on likelihood ratio (LR) to monitor simple linear profiles. Niaki et al. [4] proposed a control chart based on the generalized linear test (GLT) to monitor coefficients of a simple linear profile and an R-chart to monitor the error variance. Saghaei et al. [5] applied cumulative sum (CUSUM) control charts to monitor simple linear profiles in Phase II. Khedmati and Niaki [6] proposed an approach to monitor simple linear profile in multistage processes. In their proposed approach, all parameters in all stages are simultaneously monitored by one statistic at a time. The proposed approach can identify the out-of-control stages and parameters as well. Khedmati and Niaki [7] also proposed a new control scheme for Phase II monitoring of simple linear profiles in multistage processes which is based on U transformation applied to remove the effect of the cascade property. Gupta et al. [8] compared the performance of two monitoring schemes for Phase II monitoring of simple linear profiles. Kazemzadeh et al. [9] proposed variable sampling interval (VSS) schemes to monitor simple linear profiles in Phase II. De Magalhaes et al. [10] proposed a model for the statistical design of a VSS chi-square control chart to

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monitor linear profiles. Mahmoud & Woodall [11] investigated the Phase I analysis of data in simple linear profiles.

In the literature of SPC, Phase I and Phase II need to be distinguished. In Phase II monitoring, the process parameters are presumed to be known. Before Phase II monitoring, we need to analyze Phase I to ensure that the process is statistically in-control (IC) and to estimate process parameters. However, there is not always enough data to perform Phase I analysis and estimate the parameters. The self-starting control charts start monitoring the process without the need for large amount of preliminary observations. These control charts are used when production process is slow or the cost of out-of-control production at the beginning of the process is high. Self-starting method updates the parameter estimates with each new observation and simultaneously checks for out-of-control condition. Hawkins [12] proposed a self-starting cumulative sum (CUSUM) control chart for location and scale parameters, by using some theoretical properties of residuals independency. Sullivan and Jones [13] considered a self-starting control chart for monitoring multivariate individual observations. The proposed control chart uses the deviation of each observation vector from the average of all previous observations. Li et al. [14] proposed a self-starting control chart to monitor process mean and variance simultaneously based on likelihood ratio test (LRT) method and EWMA procedure. Cappizi and Masarotto [15] suggested a self-starting control chart which uses sequential observations to both update the parameter estimates and check for OC condition. In fact, they introduced a charting procedure that updates the reference pattern of a cumulative score (CUSCORE) control chart using an adaptive EWMA. Li et al. [16] proposed a self-starting control chart for monitoring high-dimensional short run processes. The proposed control chart solved a key challenge about traditional Hotelling's T^2 chart with high dimensionality measurements. The problem was that monitoring could not begin until the number of observations exceeds the dimensionality of the measurement.

In addition to problems noted above about the lack of sufficient initial samples and unknown parameters, practitioners are interested in developing some kinds of control charts which can simultaneously monitor processes mean and variability. In fact, the quality engineers are interested in having a single control chart instead of two or more. Zhang et al. [17] proposed a single control chart based on the combination of EWMA procedure and generalized likelihood ratio (GLR) test statistic for joint monitoring of both the process mean and variance. Zhang et al. [18] suggested a new single control chart based on the combination of EWMA control chart and the GLR test for joint monitoring of multivariate process mean and variability. Sheu et al. [19] proposed maximum chi-square generally weighted moving average (MCSGWMA) control chart based on the combination of two generally weighted moving average (GWMA) control charts into a single one. Ghashghaei et al. [20] investigated the effect of measurement errors on joint monitoring of process mean and variance when simple random sampling (SRS) and ranked set sampling (RSS) procedures are used in the process. Maleki et al. [21] proposed a new control chart for simultaneous monitoring of multivariate process mean vector and covariance matrix in the presence of measurement errors with linearly increasing variance under additive covariate model.

This paper is motivated from the research work of Zou et al. [22]. They proposed a self-starting control chart based on recursive residuals to monitor simple linear profiles. The proposed control chart can detect shifts in the mean vector of a simple linear profile (intercept and slope), and/or the variability of a simple linear profile (error term variance). The aim of this paper is developing a Max-CUSUM control chart for simultaneous monitoring of the regression parameters and error variance of simple linear profile in Phase II as well as a self-starting Max-CUSUM (SSMax-CUSUM) control chart for simultaneous monitoring of mean and variance of a simple linear profile. The proposed control charts have the identification feature of determining the out-of-control source of variation as well. The performance of the proposed control charts is evaluated in terms of ARL criterion and compared with SS control chart proposed by Zou et al. [22].

The remainder of this paper is organized as follows: in the next section, we present a brief introduction of simple linear profiles. In Section 3, we present our proposed control charts, Max-CUSUM and SSMax-CUSUM for monitoring simple linear profiles. The simulation studies, results and comparisons are presented in Section 4. In Section 5, we apply the SSMax-CUSUM control chart in a real world example. The concluding remarks are given in the final Section.

2. SIMPLE LINEAR PROFILE MODEL

If (x_i, y_{ij}) is the jth random sample observed over the time, then when the process is in-control, the relationship between response variable y_{ij} and the explanatory variable x_i is presumed to be as follows:

$$y_{ij} = B_0 + B_1 x_i + \varepsilon_{ij}, \quad ; \quad i = 1, 2, ..., n \ , \tag{1}$$

where ε_{ij} is an independent and identically distributed (I.I.D) standard normal random variable. The regression parameters B_0 , B_1 and σ^2 in jth profile are estimated by b_{0j} , b_{1j} and MSE_j , respectively:

$$b_{1j} = \frac{S_{xy(j)}}{S_{xx}}, \tag{2}$$

$$b_{0j} = \bar{y}_j - b_{1j}\bar{x}, \tag{3}$$

$$MSE_j = \frac{1}{n-2}\sum_{i=1}^{n}(y_{ij} - b_{1j}x_i - b_{0j})^2, \tag{4}$$

where $\bar{y}_j = (1/n)\sum_{i=1}^{n} y_{ij}$, $\bar{x} = (1/n)\sum_{i=1}^{n} x_i$, $S_{xx} = \sum_{i=1}^{n}(x_{ij} - \bar{x})^2$, and $S_{xy}(j) = \sum_{i=1}^{n}(x_i - \bar{x})y_{ij}$.

3. PROPOSED CONTROL CHARTS

According to the previous sections and the aim of this paper, in this section we develop control charts for simultaneous monitoring of mean and variance of simple linear profiles based on the condition that the profile parameters are known and unknown. Then, in order to check the constancy of the regression relationship over time, we should do the following hypothesis tests in the proposed control charts:

$$\begin{cases} H_0 : \delta_0 = 0,\ \delta_1 = 0,\ \gamma = 1 \\ H_1 : \text{otherwise} \end{cases}, \tag{5}$$

where δ_0, δ_1 and γ are shifts values which may occur in intercept, slope and standard deviation of the simple linear regression model, respectively.

Since the importance of joint monitoring in practical applications in industries is clear, so the engineers of quality control (QC) department of a manufacturing unit prefer to have only one control chart instead of two or more for monitoring the process. Cheng and Thaga [23] proposed a new kind of CUSUM procedure called Max-CUSUM chart for simultaneous monitoring of mean and variance of a univariate process. In this section, we use the Max-CUSUM procedure to monitor simple linear profile in Phase II and under the condition that the parameters are unknown a priori. In this situation that there is no initial data and information about the process, according to the literature, we should use self-starting procedure to monitor simple linear profiles.

3. 1. Max-CUSUM Control Chart

Let $e_j = (e_{j1}, e_{j2}, ..., e_{jn})$; $j = 1, 2, 3, ...$ denote the sequence of residuals of size n. Residual is the difference between the observed value of the response variable (y_{ij}) and the corresponding predicted value $(\hat{y}_{ij})$ as given in Equation (6). The residuals are independent and identically distributed and follow normal distribution with mean 0 and variance σ_ε^2. Here, we use Max-CUSUM control chart and accommodate it with the simple linear profile structure to monitor residuals.

$$e_{ij} = y_{ij} - B_0 - B_1 x_i \quad ; \quad \begin{array}{l} i = 1, 2, ..., n \\ j = 1, 2, 3.... \end{array} \tag{6}$$

In order to monitor residuals, we need $\bar{e}_j = (1/n)\sum_{i=1}^{n} e_{ij}$ and $S_{e_j}^2 = \frac{1}{n-1}\sum_{i=1}^{n}(e_{ij} - \bar{e}_j)^2$ as mean and variance of jth sample, respectively. These estimators are unbiased and independent, and also follow different distributions. Equations (7) and (8) are used to transform the distribution of the mentioned statistics, $\bar{e}_j$ and $S_{e_j}^2$, to standard normal distribution, respectively.

$$Z_j = \frac{\sqrt{n}\,(\bar{e}_j)}{\sigma_e}, \tag{7}$$

$$F_j = \Phi^{-1}\left\{ H\left[\frac{(n-1)S_{e_j}^2}{\sigma_{e_j}^2}; n-1 \right] \right\}, \tag{8}$$

where Z_j has standard normal distribution and $H[X;\upsilon] = P(X \le x)$ is chi-square cumulative distribution function of X with υ degrees of freedom, Φ^{-1} is the inverse of the standard normal cumulative distribution function. Now Z_j and F_j have the same distribution, so the Max-CUSUM control chart can be applied. At first, we apply the traditional CUSUM statistics as follows:

$$C_j^+ = \max(0, Z_j - k_1 + C_{j-1}^+), \tag{9}$$

$$C_j^- = \max(0, -Z_j - k_1 + C_{j-1}^-), \tag{10}$$

and

$$S_j^+ = \max(0, F_j - k_2 + S_{j-1}^+), \tag{11}$$

$$S_j^- = \max(0, -F_j - k_2 + S_{j-1}^-), \tag{12}$$

where $C_0 = 0$ and $S_0 = 0$ are starting points and k_1 and k_2 are reference values for CUSUM control charts. Combining C_j and S_j defines statistic for a single control chart as:

$$M_j = Max\left\{ C_j^+, C_j^-, S_j^+, S_j^- \right\}. \tag{13}$$

Since M_j is the maximum of C_j^+, C_j^-, S_j^+ and S_j^- which are based on four cumulative sum (CUSUM) statistics, it is natural to name the control chart based on M_j, Max- CUSUM control chart. A large value of M_j means that mean and/or variance of the processes have shifted. Because M_j is non-negative, only an upper control limit (*UCL*) is used for monitoring purposes. If

$M_j > UCL$, the control chart triggers an out-of-control alarm, where $UCL > 0$ is chosen to achieve a specified in-control ARL.

3. 2. Self-Starting Max-CUSUM Control Chart

Recursive residuals first were applied by Brown et al. [24] to test the constancy of regression relationships over time. Also, it has been shown that the recursive residuals are useful in a variety of applications in linear models. The use of these kinds of residuals in data analysis is attractive when a relevant ordering of the observations exists. Recursive residuals at first were proposed for data which has natural ordering such as time series data, but they have been effectively used in tests for structural change (i.e., change in regression coefficients), serial correlation, heteroscedasticity, and functional misspecification.

In this section, we use recursive residuals to design a self-starting control chart. As noted above, self-starting control charts uses moving statistics which can update the estimations of parameters during the process while it is checking the out-of-control state simultaneously. Now suppose that there are $m-1$ IC historical data and $m, m+1, \ldots$ future sample of size n. If we pool all the $m-1$ IC historical and $m, m+1, \ldots$ future data in one sample, i.e.

$\{(x_i, y_{ij}), i = 1,2,\ldots,n, j = 1,2,\ldots,m-1,m,m+1\ldots\}$, then we can calculate the standardized recursive residuals for each future sample as Zou et al. [22] used in their study.

$$e_{ij} = \left(y_{(j-1)n+i} - \mathbf{z}' \beta_{(j-1)n+i-1}\right) \div \left[S_{(j-1)n+i-1} \times \left(1 + \mathbf{z}_i' \left(\mathbf{X}_{(j-1)n+i-1}' \mathbf{X}_{(j-1)n+i-1}\right)^{-1} \mathbf{z}_i\right)\right]^{1/2} \tag{14}$$

$i = 1,2,\ldots,n \quad , \quad j = m, m+1, \ldots$

where

$$\mathbf{z}'_i = (1, x_i), \tag{15}$$

$$\mathbf{y}'_{(j-1)n+i-1} = (y_1, y_2, y_3, \ldots, y_{(j-1)n+i-1}), \tag{16}$$

$$\mathbf{X}'_{(j-1)n+i-1} = \overbrace{(z_1, z_2, \ldots, z_n, z_1, z_2, \ldots, z_n, \ldots, z_1, z_2, \ldots, z_{i-1})}^{(j-1)\times n} \tag{17}$$

$$\boldsymbol{\beta}_t = (\mathbf{X}'_t \mathbf{X}_t)^{-1} \mathbf{X}'_t \mathbf{y}_t, \tag{18}$$

$$S_t = \frac{1}{t-2} (\mathbf{y}_t - \mathbf{X}_t \boldsymbol{\beta}_t)'(\mathbf{y}_t - \mathbf{X}_t \boldsymbol{\beta}_t) \cdot \tag{19}$$

And for simplicity let $y_{(j-1)n+i} = y_{ij}$, $i = 1,2,\ldots,n$, $j = 1,2,\ldots$.

As mentioned above, the recursive residual is a moving equation. The e_{ij} value of each observation depends on estimated regression parameters ($\boldsymbol{\beta}$), standard deviation (S) and the $\mathbf{X}'\mathbf{X}$ value of previous observations, so to avoid the high volume of calculations to reach each observation's e_{ij}, Zou et al. [22] used the following recursive formulas:

$$(\mathbf{X}_t' \mathbf{X}_t)^{-1} = (\mathbf{X}_{t-1}' \mathbf{X}_{t-1})^{-1} - \frac{(\mathbf{X}_{t-1}' \mathbf{X}_{t-1})^{-1} \mathbf{z}_i \mathbf{z}_i' (\mathbf{X}_{t-1}' \mathbf{X}_{t-1})^{-1}}{1 + \mathbf{z}_i' (\mathbf{X}_{t-1}' \mathbf{X}_{t-1})^{-1} \mathbf{z}_i}, \tag{20}$$

$$\boldsymbol{\beta}_t = \boldsymbol{\beta}_{t-1} + (\mathbf{X}_{t-1}' \mathbf{X}_{t-1})^{-1} \mathbf{z}_i (\mathbf{y}_t - \mathbf{z}_i' \boldsymbol{\beta}_{t-1}), \tag{21}$$

$$S_t^2 = \frac{(t-3) S_{t-1}^2 + (e_{ij})^2}{t-2}, \tag{22}$$

where $t = (j-1)n + i$.

Brown et al. [24] showed that under the in-control linear model, e_{ij} has *Student-t* distribution with $(j-1)n + i - 3$ degrees of freedom. Also, it is proved by Lehmann et al. [25] that the e_{ij}'s are statistically independent. Hence, by using a transformation, the Q_{ij} statistic which is called *Q*-statistic by Quesenberry [26] is obtained as follows:

$$Q_{ij} = \Phi^{-1}\left[T_{(j-1)n+i-3}(e_{ij})\right] \tag{23}$$

where Φ^{-1} denotes the inverse of CDF of standard normal random variable, $T_\nu(.)$ is the CDF of the *Student-t* distribution with ν degrees of freedom. So, $\{Q_{ij}, i = 1,2,\ldots,n, \; j = 1,2,\ldots,m-1,m,m+1\ldots\}$ is a sequence of random variables which are independent and follow standard normal distribution.

When an assignable cause occurs after some subgroups (τ subgroups), the distribution of *Q*-statistics when $j = \tau+1, \tau+2, \ldots$ is different from their distribution when $j = 1,2,\ldots,\tau$. This difference is used to detect assignable causes in the process.

For sequence of Q_{ij}'s in each sample (jth sample), $\bar{Q}_j$ and $S^2_{Q_j}$ are obtained as follows:

$$\bar{Q}_j = (1/n) \sum_{i=1}^{n} Q_{ij} \tag{24}$$

$$S^2_{Q_j} = [1/(n-1)] \sum_{i=1}^{n} \left(Q_{ij} - \bar{Q}_j\right)^2 \tag{25}$$

$$G_j = \Phi^{-1}\left\{ H\left[\frac{(n-1) S^2_{Q_j}}{\sigma^2_{Q_j}}; n-1\right]\right\}. \tag{26}$$

Now similar to the previous subsection, we have the same situation and should transform the distribution of $\bar{Q}_j$ and $S^2_{Q_j}$ to standard normal distribution and then apply the Max-CUSUM control chart.

$$U_j^+ = \max(0, \sqrt{n}\bar{Q}_j - k_1 + U_{j-1}^+), \tag{27}$$

$$U_j^- = \max(0, -\sqrt{n}\bar{Q}_j - k_1 + U_{j-1}^-), \tag{28}$$

and

$$V_j^+ = \max(0, G_j - k_2 + V_{j-1}^+), \tag{29}$$

$$V_j^- = \max(0, -G_j - k_2 + V_{j-1}^-), \tag{30}$$

where $U_0 = 0$ and $V_0 = 0$ are starting points and k_1 and k_2 are reference values. Hence, the final statistics is obtained as follows:

$$M_j = \text{Max}\left\{U_j^+, U_j^-, V_j^+, V_j^-\right\}. \tag{31}$$

For some given IC ARL and sample size $n = 3,4,...,10$, the control limits for self-starting Max-CUSUM chart are tabulated in Table 1.

3. 3. Diagnostic Procedure As a diagnosing procedure for self-starting Max-CUSUM control chart, the following algorithm is proposed to determine the source and the direction of the shift:

Case 1: if $M_j = |U_j| > \text{UCL}$ and $|V_j| \le \text{UCL}$, then we have a shift in mean of the process. The shift is increasing if $U_j > 0$ and it is decreasing if $U_j < 0$.

Case 2: if $|U_j| \le \text{UCL}$ and $M_j = |V_j| > \text{UCL}$, then a shift has been only occuring in variability. The shift is increasing if $V_j > 0$ and it is decreasing if $V_j < 0$.

Case 3: if $|U_j|$ and $|V_j|$ is greater than UCL, then the signal has occurred due to simultaneous changes in process mean and variance. The change direction in location and scale of the process is determined by aforesaid methods in Cases 1 and 2.

TABLE 1. Control limits of SSMax-CUSUM chart when $k_1 = 1$ and $k_2 = 1.5$

	IC ARL			
n	**100**	**200**	**300**	**370**
3	1.575	1.889	2.085	2.189
4	1.585	1.898	2.095	2.197
5	1.594	1.908	2.106	2.207
6	1.602	1.917	2.115	2.207
7	1.604	1.918	2.210	2.208
8	1.603	1.917	2.201	2.209
9	1.601	1.916	2.192	2.207
10	1.598	1.913	2.117	2.212

4. PERFORMANCE EVALUATION AND COMPARISONS

In this section, we evaluate the performance of the proposed control charts including Max-CUSUM and self-starting Max-CUSUM and compare them with the performance of self-starting (SS) control chart proposed by Zou et al. [22]. Hence, we assess the OC ARL performance of the self-starting Max-CUSUM control chart under different values of τ for IC samples before a shift occurs. In this study, IC samples are including historical samples and future in-control samples (before shift).

Tables 2, 3 and 4 represent the results of simulation studies on the performance of the proposed control charts. The IC ARL in these tables is equal to 200. Similar to the work of Kang and Albin [1], the parameters of simple linear profile is $B_0 = 3$, $B_1 = 2$, $\sigma^2 = 1$ and $x_i = 2,4,6,8$. The control limits are set equal to 1.898 and 1.925 for self-starting Max-CUSUM and Max-CUSUM, respectively and the smoothing constant λ is equal to 0.2. The results are obtained by 10,000 simulation runs. Also, the OC ARLs of self-starting Max-CUSUM chart with $\tau = 3, 20, 50, 100, 300 \text{ and } 500$ are tabulated in aforesaid tables. Moreover, in all simulation runs, the reference value k_1 is equal to 1 (half of shift interval in the intercept) and k_2 is equal 1.5 (half of shift interval in the variance).

The results in Tables 2, 3 and 4 showed that the self-starting Max-CUSUM control chart performs almost better than SS control chart under moderate and large shifts in intercept, slope and variance. Since the self-starting Max-CUSUM control chart uses a recursive statistic, as the number of reference samples observed before occurring an increasing shift, the performance of the self-starting Max-CUSUM chart improves in detecting small shifts. The results showed that when $\tau = 500$, the performance of self-starting Max-CUSUM control chart is almost similar to the performance of Max-CUSUM chart which is designed in Phase II and better than SS control chart.

In order to better assess the performance of the self-starting Max-CUSUM control chart against SS control chart, we consider the change point $\tau = 3$ which is also tabulated in Tables 2, 3 and 4. The results also represent that the self-starting Max-CUSUM control chart performs better than SS chart under shifts in intercept and slope. However, the SS control chart performs better than the self-starting Max-CUSUM under shift in standard deviation. Table 5 represents OC ARLs for self-starting Max-CUSUM, Max-CUSUM and SS control charts in the situation that simultaneous shifts occur in intercept and variance of the considered linear profile $y_{ij} = 3 + 2x_i + \varepsilon_{ij}$.

TABLE 2. Out-of-control ARLs comparisons of SSMax-CUSUM, SS charts with different values of τ and Max-CUSUM control charts under shifts from β_0 to $\beta_0+\delta_0\sigma$

δ_0	**Charts**	$\tau=3$	$\tau=20$	$\tau=50$	$\tau=100$	$\tau=300$	$\tau=500$	**Max-CUSUM**
0.2	SSMax-CUSUM	**185.83**	**133.66**	**100.52**	**76.84**	**57.49**	**52.62**	48.66
	SS	215.75	161.2	125.4	94.8	59.9	53.8	
0.4	SSMax-CUSUM	**166.30**	**68.60**	33.79	21.70	15.86	15.33	14.58
	SS	199.27	80.2	**30.0**	**18.2**	**14.6**	**14.1**	
0.6	SSMax-CUSUM	**141.96**	23.25	9.45	**7.40**	**6.74**	**6.67**	6.62
	SS	176.06	**22.6**	**8.7**	7.7	7.1	7.1	
0.8	SSMax-CUSUM	**111.54**	7.56	**4.31**	**4.02**	**3.89**	**3.85**	3.84
	SS	140.81	**7.1**	5.1	4.9	4.7	4.6	
1.0	SSMax-CUSUM	**78.69**	**3.31**	**2.82**	**2.73**	**2.66**	**2.65**	2.66
	SS	98.85	4.3	3.7	3.6	3.5	3.5	
1.2	SSMax-CUSUM	**49.13**	**2.31**	**2.10**	**2.08**	**2.04**	**2.03**	2.04
	SS	63.86	3.2	3.0	2.9	2.8	2.8	
1.4	SSMax-CUSUM	**25.02**	**1.80**	**1.73**	**1.68**	**1.68**	**1.67**	1.67
	SS	34.39	2.7	2.5	2.5	2.4	2.4	
1.6	SSMax-CUSUM	**9.92**	**1.52**	**1.46**	**1.44**	**1.42**	**1.43**	1.42
	SS	15.58	2.3	2.2	2.2	2.1	2.1	
1.8	SSMax-CUSUM	**4.70**	**1.33**	**1.26**	**1.25**	**1.24**	**1.25**	1.25
	SS	7.08	2.1	2.0	2.0	1.9	1.9	
2	SSMax-CUSUM	**2.38**	**1.18**	**1.15**	**1.14**	**1.13**	**1.13**	1.14
	SS	3.91	1.9	1.8	1.8	1.8	1.8	

TABLE 3. Out-of-control ARLs comparisons of SSMax-CUSUM, SS charts with different values of τ and Max-CUSUM control charts under shifts from β_1 to $\beta_1+\delta_1\sigma$

δ_1	**Charts**	$\tau=3$	$\tau=20$	$\tau=50$	$\tau=100$	$\tau=300$	$\tau=500$	**Max-CUSUM**
0.025	SSMax-CUSUM	**191.43**	**158.31**	**133.12**	**111.00**	**93.64**	**85.33**	81.10
	SS	218.75	181.7	166.9	152.3	117.3	108.3	
0.05	SSMax-CUSUM	**181.03**	**117.11**	**78.43**	**58.11**	39.03	37.34	35.13
	SS	209.33	144.9	95.4	62.6	**38.7**	**34.5**	
0.075	SSMax-CUSUM	**169.60**	**78.07**	37.58	26.07	18.92	18.19	16.98
	SS	204.49	92.4	**37.3**	**22.0**	**16.6**	**15.8**	
0.1	SSMax-CUSUM	**154.90**	**45.17**	16.96	12.16	10.02	9.68	9.25
	SS	188.59	46.6	**14.5**	**10.7**	**9.6**	**9.4**	
0.125	SSMax-CUSUM	**139.83**	22.81	**5.22**	**6.78**	**6.21**	**6.11**	6.05
	SS	172.82	**19.7**	7.9	7.1	6.7	6.6	
0.15	SSMax-CUSUM	**123.06**	9.84	**5.11**	**4.61**	**4.34**	**4.34**	4.30
	SS	150.47	**9.0**	5.6	5.3	5.1	5.1	
0.175	SSMax-CUSUM	**98.59**	**5.17**	**3.59**	**3.46**	**3.31**	**3.30**	3.27
	SS	121.32	5.8	4.5	4.3	4.1	4.1	
0.2	SSMax-CUSUM	**76.61**	**3.43**	**2.85**	**2.74**	**2.63**	**2.34**	2.65
	SS	99.76	4.3	3.7	3.6	3.5	3.5	
0.225	SSMax-CUSUM	**59.47**	**2.69**	**2.32**	**2.26**	**2.22**	**2.22**	2.23
	SS	76.96	3.6	3.2	3.1	3.0	3.0	
0.25	SSMax-CUSUM	**42.63**	**2.17**	**2.02**	**1.96**	**1.94**	**1.94**	1.91
	SS	59.15	3.1	2.8	2.8	2.7	2.7	

TABLE 4. Out-of-control ARLs comparisons of SSMax-CUSUM, SS charts with different values of τ and Max-CUSUM control chart under shifts from σ to $\gamma\sigma$

γ	Charts	$\tau=3$	$\tau=20$	$\tau=50$	$\tau=100$	$\tau=300$	$\tau=500$	Max-CUSUM
1.2	SSMax-CUSUM	92.35	**79.18**	**64.77**	59.46	48.12	42.95	41.81
	SS	80.52	116.5	73.3	**49.0**	**33.0**	**31.2**	
1.4	SSMax-CUSUM	50.47	**32.16**	22.36	17.88	14.75	14.03	13.64
	SS	37.16	49.0	**18.5**	**12.1**	**10.3**	**9.9**	
1.6	SSMax-CUSUM	30.66	**13.79**	9.12	7.73	6.86	6.71	6.65
	SS	21.94	18.1	**7.1**	**6.1**	**5.6**	**5.5**	
1.8	SSMax-CUSUM	20.11	**7.00**	5.14	4.46	4.19	4.17	4.20
	SS	13.86	7.4	**4.4**	**4.0**	**3.8**	**3.8**	
2	SSMax-CUSUM	14.25	**4.37**	3.45	3.19	3.01	3.00	3.02
	SS	9.93	4.3	**3.3**	**3.0**	**2.9**	**2.9**	
2.2	SSMax-CUSUM	10.24	**3.15**	**2.61**	**2.48**	**2.39**	**2.38**	2.37
	SS	7.68	3.1	2.6	2.5	2.4	2.4	
2.4	SSMax-CUSUM	7.83	**2.49**	**2.19**	**2.09**	**2.06**	**2.03**	2.03
	SS	6.39	2.5	2.2	2.1	2.1	2.1	
2.6	SSMax-CUSUM	6.04	**2.09**	**1.89**	**1.80**	**1.78**	**1.77**	1.79
	SS	5.14	2.2	1.9	1.9	1.8	1.8	
2.8	SSMax-CUSUM	4.97	**1.83**	**1.72**	**1.65**	**1.64**	**1.63**	1.64
	SS	4.49	1.9	1.7	1.7	1.7	1.6	
3	SSMax-CUSUM	4.13	**1.68**	**1.55**	**1.54**	**1.47**	**1.45**	1.51
	SS	3.91	1.7	1.6	1.6	1.5	1.5	

TABLE 5. The simulated out-of-control ARL values for SSMax-CUSUM (with $\tau=20$), Max-CUSUM and SS (with $\tau=20$) control charts under simultaneous shifts in intercept from β_0 to $\beta_0+\delta_0\sigma$ and standard deviation from σ to $\gamma\sigma$ (in-control ARL=200)

γ	Control Charts	δ_0									
		0.1	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1**
1.1	SSMC	102.53	78.80	62.03	42.74	26.58	15.89	9.80	6.22	4.13	3.27
	MC	49.96	30.64	18.44	12.08	8.58	6.10	4.73	3.78	3.11	2.62
	SS	111.97	94.00	73.70	48.18	29.38	15.49	9.04	6.54	5.08	4.26
1.2	SSMC	65.43	52.43	38.95	27.95	19.52	12.53	7.94	5.75	4.15	3.34
	MC	28.15	20.23	13.62	10.16	7.40	5.49	4.38	3.58	3.07	2.62
	SS	60.68	53.32	43.34	29.51	19.20	12.39	8.59	6.33	4.96	4.24
1.3	SSMC	41.85	34.27	26.49	19.39	13.67	9.60	7.00	5.14	3.92	3.15
	MC	17.65	13.55	10.26	8.00	6.17	5.02	4.16	3.41	2.97	2.61
	SS	33.68	30.41	24.77	18.60	14.15	9.84	7.37	6.06	4.92	4.22
1.4	SSMC	28.40	22.63	17.50	13.65	10.00	7.64	5.97	4.57	3.73	3.14
	MC	11.60	9.38	7.90	6.34	5.31	4.43	3.78	3.29	2.87	2.53
	SS	20.14	18.10	15.65	13.08	10.43	8.20	6.68	5.62	4.85	4.12
1.5	SSMC	18.52	15.14	12.44	9.74	7.81	6.39	5.02	4.18	3.47	2.97
	MC	8.15	7.03	6.16	5.28	4.61	3.95	3.52	3.02	2.75	2.46
	SS	12.76	12.10	10.83	9.20	8.23	6.74	5.97	5.02	4.46	3.91

The IC ARL is set equal to be 200 and the number of IC samples for SSMax-CUSUM and Max-CUSUM control chart (τ) is equal to 20. The range of shifts for intercept and variance are 0.1 to 1 and 1.1 to 1.5 with step size of 0.1, respectively. Since the Max-CUSUM control chart applied in Phase II, as a result, it totally performs better than two other control charts. However, the proposed self-starting Max-CUSUM control chart almost roughly performs better than SS chart.

The Max charts can diagnose the parameters responsible for out-of-control signal (shift in mean, variance or both). In Table 6, there are three rows named U, V and UV that represent the mean shifts, variance shifts and simultaneous shifts, respectively. Table 6 presents the percentage of times that the self-starting Max-CUSUM chart diagnose the parameter responsible for signal when the shift actually occurs in that parameter. For example, when there is a 0.125 shift of size in the slope, the self-starting Max-CUSUM control chart diagnoses that in 98.94%, 1.04% and 0.02% of the times, shift has occurred in the regression parameters, error variance and both, respectively. The results showed that the performance of diagnosing procedure in the self-starting Max-CUSUM control chart under different shifts in intercept, slope and standard deviation is excellent.

5. AN ILLUSTRATIVE EXAMPLE

In this section, we adopt a case study from leather industry described by Amiri et al. [27]. As we know leather is an important and useful material for production of shoes. The importance of leather is due to its effect on customer satisfaction and their feet comfort in the shoes. Leather industry has numerous processes if a high quality leather is to be produced. The process investigated by Amiri et al. [27] is dying process.

When the feet are in shoes, the temperature rises and the feet begin sweating. The result of this sweating is that the color of the leather stains the socks and makes them dirty. Therefore, for this reason, the dyeing process is important. In the dyeing process, there is a relationship between color effluent and temperature which is the most important quality characteristic in this process and should be monitored over time. For in-control situation the profiles are similar, but when an assignable cause occurs we have OC profiles, so the relationship between color effluent and temperature is not stable over time.

The experiment and the corresponding data gathering are described in detail in Amiri et al. [27]. We use the outputs of their experiment in our study. There are 11 in-control profiles which are observed from laboratory. The results showed that there is a simple linear regression model between color effluent and temperature.

The IC simple linear regression profile is $y_{ij} = -0.0509 + 0.0034x_i + \varepsilon_{ij}$, $x_i = 25, 32, 39, 46, 53$ and ε_{ij} 's are identically independent distributed standard normal random variable. For given IC ARL=200, the UCL value is set to be 0.263.

TABLE 6. The performance of diagnosing procedure in SSMax-CUSUM (accuracy percent) under individual shifts in the intercept, slope and the standard deviation with $\tau = 20$ (in-control ARL=200)

Intercept shifts from β_0 to $\beta_0 + \delta_0\sigma$										
δ_0	**0.2**	**0.4**	**0.6**	**0.8**	**1**	**1.2**	**1.4**	**1.6**	**1.8**	**2**
U	93.22%	97.04%	99.36%	99.90%	99.96%	99.96%	100%	100%	100%	100%
V	6.78%	2.96%	0.64%	0.1%	0.04%	0.02%	0%	0%	0%	0%
UV	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
Slope shifts from β_1 to $\beta_1 + \delta_1\sigma$										
δ_1	**0.025**	**0.05**	**0.075**	**0.1**	**0.125**	**0.15**	**0.175**	**0.2**	**0.225**	**0.25**
U	92.68%	94.28%	96.40%	98.04%	98.94%	99.54%	99.74%	99.72%	99.74%	99.78%
V	7.32%	5.72%	3.58%	1.92%	1.04%	0.44%	0.26%	0.24%	0.14%	0.16%
UV	0%	0%	0.02%	0.04%	0.02%	0.02%	0%	0.04%	0.02%	0.02%
Standard deviation shifts from σ to $\gamma\sigma$										
γ	**1.2**	**1.4**	**1.6**	**1.8**	**2**	**2.2**	**2.4**	**2.6**	**2.8**	**3**
U	76.92%	58.26%	42.58%	31.32%	25.68%	19.86%	18.28%	15.68%	13.80%	13.80%
V	22.82%	41.08%	55.82%	66.34%	70.10%	74.26%	74.06%	75.34%	76.26%	74.94%
UV	0.26%	0.66%	1.60%	2.34%	4.22%	5.88%	7.66%	8.98%	9.94%	11.26%

The IC profiles presented in Table 7 are used as historical data to estimate the initial values of B_0, B_1 and variance used in self-starting procedure of Max-CUSUM control chart. We generate 18 samples of size $n=5$ with a sustained shift in the intercept of the simple linear profile from $B_0=-0.0509$ to $B_0=-0.0489$ in sample 10 and compute the statistic values of the proposed self-starting control chart. The statistics of SSMax-CUSUM related to 18 samples are tabulated in Table 8. The results showed that the self-starting Max-CUSUM control chart detects the shift in 14th sample which shows the satisfactory performance of the proposed self-starting Max-CUSUM control chart.

TABLE 7. In-control dataset of the leather color effluent under 5 different temperatures

Temp / profile	25	32	39	46	53
1	0.0218	0.0288	0.0908	0.1011	0.1257
2	0.0302	0.0542	0.0718	0.1172	0.1313
3	0.0288	0.0287	0.0858	0.0931	0.1355
4	0.0306	0.0757	0.0101	0.1162	0.1285
5	0.0488	0.0281	0.0855	0.1181	0.1188
6	0.0310	0.0944	0.0716	0.1192	0.1497
7	0.0231	0.0763	0.0809	0.1399	0.1571
8	0.0455	0.0925	0.1511	0.0875	0.1410
9	0.0209	0.0475	0.1023	0.1265	0.1230
10	0.0578	0.0223	0.1156	0.1126	0.0920
11	0.0463	0.0644	0.0868	0.0788	0.1063

TABLE 8. The statistics of SSMax-CUSUM control chart with 9 in-control samples and a shift in the intercept from sample 10 (in-control ARL=200)

j	$\bar{Q}_j$	$S^2_{Q_j}$	U^+ / U^-	V^+ / V^-	Self-starting Max-CUSUM
1	0.104	-2.909	0.209	0.00	0.209
2	-0.003	-3.328	0.177	0.00	0.177
3	-0.006	-4.440	0.138	0.00	0.138
4	-0.029	-4.207	0.048	0.00	0.048
5	0.017	-4.138	0.061	0.00	0.061
6	0.004	-3.897	0.045	0.00	0.045
7	-0.016	-4.646	0.000	0.00	0.000
8	0.009	-4.328	0.000	0.00	0.000
9	0.026	-3.837	0.033	0.00	0.033
10	0.000	-4.713	0.008	0.00	0.008
11	0.056	-3.754	0.107	0.00	0.107
12	0.026	-4.119	0.141	0.00	0.141
13	0.041	-4.327	0.207	0.00	0.207
14	0.048	-3.965	0.290	0.00	**0.290**
15	0.042	-4.093	0.359	0.00	0.359
16	0.062	-4.316	0.472	0.00	0.472
17	0.012	-4.982	0.473	0.00	0.473
18	0.020	-3.789	0.493	0.00	0.493
					UCL=0.263

6. CONCLUSION AND FUTURE RESEARCHES

In this paper, we applied a Max-CUSUM control chart to monitor the parameters of a simple linear profile simultaneously in Phase II. In addition, we proposed a self-starting Max-CUSUM control chart based on recursive residuals to monitor the parameters of a simple linear profile when there is not enough data at the start-up stages for satisfactory estimation. Moreover, the proposed control charts are able to diagnose if the mean or variance of the simple linear profile has changed. The results of simulation studies showed that the proposed control charts almost perform well under moderate and large shifts. Note that more IC samples before a shift are needed if one is interested to increase the sensitivity of the proposed self-starting control chart to detect small shifts. Finally, the application of the proposed self-starting Max-CUSUM control chart is illustrated through a real case in the leather industry.

Developing a self-starting control chart to monitor multivariate multiple linear regression profiles can be considered as a future research. In addition, one can design a self-starting control chart to monitor simple linear profiles in multistage processes (See Niaki et al. [28]).

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A Self-starting Control Chart for Simultaneous Monitoring of Mean and Variance of Simple Linear Profiles

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چکیده

در بسیاری از فرایندها در دنیای واقعی، پارامترهای فرایند در مراحل اولیه از پیش معلوم نیستند و نمونه‌های اولیه برای اجرای فاز ۱ و تخمین پارامترهای فرایند وجود ندارد. به‌علاوه، دست اندر کاران صنعت علاقمند به استفاده از یک نمودار کنترل به جای دو یا چند نمودار کنترل برای پایش میانگین و تغییر پذیری فرایند می‌باشند. در این مقاله، به بررسی یک پروفایل خطی ساده پرداخته‌ایم که در آن رابطه‌ای میان متغیر پاسخ و متغیر مستقل وجود دارد. در این مقاله یک نمودار Max-CUSUM خودآغازکننده بر پایه باقی‌مانده‌های بازگشتی برای پایش هم‌زمان بردار میانگین (شامل عرض از مبدا و شیب) و تغییر پذیری (واریانس خطا) پروفایل خطی ساده از مراحل اولیه فرایند پیشنهاد شده است. سپس، یک نمودار کنترل Max-CUSUM برای پایش پروفایل خطی ساده در فاز ۲ توسعه داده شده است. همچنین در این مقاله نمودارهای کنترل پیشنهادی با روش موجود در ادبیات از طریق مطالعات شبیه سازی مقایسه شده است. نتایج شبیه سازی نشان می‌دهد که نمودارهای کنترل پیشنهادی از عملکرد بهتری نسبت به نمودار کنترل رقیب در شیفت‌های متوسط و بزرگ از حیث متوسط طول دنباله خارج از کنترل برخوردار است. در نهایت، کاربرد نمودار کنترل خودآغازکننده از طریق یک مثال واقعی در صنعت چرم نشان داده شده است

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