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Two-tier Supplier Base Efficiency Evaluation Via Network DEA: A Game Theory Approach

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ABSTRACT

In today's competitive markets, most firms try to reduce their supply costs by selecting efficient suppliers using different techniques. Several methods can be applied to evaluate the efficiency of suppliers. This paper develops generalized network data envelopment analysis models to examine the efficiency of two-tier suppliers under cooperative and non-cooperative strategies where each tier has its own inputs/outputs and some outputs of the first tier can be fed back to the second tier. Since the proposed models become nonlinear, an efficient heuristic method is proposed as an alternative solution, which can be used instead of existing time consuming methods like parametric linear programming approach. A numerical example is presented to exhibit the implementation of the proposed models. Also, for simulated data, results of proposed heuristic method and parametric linear programming are compared to demonstrate the validity and efficiency of the proposed approach.

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1. INTRODUCTION

Supply chain management (SCM) is considered as the management of upstream/downstream relationships with suppliers and customers to deliver superior customer value to the supply chain as a whole by spending minimum expenses [1]. One of the key aspects of SCM, which plays an important role in supply chain costs reduction is supplier management. Supplier management means “organizing the optimal flow of high-quality, value-for-money materials or components to manufacture companies from a suitable set of innovative suppliers” [2]. Indeed, supplier management is the management of the upstream part of a supply chain.

Supplier evaluation is one of the most important tasks in supplier management that refers to methods, modals and techniques that firms apply to asses and select their suppliers. Supplier evaluation problem has been widely studied and various approaches have been presented to tackle the problem. Some of the basic and popular supplier evaluation and selection techniques are: data envelopment analysis, analytical hierarchical process, linear weighting models, outranking, expert systems and portfolio analysis [3].

The main objective of this paper is to develop general data envelopment analysis models for efficiency evaluation of suppliers. Data envelopment analysis (DEA) is an effective method proposed by Charnes et al. [4] to evaluate relative efficiency of a set of decision making units (DMU) that use multiple inputs to produce multiple outputs. DEA has been used in variety of industries and many theoretical developments have been reported. For example, recently Hatefi et al. [5] developed a common weight multi criteria decision analysis-data envelopment analysis (MCDA-DEA) method with assurance region for weight derivation in a pair wise comparison matrix (PCM). Torabi and Shokr applied the approach proposed by Hatefi et al. [5] for material selection problem [6]. A comprehensive review of DEA models and applications were presented by Cook and Seiford [7] and Liu et al. [8].

Traditional DEA was developed to evaluate the efficiency of a DMU as a black box without considering

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its internal structure. Some studies indicate that to investigate the efficiency of a DMU, we may not always ignore internal processes [9, 10]. In the related literature, DEA models that consider the internal structure of DMU's are called “network DEA” models [11]. Kao [12] reviewed different studies on network DEA and classified them based on the type of model used or developed and structures of considered network.

Application of game theory approach to model the relationship between components in network DEA originates from the work of Liang et al. [13]. They developed DEA-based models to evaluate the efficiency of seller-buyer supply chains when intermediate measures are considered. In their paper the relationship between buyer and seller is treated as leader-follower and also cooperative game. They developed non-linear programming models to evaluate the efficiency of two stage supply chain. The models can be treated as parametric linear programming models to obtain the best solution.

In existing literature, three types of games are applied to model the relationship between sub-DMUs, non- cooperative game [13-17], cooperative game [13-17] and bargaining game [18, 19].

First probable relationship between suppliers is to consider them as leader and follower, i.e. non-cooperative game, which is also known as Stackelberg game. In this paradigm the leader supplier maximizes its profit and efficiency and the follower supplier tries to maximize its profit subject to consider a fixed value for the efficiency of leader. Another formal relationship among suppliers is a cooperative one to improve the overall efficiency. In this approach, efficiencies of suppliers are evaluated, simultaneously. For cases where all outputs of the first stage are inputs of the second stage, Liang et al. [16] developed both cooperative and non-cooperative models. Li et al. [15] extended the models proposed by Ling et al. by assuming that inputs of the second stage include outputs of the first stage as well as additional inputs. They developed a linear model for non-cooperative game and also a non-linear model for cooperative approach. The non-linear model can be solved globally using a parametric linear programming algorithm. Chen and Yan [17] introduced three mathematical models under the concept of centralized (cooperative), decentralized (non-cooperative), and mixed centralized situation to evaluate the performance of a supply chain with one supplier and two manufacturers.

Existing studies for evaluating the efficiency of suppliers are under following criticisms:

1) Most of the methods developed in previous researches are appropriate for evaluating the efficiency of one-tier suppliers, i.e., internal relationships between suppliers of different layers in a supplier base[2] are ignored. Hence, existing approaches cannot be used to evaluate the efficiency of real-world cases with multi-tier (multi-stage, multi-layer) supplier bases.

2) In a few number of studies in the literature, a simple form of two-tier supplier base is assumed. This simple form is shown in Figure 1.

In this structure, the only outputs of stage 2 are those that are inputs of stage 1, i.e. stage 2 does not have additional outputs and stage 1 does not have additional inputs either. Furthermore, all outputs of stage 1 are sold to customer.

This simple structure may not be practical in real world applications. In many practical cases, stage 2 and stage 1 have additional outputs and inputs, respectively. Furthermore, some outputs of stage 1 may be fed back to stage 2. For example, in Hi-Tech industries, because of rapid growth, some unsold products are fed back to stage 1 for upgrading. Another example of such structure is automobile industry, where there is a close relationship between different suppliers of an automobile manufacturer. Supplier base of an automobile manufacturer mainly consists of raw material producer and automobile parts manufacturer. Raw material producer, sales metal plates in standard forms to parts manufacturer and scraps are flew back from parts manufacturer to raw material producer for remanufacturing (see illustrative example).

3) In papers that multi-tier supplier base and cooperation between suppliers are assumed, parametric linear programming is the common approach to solve the resulted non-linear models. This approach is time consuming especially when the number of DMUs is large.

This paper contributes to the current strand of literature in the field of supplier evaluation by developing general network DEA models under cooperation and non-cooperation conditions. The contributions of this paper can be categorized as follows:

1) Mathematical models based on network DEA approach are developed to evaluate the efficiency of two-tier supplier bases with input and output data in tier 2, input and output data in tier 1, intermediate measures between tier 2 and tier1and feedback measures from tier 1 to tier 2. All or some of these measures are ignored in literature for simplicity [15, 16].

Hence, proposed models are general forms that can be used for a wide range of applications. Models are developed under cooperation between suppliers and non-cooperation conditions.

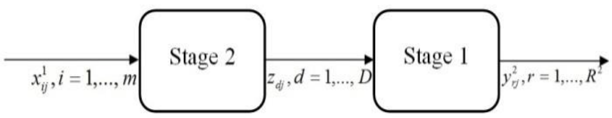

Figure 1. Common form of two-stage structures

[2] A group of suppliers which are in relationship with the main firm.

2) For cases that developed models are in nonlinear form, a novel efficient heuristic method is proposed that can be used instead of parametric linear programming method.

This paper is structured as follows: problem definition and network DEA models under cooperative and non-cooperative situations are presented in section 2. The proposed heuristic method to solve the cooperative model is illustrated in section 3. In section 4, the relationship between efficiencies in different mechanisms and also efficiencies of tiers are presented through two theorems. The models are verified via numerical examples in section 5. In last section conclusions are given.

2. PROBLEM STATEMENT AND MODEL

In this paper it is considered that a company wishes to evaluate relative efficiency of n two-tier supplier bases and chooses the most efficient one as its supplier base. Each supplier has its inputs, outputs and also intermediate flow between suppliers in different tiers is considered. Furthermore, the situation that outputs of first tier can be fed back to second tier as input is considered. This structure is depicted in Figure2.

It is assumed that in each two-tier supplier base j $(j=1,\dots,n)$, supplier of tier k $(k=1,2)$ has m^k inputs and R^k outputs. Also supplier of tier 2 has D intermediate outputs that are inputs of supplier of first tier. Furthermore, first tier supplier has G outputs that flow back to the tier 2. Parameters and variables used in presented models are as follows:

o	Index of under evaluation supplier base (o=1,…,n)
k	Index of tiers number in supplier base (k=1,2)
m^k	Number of inputs of tier k
R^k	Number of outputs of tier k
D	Number of intermediate measures between tier 2 and tier 1
G	Number of feedbacks from tier 1 to tier 2
x_{ij}^k	ith input of tier k for jth supplier base
y_{rj}^k	rth output of tier k for jth supplier base
z_{dj}	dth intermediate measure for jth supplier base
f_{gj}	gth feedback for jth supplier base
u_r^k	Weight assigned to the rth output of tier k (decision variable)
v_i^k	Weight assigned to the ith input of tier k (decision variable)
h_d	Weight assigned to the dth intermediate measure (decision variable)
w_g	Weight assigned to the gth feedback (decision variable)

Based on the constant return to scale CCR model [4], models 1 and 2 can be established to compute the efficiency of supplier 1 (e^1) and supplier 2 (e^2), respectively.

$$\begin{aligned}
&\max \quad e_o^1=\frac{\sum_{r=1}^{R^1}u_r^1y_{ro}^1+\sum_{g=1}^{G}w_g f_{go}}{\sum_{d=1}^{D}h_d z_{do}+\sum_{i=1}^{m^1}v_i^1x_{io}^1}\\
&\textit{subject to}:\\
&\qquad \frac{\sum_{r=1}^{R^1}u_r^1y_{rj}^1+\sum_{g=1}^{G}w_g f_{gj}}{\sum_{d=1}^{D}h_d z_{dj}+\sum_{i=1}^{m^1}v_i^1x_{ij}^1}\le 1 \qquad j=1,\dots,n\\
&\qquad u_r^1,w_g,h_d,v_i^1\ge 0 \qquad \forall r,g,d,i
\end{aligned}\tag{1}$$

$$\begin{aligned}
&\max \quad e_o^2=\frac{\sum_{r=1}^{R^1}u_r^2y_{ro}^2+\sum_{d=1}^{D}h_d z_{do}}{\sum_{g=1}^{G}w_g f_{go}+\sum_{i=1}^{m^1}v_i^2x_{io}^2}\\
&\textit{subject to}:\\
&\qquad \frac{\sum_{r=1}^{R^2}u_r^2y_{rj}^2+\sum_{g=1}^{G}h_d z_{dj}}{\sum_{g=1}^{G}w_g f_{gj}+\sum_{i=1}^{m^2}v_i^2x_{ij}^2}\le 1 \qquad j=1,\dots,n\\
&\qquad u_r^2,w_g,h_d,v_i^2\ge 0 \qquad \forall r,g,d,i
\end{aligned}\tag{2}$$

2. 1. Non-cooperative Model (Stackelberg Game Model)

In cases that one supplier is more important than the other one, the leader optimizes its profit and the follower determines its efficiency based on the information from leader. Considering supplier of tier 1 (supplier 1) as leader and supplier of tier 2 (supplier 2) as follower, the efficiency of supplier 1 can be computed using the following linear programming model. In a similar manner supplier 2 can be considered as leader.

$$\begin{aligned}
&\max \quad e_o^{1*}=\sum_{r=1}^{R^1}u_r^1y_{ro}^1+\sum_{g=1}^{G}w_g f_{go}\\
&\textit{subject to}:\\
&\qquad (\sum_{r=1}^{R^1}u_r^1y_{rj}^1+\sum_{g=1}^{G}w_g f_{gj})-(\sum_{d=1}^{D}h_d z_{dj}+\sum_{i=1}^{m^1}v_i^1x_{ij}^1)\le 0 \qquad j=1,\dots,n\\
&\qquad \sum_{d=1}^{D}h_d z_{do}+\sum_{i=1}^{m^1}v_i^1x_{io}^1=1\\
&\qquad u_r^1,w_g,h_d,v_i^1\ge 0 \qquad \forall r,g,d,i
\end{aligned}\tag{3}$$

The follower (supplier 2) will maximize its efficiency such that efficiency of supplier 1 remains at e_o^{1*}. Hence, model (4) can be applied to measure the efficiency of follower.

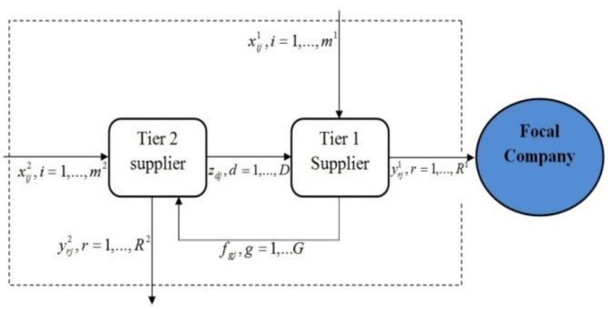

Figure 2. Structure of supplier base *j*

$$
\begin{aligned}
\max \quad & e_o^2 = \frac{\sum_{r=1}^{R^1} u_r^2 y_{ro}^2 + \sum_{d=1}^{D} h_d z_{do}}{\sum_{g=1}^{G} w_g f_{go} + \sum_{i=1}^{m^1} v_i^2 x_{io}^2} \\
\textit{subject to}: & \\
& \frac{\sum_{r=1}^{R^2} u_r^2 y_{rj}^2 + \sum_{g=1}^{G} h_d z_{dj}}{\sum_{g=1}^{G} w_g f_{gj} + \sum_{i=1}^{m^2} v_i^2 x_{ij}^2} \le 1 \qquad j = 1,\ldots,n \\
& \frac{\sum_{r=1}^{R^1} u_r^1 y_{rj}^1 + \sum_{g=1}^{G} w_g f_{gj}}{\sum_{d=1}^{D} h_d z_{dj} + \sum_{i=1}^{m^1} v_i^1 x_{ij}^1} \le 1 \qquad j = 1,\ldots,n \\
& \frac{\sum_{r=1}^{R^1} u_r^1 y_{ro}^1 + \sum_{g=1}^{G} w_g f_{go}}{\sum_{d=1}^{D} h_d z_{do} + \sum_{i=1}^{m^1} v_i^1 x_{io}^1} = e_o^{1*} \\
& u_r^2, w_g, h_d, v_i^2, u_r^1, v_i^1 \ge 0 \qquad \forall r,g,d,i
\end{aligned} \tag{4}
$$

Model (4) is a nonlinear model and can be converted to the following linear programming model.

$$
\begin{aligned}
\max \quad & e_o^{2*} = \sum_{r=1}^{R^1} u_r^2 y_{ro}^2 + \sum_{d=1}^{D} h_d z_{do} \\
\textit{subject to}: & \\
& (\sum_{r=1}^{R^2} u_r^2 y_{rj}^2 + \sum_{g=1}^{G} h_d z_{dj}) - (\sum_{g=1}^{G} w_g f_{gj} + \sum_{i=1}^{m^2} v_i^2 x_{ij}^2) \le 0 \qquad j = 1,\ldots,n \\
& (\sum_{r=1}^{R^1} u_r^1 y_{rj}^1 + \sum_{g=1}^{G} w_g f_{gj}) - (\sum_{d=1}^{D} h_d z_{dj} + \sum_{i=1}^{m^1} v_i^1 x_{ij}^1) \le 0 \qquad j = 1,\ldots,n \\
& \sum_{g=1}^{G} w_g f_{go} + \sum_{i=1}^{m^1} v_i^2 x_{io}^2 = 1 \\
& (\sum_{r=1}^{R^1} u_r^1 y_{ro}^1 + \sum_{g=1}^{G} w_g f_{go}) - e_o^{1*}(\sum_{d=1}^{D} h_d z_{do} + \sum_{i=1}^{m^1} v_i^1 x_{io}^1) = 0 \\
& u_r^2, w_g, h_d, v_i^2, u_r^1, v_i^1 \ge 0 \qquad \forall r,g,d,i
\end{aligned} \tag{5}
$$

The efficiency of the whole supplier base will be: $e_o^{NC*} = e_o^{1*} * e_o^{2*}$. In situation that supplier 2 is leader and supplier 1 is follower, the modeling process will be the same.

2. 2. Cooperative Model According to CCR model, the following cooperative model can be established:

$$
\begin{aligned}
\max \quad & e_o^{C*} = e_o^1 * e_o^2 = \frac{\sum_{r=1}^{R^1} u_r^1 y_{ro}^1 + \sum_{g=1}^{G} w_g f_{go}}{\sum_{d=1}^{D} h_d z_{do} + \sum_{i=1}^{m^1} v_i^1 x_{io}^1} * \frac{\sum_{r=1}^{R^1} u_r^2 y_{ro}^2 + \sum_{d=1}^{D} h_d z_{do}}{\sum_{g=1}^{G} w_g f_{go} + \sum_{i=1}^{m^1} v_i^2 x_{io}^2} \\
\textit{subject to}: & \\
& \frac{\sum_{r=1}^{R^1} u_r^1 y_{rj}^1 + \sum_{g=1}^{G} w_g f_{gj}}{\sum_{d=1}^{D} h_d z_{dj} + \sum_{i=1}^{m^1} v_i^1 x_{ij}^1} \le 1 \qquad j = 1,\ldots,n \\
& \frac{\sum_{r=1}^{R^2} u_r^2 y_{rj}^2 + \sum_{g=1}^{G} h_d z_{dj}}{\sum_{g=1}^{G} w_g f_{gj} + \sum_{i=1}^{m^2} v_i^2 x_{ij}^2} \le 1 \qquad j = 1,\ldots,n \\
& u_r^1, w_g, h_d, v_i^1, u_r^2, v_i^2 \ge 0 \qquad \forall r,g,d,i
\end{aligned} \tag{6}
$$

Model (6) is a non-linear programming model and cannot be converted into linear form but a parametric linear programming approach can be applied to solve this model (see Appendix A). In next section an efficient heuristic method is proposed which in many cases can obtain the global solution.

3. SOLUTION METHOD

The non-cooperative model is a linear programming model and can be solved globally. But the cooperative model is in nonlinear form and optimal solution is not guaranteed. A parametric linear programming approach can be applied to obtain global solution of this model. As the parametric linear programming approach is very time consuming, in this paper a heuristic method is developed to solve model (6). This approach is an efficient method.

- **Step 1.** Calculate maximum achievable efficiency of suppliers (e_o^{1+}, e_o^{2+}) via following linear models:

$$
\begin{aligned}
\max \quad & e_o^{1+} = \sum_{r=1}^{R^1} u_r^1 y_{ro}^1 + \sum_{g=1}^{G} w_g f_{go} \\
\textit{subject to}: & \\
& (\sum_{r=1}^{R^1} u_r^1 y_{rj}^1 + \sum_{g=1}^{G} w_g f_{gj}) - (\sum_{d=1}^{D} h_d z_{dj} + \sum_{i=1}^{m^1} v_i^1 x_{ij}^1) \le 0 \qquad j = 1,\ldots,n \\
& (\sum_{r=1}^{R^2} u_r^2 y_{rj}^2 + \sum_{g=1}^{G} h_d z_{dj}) - (\sum_{g=1}^{G} w_g f_{gj} + \sum_{i=1}^{m^2} v_i^2 x_{ij}^2) \le 0 \qquad j = 1,\ldots,n \\
& \sum_{d=1}^{D} h_d z_{do} + \sum_{i=1}^{m^1} v_i^1 x_{io}^1 = 1 \\
& u_r^1, w_g, h_d, v_i^1, u_r^2, v_i^2 \ge 0 \qquad \forall r,g,d,i
\end{aligned} \tag{7}
$$

$$
\begin{aligned}
\max \quad & e_o^{2+} = \sum_{r=1}^{R^1} u_r^2 y_{ro}^2 + \sum_{d=1}^{D} h_d z_{do} \\
\textit{subject to}: & \\
& (\sum_{r=1}^{R^1} u_r^1 y_{rj}^1 + \sum_{g=1}^{G} w_g f_{gj}) - (\sum_{d=1}^{D} h_d z_{dj} + \sum_{i=1}^{m^1} v_i^1 x_{ij}^1) \le 0 \qquad j = 1,\ldots,n \\
& (\sum_{r=1}^{R^2} u_r^2 y_{rj}^2 + \sum_{g=1}^{G} h_d z_{dj}) - (\sum_{g=1}^{G} w_g f_{gj} + \sum_{i=1}^{m^2} v_i^2 x_{ij}^2) \le 0 \qquad j = 1,\ldots,n \\
& \sum_{g=1}^{G} w_g f_{go} + \sum_{i=1}^{m^1} v_i^2 x_{io}^2 = 1 \\
& u_r^1, w_g, h_d, v_i^1, u_r^2, v_i^2 \ge 0 \qquad \forall r,g,d,i
\end{aligned} \tag{8}
$$

- **Step 2.** Solve models (9) and (10).

$$
\begin{aligned}
\max \quad & e_o^{C1} = e_o^{1+} * (\sum_{r=1}^{R^1} u_r^2 y_{ro}^2 + \sum_{d=1}^{D} h_d z_{do}) \\
\textit{subject to}: & \\
& (\sum_{r=1}^{R^1} u_r^1 y_{rj}^1 + \sum_{g=1}^{G} w_g f_{gj}) - (\sum_{d=1}^{D} h_d z_{dj} + \sum_{i=1}^{m^1} v_i^1 x_{ij}^1) \le 0 \qquad j = 1,\ldots,n \\
& (\sum_{r=1}^{R^2} u_r^2 y_{rj}^2 + \sum_{g=1}^{G} h_d z_{dj}) - (\sum_{g=1}^{G} w_g f_{gj} + \sum_{i=1}^{m^2} v_i^2 x_{ij}^2) \le 0 \qquad j = 1,\ldots,n \\
& (\sum_{r=1}^{R^1} u_r^1 y_{ro}^1 + \sum_{g=1}^{G} w_g f_{go}) - e_o^{1+}(\sum_{d=1}^{D} h_d z_{do} + \sum_{i=1}^{m^1} v_i^1 x_{io}^1) = 0 \\
& \sum_{g=1}^{G} w_g f_{go} + \sum_{i=1}^{m^1} v_i^2 x_{io}^2 = 1 \\
& u_r^1, w_g, h_d, v_i^1, u_r^2, v_i^2 \ge 0 \qquad \forall r,g,d,i
\end{aligned} \tag{9}
$$

$$
\begin{aligned}
\max \quad & e_o^{C2} = e_o^{2+} * (\sum_{r=1}^{R^1} u_r^1 y_{ro}^1 + \sum_{g=1}^{G} w_g f_{go}) \\
\textit{subject to}: & \\
& (\sum_{r=1}^{R^1} u_r^1 y_{rj}^1 + \sum_{g=1}^{G} w_g f_{gj}) - (\sum_{d=1}^{D} h_d z_{dj} + \sum_{i=1}^{m^1} v_i^1 x_{ij}^1) \le 0 \qquad j = 1,\ldots,n \\
& (\sum_{r=1}^{R^2} u_r^2 y_{rj}^2 + \sum_{g=1}^{G} h_d z_{dj}) - (\sum_{g=1}^{G} w_g f_{gj} + \sum_{i=1}^{m^2} v_i^2 x_{ij}^2) \le 0 \qquad j = 1,\ldots,n \\
& (\sum_{r=1}^{R^1} u_r^2 y_{ro}^2 + \sum_{d=1}^{D} h_d z_{do}) - e_o^{2+}(\sum_{g=1}^{G} w_g f_{go} + \sum_{i=1}^{m^1} v_i^2 x_{io}^2) = 0 \\
& \sum_{d=1}^{D} h_d z_{do} + \sum_{i=1}^{m^1} v_i^1 x_{io}^1 = 1 \\
& u_r^1, w_g, h_d, v_i^1, u_r^2, v_i^2 \ge 0 \qquad \forall r,g,d,i
\end{aligned} \tag{10}
$$

- **Step3.** $e_o^{C*} = \max\{ e_o^{C1}, e_o^{C2} \}$

4. EFFICIENCY ANALYSIS

In this section, the relationship between efficiency scores in different mechanisms and efficiency scores of tiers are investigated.

4. 1. Efficiency In Different Mechanisms

Theorem 1. The efficiency of supplier base under different mechanisms can be expressed as:

$$e^{NC^*} \le e^{C^*}$$

Proof. Suppose $\lambda 1 = \{u_r^{2*}, w_g^*, h_d^*, v_i^{2*}, u_r^{1*}, v_i^{1*}\}$ is an optimal solution to model (4). Accordingly, the efficiency of the system is $e_o^{NC*} = e_o^{1*} * e_o^{2*}$. Note that $\lambda 1$ is also a feasible solution to model (6). Thus we have $e^{NC^*} \le e^{C^*}$. Similarly, it can be proven that when supplier 2 is leader, optimal efficiency of non-cooperative model is smaller or equal to cooperative mechanism. However, the relationship between CCR model and game models depends on parameters.

4. 2. Tiers Efficiency

Theorem2. $e_1^{S1^*} \ge e_2^{S1^*}$ and $e_2^{S2^*} \ge e_1^{S2^*}$. Where $e_1^{S1^*}$ and $e_2^{S1^*}$ are the optimal efficiency of supplier 1 when supplier 1 is leader and when supplier 2 is leader, respectively. And $e_2^{S2^*}$ and $e_1^{S2^*}$ are the optimal efficiency of supplier 2 when supplier 2 is leader and when supplier 1 is leader, respectively.

Proof. Let $\lambda 2 = \{u_r^{2*}, w_g^*, h_d^*, v_i^{2*}, u_r^{1*}, v_i^{1*}\}$ be the optimal efficiency of supplier 1, when supplier 2 is leader. This solution is feasible for model (3). Denote that model (3) calculates the optimal efficiency of supplier 1, when its leader. Hence $e_1^{S1^*} \ge e_2^{S1^*}$ In a similar manner we have $e_2^{S2^*} \ge e_1^{S2^*}$.

5. ILLUSTRATIVE EXAMPLE

An automobile manufacturer wishes to choose its supplier base. 20 two tier supplier bases with same structure as depicted in Figure 3 are considered. Supplier of tier 2 produces raw materials from mineral stones in standard plate forms. Numbers of employees and operation costs are inputs of tier 2 supplier. This supplier sales steel and aluminum plates in standard form to supplier of tier 1 and resin and graphite to other customers. Supplier of tier 1 is an automobile parts manufacturer that uses steel and aluminum plates to produce automobile parts. Number of employees is another input of supplier 1. During manufacturing process, some scraps are produced. Supplier 1 flows back these scraps to supplier 2 for remanufacturing. Data related to numerical example are listed in Table 1.

In Table 2 and Figure 4, the efficiencies of 20 supplier bases under cooperative and non cooperative mechanisms are compared and results of CCR model are illustrated. In Table 2, e^{S1^*} and e^{S2^*} are optimal efficiencies of supplier 1 and supplier 2 without considering any relationship with other supplier.

As shown in Figure 4 for all supplier bases, efficiency under non cooperative mechanism is smaller or equal to efficiency score in cooperative approach (Theorem 1).

The cooperative method can be solved globally via parametric linear programming method which is an iterative method and takes a long time (especially for large number of DMUs). Table 3 shows that in this example for 85% of DMUs the heuristic method obtains the global solution in 6.756 seconds while achieving optimal solutions taking 178.837 seconds in parametric linear programming method.

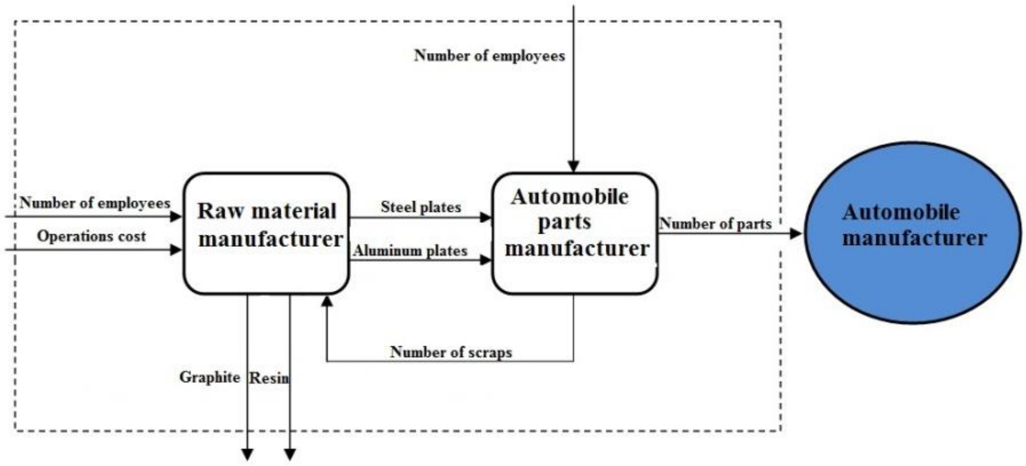

Figure 3 Structure of supplier bases of numerical example

TABLE 1. Input and output data in example

Supplier base	Inputs of supplier 2		Output of supplier 2		Intermediate measures		Input of supplier 1	Output of supplier 1	Feedback measure
1	722	2178.7314	1416.1525	1179.8818	6817.8275	975.8726	4333	380	30
2	4492	2670.2533	1665.3914	5650.5186	3676.4582	86.2710	1754	630	65
3	2799	5627.7632	1039.4028	6345.7378	2209.2294	3495.6749	11744	3148	139
4	4927	6823.7981	3673.3145	305.2578	3975.9427	72.3829	5524	846	218
5	7536	984.5457	1465.4060	1241.4379	2790.1500	1709.0679	225	1441	145
6	274	3560.8175	1871.7617	1282.1199	384.2895	695.7482	5123	2208	629
7	1626	5118.7233	2710.8209	3162.1815	1202.8482	78.7921	998	4456	283
8	3217	176.8531	7002.0519	3738.5526	987.3822	359.8143	2525	3516	289
9	3101	2020.0792	3272.6638	155.0898	1837.6022	897.9666	1459	252	768
10	692	596.0754	2374.0731	1298.3391	267.7575	82.7818	5331	1757	234
11	647	572.1940	2278.5883	3708.8506	6366.2410	3936.3700	105	2841	696
12	4535	2579.7147	143.6022	3344.9554	1769.7033	4619.4832	1200	1048	801
13	3266	2502.1732	5558.0972	5320.0198	2897.9221	875.4286	925	276	214
14	77	460.2188	2545.4530	3871.8620	4837.4270	3858.2487	117	116	347
15	7259	381.3415	7565.3368	1139.3762	3473.1564	344.2627	2267	1018	30
16	4032	28.9787	5946.1243	2285.3894	6836.7164	1162.5357	2311	2614	73
17	653	4046.7349	4648.9406	9930.1462	2622.7159	1557.5466	1188	3472	78
18	2724	3588.6361	777.0623	23.2250	2974.7635	8356.2539	4463	5677	129
19	3543	2500.8092	4663.9877	6174.4482	5780.0382	89.8057	1059	1408	148
20	3789	370.6990	1850.4613	65.2789	1268.6359	7767.1498	3748	5155	39

TABLE 2. Supply base efficiency under different relationship mechanisms

Supplier bases	e^{NC^*} Supplier 1 as leader	e^{NC^*} Supplier 2 as leader	e^{C^*}	e^{CCR^*}	e^{S1^*}	e^{S2^*}
1	0.0058	0.0079	0.0153	0.1046	0.0520	1.0000
2	0.2232	0.0243	0.2232	0.0428	1.0000	0.4328
3	0.1304	0.0527	0.1304	0.2378	1.0000	0.5321
4	0.0818	0.0227	0.0818	0.0378	1.0000	0.2881
5	0.1336	0.0627	0.1338	0.1286	0.8996	0.2653
6	0.2069	0.0063	0.2069	1.0000	1.0000	0.3420
7	0.0911	0.0180	0.0911	0.5399	1.0000	0.3283
8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
9	0.1996	0.0234	0.1996	0.0233	1.0000	0.2950
10	0.5377	0.0098	0.5377	0.5901	1.0000	0.8025
11	0.9215	0.9215	0.9215	1.0000	1.0000	0.9215
12	0.1402	0.0343	0.1542	0.0728	1.0000	0.2876
13	0.0604	0.0066	0.0604	0.0215	0.2088	0.6838
14	0.6329	0.6329	0.6329	0.2547	0.6329	1.0000
15	0.1344	0.0166	0.1344	0.1310	0.2760	1.0000
16	0.5602	0.5602	0.5602	1.0000	0.5602	1.0000
17	0.3032	0.1080	0.3032	0.8867	1.0000	1.0000
18	0.0541	0.0470	0.1106	0.4608	0.7481	1.0000
19	0.2299	0.0373	0.2299	0.1073	0.8653	0.7596
20	0.2226	0.4838	0.4838	1.0000	1.0000	1.0000

To investigate the validity of proposed heuristic method, 25 cases are randomly generated. Input and output measures are random numbers distributed uniformly between 1 and 10000.

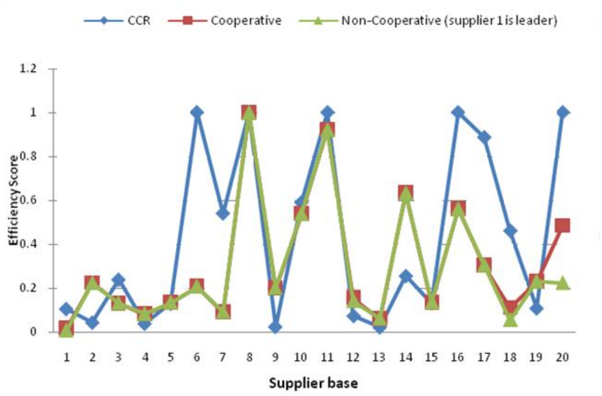

Figure4. Supply base efficiency under different relationship mechanisms

TABLE 3. Results of proposed heuristic method and the parametric linear programming approach

Supplier base	Proposed heuristic method	Parametric linear programming method
1	0.0079	0.0153
2	0.2232	0.2232
3	0.1304	0.1304
4	0.0818	0.0818
5	0.1338	0.1338
6	0.2069	0.2069
7	0.0911	0.0911
8	1.0000	1.0000
9	0.1996	0.1996
10	0.5377	0.5377
11	0.9215	0.9215
12	0.1402	0.1542
13	0.0604	0.0604
14	0.6329	0.6329
15	0.1344	0.1344
16	0.5602	0.5602
17	0.3032	0.3032
18	0.0541	0.1106
19	0.2299	0.2299
20	0.4838	0.4838
Computational time (Sec)	6.756	178.837

Number of DMUs in each case is a random number between 10 and 30 and number of inputs and outputs for each supplier is also a random number between 1 and 4. For each case, results of heuristic method and parametric linear programming $(\Delta\varepsilon = 0.001)$ are compared. Results are shown in Table 4.

Table 4 shows that for different DMUs at least for about 85% of DMUs, the heuristic method reaches the optimal solution in a very short time.

TABLE 4. Validity of proposed heuristic method

Case number	Number of DMUs	Computational time (Sec)		Percentage of optimal solutions obtained via heuristic method
		Parametric approach	Heuristic method	
1	23	134.090	6.279	95.65
2	12	135.193	5.243	100
3	21	141.773	5.989	90.47
4	25	167.635	6.217	100
5	21	181.798	5.541	90.47
6	13	157.621	5.836	100
7	14	163.511	6.222	100
8	19	145.233	5.805	89.47
9	22	143.272	6.675	100
10	14	85.602	3.426	85.71
11	11	129.356	4.981	100
12	20	201.542	9.285	100
13	29	214.940	8.501	93.10
14	21	247.549	9.567	100
15	19	224.052	8.614	94.73
16	13	151.849	5.972	100
17	29	352.628	13.602	96.55
18	16	160.482	6.945	93.75
19	27	282.754	12.603	96.29
20	19	204.979	8.399	100
21	25	270.818	11.487	92
22	24	286.372	11.241	100
23	14	151.069	6.563	100
24	17	125.287	5.775	88.23
25	27	331.755	12.609	92.59

6. CONCLUSIONS

Network data envelopment analysis can be applied by firms to evaluate the efficiency of potential supplier bases and choose the most efficient one to reduce their supply costs. The current paper develops network data envelopment analysis models to investigate efficiency of a generalized form of two tier supplier bases. Different relationship mechanisms are considered in this paper: cooperation and leader-follower game. As under cooperation mechanism the resulted model is nonlinear, a novel heuristic method is proposed that can be used instead of existing methods. It is shown that the proposed method can obtain the optimal solution for more than 85% of supplier bases.

The proposed models can be extended for other relationship mechanisms such as bargaining. Another potential extension of this paper is evaluating efficiency of supplier bases when input and output data are uncertain. Supplier bases with undesirable outputs provide another opportunity for extending this study.

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8. APPENDIX A: PARAMETRIC LINEAR PROGRAMMING APPROACH

Model (6) is a nonlinear model and can't be solved globally but parametric linear programming approach can be applied to solve this model. To achieve this aim, efficiency of one of suppliers can be considered as a parameter and efficiency of another supplier will be variable. Consider the following model:

$$
\begin{aligned}
&\max \quad e_o^{C*} = e_o^1 * \sum_{r=1}^{R^1} u_r^2 y_{ro}^2 + \sum_{d=1}^{D} h_d z_{do} \\
&\textit{subject to:} \\
&\qquad (\sum_{r=1}^{R^1} u_r^1 y_{rj}^1 + \sum_{g=1}^{G} w_g f_{gj}) - (\sum_{d=1}^{D} h_d z_{dj} + \sum_{i=1}^{m^1} v_i^1 x_{ij}^1) \le 0 \qquad j = 1,\dots,n \\
&\qquad (\sum_{r=1}^{R^2} u_r^2 y_{rj}^2 + \sum_{g=1}^{G} h_d z_{dj}) - (\sum_{g=1}^{G} w_g f_{gj} + \sum_{i=1}^{m^2} v_i^2 x_{ij}^2) \le 0 \qquad j = 1,\dots,n \\
&\qquad \sum_{g=1}^{G} w_g f_{go} + \sum_{i=1}^{m^1} v_i^2 x_{io}^2 = 1 \\
&\qquad u_r^1, w_g, h_d, v_i^1, u_r^2, v_i^2 \ge 0 \qquad \forall r, g, d, i
\end{aligned}
\tag{11}
$$

In model (11), efficiency of supplier 1 (e_o^1) is a parameter. The maximum efficiency of supplier 1 can be obtained by solving model (7). Let $e_o^1 = e_o^{1+} - k\Delta\varepsilon$. $\Delta\varepsilon$ is step size. To obtain more precise results, a smaller step size should be selected. k is an integer number between 0 and $e_o^{1+} / \Delta\varepsilon$. To solve model (11), in first iteration $k=0$. Now given e_o^1, model (11) can be solved optimally as a linear programming model. In each iteration we increase k and obtain $e_o^{C*}(k)$. Finally $e_o^{C*} = \max_k e_o^{C*}(k)$ .If we consider efficiency of supplier 1 as variable and efficiency of supplier 2 as parameter, same result will be obtained.

Two-tier Supplier Base Efficiency Evaluation Via Network DEA: A Game Theory Approach

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چکیده

امروزه شرکت‌ها در تلاش‌اند تا با انتخاب تامین‌کنندگانی مناسب و با کارایی بالا، هزینه‌های تامین خود را تا حد امکان کاهش دهند. روش‌های مختلفی به منظور ارزیابی مجموعه تامین‌کنندگان وجود دارد. در این مقاله مدل‌هایی جامع مبتنی بر تحلیل پوششی داده‌ها به منظور ارزیابی مجموعه‌های دوسطحی تامین‌کنندگان با در نظر گرفتن دو مکانیزم محتمل بین سطوح مختلف مجموعه تامین‌کنندگان (مکانیزم‌های همکاری و عدم همکاری) توسعه داده شده‌اند. ساختار در نظر گرفته شده برای مجموعه تامین‌کنندگان بدین صورت است که هر سطح دارای ورودی و خروجی است و برخی از خروجی‌های سطح دوم ورودی سطح اول است. همچنین برخی از خروجی‌های سطح اول می‌تواند به عنوان ورودی به سطح دوم بازگردانده شود. نوآوری این مقاله ساختار جامع در نظر گرفته شده برای مجموعه تامین‌کنندگان، و نیز ارائه یک روش ابتکاری برای رسیدن به جواب مطلوب برای مدل‌های غیرخطی توسعه داده شده است. در این مقاله، کاربرد مدل‌های پیشنهادی با استفاده از یک مثال عددی در صنعت خودروسازی نشان داده شده است. همچنین برای داده های شبیه‌سازی شده، روش ابتکاری ارائه شده با روش برنامه‌ریزی پارامتریک مقایسه و اعتبارسنجی گردیده و ارجحیت آن نسبت به روش برنامه ریزی پارامتریک نشان داده شده است.

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