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Capacitated Single Allocation P-Hub Covering Problem in Multi-modal Network Using Tabu Search

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ABSTRACT

The goals of hub location problems are finding the location of hub facilities and determining the allocation of non-hub nodes to these located hubs. In this work, we discuss the multi-modal single allocation capacitated p-hub covering problem over fully interconnected hub networks. Therefore, we provide a formulation to this end. The purpose of our model is to find the location of hubs and the hub links between them at a selected combination of modes for each origin-destination. Furthermore, it determines the allocation of non-hub nodes to the located hubs at the best mode for each allocation such that the travel time between any origin–destination pair is not greater than a given time bound. In addition, the capacity of hub nodes is considered. Six valid inequalities are presented to tighten the linear programming lower bound. We present a heuristic tabu search based algorithm and test the performance of it on the Australian Post (AP) data set.

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1. INTRODUCTION

A hub location problem is applied when it is cost-effective to join and distribute the flow at a centralised location called a hub node. Flow concentration and consolidation take place on the arcs that unify hub nodes, called hub links. These links allow for a more efficient transportation of origin-destination (O-D) flows. It is also possible to remove many expensive direct links between O-D pairs. The advantage of using hubs is that by consolidating the flow, economies of scale can be achieved, such that transferring flow between hubs is cheaper than the cost of moving flow directly between non-hubs. Non-hubs can be connected to one or more hubs, depending on whether we deal with a single or multiple allocation strategy. It is usually assumed that the hubs are fully interconnected, while non-hubs are not connected to each other. Therefore, all flows have to be routed via at least one hub. Hence, such a logistic network consists of two parts, called the "hub-and-spoke" network.

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This research follows the mentioned assumptions. Commodities are shipped in the hub-and-spoke network in three phases: 1) Collecting: they move along their origin nodes to the assigned hub nodes, 2) Transferring: commodities go through the hub arcs if necessary, and 3) Distributing: commodities depart from the hubs and arrive in destination nodes. The unit cost factor of the transferring phase is α. The transferring cost factor α is usually less than the collecting and distributing cost factors because of economies of scale from the gathering of flows at the hubs and the transfer of these bundled flows through the hub arc [1]. Typical applications of hub locations include airline passenger travel [2], telecommunication systems [3] and postal networks [4].

The hub location problem was originally introduced by O'Kelly [5]. Later, a quadratic model formulation for a single allocation hub location problem was provided [6]. The first model objective was minimising the total cost of flows. The rest of the literature was about linearising and closing the formulation for real-world problems. Readers interested in the hub network problem can study Alumur and Kara [7] and Farahani et al. [8] which contains some recent trends on hub location. Campbell [9] categorised this problem into three important cases: hub median, hub centre and hub covering problem. Another classification in hub location is related to the capacity of hub nodes or arcs. Here, multi-modal system as one of important issues in transportation system is studied. Multi-modal transport, which is taking advantage of combination of transport modes for a trip between which a transfer is necessary, seems an interesting approach to solving transportation problems with respect to the deteriorating accessibility of city centres, recurrent congestion, and environmental impact [10]. Interested, readers in the multi-modal transportation study the SteadieSeifi et al. [11] and therein references.

Trade hubs are one of real applications of the multi-modal transportation system. The trade success of each country depends on its trade infrastructure, and the main component is transportation facilitators, so trade growth in each country can occur if the trade hubs are designed and developed properly. In contrast, trade hubs connect most trade routes with some facilitation to decrease the total transportation cost with the lowest delivery time, so according to its geographic position, each location should employ different modes of transportation systems. Hence, most of trade hubs use multi-modal transportation systems, and it is obvious that the goods can be imported by train from the origin country and re-exported by the sea to the destination country. Multi-modal transportation systems in the real trade hubs show that we need to consider the transportation type in the hub location problems, which is why we call it a multi-modal problem.

The remainder of the paper is organised as follows. In Section 2, the related literature is reviewed, and then the problem is stated. In Section 3, the model is introduced in details. In the next section, a tabu search-based algorithm is proposed to handle the model. In Section 5, we present the results of computational tests performed to analyse valid inequalities and the proposed algorithm performances. The paper ends with some conclusions drawn from this research.

2. RELATED WORKS AND PROBLEM DESCRIPTION

In this section, first, we review the literature of the hub covering problem, then the capacitated hub location problem is considered. Also, the problem charasteristics are described.

2. 1. Hub Covering Problem Campbell [9] provided the hub set and hub maximal covering problem with single and multiple allocation. Kara and Tansel [12] advanced the single allocation hub set covering problem formulation and its linearisation at a later time. A new formulation for the single allocation hub set covering problem was proposed by Wagner [13]. Moreover, a formulation for a multiple allocation hub set covering problem with quantity-independent discount factors has been introduced. Ernst et al. [14] suggested a novel formulation based on the coverage radius concept. This novel idea improved the single and multiple allocation hub set covering problems. Hamacher and Meyer [15] contemplated similarities and differences in the various formulations for hub covering problems, and analysed the feasibility polyhedron of this problem. Weng et al. [16] designed a new formulation for multiple allocation hub maximal covering problems. In addition, two artificial intelligence heuristic methods based on the genetic algorithm and tabu search to solve this model have been employed. Subsequently, Tan and Kara [17] provided hub covering formulations for cargo delivery systems and introduced a Turkish dataset.

Other work by Weng and Wang [18] improves the multiple allocation hub set covering model. Scatter search and genetic algorithm has been used for solving. Qu and Weng [19] proposed a path relinking method to solve the multiple allocation hub maximal covering problem. This method has been applied to Chinese hub airports and AP dataset. Calik et al. [20] studied the single allocation hub covering problem over incomplete hub networks and proposed an integer programming formulation for it. In addition, an efficient heuristic tabu based search has been presented. Ishfaq and Sox [21] presented intermodal hub location model in the logistic network. This model encompassed the dynamics of individual modes of transportation through transportation costs, modal connectivity costs, and fixed location costs under service time coverage requirements. Karimi and Bashiri [22] expressed hub set and maximal covering with single and multiple allocation strategies. These models decide the location of hubs and allocation of non-hub nodes to the located hub nodes subjected to pre determined travel time between two nodes in origin-destination. Two heuristic procedures to handle these models in a reasonable solution quality and computational time have been used. Recently, Alumur et al. [23] considered transportation travel times and costs together. They also let various modes between hubs. Different types of service time between O-Ds have been assured. In fact, a type of hub set covering was designed in their model. Unlike our work, they implemented multi-modal concept just on hub links. In addition, Alumur et al. [24] considered time-definite deliveries in an hierarchical hub location model. Their model allowed two modes between hubs. Unlike our work, their hirerachical multi-modal hub covering just has been taken into account on hub links. Ghodratnama et al. [25] presented a fuzzy possibilistic model for a multiple allocation hub covering location problem. Two objectives have been considered in their modelling. The first objective was to minimize the total costs of opening, reopening, activating, covering and transporters purchasing. The second was the sum of the times of travelling commodities from the origin to destination via hubs. Davari et al. [26] suggested an incomplete hub covering problem. Their model considered the imprecise flow between nodes. Mohammadi et al. [27] developed a new stochastic multi-modal model for the hub covering location with two objectives. Unlike our suggested model, the employed mode which was used between O-Ds was not determined with their model. However, just a new variable was used to find the number of required mode. Moreover, Setak and Karimi [28] formulated the gradual hub covering location problem. They employed a tabu search based heuristic algorithm to handle the large instances.

2. 2. Capacitated Hub Location Aykin [29] presented the capacitated hub-and-spoke network design problem in which hubs have a fixed capacity for directing flows between the nodes served by the system. The problem was formulated under a networking policy allowing both direct (nonstop) and hub-connected (one-hub-stop and two-hub-stop) services between the nodes. A branch and bound algorithm and a heuristic procedure partitioning the set of solutions on the basis of hub locations have been proposed. Campbell [9] proposed the single and multiple allocation capacitated hub location problem.

Other work by Aykin [30] analyses a capacitated hub location problem with fixed costs and a given number of hubs to be located. Two policies, which called strict and non-strict (direct connections are allowed) have been studied. Moreover, a counting algorithm and a tabu search-based greedy interchange heuristic have been proposed. Two new formulations for the capacitated single allocation hub location problem have been provided by Ernst and Krishnamoorthy [31]. Two heuristic approaches to cope with these models have been applied. The first one was tabu search, and the other one was the random descent. Optimal solutions by using an LP-based branch and bound method with the initial upper bound supplied by these heuristics have been achieved. Moreover, the algorithm has been evaluated on the AP data set, because the CAB data set does not contain fixed costs and capacities.

Subsequently, Ebery et al. [32] suggested the multiple allocation version of the capacitated hub location problem based on the Ernst and Krishnamoorthy [31] models. They constructed an efficient heuristic algorithm using the shortest paths. A model for the capacitated one-stop multiple allocation hub location problem has been presented by Sasaki and Fukushima [33]. This model considered capacity constraints both on hubs and arcs. Marin [34] presented splittable capacitated multiple allocation hub location problems and useful properties of the optimal solutions that can be used to speed up the resolution. Rodriguez and Salazar [35] proposed a model of the capacitated hub location problem in which the arcs linking hubs are not assumed to make a complete graph. A mixed integer linear programming formulation and two branch-and-cut algorithms based on decomposition techniques have been proposed. Randall [36] used an ant colony to solve the capacitated single allocation hub location problem. Costa et al. [37] introduced a second objective function to the model besides the traditional cost minimising function that tried to minimise the time to process the flow entering the hubs.

Afterward, Contreras et al. [38] proposed a Lagrangean relaxation to obtain tight upper and lower bounds for the capacitated single allocation hub location problem. Kratica et al. [39] presented a modified mixed integer linear programming formulation for the single allocation capacitated hub location problem. This modified formulation contains fewer variables and constraints compared with the existing problem formulations in the literature. Two evolutionary algorithms that use binary encoding and standard genetic operators adapted to the problem have been provided. Correia et al. [40] suggested a multiple capacity level hub location problem, which is a novel idea in the hub location problem. The classical formulation that Ernst and Krishnamoorthy [31] introduced was incomplete. Correia et al. [41] presented a note on this formulation and showed that the solution of the previous model is not correct. After one decade of study on the capacitated hub location problem, this model has been improved and corrected.

2. 3. Problem Definition In the previous research, single and multiple allocations, capacitated and uncapacitated, hub covering, hub centre and hub median problems have been discussed. Furthermore, transportation types between a non-hub to a hub, hub to hub and hub to non-hub nodes are determined before solving the problem. This paper considers the combination of transportation modes as a decision in solving the problems. Indeed, a mode is the transportation manner of operation in the hub-and-spoke models. Our proposed model distinguishes the best combination of modes with minimum transhipment cost for each O-D route in the multi-modal structure. Unlike the previous research which describes the single mode or two modes ([24, 42, 43]), we suggest multi-modal hub location problem on all links between O-Ds. Actually, as explained before, recently, Alumur et al. [23] consider multi-modal nature just in hub links. Moreover, Sedehzadeh and Tavakkoli-Moghaddam [44] present a capacitated single allocation multi-modal tree p-hub median location problem in which different transportation modes can be established between hub nodes. Zamani and Razmi [45] propose a single allocation p-hub location-routing problem with multimodality in hub link.

In this research, we provide the Multi-Modal Single Allocation Capacitated p-Hub Covering Problem (MMSACpHCP) model. This model merges the two important concepts in the hub location problem: capacity and covering. Moreover, selection of the combination of modes is a decision variable in the proposed model to create an efficient route in the hub-and-spoke network. MMSACpHCP is classified as a multi-modal design (see Figure 1). The scope and features of MMSACpHCP based on the studied literature are shown in this figure.

Kara and Tansel [46] and Ernst et al. [14] proved that hub covering problems can be categorised by NP-Hard class. We propose six valid inequalities to achieve tighten lower bound. Moreover, to cope with the MMSACpHCP model, we present a tabu search-based algorithm and test this method for a well-known AP data set.

3. MATHEMATICAL FORMULATION

This section is dedicated to describing the MMSACpHCP model formulation. We consider the time coverage in this model. If the time for the path from i to j via k and m is not greater than a specific value (β), then O-D (i, j) is covered by hubs k and m. We assume that $N=\{1,2,...n\}$ is node set with n nodes and $L=\{1,2,...l\}$ combination of modes. The triangle inequality is supposed for the all arcs. The mathematical model locates hubs from the node set and allocates the non-hub nodes to these hubs, depending on the capacity of the hub nodes and the coverage time bound. Moreover, the combination of modes for each O-Ds route is determined. So, the hub network is constructed. The model forces the travel time between any O-D pair to be less than a given time bound. The definition of the model decision variables and parameters are as follows:

Parameters:

$\alpha \in [0,1]$ is discount factor for the hub to hub link. β is time bound for each link between two nodes. W_{ij} is flow from node i to j. F_k is the fixed cost of a hub at node k. C_{ik}^{l} is the cost of transshipment from node i to k at combination of modes l, also, $C_{ikmj}^{l}=C_{ik}^{l}+\alpha C_{km}^{l}+C_{mj}^{l}$, $C_{ikmj}^{l}=C_{jmki}^{l}$ is the transshipment cost from nodes i to j via hubs k and m with combination of modes l. T_{ik}^{l} is the transportation time between nodes i and k at combination of modes l, moreover, $T_{ikmj}^{l}=T_{ik}^{l}+\alpha T_{km}^{l}+T_{mj}^{l}$, $T_{ikmj}^{l}=T_{jmki}^{l}$ is the travel time between nodes i and j via hubs k and m with combination of modes l. Γ_k is the capacity of the hub installed at node k. p is the number of required hubs.

V_{ikmj}^{l} is the coverage matrix. This matrix is calculated as following:

$$V_{ikmj}^{l}=\begin{cases}1 & T_{ikmj}^{l}\le\beta \\ 0 & o.w\end{cases} \qquad \forall i,k,m,j,l$$

Variables:

$X_{ikmj}^{l}=1$ if node i sends its flow via hub k and m to node j by combination of modes l ; otherwise $X_{ikmj}^{l}=0$. These types of variables in MMSACpHCP are continous. Rely on the formulation constraints, they just can take binary value. $Z_{ik}=1$ if non-hub node i allocates to hub node k; otherwise $Z_{km}=0$. $Z_{kk}=1$ means node k is a hub.

Taking into account the above definitions, the formulation that we propose for the MMSACpHCP is the following:

$$\min\sum_{l}\sum_{i}\sum_{k}\sum_{m}\sum_{j}W_{ij}C_{ikmj}^{l}X_{ikmj}^{l}+\sum_{k}F_kZ_{kk} \tag{1}$$

$$\sum_{k}Z_{kk}=p \tag{2}$$

$$\sum_{k}Z_{ik}=1 \qquad \forall i \tag{3}$$

$$\sum_{k}\sum_{m}\sum_{l}X_{ikmj}^{l}V_{ikmj}^{l}\ge 1 \qquad \forall i, j\ne i \tag{4}$$

$$\sum_{i}\sum_{j}W_{ij}Z_{ik}\le\Gamma_kZ_{kk} \qquad \forall k \tag{5}$$

$$\sum_{l}\sum_{j}\sum_{m}(W_{ij}X_{ikmj}^{l}+W_{ji}X_{jmki}^{l})=\sum_{j}(W_{ij}+W_{ji})Z_{ik} \qquad \forall i,k \tag{6}$$

$$X_{ikmj}^{l}\ge 0 \qquad \forall i,k,m,j,l \tag{7}$$

$$Z_{ik}\in\{0,1\} \qquad \forall i,k \tag{8}$$

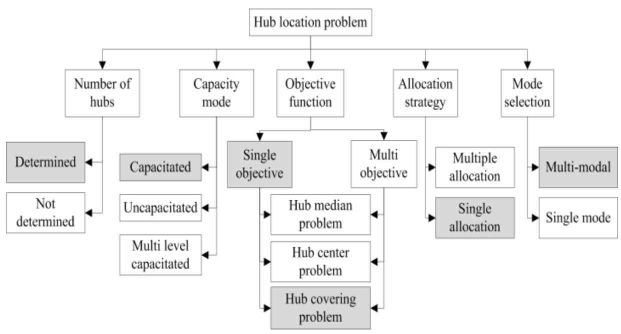

Figure 1. The scope of MMSACpHCP.

In this formulation, the objective function minimises the overall cost, which consists of two parts: the cost for routing the flow in the network and the cost for installing the hubs. The required hubs are satisfied by Constraint (2). Constraints (3) ensure that all nodes are allocated to a single hub. Constraints (4) ensure that the transportation time between each O-D pair does not exceed the time bound (β). Constraints (5) refer to the capacity of the hubs; also they guarantee that non-hub nodes can only be allocated to installed hubs. Constraint (6) involves divergence equations at node k for flow associated with node i. Constraints (7) and (8) are domain constraints. An example solution of this model is depicted in Figure 2.

The solution in Figure 2 has two combinations of modes defined in the figure. In addition, covering and capacity constraint should be considered in this solution. As stated by this figure, combination (1) is made by two types (truck and train), and combination (2) includes three types (truck, airplane and train). According to its solution, $X_{1234}^{1}=1$ implies that the first combination of modes can be selected for O-D (1, 4). Moreover, $X_{1234}^{2}=1$ infers that the second combination of modes can be chosen for this O-D. If combination (1) is selected for the route between O-D (1, 4), the combination of modes for (1, 4) is the same as (4, 1), since we suppose that the triangle inequality and symmetric data for travel cost and time between each two nodes.

Generally, the model contains n^2 binary variables, $n^4 l$ positive variable and $2n^2+2n-n+1$ constraints, where n and l are the number of nodes and combination of modes, respectively. These variables and constraints will make it extremely challenging to find the optimal solution, especially for large values of n. Therefore, in Section 4, we introduce a tabu search-based heuristic to solve the MMSACpHCP model for realistic network dimensions at a reasonable computational time.

3. 1. Valid Inequalities

One of the most efficacious methods to tighten the linear relaxations of the formulations is valid inequalities. These inequalities release some fractional solutions from the solution space such that a stronger lower bound can be achieved from linear programming (LP). In this paper, we employ the following six valid inequalities.

Constraints (9) and (10) are two sets of valid inequalities that caused some enrichment for the LP. These constraints state when the nodes k and m are both hub nodes, the total flow departing from k and/or destined to j should be routed via a hub link.

$$\sum_{l}\sum_{j\neq k} X_{kkmj}^{l} \geq Z_{kk}+Z_{mm}-1 \qquad \forall k, m\neq k \qquad (9)$$

$$\sum_{l}\sum_{j\neq k} X_{jmkk}^{l} \geq Z_{kk}+Z_{mm}-1 \qquad \forall k, m\neq k \qquad (10)$$

Another valid inequality is found on some variables characteristic of MMSACpHCP. By constraints (11), we ensure that node i is routed to hub node k by one type of transportation mode, when non-hub node i is allocated to hub node k.

$$\sum_{l} X_{ikkk}^{l} = Z_{ik} \qquad \forall i,k \qquad (11)$$

Constraints (12) are presented as a different valid inequality. By these constraints, when node k is a hub node, the summation of all hub links which applied one type of modality and connected to it should be at least equal to it. It must be noted that constraints (12) are valid when $p>1$.

$$\sum_{l}\sum_{m} X_{kkmm}^{l} \geq Z_{kk} \qquad \forall k \qquad (12)$$

As for the constraints (13), note that at least one O-D route is directed through selected hubs, when node k and m simultaneously are hub nodes and connected via one mode.

$$\sum_{l}\sum_{i}\sum_{j\neq i} X_{ikmj}^{l} \geq 2\sum_{l} X_{kkmm}^{l} \qquad \forall k,m \qquad (13)$$

The last valid inequality for MMSACpHCP is based on the number of non-hub allocation to hub nodes. When a node is a hub node, at most $N-p+1$ non-hub nodes can be allocated to it. The constraints (14) describe it.

$$\sum_{i} Z_{ik} \leq (N-p+1) Z_{kk} \qquad \forall k \qquad (14)$$

One of the properties of MMSACpHCP is the minimum number of hub nodes. This property can be calculated based on the capacity and flow of the nodes. First, we sort the capacity in descending order and call it capsort. The following procedure in Figure 3 finds the minimum number of hubs. We apply this property to define the p in tested instances.

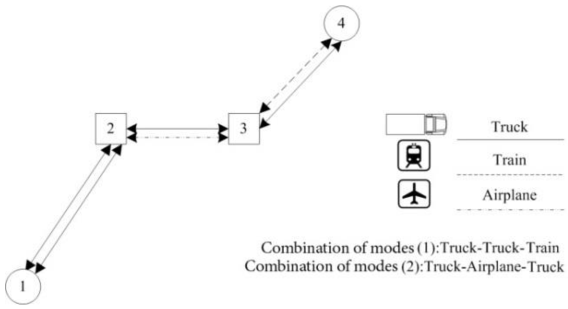

Figure 2. A feasible solution of MMSACpHCP.

Min number of hub=1;

While $\sum_i \sum_j W_{ij} / \sum_n^{Min\ number\ of\ hub} capsort_n > 1$

Min number of hub= Min number of hub+1;

End

Figure 3. Procedure for finding the minimum number of hubs

4. TABU SEARCH-BASED HEURISTIC ALGORITHM

For large scale instances, NP-hard problems are in most cases only tractable by heuristics. The renowned tabu search heuristic methodology is applied in order to avoid trapping at a local optimal solution. This approach has found vast acceptance for solving combinatorial optimisation problems for two reasons: it provides relatively good solutions without consuming much CPU time, and it is easy to execute in most situations.

TS scientific method was selected to depend on the earlier literature for p-hub location problem. Skorin-Kapov et al. [47] showed that TS can find high-quality solutions, which have less than 1% optimality gaps for the p-hub median problem. MMSACpHCP objective function and p-hub median problem are alike. TS handles the solution approach to run away from a local optimum and to move to former unexamined areas of the solution space [48]. A short-term memory is employed, which entered a node previously managed shifts. Such shifts are believed tabu for some period of time. This assists one to escape re-inspecting solutions that were lately seen. To veto a tabu move, an aspiration criterion is applied, so that not to miss a better solution throughout the search. A long-term memory list is used to begin the search process another time from an antecedent unvisited part of the solution space [49, 50].

Two aspects of TS are intensification face and diversification face. TS commences at a randomly generated initial solution and included with a set of hub nodes (p). We run the model with four types of neighbourhoods. A pair-wise exchange of a non-hub node with a hub node produces the neighbourhood solutions of the current solution. This interchange is called a shift or move. As a move is made, its qualities such as entering a node and exiting node are listed in a tabu list. The search moves from one solution to the next and terminated when local optimum is procured, in the intensification aspect. After it is completed, TS takes part in the diversification aspect. In this aspect, the TS restarts the search procedure from a new solution.

The pseudo-code of TS developed for this study is shown in Figure 4. The TS parameters are set at the beginning of procedure. The while loop in line 5 executes the main body of TS. The search begins with a randomly random??? solution in which hub locations are picked out. In sequential iterations, the set of hub locations change under pair-wise interchange. Pair-wise swap exchanges two hub and non-hub nodes. As we have n nodes and p hub nodes, $(n-p)^p$ pair-wise swap should be done. From a given set of hub locations, the optimal solution can be generated by finding the hub-spoke structure which minimises the cost of hub-and-spoke network. The structure must be feasible according to the service time coverage necessities and capacity of chosen hub nodes. Two important characteristics of our TS based approaches are explained in the following.

Combination of modes selection. We select the best combination that relies on the coverage constraint and cost. First V_{ikmj}^l and C_{ikmj}^l are generated. Subsequently, the best combination (BM_{ikmj}) for each O-D is selected as follows: $BM_{ikmj} = \left\{ (i,k,m,j) \mid \min_l V_{ikmj}^l C_{ikmj}^l \right\}$. According to the chosen combination for each O-D, the allocation procedure can be done.

Allocation strategy. In the TS approach, a heuristic procedure to allocation non-hub nodes to selected hub nodes in each swap is suggested. We define total cost (TC_{ik}) for each non-hub and its potential hub as $TC_{ik} = \sum_{l \in L} \sum_{j \in N} \sum_{m \in H} w_{ij} C_{ikmj}^l \quad \forall i \in N, k \in H$, where N is set of nodes, H set of selected hub and L set of combination of modes. Then, for each non-hub, a hub node can be found by minimising the total cost as follows: $IX = \left\{ (i \in N, k \in H) \mid \min_k TF_{ik} = TF_{ik} \right\}$, where IX is an index. Hence, the single allocation strategy obtains using $allocate\ (i \in IX) = hub(k \in IX)$. The pseudo code of the tabu search algorithm is described in Figure 4.

In the absence of optimal solutions to use as benchmarks, upper bounds or lower bounds are needed. These lower bounds are required to be tight for the solution evaluation to be accurate in MMSACpHCP. The lower bounds of MMSACpHCP are gained by using LP for it with the suggested valid inequality. Parameters were chosen after extensive computational testing and analysis both on solution quality and computational efficiency. To test the performance of TS, we execute it on some benchmarks in the next section.

5. COMPUTATIONAL EXPERIMENTS

In this section, the performance of valid inequalities, the tabu search-based algorithm and time bound under various conditions is considered. A number of standard problem benchmarks are applied.

```
maxriter←(p-1)n; maxcount←(n-p)^p; listsize ←⌊n/3⌋,
current solution = generate feasible random solution ();
best solution = current solution;
best current solution←∞
while (runcount < maxiter) {
 count = 0;
 for (count) to (maxcount) do {
 count← count+1;
 generate a pair-wise shift;
        if (shift ∉ tabulist){
        update current solution;
        check the feasibility;
               if (current solution < best current solution ) {
               update best current solution;
               best count←count;
               }
        }
 }
 best solution← best current solution;
 runcount← runcount+1;
 if (current solution < best solution){
 update best solution;
 }
 clear tabu list;
 update tabulist for best count ;
 }
report best solution;
```

Figure 4. Pseudo code of the proposed tabu search-based algorithm.

The most well-known and established data set for testing the performance of algorithms for hub location problems is the CAB data set. However, the CAB data set cannot be used in the MMSACpHCP model that we are attempting to solve, because the data set does not include capacity restrictions and fixed costs on the nodes. We apply the AP dataset, which is derived from a real application to a postal delivery network. It includes 200 nodes, which portray postcode districts; the direction of their coordinates and flow volumes (mail flow); in addition, here, $\alpha = 0.75$. A feature of this data is that the flow matrix is not symmetrical (i.e., $W_{ij} \neq W_{ji}$ and $W_{ii} \neq 0$). We employ near to half of the time bounds which Ernst et al. [14] calculated them.

Results for the AP data set for problems of size n = 10, 20 are given. All the tests were executed in MATLAB 7.8 (for TS) and GAMS 23.5 with CPLEX 12 solver. They were run on an experimental computer that is equipped with 2.99 GB of RAM and a Pentium microprocessor running at 2.53 GHz. For the larger problems (n>20), our machine cannot run the model, since they employ much memory. So, the exact solution cannot be reached.

The AP data set also includes capacities and fixed costs on the nodes. Two types of fixed costs: tight (T) and loose (L) are considered. We also consider two types of capacities: tight (T) and loose (L). For each benchmark, four types of problems LL, LT, TL and TT are used. Since the AP data set does not consider multi-modal data, we take into account four combination of modes for these benchmarks. We use some coefficients for travel time and cost of combination (coefficient for travel time of mode (φ), coefficient for cost of mode (η)) for each link between O-D, so three coefficients pairs must be applied for each O-D. The following assumptions are embedded in table 1 for AP data set. The following calculations are run for the dataset:

$T^{l}_{ikmj|i\geq j} = \varphi_1 T_{ik} + \varphi_2 \alpha T_{km} + \varphi_3 T_{mj}$, $T^{l}_{jmki|i\geq j} = T^{l}_{ikmj|i\geq j}$,

$C^{1}_{ikmj|i\geq j} = \eta_1 C_{ik} + \eta_2 \alpha C_{km} + \eta_3 C_{mj}$ and $C^{l}_{jmki|i\geq j} = C^{l}_{ikmj|i\geq j}$

Moreover, we defined two time bounds for each benchmark instance. The first coverage limit (i.e., lower one) is tighter than the second one for each test.

5. 1. Valid Inequality Performance Comparisons among valid inequalities are done in the first part of the computational study. Seven forms of MMSACpHCP are run with respect to the valid inequalities. The first form relies just on pure linear relaxation of MMSACpHCP, called LR. Valid inequalities (9-14) are added to the model, and implemented in the second, third, fourth, fifth and sixth forms, respectively. Finally, MMSACpHCP with all valid inequalities are placed as the last form. The results of linear relaxation objective function gap (i.e., $\left(1-\left(relaxed\ solution / Optimal\right)\right)\times 100$) are given in Tables 2 for each instance size.

An inclusive overview of the instances results runs on AP is provided in the last row (i.e., “average”) of Table 2. As a general trend, in average, using all valid inequalities (i.e., column “All”) give the best lower bound resulting, and tighter than any other implementation. In fact, using all valid inequalities lifts %1.251 lower bound of pure linear relaxation of MMSACpHCP. Moreover, valid inequality (11) is the most effective inequality in tightening the model.

5. 2. TS Performance TS solution approaches are run ten times for instances and the average values are reported, in this subsection. According to Table 3, the CPLEX consumes much more average computational time than the proposed tabu search-based algorithm (average CPU time for CPLEX and TS are 335.1435 and 38.76856, respectively).

Figure 5 shows the meaningful improvement in computational time to solve MMSACpHCP with TS for AP20. We let CPLEX use the same time as TS.

TABLE 1. Coefficient of combinations.

Combination \ Coefficients	(φ_1,η_1)	(φ_2,η_2)	(φ_3,η_3)
1	(1,1)	(1,1)	(1,1)
2	(0.3,1.7)	1	(1,1)
3	(0.3,1.7)	(0.3,1.7)	(1,1)
4	(0.3,1.7)	(0.3,1.7)	(0.3,1.7)

Then, the qualities of gaps for TS and CPLEX are compared (see Figure 6). In Figures 5 and 6, the odd and even numbers in horizontal axis express $p=2$ and $p=3$, respectively. In Figure 6, 1 to 16 relates to the AP 10, and the remainders are for AP 20.

TS has a reasonable objective function value (OFV) for MMSACpHCP. The tabu search-based algorithm performs better when the problem constraints are loose, and the number of nodes is increased. Moreover, it can be used for all MMSACpHCP problems. The TS achieves a small gap for the tested instances. The gap for TS and CPLEX in the same time as TS is defined as $((Final\ \ solution\ /\ Optimal)-1)\times 100$ (i.e., the gap in CPLEX vs. TS column).

TABLE 2. The GAP (%) performances of the implementations of MMSACpHCP.

Name	Min p		p	Optimal	LR	(9)-(10)	(11)	(12)	(13)	(14)	All
10LL	1	17.906	2	160528.656	23.931	23.931	22.932	23.785	23.733	23.748	22.449
			3	171221.744	16.923	16.754	15.900	16.820	16.671	16.809	15.405
		19.696	2	158635.139	24.442	24.442	23.503	24.280	24.356	24.248	23.064
			3	170198.895	17.761	17.587	16.899	17.686	17.552	17.624	16.294
10LT	2	17.906	2	172987.944	22.416	22.416	21.863	22.186	22.416	22.416	21.571
			3	184392.261	18.708	18.540	17.562	18.521	18.704	18.708	17.185
		19.696	2	168264.716	21.467	21.467	20.861	21.289	21.467	21.467	20.696
			3	178942.801	17.236	17.149	16.001	17.102	17.235	17.236	15.715
10TL	1	17.906	2	163207.245	18.978	18.930	17.553	18.683	18.978	17.583	16.406
			3	183789.823	16.788	16.636	14.564	16.629	16.788	15.920	13.608
		19.696	2	159788.123	18.860	18.845	17.328	18.605	18.860	17.578	16.220
			3	180980.361	16.346	16.209	14.645	16.228	16.308	15.623	13.606
10TT	2	17.906	2	174312.449	18.989	18.959	18.243	18.698	18.989	18.989	18.054
			3	189759.246	15.806	15.423	13.460	15.526	15.806	15.806	13.233
		19.696	2	168808.940	17.588	17.564	16.775	17.312	17.588	17.588	16.557
			3	186812.528	15.289	15.041	13.385	15.111	15.289	15.289	13.129
20LL	1	26.001	2	153809.395	24.202	24.202	23.664	24.190	24.191	23.973	23.442
			3	170682.183	19.464	19.444	18.920	19.457	19.439	19.316	18.724
		28.601	2	148229.909	22.414	22.413	21.873	22.396	22.404	22.169	21.652
			3	167002.748	18.572	18.549	18.076	18.563	18.555	18.427	17.879
20LT	2	26.001	2	160067.477	16.255	16.251	15.563	16.202	16.202	16.255	16.255
			3	177836.354	16.334	16.325	15.549	16.313	16.313	16.334	16.334
		28.601	2	156176.110	15.916	15.899	15.185	15.864	15.864	15.916	15.916
			3	173556.859	15.093	15.073	14.256	15.047	15.047	15.065	15.093
20TL	1	26.001	2	159660.636	20.170	20.156	19.618	20.150	20.150	20.170	19.670
			3	181854.962	16.529	16.492	15.968	16.522	16.522	16.525	16.281
		28.601	2	153820.299	18.246	18.219	17.823	18.227	18.227	18.246	17.765
			3	176741.318	15.079	15.047	14.543	15.073	15.073	15.079	14.874
20TT	2	26.001	2	202052.196	24.665	24.648	24.250	24.641	24.641	24.665	24.665
			3	208080.423	17.584	17.571	16.974	17.570	17.570	17.584	17.584
		28.601	2	197988.334	24.658	24.633	24.310	24.638	24.638	24.658	24.658
			3	202908.250	16.913	16.886	16.291	16.901	16.901	16.912	16.913

5. 3. Time Bound Performance When the instance is loose for time bound, the CPLEX average CPU time is 356.7238 and for tight time bound, it is 313.56325. We conclude that the loose time bound consumes much more time to solve the problem.

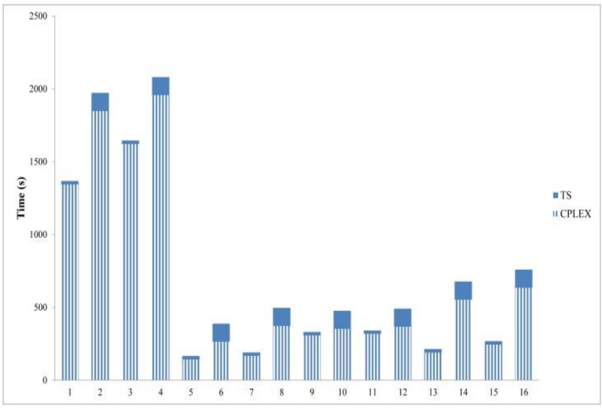

Figure 5. TS and CPLEX computational times (s) comparison for AP 20.

Moreover, the objective function values for tight time bound are more than the loose ones. It is observed that the solution space will be more restricted when the time bound is tight. So, the time can be reduced to find the optimal solution and its cost at least is the same as loose time bound.

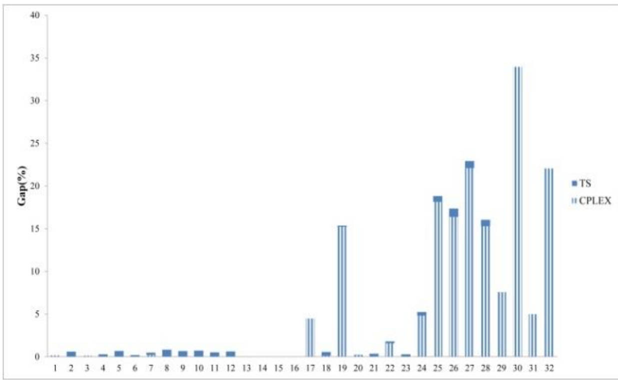

Figure 6. CPLEX quality gap vs. TS, in the same computational time as TS.

TABLE 3. Computational comparison of CPLEX and the tabu search-based algorithm.

I	Min *p*	*β*	*p*	CPLEX		CPLEX vs. TS (d)		
				Optimal	Time(s)	Gap (%) CPLEX	Gap (%) TS	Time(s)
10LL	1	17.906	2	160528.656	2.459	0.132	0.000	1.339
			3	171221.744	3.006	0.000	0.598	5.982
		19.696	2	158635.139	5.753	0.121	0.000	1.376
			3	170198.895	4.468	0.000	0.282	5.942
10LT	2	17.906	2	172987.944	2.156	0.000	0.682	1.284
			3	184392.261	2.660	0.000	0.193	5.901
		19.696	2	168264.716	3.995	0.237	0.237	1.291
			3	178942.801	2.842	0.000	0.825	5.994
10TL	1	17.906	2	163207.245	1.366	0.000	0.661	1.344
			3	183789.823	1.884	0.000	0.721	6.003
		19.696	2	159788.123	2.035	0.001	0.518	1.332
			3	180980.361	2.170	0.000	0.621	5.989
10TT	2	17.906	2	174312.449	1.549	0.000	0.000	1.297
			3	189759.246	1.731	0.000	0.000	5.916
		19.696	2	168808.940	2.098	0.000	0.000	1.256
			3	186812.528	2.251	0.000	0.000	5.959
20LL	1	26.001	2	153809.395	1344.149	4.440	0.001	23.305
			3	170682.183	1847.487	0.126	0.431	124.851
		28.601	2	148229.909	1621.665	15.237	0.120	23.415
			3	167002.748	1957.004	0.171	0.041	124.162
20LT	2	26.001	2	160067.477	142.278	0.051	0.303	23.523
			3	177836.354	263.971	1.573	0.214	123.814
		28.601	2	156176.110	167.401	0.041	0.240	23.312
			3	173556.859	371.940	4.822	0.421	124.501
20TL	1	26.001	2	159660.636	307.239	18.154	0.665	23.728
			3	181854.962	352.523	16.391	0.964	124.099
		28.601	2	153820.299	318.135	22.088	0.842	23.058
			3	176741.318	366.663	15.296	0.734	124.081
20TT	2	26.001	2	202052.196	189.374	7.537	0.010	24.321
			3	208080.423	553.180	33.946	0.000	123.915
		28.601	2	197988.334	244.359	4.981	0.000	23.712
			3	202908.250	634.802	22.056	0.000	124.592

6. CONCLUSIONS

The MMSACpHCP model has a wide range of applications in trade hubs networks when multi-modal transportation is considered. In this paper, in addition to modelling the multi-modal single allocation capacitated p-hub covering problem, to solve realistically sized problems, we provide a tabu search-based heuristic algorithm. Moreover, family of valid inequalities to strengthen the formulation are proposed. The innovation of this study is multi-modality on all links with the capacity constraint on the hub nodes under the predetermined time bound. In multi-modal hub-and-spoke network, each O-D can be connected by combination of modes which is including at most three types of transportation means. Constructing feasible solutions for the MMSACpHCP model in contrast to the other hub location problems, particularly with tight time bounds is difficult. Consequently, we apply a heuristic approach to the allocation non-hub to hub nodes and construct feasible solutions. We tested the TS algorithm on the well-known AP data set. We compared the performance of the TS with CPLEX and found that TS obtained more efficient solutions. The computational times are better than CPLEX in the tested instances. The outcomes also present that using all suggested valid inequalities have a good performance on MMSACpHCP.

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Capacitated Single Allocation P-Hub Covering Problem in Multi-modal Network Using Tabu Search

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چکیده

هدف از مسئله مکان‌یابی محور، یافتن مکان‌های محور و تخصیص غیرمحورها به این مکان‌هاست. در این تحقیق، ما مسئله مکان‌یابی p-محور پوششی ظرفیت‌دار در حالت تخصیص یگانه و چندوجهی در حالت شبکه کامل محوری را مورد بحث قرار دادیم. به همین جهت، یک فرمول‌بندی برای این مسئله ارائه شده است. هدف مدل ما یافتن مکان‌های محور و ارتباطات محوری آن‌ها با توجه به وجه حمل و نقلی برای هر مبدأ–مقصد است. علاوه بر این، تخصیص غیر محورها به محورها در بهترین وجه حمل و نقلی تعیین خواهد شد تا زمان سفر از هر مبدأ به هر مقصد بیشتر از یک حد زمانی داده شده، نشود. همچنین، یک الگوریتم جستجوی ممنوعه ابتکاری نیز ارائه شده که عملکرد این الگوریتم بر روی مجموعه داده‌های پست استرالیا مورد بررسی قرار گرفته است..

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