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Introducing the Time Value of Money in a Non-consignment Vendor Managed Inventory Model

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A B S T R A C T

Vendor managed inventory (VMI) is an integrated approach for buyer–vendor coordination, according to which the vendor (supplier or manufacturer) decides on the appropriate buyer's (retailer's) inventory levels. The time value of money has not traditionally been considered in evaluating VMI supply chain's total inventory cost in any studies up to now. Therefore, in the present study a new model for two-echelon single-manufacturer multi-retailer supply chain under non-consignment VMI program by considering time value of money is proposed. In order to take the time value of money into consideration, the present value of each inventory cost is evaluated in a single period and generalized to infinity horizon and then is transformed to the equivalent annual cost. This model also explicitly includes contractual agreements between the manufacturer and each retailer. Under this type of contracts, an upper bound on each retailer's inventory level is placed such that the manufacturer is penalized for items exceeding this bound. At the end, a sensitivity analysis is conducted to study effects of key parameters on the optimal solution and validate the proposed model.

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1. INTRODUCTION

Vendor managed inventory (VMI) is a partnership between a supplier (often a manufacturer) and a buyer (described here as a retailer) whereby the supplying organization makes inventory replenishment decisions on behalf of the buyer [1-3]. As participants of a VMI program, the buyer may benefit from cost saving or profit increasing, while the supplier may benefit by integrating his operational decisions of production and supply so as to attain economies of scale and flexible deliveries in the distribution process [4-6].

Although VMI programs bring benefits to participants, there are potential challenges in implementing VMI programs [4]. For example, under a non-consignment VMI process, the downstream buyer pays for an item as soon as he receives it, thus, the buyer is the owner of the inventory in his site and incurs holding cost. In this case, it is in the benefit of the supplier to push a lot of inventory downstream to save on his holding and dispatching costs. As a result, the buyer incurs a considerable holding cost. To overcome this deficiency, two common approaches have been proposed: Consignment VMI (supplier owned inventory): This is a modification of non-consignment VMI in which the supplier owns the items in the buyers' warehouses until they are sold [7].

Contractual agreements: Under this kind of contracts, an upper bound on the buyer's inventory level is set during his negotiations with supplier such that the supplier agrees to pay a penalty cost per unit to the buyer for every unit of the buyer's inventory that is more than the upper bound [8].

In recent years, contract design for VMI programs are recognized to be an important issue, but only a few researches have been published on it [4] such that the contractual agreements have not been explicitly reflected in VMI analytical models presented in the VMI literature except for Fry et al. [8], Shah and Goh [9], and Darvish and Odah [10]. Also, to the best of the authors' knowledge, the time value of money has not traditionally been a consideration in evaluating the VMI supply chain's total inventory cost in any studies up to

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now; this, however, is necessary to be considered because each party in the supply chain (SC) incurs inventory costs at different time. In order to cover these research gaps, efforts are made in this paper to investigate a decision problem of a two-echelon single-manufacturer multi-retailer supply chain under the contractual agreements by considering the time value of money.

The outline of the remainder of the paper is as follows: a review of the literature is presented in Section 2. We define the problem in Sections 3. Section 4 deals with assumptions and notations, and then presentation of the model. In Section 5, a sensitivity analysis is conducted to study effects of key parameters on the optimal solution and validate the proposed model. Finally, summery and conclusions are provided in Section 6.

2. LITERATURE REVIEW

There are two main research streams with regard to the subject under study in this paper: single-vendor and multi-buyer coordination models under VMI programs and contract designs for VMI programs. We review the works that are important to our problem in the following two sections:

2. 1. Two-echelon Single-vendor Multi-buyer Supply Chains (TSVMBSC) under VMI Programs

Woo et al. [11] modeled an integrated inventory system where a single vendor purchases and processes raw materials in order to deliver finished items to multiple buyers. In their study, the vendor and all the buyers are willing to invest in reducing the ordering cost in order to decrease their joint total cost. Zhang et al. [12] extended the work of Woo et al. [11] by relaxing their assumption of a common cycle time for the vendor and all the buyers.

Jasemi [13] considered a TSVMBSC and compared performances of a VMI system with a traditional one. He also made a pricing system for profit sharing between parties. Nachiappan and Jawahar [14] formulated a TSVMBSC under VMI mode of operation as a non-linear integer programming problem (NIP), and then proposed a Genetic Algorithm based heuristic for solve it. Also, Sue-Ann et al. [15] considered the problem of Nachiappan and Jawahar [14] and presented a hybrid of Genetic Algorithm and Artificial Immune System (GA–AIS) to find the optimal solution.

Taleizadeh et al. [16] developed a TSVMBSC model of VMI system in which both the raw material and the finished product had different deterioration rates. Pasandideh et al. [17] proposed a bi-objective mathematical model for a single manufacturer multi retailers VMI supply chain in which both the manufacturer and retailers' profits was maximized. Then, the bi-objective problem was formulated as a lexicographic max–min problem in order to find a Pareto optimal solutions. Sadeghi et al. [18] presented a combinatorial optimization model for TSVMBSC under VMI program with fuzzy demand.

2. 2. Contract Designs for VMI Programs

In recent years, contract designs for VMI programs are recognized to be an important issue, but only a few researches have been published [4]. For example, Cachon and Lariviere [19] analyzed VMI contracts with revenue sharing, in which a manufacturer facing an uncertain demand offers various contracts to a component supplier.

Yu et al. [20] studied a single-manufacturer multi-retailer VMI supply chain, and discussed how a manufacturer can take advantage of his retailers' market-related information for increasing his own profit by using a Stackelberg game and improved it using a cooperative contract.

Wong et al. [21] studied a sales rebate contract to help coordinate a TSVMBSC under VMI program. Guan and Zhao [4] dealt with contracts for TSVSBSC under consignment and non-consignment VMI programs. They designed a revenue sharing contract for consignment VMI and a franchising contract for non-consignment VMI.

Yao et al. [1] showed how a manufacturer uses an incentive contract with a distributor in a VMI program to gain market share as well as how the manufacturer inspirits the distributor's efforts to convert potential lost sales. Lee and Cho [22] developed a model of designing a vendor-managed inventory (VMI) contract with consignment stock and stockout-cost sharing in a (Q, r) inventory system between a supplier and a retailer.

Also, to the best of our knowledge, there are only three studies in the VMI literature that include the contractual agreements: Fry et al. [8] investigated a TSVSBSC under the contractual agreements. Shah and Goh [9] modeled a VMI problem in a context of supply hub with a single retailer where a contractual agreement is explicitly included. Darwish and Odah [10] developed a model that explicitly incorporates this contract into a TSVMBSC.

Furthermore, the time value of money has not traditionally been a consideration in evaluating the VMI supply chain's total inventory cost in any studies up to now; this, however, is necessary to consider because each party in the SC incurs inventory costs at different time. Therefore, the contribution of this paper is to cover these research gaps in a two-echelon single-manufacturer multi-retailer supply chain under the contractual agreements. To the best of the authors' knowledge, the present study is the first research that introduces the time value of money in VMI modeling.

3. PROBLEM DEFINITION

Consider a non-deteriorating single-item two-echelon single-manufacturer multi-retailer supply chain in which shortages aren't allowed. The manufacturer produces inventory by constant rate p. When on hand inventory at the manufacturer becomes a natural multiple of a total replenishment quantity of all the retailers (nq), the production is paused. At this moment, the manufacturer dispatches a total replenishment quantity (q) to all the retailers instantaneously. As a result, the manufacturer's inventory level decreases to $(n-1)q$ and inventory level for each retailer increases by q_j.

Based on VMI systems, it is assumed that the manufacturer replenishes the retailers at the same time. This is a reasonable assumption in VMI programs because the manufacturer makes decisions regarding the replenishment timing and amount. Each retailer consumes his inventory by constant rate D_j. According to assumption of the simultaneous replenishment for all the retailers, inventory level for each of the retailers becomes zero at the same time. At this moment, the manufacturer dispatches the total replenishment quantity (q) to all the retailers again instantaneously. This replenishment cycle is repeated until the manufacturer's inventory level reaches zero. Afterwards, for $(\frac{q}{D}-\frac{nq}{p})$ time units, inventory level of the manufacturer remains zero and then by resuming the production at the rate p, a new cycle for the manufacturer is started (Figure 1).

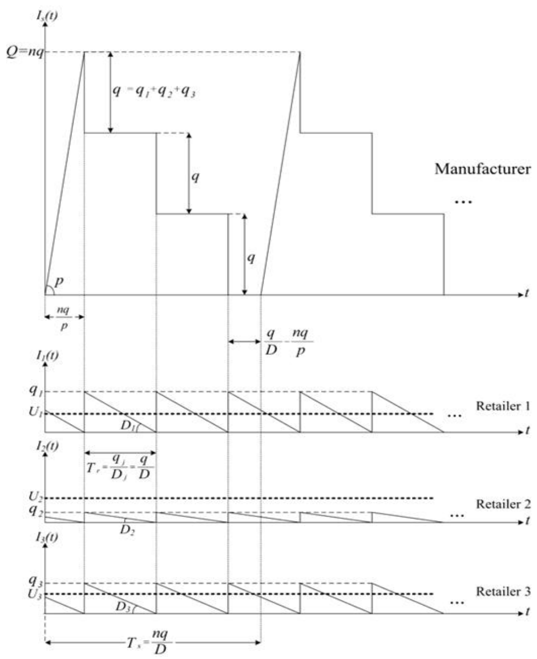

Figure 1. Inventory level against time for a single manufacturer and three retailers

While VMI can be implemented in conjunction with consignment, our paper focuses on non-consignment VMI setting in order to isolate impacts of the transfer of replenishment decisions to the manufacturer. Under non-consignment VMI environment, it is in the benefit of the manufacturer to push a lot of inventory downstream to save on the holding and dispatching costs. In order to prevent this trend, non-consignment VMI program includes contractual agreements between the manufacturer and each retailer. Under each contract, an upper bound U_j is set on the retailer j's inventory level such that the manufacturer pays a penalty cost per unit per unit time (π_j) to this retailer for every unit of the retailer j's inventory that is more than the upper bound U_j.

The aim of this problem is to find optimal operating policies for the manufacturer and the retailers by considering the time value of money such that total annual cost of the whole supply chain is minimized.

4. MATHEMATICAL MODEL

4. 1. Assumptions and Notations The mathematical model in this research was developed on the basis of the following assumptions:

1) A two-echelon single-manufacturer multi-retailer supply chain with a non-deteriorating single-item under non-consignment VMI is considered.
2) Planning horizon is assumed to be infinite.
3) In order to take the time value of money into consideration, each of the inventory costs is discounted by a continuous compounding discount rate.
4) Demand and production rates are deterministic and constant.
5) The production rate is finite and greater than the sum of all retailers' demand rates.
6) Shortages are not allowed.
7) The lead time is zero.
8) The manufacturer replenishes the retailers at the same time.
9) Each retailer can replenish more than once during the manufacturer's cycle time.
10) There are no constraints on the capacity of warehouses, number of orders, production resources, and investment involved in inventory.
11) Based on VMI systems, it is assumed that the manufacturer replenishes the retailers at the same time.

The following notations are used in the developed model:

r: discount rate,

m: number of retailers,

p: production rate for manufacturer,

D_j: demand rate for retailer j,
$D = \sum_{j=1}^{m} D_j$: total demand rate for all retailers,
A_s: setup cost per production run for manufacturer,
A_j: ordering cost for retailer j per order,
h_s: manufacturer's holding cost per unit product per unit time,
h_j : holding cost for retailer j per unit product per unit time,
π_j: over stock penalty cost for retailer j per unit product per unit time,
U_j: upper limit on the inventory level of retailer j,
T_j: cycle time for retailer j,
T_r: common cycle time of retailers,
T_s: manufacturer's cycle time,
q_j: replenishment quantity of retailer j,
$q = \sum_{j=1}^{m} q_j$: total replenishment quantity of all retailers,
n: number of shipments received by a retailer during the manufacturer's cycle time,
$Q = nq$: manufacturer's production quantity,
S: set of all retailers whose upper limit is exceeded,
$\bar{S}$: set of all retailers whose upper limit is not exceeded (complement of the set S),
$I_s(t)$: manufacturer's inventory level in terms of time,
$I_j(t)$: retailer j's inventory level in terms of time,
OC_j : equivalent annual ordering cost of retailer j,
OC_r : total equivalent annual ordering cost of all retailers,
HC_j : equivalent annual holding cost of retailer j,
HC_r : total equivalent annual holding cost of all retailers,
OC_s : equivalent annual setup cost of manufacturer,
HC_s : equivalent annual holding cost of manufacturer,
PC_j : equivalent annual penalty cost of manufacturer for violating the upper limit of retailer j,
PC_r : total equivalent annual penalty cost of manufacturer for violating the upper limits of all retailers,
AC_s : total equivalent annual cost of manufacturer,
AC_j : total equivalent annual cost of retailer j,
AC_r : total equivalent annual cost of all retailers,
AC: total equivalent annual cost of the whole supply chain,
M: a very large positive number, and
y_j: a binary variable equal to 1 if the retailer j be a member of the set S, and 0 otherwise.

4. 2. Modeling According to the simultaneous replenishment assumption for all the retailers we have:

$$T_i = T_j = T_r = \frac{q_i}{D_i} = \frac{q_j}{D_j} = \frac{q}{D} \quad \forall i,j \in \{1,2,\dots,m\} \Rightarrow$$
$$q_j = \frac{q}{D} D_j \quad \forall j \in \{1,2,\dots,m\}. \tag{1}$$

Based on Figure 1, the manufacturer's cycle time will be:

$$T_s = \frac{nq}{D}. \tag{2}$$

According to Figure 1, the manufacturer's inventory level in terms of time during the time interval $\left[0,\frac{nq}{p}\right]$ is $I_s(t) = pt$. So, in this interval, the manufacturer's holding cost at time t in a very small time interval dt can be obtained as follows:

$$h_s pt dt. \tag{3}$$

By discounting this cost to time zero, the present value of Equation (3) will be:

$$h_s pt e^{-rt} dt. \tag{4}$$

By integrating Equation (4) on the time interval $\left[0,\frac{nq}{p}\right]$, the present value of the manufacturer's holding cost during this interval is obtained as follows:

$$\int_0^{\frac{nq}{p}} h_s pt e^{-rt} dt. \tag{5}$$

According to Figure 1, the manufacturer's inventory level in terms of time during the time interval $\left[\frac{nq}{p},\frac{nq}{p}+\frac{q}{D}\right]$ is $I_s(t) = (n-1)q$. As a result, in this interval, the present value of the manufacturer's holding cost is calculated as follows:

$$\int_{\frac{nq}{p}}^{\frac{nq}{p}+\frac{q}{D}} h_s (n-1) q e^{-rt} dt. \tag{6}$$

Hence, in the time interval $\left[0,\frac{nq}{D}\right]$ (during the manufacturer's cycle time), the present value of the manufacturer's holding cost is:

$$\int_0^{\frac{nq}{p}} h_s pt e^{-rt} dt + \sum_{i=1}^{n-1} \int_{\frac{nq}{p}+(i-1)\frac{q}{D}}^{\frac{nq}{p}+i\frac{q}{D}} h_s (n-i) q e^{-rt} dt. \tag{7}$$

Considering infinite horizon, the present value of Equation (7) will be:

$$\left(\int_0^{\frac{nq}{p}} h_s pt e^{-rt} dt + \sum_{i=1}^{n-1} \int_{\frac{nq}{p}+(i-1)\frac{q}{D}}^{\frac{nq}{p}+i\frac{q}{D}} h_s (n-i) q e^{-rt} dt\right) \times (1 + e^{-rT_s} + e^{-2rT_s} + \cdots) = \left(\int_0^{\frac{nq}{p}} h_s pt e^{-rt} dt + \sum_{i=1}^{n-1} \int_{\frac{nq}{p}+(i-1)\frac{q}{D}}^{\frac{nq}{p}+i\frac{q}{D}} h_s (n-i) q e^{-rt} dt\right) \times \sum_{i=0}^{\infty} e^{-irT_s}. \tag{8}$$

Also, considering infinite horizon, Equation (9) holds between the present value (PV) and equivalent annual value (EAV) [23]:

$$EAV = r \times PV. \tag{9}$$

Therefore, the equivalent annual value of the manufacturer's holding cost is calculated according to Equation (10):

$$〖HC〗\_s^{\wedge} = (h\_s/r^{\wedge}2\ (p - pe^{\wedge}((-rnq)/p) - rnqe^{\wedge}((-rnq)/p)\ ) + (h\_s\ qe^{\wedge}((-rnq)/p))/(1 - e^{\wedge}(-r\ q/D)\ )\ (e^{\wedge}((-rnq)/D) + n - ne^{\wedge}(-r\ q/\ D) - 1)) \times (r/(1 - e^{\wedge}((-rnq)/D)\ )). \tag{10}$$

Always, jth retailer's inventory level in terms of time is $I_j(t) = (q_j - D_j t)$. For each retailer as a member of the set S, the present value of the manufacturer's penalty cost for violating the upper limit of retailer j during the time interval $\left[0, \frac{q}{D}\right]$ is equal to:

$$\int_0^{\frac{q_j - U_j}{D_j}} \pi_j (q_j - D_j t - U_j) e^{-rt} dt \quad \forall j \in S. \tag{11}$$

Therefore, during the time interval $\left[0, \frac{nq}{D}\right]$, the present value of Equation (11) will be:

$$\left(\int_0^{\frac{q_j - U_j}{D_j}} \pi_j (q_j - D_j t - U_j) e^{-rt} dt\right) \times \left(1 + e^{-r\frac{q_j}{D_j}} + e^{-2r\frac{q_j}{D_j}} + \dots + e^{-(n-1)r\frac{q_j}{D_j}}\right) = \left(\int_0^{\frac{q_j - U_j}{D_j}} \pi_j (q_j - D_j t - U_j) e^{-rt} dt\right) \times \sum_{i=0}^{n-1} e^{-ir\frac{q_j}{D_j}} \quad \forall j \in S. \tag{12}$$

Using the previous approach, the total equivalent annual penalty cost of the manufacturer is calculated in Equation (13):

$$\sum_{j=1}^{m} PC_j = \sum_{j=1}^{m} \left(\int_0^{\frac{q_j - U_j}{D_j}} \pi_j (q_j - D_j t - U_j) e^{-rt} dt\right) \times \sum_{i=0}^{n-1} e^{-ir\frac{q_j}{D_j}} \times \sum_{i=0}^{\infty} e^{-irT_s} \times r = \sum_{j=1}^{m} \frac{\pi_j D_j}{r} \left(e^{-r\left(\frac{q}{D} - \frac{U_j}{D_j}\right)} + r\left(\frac{q}{D} - \frac{U_j}{D_j}\right) - 1\right) \times \left(\frac{1}{1 - e^{-r\frac{q}{D}}}\right) \quad \forall j \in S. \tag{13}$$

Considering the production setup cost at the beginning of the manufacturer's cycle, equivalent annual value of this cost is obtained as follows:

$$OC_s = A_s \times \sum_{i=0}^{\infty} e^{-irT_s} \times r = A_s \times \left(\frac{r}{1 - e^{\frac{-rnq}{D}}}\right). \tag{14}$$

If we consider the ordering cost for each retailer at the beginning of replenishment, the equivalent annual value of this cost for retailer j will be:

$$OC_j = A_j \times \sum_{i=0}^{n-1} e^{-ir\frac{q_j}{D_j}} \times \sum_{i=0}^{\infty} e^{-irT_s} \times r = A_j \times \left(\frac{r}{1 - e^{-r\frac{q}{D}}}\right) \quad \forall j \in \{1, 2, \dots, m\}. \tag{15}$$

The present value of holding cost for retailer j during T_r is:

$$\int_0^{\frac{q_j}{D_j}} h_j (q_j - D_j t) e^{-rt} dt \quad \forall j \in \{1, 2, \dots, m\}. \tag{16}$$

Using the previous approach, the equivalent annual value of this cost will be:

$$HC_j = \left(\int_0^{\frac{q_j}{D_j}} h_j (q_j - D_j t) e^{-rt} dt\right) \times \sum_{i=0}^{n-1} e^{-ir\frac{q_j}{D_j}} \times \sum_{i=0}^{\infty} e^{-irT_s} \times r = \left(\frac{h_j D_j}{r}\left(e^{-r\frac{q}{D}} + r\frac{q}{D} - 1\right)\right) \times \left(\frac{1}{1 - e^{-r\frac{q}{D}}}\right) \quad \forall j \in \{1, 2, \dots, m\}. \tag{17}$$

In both consignment and non-consignment VMI, the manufacturer incurs the retailers' ordering cost. Also, unlike consignment VMI, in non-consignment VMI, each retailer incurs his holding cost. So, according to the problem under study, the manufacturer's costs involved in this model are the retailers' ordering cost, the manufacturer's holding, setup, and penalty costs. Therefore, the total equivalent annual cost of the manufacturer under non-consignment VMI system is obtained from the following equation.

$$AC_s = HC_s + \sum_{\forall j \in S} PC_j + OC_s + \sum_{j=1}^{m} OC_j = HC_s + PC_r + OC_s + OC_r. \tag{18}$$

Also, according to our problem, only cost incurred by each retailer is his holding cost. So, the equivalent annual cost of retailer j will be:

$$AC_j = HC_j \quad \forall j \in \{1, 2, \dots, m\}. \tag{19}$$

Finally, the equivalent annual value of the total inventory cost for the whole supply chain will be resulted from the sum of Equations (18) and (19) as follows:

$$AC = AC_s + \sum_{j=1}^{m} AC_j = AC_s + AC_r. \tag{20}$$

Now, to model the proposed problem, we should consider Constraints (21) and (22):

$$q_j \geq U_j \quad \forall j \in S, \tag{21}$$

$$q_j \leq U_j \quad \forall j \in \bar{S}. \tag{22}$$

They are included in order to ensure that penalty is incurred only for the retailers whose bound is violated. To decrease the number of variables of the model, we can rewrite Constraints (21) and (22) using Equation (1) as follows:

$$q \geq \frac{U_j}{D_j} D \qquad \forall j \in S, \tag{23}$$

$$q \leq \frac{U_j}{D_j} D \qquad \forall j \in \bar{S}. \tag{24}$$

Now, the final model is formulated as a mixed integer non-linear programming (MINLP). The decision variables are n, q, and y_j. We present this formulation below:

$$min\ AC(y_j, q, n) = \Bigg[A_s + \frac{h_s}{r^2}\left(p - pe^{\frac{-rnq}{p}} - rnqe^{\frac{-rnq}{p}}\right) + \frac{h_s q e^{\frac{-rnq}{p}}}{1-e^{-r\frac{q}{D}}}\left(e^{\frac{-rnq}{D}} + n - ne^{-r\frac{q}{D}} - 1\right) + \sum_{j=1}^{m} \frac{\pi_j D_j}{r^2}\left(e^{-r\left(\frac{q}{D}-\frac{U_j}{D_j}\right)} + r\left(\frac{q}{D} - \frac{U_j}{D_j}\right) - 1\right) \times \left(\frac{1-e^{\frac{-rnq}{D}}}{1-e^{-r\frac{q}{D}}}\right) \times y_j + \sum_{j=1}^{m}\left(A_j + \frac{h_j D_j}{r^2}\left(e^{-r\frac{q}{D}} + r\frac{q}{D} - 1\right)\right) \times \left(\frac{1-e^{\frac{-rnq}{D}}}{1-e^{-r\frac{q}{D}}}\right)\Bigg] \times \left(\frac{r}{1-e^{\frac{-rnq}{D}}}\right). \tag{25}$$

$s.t.$

$$q \geq \frac{U_j}{D_j} D - M(1 - y_j) \qquad \forall j \in \{1,2,\dots,m\}, \tag{26}$$

$$q \leq \frac{U_j}{D_j} D + M y_j \qquad \forall j \in \{1,2,\dots,m\}, \tag{27}$$

$$n \leq \frac{p}{D}, \tag{28}$$

$$q \geq 0, n \in \mathbb{N}, y_j \in \{0,1\} \qquad \forall j \in \{1,2,\dots,m\}. \tag{29}$$

The objective function (25) minimizes the total equivalent annual cost of the whole VMI supply chain. Constraint (26) is activated when $y_j = 1$. In this case, this constraint acts for members of the set S according to relation (23).

Constraint (27) is activated when $y_j = 0$. In this case, this relation acts according to relation (24) for members of the set $\bar{S}$. Constraints (26) and (27) ensure that penalty is incurred only for the retailers whose bound is violated. Constraint (28) ensures that shortages are not allowed (assumption 3); otherwise, according to relation (30), the manufacturing time $\left(\frac{nq}{p}\right)$ will become more than the common cycle time for retailers $\left(\frac{q}{D}\right)$ and they will face shortages.

$$n > \frac{p}{D} (violating\ constraint\ (28)) \Rightarrow \frac{n}{p} > \frac{1}{D} \Rightarrow \frac{nq}{p} > \frac{q}{D}. \tag{30}$$

5. SENSITIVITY ANALYSIS

In this section, a sensitivity analysis is conducted to study the effects of parameters h_s on the optimal solution. In addition to performance description of the proposed model, this analysis also includes validation of the model. Unless specified otherwise, the model parameters are given below and in Table 1:
$p = 600$, $A_s = 130$, $h_s = 3$, and $r = 0.2$.

To study the effects of the parameter h_s, we obtained optimal solutions for selected values of h_s ranging from 0 to 24 with an increment of 4. The results are summarized in Table 2. Intuitively, from a continuous cash flow perspective, a holding cost is dependent on inventory levels at any time. So, when h_s increases, it is better to decrease the manufacturer's inventory level. Hence, value of Q decreases as h_s increases according to Table 2.

TABLE 1. Parameters of retailers

Retailer	D_j	h_j	A_j	U_j	π_j
A	60	7	15	15	2
B	140	5	12	14	3
C	50	6	13	20	4

TABLE 2. Effect of h_s

h_s	**S**	**Q**	**n**	**q**	AC^{VMI}	T_r	T_s	HC_s^{VMI}	OC_s^{VMI}	PC_r	OC_r^{VMI}	HC_r^{VMI}
0	{B,A}	166.3	2	83.15	607.822	0.333	0.665	0	208.718	36.033	124.309	238.765
4	{B,A}	134.57	2	67.285	764.192	0.269	0.538	141.232	254.744	22.755	152.658	192.803
8	{B}	115.819	2	57.909	894.783	0.232	0.463	242.588	293.811	15.936	176.714	165.733
12	{B}	103.068	2	51.534	1008.92	0.206	0.412	323.352	328.504	11.632	198.074	147.363
16	{B,A}	76.955	1	76.955	1080.89	0.308	0.308	260.01	435.461	30.633	133.988	220.793
20	{B,A}	72.914	1	72.914	1144.11	0.292	0.292	307.732	458.856	27.249	141.186	209.089
24	{B,A}	69.453	1	69.453	1204.14	0.278	0.278	351.536	481.064	24.452	148.02	199.073

Surface AC^{VMI} in terms of h_s and q when $n = 1$ and $n = 2$ are represented in Figures 2 and 3, respectively. In $h_s = 22$ and $h_s = 8$, the optimal solutions are depicted in red (Figure 2) and black points (Figure 3), respectively. Comparing these two surfaces shows the necessity of decrease in n when h_s increases. As h_s increases, the state $n = 1$ (Figure 2) has a much smaller increase in the objective function (AC) than the state $n = 2$ (Figure 3). Moreover, as n and Q decrease, the production runs and the manufacturer's setup cost (OC_s) increase obviously.

It also seems that it is better to increase the total replenishment quantity (q) when h_s increases so that the manufacturer's holding cost decreases. But, Table 2 shows that q has a descending trend in equal values of n. This is true because the decrease in the manufacturer's penalty cost (PC_r) and the retailers' holding cost (HC_r) not only compensates the increase in the manufacturer's holding cost (HC_s) and the retailers' ordering cost (OC_r), but also reduces the total cost of the VMI system (AC).

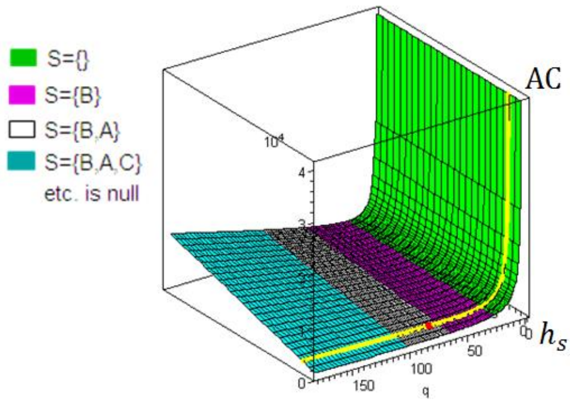

Figure 2. Surface AC in terms of h_s and q when $n = 1$

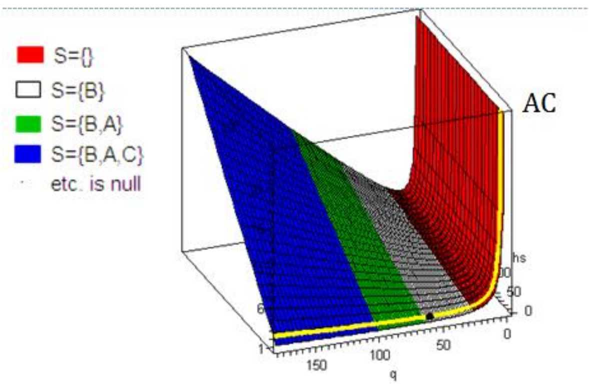

Figure 3. Surface AC in terms of h_s and q when $n = 2$

6. SUMMERY AND CONCLUSIONS

In the present research, we proposed a new model for a two-echelon single-manufacturer multi-retailer supply chain under non-consignment VMI and contractual agreements between the manufacturer and each retailer by considering the time value of money. Under this type of contracts, each retailer is protected by an upper bound on his inventory level, such that the manufacturer is penalized for quantity dispatched that is more than upper bound. In order to take the time value of money into consideration, the present value of each inventory cost was evaluated in a single period and generalized to infinity horizon and then transformed to the equivalent annual cost. Sensitivity analysis was conducted to study the effects of the parameters on the optimal solution and validate the proposed model. The most important results are presented as follows:

In case where penalty cost rate for a retailer is very low and the other one where upper bound of this retailer is high same optimal solutions exist. In both cases, the constraint related to upper bound on the retailer's inventory will be redundant.

- As penalty cost rate for a retailer is very high, the constraint related to over-stock limit of this retailer acts as a capacity constraint. This result can extend the proposed model in this paper to other applications where retailers have to manage the physical constraints of limited warehouse.
- The sum of retailers' ordering and holding costs and manufacturer's penalty cost are nearly constant for different values of production rate.
- As manufacturer's production rate increases, the total replenishment quantity has a fluctuating behavior and the sensitivity of the optimal solution will become less. Also, when this rate is sufficiently large, the total annual cost of the whole system approaches to a value which is not dependent on it.

The model can be further extended to some more practical situations, such as considering multi-manufacturer, multi-item and shortages, taking the raw material supply into account, and etc. We will consider these problems in the near future.

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Introducing the Time Value of Money in a Non-consignment Vendor Managed Inventory Model

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چکیده

مدیریت موجودی توسط فروشنده (VMI) یک رویکرد یکپارچه برای هماهنگی بین خریدار–فروشنده است که طبق آن فروشنده (تأمین‌کننده یا تولیدکننده) تصمیمات مربوط به بازپرسازی خریدار (خرده‌فروش) را اتخاذ می‌کند. تابحال ارزش زمانی پول در ارزیابی مجموع هزینه‌های زنجیره‌ی تأمین تحت VMI لحاظ نشده است. از این رو در مطالعه‌ی حاضر یک مدل جدید برای زنجیره‌ی تأمین دوسطحی با یک تولیدکننده و چند خرده‌فروش تحت برنامه‌ی VMI غیرامانتی با درنظر گرفتن ارزش زمانی پول ارائه می‌شود. به‌منظور وارد کردن ارزش زمانی پول ابتدا ارزش فعلی هزینه‌ها در طی یک دوره محاسبه و سپس به افق بی نهایت تعمیم و بعد به هزینه‌ی همسنگ سالیانه تبدیل می‌شود. همچنین این مدل به‌طور مشخص موافقت‌نامه‌های قراردادی بین تولیدکننده و هر خرده‌فروش را شامل می‌شود. تحت این نوع قراردادها یک حد بالا روی سطح موجودی خرده‌فروش تعیین می‌شود به‌طوری‌که تولیدکننده برای هر واحد محصولی که از این حد تجاوز می‌کند، جریمه می‌شود.در پایان آنالیز حساسیت برای مطالعه‌ی تأثیر پارامترهای کلیدی روی جواب بهینه و معتبرسازی مدل انجام می‌شود.

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