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Towards an Analytical Model for Film Cooling Prediction using Integral Turbulent Boundary layer

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ABSTRACT

The objective of this work is to develop deep theoretical methods that are based on the solution of the integral boundary layer equations for investigating film cooling in liquid rocket engine. The integral model assumes that heat is transferred from hot free stream gas to the liquid film both by convection and radiation. The mass is transferred to the free srteam gas by the well-known blowing process. Downstream of the liquid film, the gas effectiveness is obtained by solving boundary layer integral equations. It incorporates a differential model for calorimeter mixing between liquid vapors in the boundary layer with the free stream gas entrained in the boundary layer. Comparisons with existing theoretical and experimental results indicate the film coating trends were well predicted by the present integral model proposed by us.

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NOMENCLATURE

A	Surface area (m^2)	**Greek symbols**	
C_p	Specific heat at constant pressure (J/kg.K)	δ	Velocity Boundary layer Thickness (m)
G_B	Blowing parameter (-)	δ^*	Displacement Boundary layer Thickness (m)
G_v	Liquid vapor rate (kg/s)	δ_E	Energy Boundary layer Thickness (m)
h_{lv}	Latent heat of vaporization of coolant (J/kg)	δ_M	Momentum Boundary layer Thickness (m)
K_t	Thermal conductivity (W/m.K)	δ_T	Temperature Boundary layer Thickness (m)
$\dot{m}$	Mass flow rate (kg/s)	γ	Specific Heat ratio (-)
m_c	Liquid film coolant flow rate per circumference (kg/s)	η	Film cooling effectiveness (-)
$\dot{m}_{ent}$	Mass flow rate entrained to the boundary layer (kg/s)	φ	Boundary layer Velocity Profile (-)
P	Static Pressure (N/m^2)	ζ	Shape factor (-)
P_p	Pressure gradient parameter ,$((\delta/\tau_w)\times(dp/dx)$ (Pas/m)	μ	Eddy viscosity ($N.s/m^2$)
q	Heat flux (W/m^2)	ρ	Gas density at the edge of boundary layer (kg/m^3)

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r	Radius (m)	τ	wall shear stresses (N/m^2)
T	Static Temperature (^{o}C)	τ_t	wall turbulent shear stresses (N/m^2)
T_o	Stagnation Temperature (^{o}C)	**Subscripts**	
T_r	Recovery Temperature (^{o}C)	AW	adiabatic wall condition
t	Static Temperature in the Boundary layer (^{o}C)	bl	boundary layer
t_o	Stagnation Temperature in the boundary layer (^{o}C)	c	Coolant
t_{slot}	Coolant slot height (m)	conv	Convective
u	Velocity component in the z-direction (m/s)	e	condition at free stream
v	Velocity component normal to the wall (m/s)	o	stagnation condition
z	Distance along axis of symmetry (m)		
Dimensionless quantities		R	due to radiation
M	Mach Number	ref	reference condition
Pr	Prandtl number	t	turbulent
St	Stanton Number	v	Vapor
Re	Reynolds number	W	wall condition
$Re_{\delta E}$	Reynolds Number based on Energy Thickness	**Superscripts**	
$Re_{\delta M}$	Reynolds Number based on Momentum Thickness	$\bar{( )}$	time average

1. INTRODUCTION

In the liquid propellant rocket engines with the regenerative cooling system, hot combustion gases are directly in contact with the walls. This leads to gas side wall temperature rising if the coolant is not sufficient to provide adequate cooling and wall material will soften and even might fail. However, introducing film cooling as a support cooling method, this problem could be circumvented. Heat conduction of the liquid film is a significant technique for controlling the wall temperatures in forced convection cooling of rocket devices at high heat fluxes [1]. In film cooling, a thin layer of cooling fluid is maintained over the inner surface of the wall which separates the wall from the hot combustion gases. Same cooling film may be introduced through orifices in the chamber wall near the injector, and sometimes in planes toward the throat. Recently, Sangkwon [2] reported four promising design paradigms for injection of coolant over as flat plate of the combustion chamber.

Rocket engines operate mostly in turbulent boundary layer regime, and consequently there is considerable interest in calculating heat transfer through the turbulent boundary layer. To obtain an accurate solution for turbulent boundary layer, flow is quite a complex problem because of the nonlinearities involved. The solution of differential equations modeling of the turbulent boundary layer, in most cases, is very cumbersome and time-consuming to calculate even a numerical solution. In literature, Shih and Sultanian [3] and Durbin and Shih [4] reported on the Computational fluid dynamic (CFD) analysis of film cooling which were carried out into four categories: (i) direct numerical simulation (DNS), (ii) large-eddy simulations (LES), (iii) Reynolds-averaged Navier-Stokes (RANS) equations, and (iv) combination of the above such as LES away from walls and RANS next to walls. Many scientists [5-7] have reported that computational techniques for turbulent flow representing film cooling are expensive, and their results may not converge. Acharay et al. [8] applied CFD methods on film cooling based on steady Reynolds-averaged Navier-Stokes (RANS), and they reported less accurate results. Rozati [9] presented numerical investigation conducted to study leading edge film cooling with Large Eddy Simulation (LES). He reported the effect of coolant blowing ratios on the adiabatic effectiveness and heat transfer coefficient. A study on Full-coverage film cooling, both experimental and numerical, with the objective of quantifying the local heat transfer augmentation and adiabatic film cooling effectiveness for four different surfaces, was carried out by Natsui [10]. Still, CFD techniques for analyzing the film cooling may not be a fully trustworthy method.

The analytical calculations of the compressible turbulent boundary layer and heat transfer under the influence of pressure gradient have been presented by many researchers like White [11] and Schlichting [12]. Shembharkar and Pai [13] developed a numerical method using the Prandtl Mixing Length turbulence model for the hot gas, coupled with one-dimensional laminar Couette model in the liquid film, to analyze the film cooling with a liquid coolant. Theoretical analysis of liquid film cooling was also presented by Grisson [14]. although he considered only a simple one-dimensional model, a satisfactory comparison with the existing data was observed. He provided a theoretical study of the gas film to modify standard gaseous film cooling correlation using a differential form to predict the wall temperature as the vapor film mixes with the free stream gas. Warner and Emmons [15] made an experimental determination of the effect of changes in temperature, pressure and Reynolds number of a hot gas stream flow over the liquid film (water), using different lengths of cylindrical rocket motor combustion chamber. Liquid film cooling was also investigated experimentally by Kinney et al. [16]. In their work, an investigation was conducted using two tubes, of diameter 2 and 4 inches, having the smooth inner surface, using water as coolant film. Gas film cooling has also been investigated theoretically by Stollery and El-Ehwany [17] using boundary layer model for correlating experimental data, to obtain the most accurate expression for gas film effectiveness η. Experimental results for film cooling effectiveness are provided by Seban [18] using air as cooling film, which is injected tangentially through the single slot into a turbulent boundary layer on a flat plate. Film cooling employed to cool the rocket engine was experimentally studied since 1950. However, no general analysis has been developed yet. Since then film influence was treated nearly exclusively by a collection of several empirical methods in most published theoretical investigation

In this context, the governing equations for fluid flow and heat transfer for film cooling are solved analytically by using momentum and energy integral method for achieving considerable degree of accuracy in results. The liquid coolant is assumed to be injected tangentially along a surface that is exposed to a hot turbulent gas stream where the cooling takes place by evaporation of the liquid film. The film becomes thin as it flows downstream along the surface and only after the liquid film has evaporated, the cooling process will continue to protect the wall by a gaseous film further downstream. The gas film then loses effectiveness as its progress downstream by heat transfer and turbulent mixing with the hot combustion gases.

The model assumes that heat is transferred from hot free steam gas to the liquid film both by convection and radiation, and the mass is transferred to the free steam gas by the well-known blowing process. Downstream of the liquid film, the gas effectiveness is obtained by solving boundary layer integral equations, which incorporate a differential model for calorimeter mixing between liquid vapors in the boundary layer with the free stream gas entrained in the boundary layer. The obtained results from this model were validated against the experimental and numerical results reported in the literature. The validation shows that film cooling trends are well predicted by the present integral model.

2. MATHEMATICAL MODEL

Figure 1 illustrates a schematic diagram of the film cooling process protecting the wall exposed to a hot gas stream. A liquid film coolant is formed due to tangential injection along the surface. It should be noted that when the liquid film just is in touch with hot gases the temperature of the liquid rises up to the saturation temperature, at this point mass transfer from the coolant film to the hot gases occurs as the coolant vaporizes. The liquid film thickness will decrease as flow downstream along the wall up to the position at which m_c= 0 and this point is known as liquid film coolant length *Lc*.

To model this problem a number of speculative assumptions are considered. These assumptions are qualitatively proper, and assume that they do not affect significantly. The assumptions considered in this work are summarized as following.

1. The flow is axis-symmetric and steady.
2. The flow outside the boundary layer is one dimensional inviscid flow.
3. The liquid film is considered to be stable one-dimensional, laminar couette flow.
4. Liquid film burnout is not considered.
5. All gases coolant remains in the boundary layer, and is in equilibrium with hot gas that entrained to the boundary layer.
6. Flowing fluid is constant property ideal gases, Flow over the wall is adiabatic and a 1/7th power turbulent profile.

The set of time mean equations of continuity, momentum and energy for a thin, steady, two dimensional and axisymmetric compressible turbulent boundary layer flows can be written as:

$$\frac{\partial}{\partial z}(\overline{\rho u}) + \frac{\partial}{\partial r}(\overline{\rho v}) + \frac{\overline{\rho u}}{\overline{r}}\frac{dr}{dz} = 0 \quad (1)$$

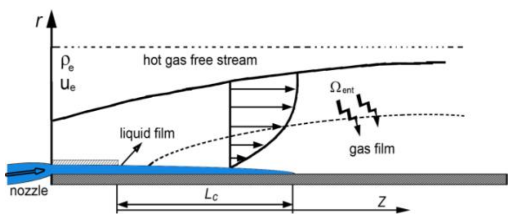

Figure 1. Schematic diagram of liquid and gas film formation in gas film cooling model.

$$\bar{\rho}\bar{u}\frac{\partial \bar{u}}{\partial z}+\bar{\rho}\bar{v}\frac{\partial \bar{u}}{\partial r}=-\frac{\partial \bar{p}}{\partial z}+\frac{\partial}{\partial r}\left(\mu_t\frac{\partial \bar{u}}{\partial r}\right) \quad (2)$$

$$\bar{\rho}C_p\left(\bar{u}\frac{\partial \bar{t}}{\partial z}+\bar{v}\frac{\partial \bar{t}}{\partial r}\right)=\bar{u}\frac{\partial \bar{p}}{\partial z}+\frac{\partial}{\partial r}\left(k_t\frac{\partial \bar{t}}{\partial r}\right)+\mu_t\frac{\partial^2 \bar{u}}{\partial r^2} \quad (3)$$

The momentum and energy equations are obtained by integrating Equations (2) and (3) over the turbulent boundary layer. In integral method we do not attempt to satisfy the differential equation for every streamline in the boundary layer. Instead the boundary layer equation is satisfied near the wall, and near the region of transition to the external flows by satisfying the boundary conditions. In the remaining region of fluid in the boundary layer only a mean value over the differential equation is satisfied, the mean being taken over the whole thickness of the boundary layer. Since the partial differential equations considered are to be expressed in terms of the temporal mean values, and so the equations can be applied to turbulent boundary layer. Appropriate boundary conditions are considered. The forms of the integral momentum and energy equations are respectively obtained as:

$$\frac{\partial \delta_M}{\partial z}+\frac{\delta_M}{\rho_e u_e^2}\frac{\partial \rho_e u_e^2}{\partial z}+\frac{\delta^*}{u_e}\frac{du_e}{dz}+\frac{\delta_M}{r}\frac{dr}{dz}=\frac{C_f}{2} \quad (4)$$

$$\frac{\partial \delta_E}{\partial z}+\frac{\delta_E}{r\rho_e u_e(T_o-T_w)}\frac{d}{dz}\left[\rho_e u_e r\left(T_o-T_w\right)\right]=St\frac{(T_{AW}-T_w)}{(T_o-T_w)} \quad (5)$$

where:

$$\frac{C_f}{2}=\frac{\tau_t}{\rho_e u_e} \quad (6)$$

$$St=\frac{q_w}{c_p\rho_e u_e}\frac{1}{\left(T_{AW}-T_w\right)} \quad (7)$$

In the present work, the integral equations are extended to the case with mass transfer at the surface by taking into account the fact that the normal component of velocity at the surface is now not zero. This allows us to investigate the whole liquid film process. These final forms of integral momentum and energy equations for the boundary layer with evaporation or transpiration at the surface obtained are:

$$\frac{\partial \delta_M}{\partial z}+\frac{\delta_M}{\rho_e u_e^2}\frac{\partial \rho_e u_e}{\partial z}+\frac{\delta^*}{u_e}\frac{du_e}{dz}+\frac{\delta_M}{r}\frac{dr}{dz}=\frac{C_f}{2}+G_B \quad (8)$$

$$\frac{\partial \delta_E}{\partial z}+\frac{\delta_E}{r\rho_e u_e\left(T_o-T_W\right)}\frac{d}{dz}\left[\rho_e u_e r\left(T_o-T_W\right)\right]= St\frac{\left(T_{AW}-T_o\right)}{\left(T_o-T_W\right)}+G_B\left(1-\left(\frac{\bar{t}_o-T_W}{T_o-T_W}\right)\right) \quad (9)$$

where,

$$G_B=\frac{v_w\rho_w}{\rho_e u_e} \quad (10)$$

In order to solve the foregone integral boundary layer equations, it is necessary to use the following assumed expressions:

i. Expressions for the time mean velocity and temperature profiles in the boundary layer.
ii. Expression of Reynolds analogy for turbulent boundary layer under pressure gradient.

It has been found in experimental analysis of turbulent layer in rocket engine that the velocity and temperature profiles are described by the 7th power law [19].

$$\frac{\bar{u}}{u_e}=\left(\frac{y}{\delta}\right)^{\frac{1}{7}} \quad (11)$$

$$\frac{\bar{t}_o-T_W}{T_o-T_W}=\left(\frac{y}{\delta_\tau}\right)^{\frac{1}{7}} \quad (12)$$

In the present investigation, we introduce what we believe is an accurate method for calculating the Reynolds analogy for a turbulent boundary layer with pressure gradient. This method was developed by Tetervin [20], in which Reynolds analogy factor obtained directly from the boundary layer momentum and energy transfer equations, by integrating these equations across the boundary layer. The expression for Pr =1 (atmospheric air at 20 ^{o}C) is given by:

$$\frac{2St}{C_f}=\frac{1}{\int_0^1\left[e^{-P_p\varphi^7(1-\varphi^7)}\right]d\varphi} \quad (13)$$

For Pr≠1 is given by

$$\frac{2St}{C_f}=\frac{Pr^{\left(-\frac{2}{3}\right)}}{\int_0^1 e^{-P_p\varphi^7(1-\varphi^7)}d\varphi} \quad (14)$$

where pressure gradient parameter P_p is given by:

$$P_p=\frac{\delta}{\tau_W}\frac{dP}{dx}=\left(\frac{\partial f}{\partial\zeta}\right)_W \quad (15)$$

where, $f=\frac{\tau(y)}{\tau_w}$ is Non-dimensional shear stress.

The skin friction coefficient C_f that based on Re_{δ_m} or Re_{δ_E} may be obtained by Cole's relation [21], or by the familiar Blasius turbulent boundary layer relation for large temperature variation:

$$C_f = \frac{0.256}{\left(Re_{\delta_M}\right)^{0.25}}\left(\frac{T_{ref}}{T_e}\right)^{-0.6} \tag{16}$$

where, T_{ref} is given by Eckert reference temperature [17-22]:

$$T_{ref} = 0.5\left(T_W + T_e\right) + 0.22\left(T_{AW} - T_e\right) \tag{17}$$

In the case of gas film cooling the adiabatic wall temperature T_{AW} is considered as the temperature that results from thermal mixing of the two gases in the boundary layer. In accordance with assumption, the increase in boundary layer temperature is only due to the entrainment of the free stream hot gases, the adiabatic wall temperature is obtained from the energy balance between the two gases in the boundary layer.

3. SOLUTION METHODOLOGY

The integral momentum and energy equations is solved simultaneously in both zones.

i. Liquid film zone: It is well known that the liquid evaporation is similar to transpiration cooling through a porous wall. When liquid film is injected to the free stream, it will touch hot gases. The total heat that will be absorbed by the film due to both convection and radiation heat flux, causing an initial temperature of the film to rise up to saturation temperature (T_v). At this point, the liquid evaporates at a rate calculated by the following formula:

$$G_v = \frac{q_{conv} + q_r}{h_{fg}} \tag{18}$$

The liquid film thickness will decrease as flows downstream along the wall up to the position at which (m_c=0). In this investigation, the liquid film blowing process is predicted by presently developed model.

By using the equations of one-dimensional theory, where the expression for ρ_e and u_e can be written in terms of Mach number, Equations (8) and (9) can be put into a form that is more convenient for computation as:

$$\frac{\partial \delta_M}{\partial z} = \left(\frac{C_f}{2} + G_B\right)\left[1 + \left(\frac{dr}{dz}\right)\right]^{\frac{1}{2}} - \left[\frac{2 - M^2 + \dfrac{\delta^*}{\delta_M}}{M\left(1 + \dfrac{\gamma - 1}{2}M^2\right)}\frac{dM}{dz} + \frac{1}{r}\frac{dr}{dz}\right] \tag{19}$$

$$\frac{\partial \delta_E}{\partial z} = St\frac{\left(T_{AW} - T_W\right)}{\left(T_o - T_W\right)}\left[1 + \left(\frac{dr}{dz}\right)^2\right]^{\frac{1}{2}} + G_B\left(1 - \left(\frac{\bar{t}_o - T_W}{T_o - T_W}\right)\right)\left[1 + \left(\frac{dr}{dz}\right)^2\right]^{\frac{1}{2}}$$
$$-\delta_E\left[\frac{1 - M^2}{M\left(1 + \dfrac{\gamma - 1}{2}M^2\right)}\frac{dM}{dz} + \frac{1}{r}\frac{dr}{dz} - \frac{1}{\left(T_o - T_W\right)}\frac{dT_W}{dz}\right] \tag{20}$$

Equations (19) and (20) are a linear first order differential equation with variable coefficients, and they are solved iteratively. The shooting method combined with the secant method for improving the guess provides us with an efficient algorithm. We begin by guessing a value for variable coefficient. Next, we integrate the system of Equations (19) and (20). Third, we compute the integral on the right-hand and left-hand side by using the trapezoidal rule. Finally, we improve the guess for variable coefficient with the goal of left hand side is equal to right hand side.

ii. Gas film zone: The gas film will lose effectiveness as flow downstream of the injection slot by heat transfer and turbulent mixing with hot combustion gases, which entrained in the boundary layer. Numerous correlations have been presented for this process. They are normally expressed in terms cooling effectiveness "η" which is defined as:

$$\eta = \frac{T_e - T_{AW}}{T_e - T_c} \tag{21}$$

η is zero when $T_{AW} = T_e$ and equal to unity when $T_{AW} = T_c$. T_{AW} is obtained by differential model derived from the mass and energy balance in a control volume in the boundary layer (Figure 1) as below:
From continuity equation:

$$\dot{m}_{bl} = \dot{m}_c + \dot{m}_{ent} \tag{22}$$

Let assume that the average temperature of boundary layer is equal to adiabatic wall temperature, and gas behave as ideal gases having constant thermos-physical properties. Hence, the adiabatic wall temperature is:

$$T_{AW} - T_e = \frac{\int_0^{\delta} \rho u C_p (T - T_e)}{\int_0^{\delta} \rho u C_p dr} \tag{23}$$

where, the C_p, the specific heat of fluid in boundary layer. The average specific heat is obtained as:

$$\bar{C}_p = \frac{\dot{m}_c C_{pc} + \dot{m}_{ent} C_{pe}}{\dot{m}_c + \dot{m}_{ent}} \tag{24}$$

The energy balance can be written as:

$$\left(\dot{m}_c + \dot{m}_{ent}\right)\bar{C}_p T_{AW} = \dot{m}_c C_{pc} T_c + \dot{m}_{ent} C_{pe} T_e \tag{25}$$

After a significant amount of algebra, the energy balance equation can be written as:

$$\frac{T_{AW} - T_e}{T_c - T_e} = \eta = \frac{1}{\left(1 + \dfrac{\dot{m}_{ent} C_{pe}}{\dot{m}_c C_{pc}}\right)} \tag{26}$$

The integral boundary layer Equations (22-26) are now solved for $\dot{m}_{bl}$ and $\dot{m}_{ent}$.

4. RESULTS AND DISCUSSIONS

Firstly, the model results were validated against the experiment data and numerical data available in literature. The most convenient available numerical results on film cooling were that obtained by Shembharkar and Pai [13], and available experimental results were obtained by Kinney et al. [16]. For comparison purpose, the same data case is to be analyzed. In investigation of reference [16], the free gas stream consisted of the products of gasoline air combustion. For property values pure air was assumed. For radiation calculation 10% CO_2 and 10% H_2O were assumed. For film cooling water was used as coolant fluid. The flow condition for given case based on information that outlined in reference [13, 16] are now summarized in a series of theoretical analysis have been carried out for air-water system to investigate the effect of coolant mass flow m_c, and free gas stagnation temperature T_o, on film cooled length and heat transfer.

The set of computations have been covering the range of coolant mass flow rates, which used in reference [16]. Figure 2 shows the variation of film cooled length L_c with the coolant mass flow rate m_c.

The present predictions are compared, with numerical predictions of reference [13], and experimental results of reference [16]. It is seen that the experimental data indicate a linear variation of the film cooled length with the coolant mass flow rate.

A series of theoretical anlysis have been carried out for air-water system to investigate the effect of coolant mass flow mc, and free gas stagnation temperature *To*, on film cooled length and heat transfer.

TABLE 1. Conditions for the case computations with (100 mm diameter chamber) reported in literature [13, 16].

Parameter	Value
P_o	2.14 [bar]
T_0	1105 [K]
$\rho_e u_e$	216 [kg/s.m^2]
γ	1.35
δ_M	1.4 [mm]
T_e	300 [K]
m_c	0.046-0.15 [kg/s.m]
t_{slot}	0.548 [mm]

The set of computations have been covering the range of coolant mass flow rates, which used in reference [16]. Figure 2 shows the variation of film cooled length L_c with the coolant mass flow rate m_c.

The present predictions are compared, with numerical predictions of reference [13], and experimental results of reference [16]. It is seen that the experimental data indicate a linear variation of the film cooled length with the coolant mass flow rate. The change in the slope of the curve for film cooled length around m_c=0.106 [kg/sec m], is attributed to droplets of liquid being torn away from the surface of the film caused by the onset of turbulent flow in the coolant film at high coolant flow rates. Whereas, the present and reference [13] computations also predict a linear variation of the film cooled length with the coolant mass flow rate. It is interesting to note that, the present predictions are closer to the experimental results than which obtained in reference [13], and almost indicate an average value between both of them. The reasons for different predictions obtained by theoretical predictions are expected to be associated with the use flow conditions of the internal flow of the experiments, and also by employing some unrealistic assumptions in the computations. The good agreement of the present computations, because the present integral boundary layer model predicts higher interface temperature T_L than the same obtained by boundary layer model in literature [13]. This higher temperature increases the evaporation rate, and subsequently shorter film cooled length is predicted. The change of slope in the experimental results at high coolant flow rates are not predicted by both, present, and by Shembharkar and Pai's [13] computations, which are based on the assumption that the coolant film is laminar and smooth.

Post Validation: Post-validation case study in order to demonstrate the present model approximate investigations were made on a rocket engine with film cooling operating under typical conditions listed in Table 2. The nozzle has throat diameter 0.045m, a contraction area ratio 8.0, an expansion area ration of 4.36, a convergent half angle of 30^o, and divergent half angle 15^o, the chamber has diameter 0.12m.

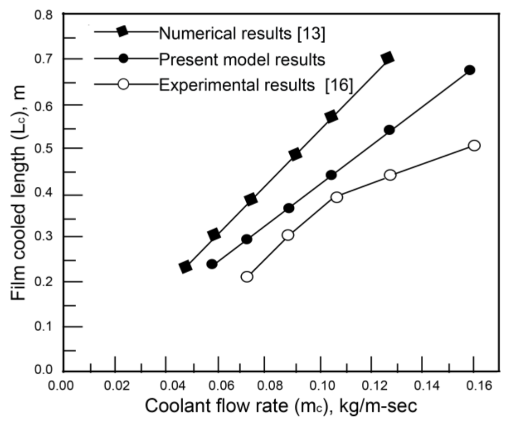

Figure 2. comparison of present computations with theoretical and experimental results.

TABLE 2. Realistic operational conditions

Parameter	Value
P_o	20 [bar]
T_0	1500-3200 [K]
M_{wt}	28.9 [kg/kmol]
γ	1.35
T_e	300 [K]
m_c	0.18-0.05 [kg/s.m]
t_{slot}	1.00 [mm]

A series of theoretical anlysis have been carried out for air-water system to investigate the effect of coolant mass flow m_c, and free gas stagnation temperature $T_{o,}$ on film cooled length and heat transfer.

Figures 3 and 4 show that, there is a considerable reduction in wall heat flux as m_c is increased. This reduction is due to liquid film vapor which lowering gas temperature next to the wall by calorimetric mixing with free stream gases that entrained in the boundary layer, and hence reducing heat transfer driving potential. The reduction in wall heat flux at the throat region for the given range of m_c is about 15%, as shown in Figure 3. In addition, interesting to note from Figure 4 that the effect of the liquid coolant mass flow rates on wall heat flux is reduced as we go farther downstream of the dry out point. This is due to reduction of gas film effectiveness.

Figure 4 also shows that wall heat flux increases as axial distance increasing. However, the slop of curve is very high from range of 270 to 350mm of axial distance. At the throat which means axial distance of 350mm on x-axis, wall heat flux is at its peak. At the throat the coolant mass flow rate is decreasing and heat flux increases.

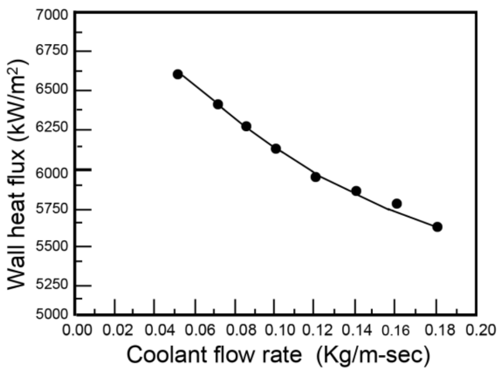

Figure 3. Wall heat flux (q_w) versus coolant flow rate per circumference (m_c) at throat region. The nominal condition for calculate the heat flux are: (a) stagnation temperature (T_o) = 2000 K, Stagnation pressure (P_o) = 20 bar, momentum boundary layer thickness (δ_m) =0.00002 mm and shape factor (H) =1.

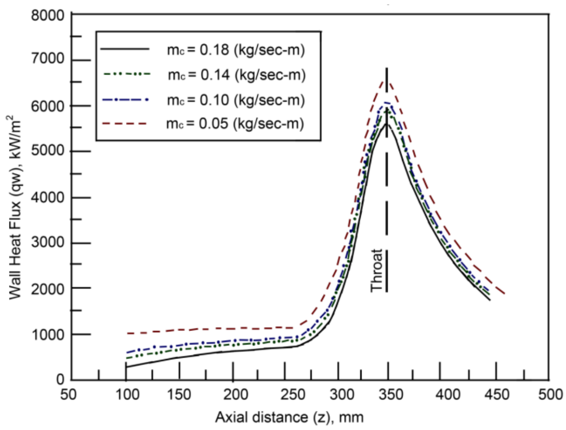

Figure 4. Heat flux for various film coolant flow rates. The nominal condition for calculate the heat flux are: (a) stagnation temperature (To) = 2000 K, Stagnation pressure (Po) = 20 bar, momentum boundary layer thickness (δ_m) =0.00002 mm and shape factor (H) =1.

We find that the thickness of the coolant fluid at a low flow rate is small which could provide less thermal resistance against the heat transfer. Figure 5 shows the variation of film cooled length with the coolant mass flow rate for the range given in Table 1. The present results predict almost linear variation of the film cooled length with the coolant flow rate. This has been obtained experimentally by Warner and Emmons [15], and theoretically by Shembharkar and Pai [13]. The effect of film waviness at high coolant flow rates do not included in the present computations, which are based on the assumptions that, liquid film is smooth and stable. Figure 6 shows gas film effectiveness distributions in the chamber and the nozzle as function of coolant mass flow rate.

In the present model gas effectiveness is obtained by solving boundary layer integral equations, which incorporates a differential model for calorimetric mixing between liquid vapors in the boundary layer with the free stream gas entrained in the boundary layer. Results indicate significant influence of increase in the coolant mass flow rate on the gas cooling effectiveness. It can be seen from the figure that, the gas film effectiveness decreases with increase in coolant flow rate, and hence reduction in adiabatic wall temperature all through the chamber and the nozzle. It is also seen from the figure that, the gas film loses its effectiveness by influx of hot gases as flows downstream of injection slot.

It is also noted that, loss in gas effectiveness is enhanced by increased mixing with free stream gas in the nozzle. The effectiveness reaches minimum at the end of supersonic region, because the adiabatic wall temperature approaches free gas static temperature, which decreases rapidly in this region.

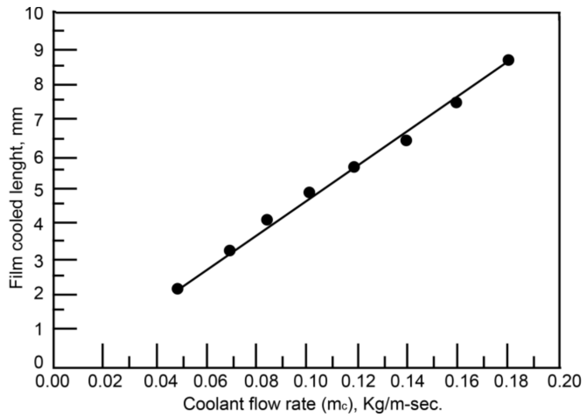

Figure 5. Liquid film cooling length various coolant flow rates. The nominal condition for calculation the heat flux are: (a) stagnation temperature (To) = 2000 K, Stagnation pressure (Po) = 20 bar, momentum boundary layer thickness (δ_m) =0.00002 mm and shape factor (H) =1

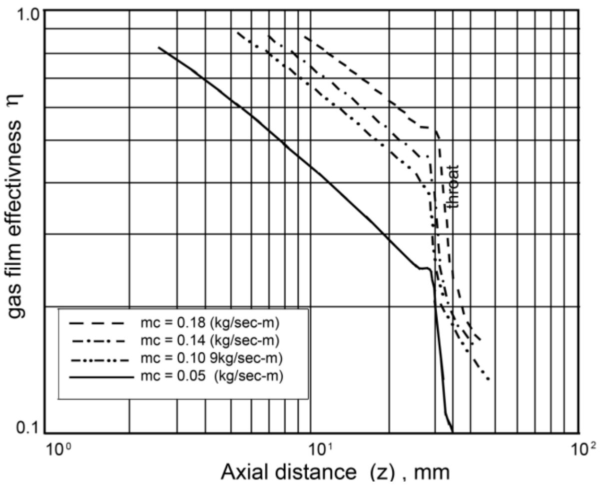

Figure 6. Gas film effectiveness for various coolant theoretical results. The nominal condition for calculate the heat flux are: (a) stagnation temperature (To) = 2000 K, Stagnation pressure (Po) = 20 bar, momentum boundary layer thickness (δ_m) =0.00002 mm and shape factor (H) =1

5. SUMMARY AND CONCLUSION

The mathematical model and the results of theoretical investigation of the film cooling in rocket engine were presented. As the theoretical method involving the use of boundary layer integral theory, momentum and energy integral equations for turbulent boundary layer were derived and utilized. The obtained results by this method were listed below:

1. The results indicate that, as liquid film coolant flow rate increased, it resulted to an increase in the film cooled length proportionally. Downstream of the liquid film, the vapor provides continued thermal protection to the wall and then is treated as a gas film cooling process.
2. The film was analyzed with integral theory, which incorporates differential form for gas mixing. Reduction in adiabatic wall temperature due to an increase in the coolant mass flow rate was observed all through the chamber and the nozzle. It was noted that, the gas film loses its effectiveness by influx of hot gases as flows downstream of the injection slot. Rapid decrease in adiabatic wall temperature due to subsonic and supersonic acceleration was also observed.
3. It was observed that, the present model predictions are closer to the experimental results than the same obtained by theoretical predictions of reference [13]. These discrepancies between the predictions and experimental results of reference [16] at high coolant mass flow rate were expected to be due to assumption that the coolant film is laminar and smooth.
4. From the results and comparisons with previous existing data, it was concluded that the integral theory is one of boundary layer theories, which can give a solution suitable for application to the film cooled rocket engines. Also, it is a theory of sufficient simplicity which can simulate the boundary layer phenomena and subsequently capable of use as part of an overall design problem.
5. Some of the novel application for the method include gas-turbine, transonic turbines, turbine blades, circular tubes, rocket engines, discrete jets, air injection through discrete holes etc. We believe that these methods are important for multiple applications in modern times of hi-performance technology. In addition, data obtained from this method help us to validate our CFD results.

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Towards an Analytical Model for Film Cooling Prediction using Integral Turbulent Boundary layer

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چکیده

هدف از این کار، توسعه روش عمیق تئوری است که بر پایه راه حل معادلات لایه مرزی جدایی ناپذیر برای بررسی خنک کننده فیلم در موتور موشک مایع است. مدل انتگرال فرض می کند که گرما از جریان گاز داغ آزاد به فیلم مایع توسط همرفت و تابش منتقل شده است. جرم بوسیله فرایند دمش شناخته شده به جریان گاز آزاد منتقل شده است. در پایین دست فیلم مایع، اثربخشی گاز با حل معادلات انتگرال لایه مرزی به دست آمده است. آن یک مدل دیفرانسیل را برای اختلاط گرماسنج بین بخارات مایع در لایه مرزی با جریان گاز آزاد در لایه مرزی ترکیب می کند. مقایسه نتایج تجربی و تئوری موجود نشان می دهد که روند پوشش دهی فیلمی به خوبی توسط مدل انتگرالی حاضر پیشنهادی توسط ما پیش بینی شده بود.

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