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RESEARCH NOTE

Simultaneous Monitoring of Multivariate Process Mean and Variability in the Presence of Measurement Error with Linearly Increasing Variance under Additive Covariate Model

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ABSTRACT

In recent years, some researches have been done on simultaneous monitoring of multivariate process mean vector and covariance matrix. However, the effect of measurement error, which exists in many practical applications, on the performance of these control charts is not well studied. In this paper, the effect of measurement error with linearly increasing variance on the performance of ELR control chart for simultaneous monitoring of multivariate process mean vector and covariance matrix is investigated. The multiple measurement approach is also extended to reduce this effect. Also, the performance of the proposed multiple measurement approach is evaluated in terms of average run length (ARL) and standard deviation run length (SDRL). Finally, the application of the proposed monitoring method is illustrated by a real data in manufacturing industry.

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1. INTRODUCTION

The control charts in the literature for monitoring the process mean and variability are usually developed separately assuming that the other parameter is in control. In practice, the control charts for monitoring both process mean and variability should be implemented together because assignable causes can affect both of them. In recent years, quality practitioners are interested in joint monitoring problem of process mean and dispersion in both univaraite and multivariate cases. Simultaneous monitoring of the process mean and process variability in univariate cases was studied by Khoo et al. [1], Guh [2], Memar and Niaki [3], Teh et al. [4], Tasias and Nenes [5], Sheu et al. [6] and Chowdhury et al. [7]. The simultaneous monitoring of multivariate process mean vector and covariance matrix has been also addressed by several authors such as Khoo [8], Hawkins and Maboudou-Tchao [9], Niaki and Memar [10], Zhang et al. [11], Ramoset al. [12], Wang et al. [13] and Maleki and Amiri [14]. For detailed information on simultaneous monitoring of the process location and dispersion refer to review paper provided by McCracken and Chakraborti [15].

In real production systems, process practitioners are faced with the error which is due to the measuring equipment called measurement error. Once the samples are taken from a process, the presence of measurement error in data collection is an inevitable issue because the measuring instruments are not entirely accurate. Recently, the effect of measurement error on control charts has been analyzed by several researchers such as Linna et al. [16], Huwang and Hung [17], Maravelakis [18], Chakraborty and Khurshid [19], Chakraborty and Khurshid [20], Ding and Zeng [21], Hu et al. [22], Haq et al. [23], Noorossana and Zerehsaz [24] and Abbasi [25].

In most researches in the literature, the variance of measurement error is considered as constant values. However, in many practical situations in industry, the variance of the measurement error component depends on the mean level of the underlying process. Montgomery and Runger [26] and Linna et al. [16] addressed some examples where the variance of

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measurement error is linearly increasing type. Recently, Dizabadi et al. [27] studied the effect of measurement error with linearly increasing variance on the performance of maximum exponentially weighted moving average and mean-squared deviation (MAX-EWMAMS) control chart. In this paper, we consider another control chart called ELR control chart for simultaneous monitoring of mean vector and covariance matrix and investigate the effect of measurement error with linearly increasing variance on this control chart. In addition, we propose the use of multiple measurements to decrease the effect of measurement error. This study is done in Phase II. Also a classical additive covariate model is considered in order to describe the relationship between the measured and actual values of quality characteristics. The rest of this paper is organized as follows: in next section, first we discuss the classical covariate model in the case of multivariate quality characteristics considering the linearly increasing variance for the error terms. After that we construct a multivariate control chart in the presence of measurements in error for simultaneous monitoring of the process mean and variability. In Section 3, we reconstruct the control chart provided in Section 2 based on multiple measurements in each sample in order to reduce the effect of measurement error. In Section 4, an illustrative example is given to demonstrate the effect of measurement error on simultaneous monitoring of multivariate process mean and variability and the capability of multiple measurement approach. The application of proposed control chart and extended remedial approach is evaluated through a real data example in Section 5. Finally, Section 6 concludes the proposed methods and provides a recommendation for future study.

2. PROBLEM DEFINITION

The monitoring procedures using control charts are classified in two major categories including Phase I and Phase II. The purpose of Phase I monitoring is providing an analysis on the preliminary data for estimating the process parameters. The main purpose of Phase II monitoring is designing a control scheme to detect different out-of-control scenarios in the process parameters. In this paper, we focus on Phase II simultaneous monitoring of process mean and variability.

Most previous works on simultaneous monitoring of the process mean and variability are focused on univaraite cases. However, there are many situations in which the overall quality of an item is determined by several (say *p*) correlated quality characteristics. Examples of such processes include the following: in a lumber manufacturing plant the quality of lumber may be monitored by measuring the stiffness and bending strength of the lumber; in a chemical industry, the process may be a function of temperature, pressure as well as viscosity; and in an automobile plant, the usefulness of an automobile part may depend on an inner diameter and outer diameter. The process is considered to be statistically in-control if all critical product quality characteristics are simultaneously in-control [28].The notations and definitions used to formulate the problem are presented in Table 1.

2. 1. Additive Measurement Error

In this section, we extend the effect of measurement error on the proposed control chart by Zhang et al. [11].

TABLE 1. The notations and definitions

Notation	Description
p	Number of quality characteristic
n	Sample size
$\mathbf{x}_{\mathbf{ijr}}$	Observation *j* in the subgroup *i* related to the original *r* quality characteristics
$\boldsymbol{\mu}_{\mathbf{X}}$	The in-control mean vector of the original quality characteristics
$\boldsymbol{\Sigma}_{\mathbf{X}}$	The in-control covariance matrix of the original quality characteristics
$\boldsymbol{\varepsilon}_{\mathbf{i}}$	Additive error term for subgroup *i* following multivariate normal distribution with mean vector of **0** and the covariance matrix $\boldsymbol{\Sigma}_{\varepsilon}$
$\boldsymbol{\Sigma}_{\varepsilon}$	The covariance matrix for error
$\mathbf{Y}_{\mathbf{i}}$	The observed value with measurement error
$\boldsymbol{\beta}$	Matrix which contains the coefficients of the original quality characteristics in the covariate model $p \times p$
$\boldsymbol{\alpha}$	Vector of constants $p \times 1$
λ	Smoothing parameter
$\mathbf{S}_{\mathbf{i}}$	The *i*th sample covariance matrix
c,d	Known constants vector

Consider a p-dimensional multivariate normal process in which the quality of a product in j^{th} sample of i^{th} subgroup is expressed as follows:

$$\mathbf{X_{ij}} = (X_{ij1}, X_{ij2}, ..., X_{ijp}), \quad (1)$$

when the process is statistically in-control, let $\boldsymbol{\mu}_x$ and $\boldsymbol{\Sigma}_x$ denote the mean vector and covariance matrix of the original quality characteristics, respectively. Due to the measurement error in real process monitoring applications, we cannot observe the original multivariate quality characteristics of interest. In this paper, a classical additive covariate model is used in order to relate the observed and original quality characteristics. The covariate model considered in this paper correspond to the ith subgroup is given according to:

$$\mathbf{Y_i} = \boldsymbol{\alpha} + \boldsymbol{\beta}\mathbf{X_i} + \boldsymbol{\varepsilon}_i, \quad (2)$$

where $\boldsymbol{\alpha}$ is a $p\times 1$ vector of intercept constants and $\boldsymbol{\beta}$ is an invertible $p \times p$ matrix which contains the slope coefficients of the original quality characteristics in the covariate model. In order to simplify the model, the matrix $\boldsymbol{\beta}$ is considered as a diagonal matrix. In Equation (2), $\boldsymbol{\varepsilon}_i$ is assumed to be a $p\times 1$ normal random vector which is independent form $\mathbf{X_i}$. Here it is assumed that the variance of the error component changes linearly with mean of X. As a result, $\boldsymbol{\varepsilon}_i$ follows a normal distribution with the mean vector of **0** and covariance matrix of $\boldsymbol{\Sigma}_\varepsilon$ ($\boldsymbol{\varepsilon}_i \sim MVN(\mathbf{0}, \boldsymbol{\Sigma}_\varepsilon)$, where

$$\Sigma_\varepsilon = \begin{pmatrix} c_1 + d_1\mu_1 & 0 & \dots & 0 \\ 0 & c_2 + d_2\mu_2 & \dots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & \dots & & c_p + d_p\mu_p \end{pmatrix}, \quad (3)$$

In Equation (3), c_r's and d_r's; $r = 1,...,p$ are assumed to be known constants. Obviously, the obtained p-dimensional vector of covariates $\mathbf{Y_i}; i = 1,2,...$ follows multivariate normal distribution with mean vector of $\boldsymbol{\alpha} + \boldsymbol{\beta}\boldsymbol{\mu}_i$ and covariance matrix of $\boldsymbol{\beta}\boldsymbol{\Sigma}_X\boldsymbol{\beta}^T + \boldsymbol{\Sigma}_\varepsilon$, i.e. $\mathbf{Y_i} \sim MVN(\boldsymbol{\alpha} + \boldsymbol{\beta}\boldsymbol{\mu}_i, \boldsymbol{\beta}\boldsymbol{\Sigma}_X\boldsymbol{\beta}^T + \boldsymbol{\Sigma}_\varepsilon)$.

2. 2. ME-ELR Control Chart

Let $\mathbf{Y_{i1}}, \mathbf{Y_{i2}}, ..., \mathbf{Y_{in}}; i = 1,2,...$ represent the i^{th} sample of size n drawn from the process in the presence of measurement error. We assume that the observation within each vector $\mathbf{Y}_{ij}$ and between vectors of $\mathbf{Y}_{ij}$'s are independent. Let $\bar{\mathbf{Y}}_i = \frac{1}{n}\sum_{j=1}^{n}\mathbf{Y_{ij}}$ and $\mathbf{S_i} = \frac{1}{n}\sum_{j=1}^{n}(\mathbf{Y_{ij}} - \bar{\mathbf{Y}}_i)'(\mathbf{Y_{ij}} - \bar{\mathbf{Y}}_i)$ be the i^{th} sample mean vector and sample covariance matrix, respectively. Simultaneous monitoring of the mean vector and covariance matrix is equivalent to the following hypothesis test [11]:

$$\begin{cases} H_0: \boldsymbol{\mu}_y = \boldsymbol{\alpha} + \boldsymbol{\beta}\boldsymbol{\mu}_X \quad \text{and} \quad \boldsymbol{\Sigma}_y = \boldsymbol{\beta}\boldsymbol{\Sigma}_x\boldsymbol{\beta}^T + \boldsymbol{\Sigma}_\varepsilon \\ H_1: \boldsymbol{\mu}_y \neq \boldsymbol{\alpha} + \boldsymbol{\beta}\boldsymbol{\mu}_X \quad \text{and} \quad \boldsymbol{\Sigma}_y \neq \boldsymbol{\beta}\boldsymbol{\Sigma}_x\boldsymbol{\beta}^T + \boldsymbol{\Sigma}_\varepsilon \end{cases} \quad (4)$$

The generalized likelihood ratio (GLR) statistic (proposed by Zhang et al. [11]) corresponding to i^{th} subgroup in the presence of measurement error (we call it ME-LR_i statistic) is computed as follows:

$$ME\text{-}LR_i = np\left(\frac{1}{p}tr(\mathbf{S_i}) - \log(|\mathbf{S_i}|)^{\frac{1}{p}} - 1\right) + n\,\|\bar{\mathbf{Y}}_i\|^2, \quad (5)$$

where $tr(.)$ is the trace operator which computes the sum of diagonal elements in a given square matrix, $|.|$ is the matrix determinant value and $\|.\|$ represents the Euclidean distance of a given vector. It can be easily statistically proved that when $n \to \infty$ then LR_i approximately follows a chi-square distribution with $p(p+3)/2$ degrees of freedom. The large value of LR_i corresponding to i^{th} sample taken, leads to rejection of the null hypothesis. Zhang et al. [11] used the EWMA procedure in construction of LR statistic and proposed ELR control chart to increase the sensitivity of this control scheme in detecting small or moderate shifts. In this paper, we extend ELR control chart which is affected by measurement error and propose the multivariate ME-ELR control chart for simultaneous monitoring of mean vector and covariance matrix of multivariate normal processes. The proposed simultaneous multivariate monitoring scheme is constructed based on two EWMA-based statistics.The EWMA-based statistics for monitoring the mean vector and covariance matrix are derived based on the sample mean vector $\bar{\mathbf{Y}}_i$ and the sample covariance matrix $\mathbf{S_i}$ according to Equations (6) and (7), respectively:

$$\mathbf{U_i} = \lambda\bar{\mathbf{Y}}_i + (1-\lambda)\mathbf{U_{i-1}}, \quad (6)$$

$$\mathbf{V_i} = \lambda\mathbf{S_i} + (1-\lambda)\mathbf{V_{i-1}}, \quad (7)$$

where $\mathbf{U_0} = \boldsymbol{\alpha} + \boldsymbol{\beta}\boldsymbol{\mu}_X$, $\mathbf{V_0} = \boldsymbol{\beta}\boldsymbol{\Sigma}_X\boldsymbol{\beta}^T + \boldsymbol{\Sigma}_\varepsilon$ and $\mathbf{S_i}$ is computed according to Equation (8):

$$\mathbf{S_i} = \frac{1}{n}\sum_{j=1}^{n}(\mathbf{Y_{ij}} - \mathbf{U_i})'(\mathbf{Y_{ij}} - \mathbf{U_i}), \quad (8)$$

where $\lambda; 0 < \lambda < 1$ denotes the smoothing parameter.In comparison with large values of λ , using the small values of this parameter can improve the performance of the proposed control chart in detecting small and moderate shifts. It is usual that the smoothing parameter be considered in the range of $[0.1, 0.25]$. Finally, the following equation is suggested for simultaneous monitoring of the mean vector and covariance matrix under measurement error with linearly increasing variance: (denoted as ME-*ELR* statistic):

$$ME\text{-}ELR_i = np\left(\frac{1}{p}tr(\mathbf{V_i}) - \log(|\mathbf{V_i}|)^{\frac{1}{p}} - 1\right) + n \,\| \mathbf{U_i} \|^2 \cdot \left\| \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right\| \quad (9)$$

Note that, the term $\| \mathbf{U_i} \|^2$ reflects the changes in the process mean whereas $\frac{1}{p}tr(\mathbf{V_i}) - \log(|\mathbf{V_i}|)^{\frac{1}{p}} - 1$ contribute to the variance changes. The control chart triggers an out-of-control signal at i^{th} sample if $ME\text{-}ELR_i > UCL$. It is worth mentioning that UCL is set to obtain a predetermined in-control ARL.

3. MULTIPLE MEASUREMENTS

A technique suggested by Linna & Woodall [29] in order to decrease the measurement error effect is taking more than one measurement in each sampled unit. Taking several measurements and averaging them leads to a more precise measurement. Moreover, the variance of the measurement error component in the average of the multiple observations becomes smaller as the number of multiple measurements increases. Therefore, ideally, if the number of multiple measurements becomes infinite, the variance of the measurement error component will approach to zero. Although, the larger number of multiple measurements leads to reducing the effect of measurement error, however, the additional cost and time are needed for these observations.

We consider the covariate model $\mathbf{Y}_{ij} = \boldsymbol{\alpha} + \boldsymbol{\beta}\mathbf{X}_i + \boldsymbol{\varepsilon}_{ij}$, for $i = 1, 2, ..., n$ and $j = 1, 2, ..., k$. Here, for each $\mathbf{X_i}$,*k* measurements are taken, where *k* is a positive integer. Note that for fixed i, j :

$$\bar{\mathbf{Y}}_{ij} \sim N\left(\boldsymbol{\alpha} + \boldsymbol{\beta}\boldsymbol{\mu}_\mathbf{X}, \boldsymbol{\beta}\boldsymbol{\Sigma}_\mathbf{X}\boldsymbol{\beta}^\mathbf{T} + \frac{\boldsymbol{\Sigma}_\varepsilon}{k}\right), \quad (10)$$

where:

$$\bar{\mathbf{Y}}_{ij} = \frac{1}{k}\sum_{l=1}^{k} \mathbf{Y_{ijl}}, \quad (11)$$

Then, for $i = 1, 2, ...$

$$\bar{\bar{\mathbf{Y}}}_i \sim N\left(\boldsymbol{\alpha} + \boldsymbol{\beta}\boldsymbol{\mu}_\mathbf{X}, \frac{1}{n}\left(\boldsymbol{\beta}\boldsymbol{\Sigma}_\mathbf{X}\boldsymbol{\beta}^\mathbf{T} + \frac{1}{k}\boldsymbol{\Sigma}_\varepsilon\right)\right) \quad (12)$$

where $\bar{\bar{\mathbf{Y}}}_i = \frac{1}{n}\sum_{j=1}^{n} \bar{\mathbf{Y}}_\mathbf{ij}$.

4. PERFORMANCE EVALUATION

Run length is defined as the number of consecutive observations which is required to be plotted on a control chart until an out-of-control condition is detected. In this section, the capability of the proposed methodology in detecting different out-of-control scenarios is studied in terms of average run length (ARL) and standard deviation run length (SDRL) criteria. In this regard, different values for parameters λ , *n*, *p* and $\boldsymbol{\Sigma}_\varepsilon$ under different shifts in the mean vector $\boldsymbol{\mu}_\mathbf{X}$ and the covariance matrix $\boldsymbol{\Sigma}_\mathbf{X}$ individually and simultaneously are considered. Consider a multivariate normal process with three quality characteristics where the correlation coefficients $\rho_{12} = 0.2$, $\rho_{13} = 0.1$ and $\rho_{23} = 0.2$ exist between the quality characteristics. Recall that $\rho_{ij}; i \neq j$ represents the correlation coefficient between i^{th} and j^{th} quality characteristics. The random samples of size *n*=5 and the smoothing parameter of $\lambda = 0.2$ are used to simulate data. The in-control mean vector and covariance matrix are supposed to be equal to $\boldsymbol{\mu}_\mathbf{x} = (4, 3, 2)'$, and $\boldsymbol{\Sigma}_\mathbf{x} = \begin{pmatrix} 4 & 0.8 & 0.2 \\ 0.8 & 4 & 0.4 \\ 0.2 & 0.4 & 1 \end{pmatrix}$, respectively.

The performances of the proposed multivariate ME-ELR control chart in terms of ARL and SDRL for detecting various shift types based on 10000 simulation replicates are shown in Table 2. Note that in each column, the value of upper control limit (H) is set such that the in-control average run length (ARL_0) approximately equals to 200. Then, we compute the out-of-control average run length (ARL_1) values.

Table 2 contains the results of multivariate ME-ELR control chart for different values of **d** when **c=0**. Note that when $\mathbf{c}, \mathbf{d} = \mathbf{0}$, multivariate ELR and ME-ELR control charts are the same ones. Comparing the first column (No error) with the other one ($\mathbf{d} \neq \mathbf{0}$) in Table 2, we can conclude that in the presence of measurement error the performance of ELR control chart under various shift magnitudes decreases.

Table 2 also shows that as the components of the vector **d** increase, the ARLs and SDRLs increase. Now, we assess the effect of parameter **c** on multivariate ME-ELR control chart at fixed value of **d=1** and summarize the ARLs and SDRLs in Table 3. Table 3 shows that components of the vector **c** increase, the effect of measurement error on detecting performance of control chart is getting larger. Additionally both Tables 2 and 3 show that in the case of measurement error with linearly increasing variance error extra precaution is needed to interpret the ARLs and SDRLs.

Table 4 represents the ARLs and SDRLs of ME-ERL control chart under different values of parameter λ. It is obvious that the best performance of multivariate ME-ELR control chart in detecting small shifts is obtained under $\lambda = 0.1$. It is also seen that multivariate ME-ELR control chart has the best performance in detecting large simultaneous shifts when $\lambda = 0.4$. Generally, in order to detect small shifts, we select small values for parameter λ while large values of parameter λ are suitable for detecting large shifts.

Figure 1 depicts the results of utilizing multiple measurements approach for different values of parameter *K* in terms of $Log(ARL_1)$ criterion. We see that as the value of parameter *K* increases, the effect of measurement error on detecting capability of multivariate ME-ELR control chart diminishes. Figure 2 also illustrates the SDRL of multivariate ME-ELR control chart under *K*=1 and multiple measurements (*K*=2 and *K*=3). The results show that as the value of *K* increases, the SDRL of multivariate ME-ELR control chart decreases.

TABLE 2. ARLs and SDRLs of multivariate ME- ELR chart under measurement error with linearly increasing variance for different values for components of vectord when *p*=3,*n*=5, $\lambda = 0.2$,c=0, $\boldsymbol{\beta} = \mathbf{I}$, $\boldsymbol{\alpha} = \mathbf{0}$ and IC *ARL*=200

		No error		**d=1**		**d=2**		**d=3**	
H		**205.85**		**255.60**		**309.80**		**364.85**	
δ	**γ**	**ARL**	**SDRL**	**ARL**	**SDRL**	**ARL**	**SDRL**	**ARL**	**SDRL**
(0.00, 0.00, 0.00)	(1.00, 1.00, 1.00)	200.69	198.28	201.13	199.70	200.88	198.27	199.16	197.53
	(1.25, 1.25, 1.25)	25.65	23.31	34.87	32.64	45.84	42.82	54.42	52.02
	(1.50, 1.50, 1.50)	9.11	7.19	12.47	10.09	16.62	14.09	21.00	18.28
	(1.75, 1.75, 1.75)	5.02	3.45	6.67	4.79	8.64	6.60	10.70	8.50
(0.25, 0.25, 0.25)	(1.00, 1.00, 1.00)	10.29	7.17	14.16	11.03	16.88	13.67	19.09	16.05
	(1.25, 1.25, 1.25)	6.29	4.18	8.24	5.87	10.02	7.57	11.44	8.79
	(1.50, 1.50, 1.50)	4.26	2.66	5.41	3.55	6.52	4.45	7.63	5.43
	(1.75, 1.75, 1.75)	3.17	1.89	3.94	2.41	4.69	2.97	5.47	3.62
(0.50, 0.50, 0.50)	(1.00, 1.00, 1.00)	3.86	1.77	5.10	2.92	6.00	3.74	6.67	4.36
	(1.25, 1.25, 1.25)	3.20	1.59	4.08	2.31	4.76	2.81	5.30	3.30
	(1.50, 1.50, 1.50)	2.68	1.39	3.30	1.81	3.88	2.21	4.36	2.62
	(1.75, 1.75, 1.75)	2.28	1.17	2.74	1.46	3.22	1.79	3.62	2.10
(0.75, 0.75, 0.75)	(1.00, 1.00, 1.00)	2.38	0.90	3.04	1.40	3.50	1.78	3.86	2.08
	(1.25, 1.25, 1.25)	2.17	0.91	2.69	1.27	3.10	1.59	3.40	1.81
	(1.50, 1.50, 1.50)	1.96	0.89	2.37	1.15	2.71	1.37	3.03	1.57
	(1.75, 1.75, 1.75)	1.79	0.84	2.12	1.02	2.42	1.22	2.69	1.38
(1.00, 1.00, 1.00)	(1.00, 1.00, 1.00)	1.75	0.61	2.18	0.88	2.48	1.11	2.71	1.30
	(1.25, 1.25, 1.25)	1.66	0.64	2.01	0.86	2.29	1.06	2.53	1.22
	(1.50, 1.50, 1.50)	1.57	0.65	1.87	0.80	2.13	0.99	2.32	1.10
	(1.75, 1.75, 1.75)	1.46	0.61	1.72	0.77	1.95	0.90	2.14	1.02

TABLE 3. ARLs and SDRLs of multivariate ME- ELR chart under measurement error with linearly increasing variance for different values for components of vector**c** when p=3, n=5, $\lambda = 0.2$, **d=1**, $\boldsymbol{\beta} = \mathbf{I}$, $\boldsymbol{\alpha} = \mathbf{0}$ and IC ARL=200

		No error		**c=0**		**c=1**		**c=2**	
		205.85		**255.60**		**272.35**		**289.78**	
δ	**γ**	**ARL**	**SDRL**	**ARL**	**SDRL**	**ARL**	**SDRL**	**ARL**	**SDRL**
(0.00, 0.00, 0.00)	(1.00, 1.00, 1.00)	200.69	198.28	201.13	199.70	200.88	199.48	200.12	199.69
	(1.25, 1.25, 1.25)	25.65	23.31	34.87	32.64	37.66	33.95	40.30	37.23
	(1.50, 1.50, 1.50)	9.11	7.19	12.47	10.09	13.29	10.94	14.63	12.31
	(1.75, 1.75, 1.75)	5.02	3.45	6.67	4.79	7.14	5.36	7.79	5.83
(0.25, 0.25, 0.25)	(1.00, 1.00, 1.00)	10.29	7.17	14.16	11.03	15.68	12.51	17.50	14.44
	(1.25, 1.25, 1.25)	6.29	4.18	8.24	5.87	9.00	6.57	9.74	7.20
	(1.50, 1.50, 1.50)	4.26	2.66	5.41	3.55	5.86	3.94	6.29	4.31
	(1.75, 1.75, 1.75)	3.17	1.89	3.94	2.41	4.19	2.61	4.48	2.81
(0.50, 0.50, 0.50)	(1.00, 1.00, 1.00)	3.86	1.77	5.10	2.92	5.58	3.25	6.07	3.73
	(1.25, 1.25, 1.25)	3.20	1.59	4.08	2.31	4.41	2.53	4.83	2.89
	(1.50, 1.50, 1.50)	2.68	1.39	3.30	1.81	3.53	1.94	3.81	2.14
	(1.75, 1.75, 1.75)	2.28	1.17	2.74	1.46	2.91	1.58	3.08	1.69
(0.75, 0.75, 0.75)	(1.00, 1.00, 1.00)	2.38	0.90	3.04	1.40	3.28	1.56	3.54	1.73
	(1.25, 1.25, 1.25)	2.17	0.91	2.69	1.27	2.88	1.40	3.08	1.53
	(1.50, 1.50, 1.50)	1.96	0.89	2.37	1.15	2.52	1.21	2.68	1.34
	(1.75, 1.75, 1.75)	1.79	0.84	2.12	1.02	2.22	1.08	2.35	1.17
(1.00, 1.00, 1.00)	(1.00, 1.00, 1.00)	1.75	0.61	2.18	0.88	2.33	0.98	2.48	1.06
	(1.25, 1.25, 1.25)	1.66	0.64	2.01	0.86	2.15	0.92	2.29	1.02
	(1.50, 1.50, 1.50)	1.57	0.65	1.87	0.80	1.98	0.89	2.08	0.94
	(1.75, 1.75, 1.75)	1.46	0.61	1.72	0.77	1.83	0.84	1.91	0.88

TABLE 4. ARLs and SDRLs of multivariate ME- ELR chart under measurement error with linearly increasing variance for different values of λ when p=3, n=5, d=1, c=1, $\boldsymbol{\beta} = \mathbf{I}$, $\boldsymbol{\alpha} = \mathbf{0}$ and IC ARL=200

		$\boldsymbol{\lambda = 0.1}$		$\boldsymbol{\lambda = 0.2}$		$\boldsymbol{\lambda = 0.3}$		$\boldsymbol{\lambda = 0.4}$	
H		**246.45**		**272.35**		**295.15**		**316.57**	
δ	**γ**	**ARL**	**SDRL**	**ARL**	**SDRL**	**ARL**	**SDRL**	**ARL**	**SDRL**
(0.00, 0.00, 0.00)	(1.00, 1.00, 1.00)	200.98	196.92	200.88	199.48	200.74	200.17	199.36	197.21
	(1.25, 1.25, 1.25)	34.01	29.92	37.66	33.95	40.87	39.10	43.67	42.00
	(1.50, 1.50, 1.50)	12.99	9.58	13.29	10.94	14.43	12.70	15.33	14.19
	(1.75, 1.75, 1.75)	7.40	4.79	7.14	5.36	7.32	5.85	7.61	6.46
(0.25, 0.25, 0.25)	(1.00, 1.00, 1.00)	14.24	9.81	15.68	12.51	18.00	15.66	20.67	18.97
	(1.25, 1.25, 1.25)	8.76	5.44	9.00	6.57	9.54	7.60	10.37	8.82
	(1.50, 1.50, 1.50)	6.08	3.54	5.86	3.94	6.00	4.37	6.16	4.83
	(1.75, 1.75, 1.75)	4.53	2.47	4.19	2.61	4.12	2.80	4.25	3.10
(0.50, 0.50, 0.50)	(1.00, 1.00, 1.00)	5.77	2.88	5.58	3.25	5.81	3.94	6.13	4.43
	(1.25, 1.25, 1.25)	4.70	2.31	4.41	2.53	4.40	2.78	4.55	3.12
	(1.50, 1.50, 1.50)	3.84	1.90	3.53	1.94	3.46	2.11	3.47	2.30
	(1.75, 1.75, 1.75)	3.19	1.52	2.91	1.58	2.85	1.69	2.78	1.79
(0.75, 0.75, 0.75)	(1.00, 1.00, 1.00)	3.54	1.49	3.28	1.56	3.19	1.67	3.25	1.89
	(1.25, 1.25, 1.25)	3.17	1.37	2.88	1.40	2.76	1.47	2.76	1.62
	(1.50, 1.50, 1.50)	2.79	1.20	2.52	1.21	2.41	1.28	2.36	1.34
	(1.75, 1.75, 1.75)	2.48	1.10	2.22	1.08	2.14	1.14	2.06	1.14
(1.00, 1.00, 1.00)	(1.00, 1.00, 1.00)	2.57	0.95	2.33	0.98	2.22	1.02	2.14	1.05
	(1.25, 1.25, 1.25)	2.40	0.93	2.15	0.92	2.03	0.95	1.98	0.99
	(1.50, 1.50, 1.50)	2.23	0.90	1.98	0.89	1.86	0.88	1.81	0.90
	(1.75, 1.75, 1.75)	2.04	0.86	1.83	0.84	1.72	0.82	1.65	0.83

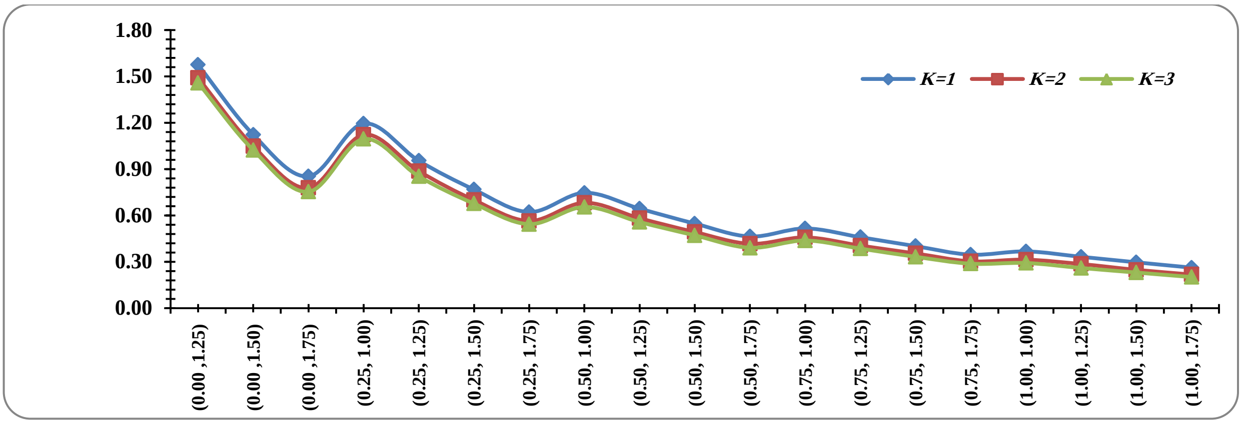

Figure 1. The values of log (ARL_1) for multiple measurements when *K*=1, *K*=2, *K*=3, *n*=5, **c=1**, **d=1** and IC ARL=200

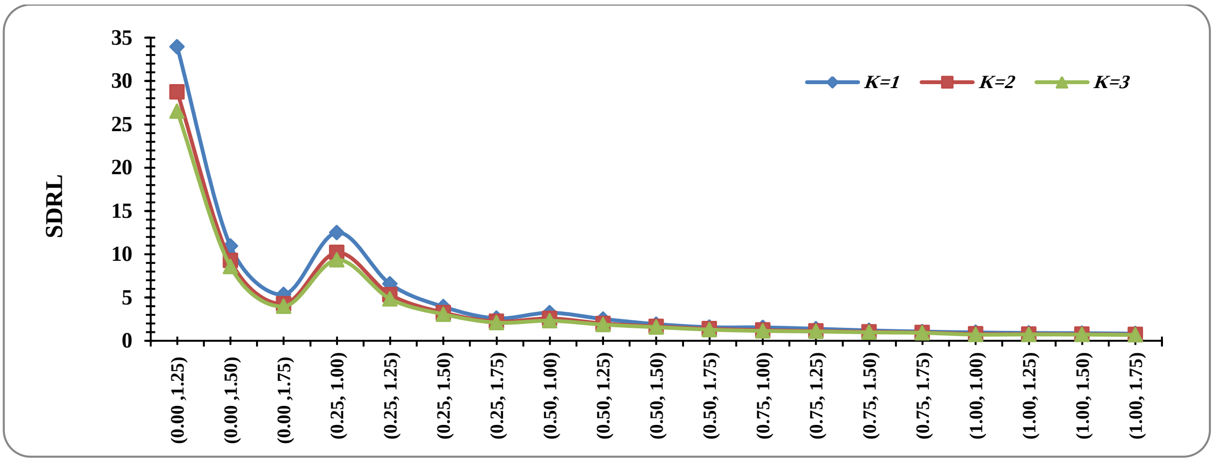

Figure 2. The values of SDRL for multiple measurements when *K*=1, *K*=2, *K*=3, *n*=5, **c=1**, **d=1** and IC ARL=200

TABLE 5. Monitoring data for process making springs

Sample	$\begin{pmatrix} X_1 \\ X_2 \end{pmatrix}$					ELR	Sample	$\begin{pmatrix} X_1 \\ X_2 \end{pmatrix}$					ELR
1	28.14	28.31	28.27	28.20	28.26	14555.81	7	28.24	28.32	28.31	28.36	28.41	14549.53
	46.32	45.79	45.88	45.88	45.80			45.90	45.83	45.69	45.78	45.72	
2	28.50	28.35	28.30	28.32	28.20	14558.44	8	28.23	28.36	28.34	28.31	28.33	14545.25
	45.85	45.91	45.80	45.91	45.93			45.75	45.89	45.66	45.84	45.74	
3	28.29	28.30	28.29	28.38	28.29	14543.01	9	28.25	28.39	28.31	28.35	28.32	14542.17
	45.83	45.75	45.75	45.52	45.58			45.59	46.10	45.87	45.57	45.87	
4	28.22	28.26	28.27	28.27	28.28	14547.52	10	28.31	28.28	28.31	28.36	28.32	14542.75
	45.81	45.99	45.78	46.02	45.85			45.70	45.75	45.78	45.89	45.90	
5	28.30	28.36	28.27	28.32	28.30	14548.44	11	28.37	28.38	28.35	28.45	28.39	14536.71
	45.77	45.94	46.04	45.77	45.65			45.82	45.35	45.76	45.81	45.88	
6	28.34	28.29	28.32	28.27	28.19	14552.96	12	28.17	28.22	28.28	28.12	28.35	14507.85
	45.77	45.93	45.77	45.92	46.04			45.30	45.25	45.73	45.81	45.88	

5. REAL DATA EXAMPLE

In this section, we illustrate the application of multivariate ME-ELR control chart and the multiple measurements on each sample point using a real data set. Table 5 contains a bivariate data set in which X_1 and X_2 are the spring inner diameter and elasticity, respectively.

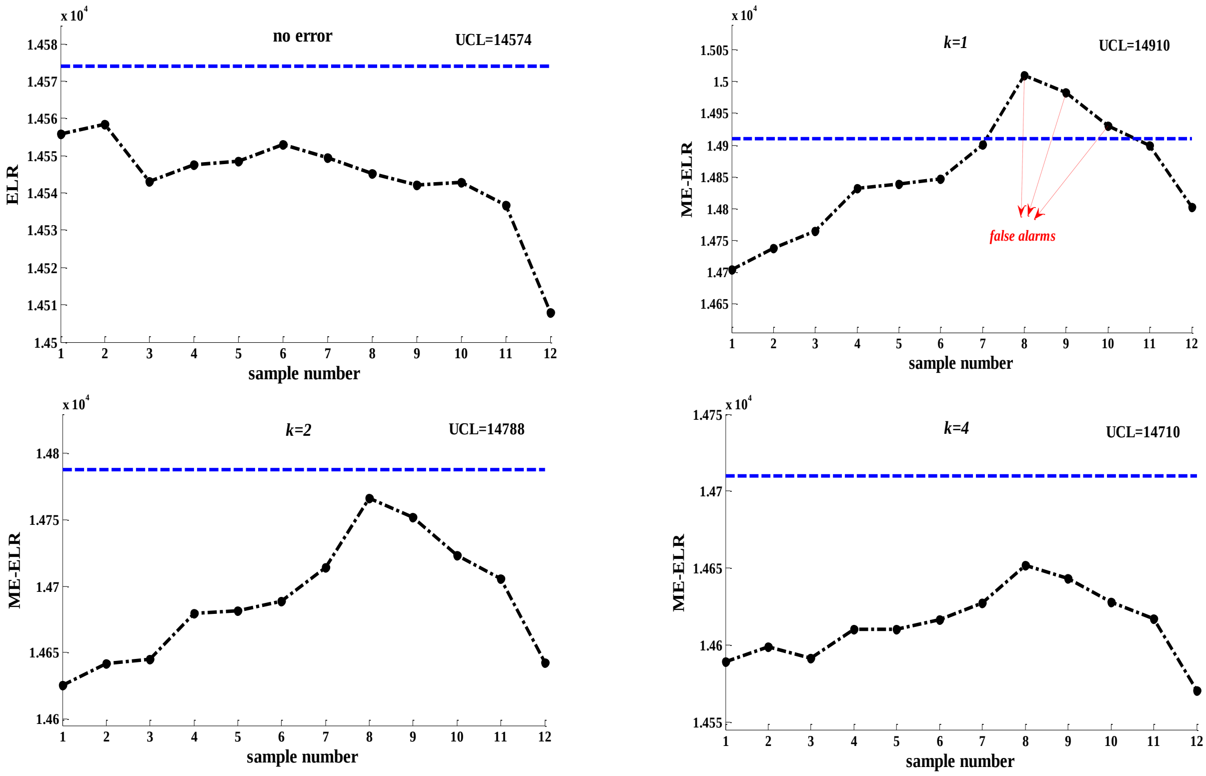

Figure 3. Comparison among multivariate ELR, ME-ELR (K=1) and ME-ELR with (K=2 and K=4) multiple measurements

The samples in Table 5 are collected every half an hour from a process of making springs in a manufacture company [30]. According to the historical data in Phase I analysis, we know that:

$$\boldsymbol{\mu}_0 = \begin{pmatrix} 28.29 \\ 45.85 \end{pmatrix}, \; \boldsymbol{\Sigma}_0 = \begin{pmatrix} 0.0035 & -0.0046 \\ -0.0046 & 0.0226 \end{pmatrix}$$

The control statistics of Table 5 under four scenarios are depicted in Figure 3. These scenarios are: (1) ELR control chart (no measurement error), (2) multivariate ME-ELR control chart (K=1), (3) multiple measurements scenario when K=2, and (4) multiple measurement scenario when K=4. Figure 3 represents that the process mean vector and covariance matrix is simultaneously in-control under ELR control chart. However, measurement error with linearly increasing variance leads to have false alarms on 8^{th}, 9^{th} and 10^{th} samples. We can also conclude that by measuring each item more than one time, the false alarms are diminished.

6. CONCLUSIONS AND A FUTURE RESEARCH

In this paper we studied the effect of measurement error with linearly increasing variance on simultaneous monitoring of multivariate process mean vector and covariance matrix. Then, we proposed multiple measurements on each sample point in order to reduce the effect of measurement error. Through a simulation study, we showed that measurement error can adversely affect the detecting performance of ELR control chart in detecting shifts in process mean vector, covariance matrix individually and simultaneously. The results of simulation studies also showed that multiple measurement approach can effectively cover the measurement error effects. We also provided a sensitivity analysis on the parameters of the proposed control chart. Finally, we illustrated the application of the proposed multivariate ME-ELR control chart and multiple measurement approach through a real data from a manufacturing process. The results of real data example showed that measurement error can increase the false alarms. Future research can investigate the effect of measurement error on multivariate control chart in multi-stage process [31, 32]. Analyzing the effect of measurement error on pattern recognition approaches [33] is also recommended for future researches.

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Simultaneous Monitoring of Multivariate Process Mean and Variability in the Presence of Measurement Error with Linearly Increasing Variance under Additive Covariate Model

RESEARCH NOTE

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چکیده

در سال‌های اخیر تحقیقاتی روی پایش همزمان بردار میانگین و ماتریس کوواریانس در فرایندهای نرمال چند متغیره انجام شده است. لیکن اثر خطای اندازه گیری که در بسیاری از کاربردهای عملی وجود دارد روی عملکرد این نوع نمودارهای کنترل به خوبی بررسی نشده است. در این مقاله اثر خطای اندازه‌گیری با واریانس خطی افزایشی روی عملکرد نمودار کنترل ELR برای پایش همزمان بردار میانگین و ماتریس کوواریانس فرایندهای نرمال چند متغیره نشان داده شده است. همچنین روش چند بار اندازه‌گیری برای کاهش اثر خطا توسعه داده شده است. همچنین برای ارزیابی عملکرد روش چندبار اندازه‌گیری از دو معیار متوسط و انحراف استاندارد طول دنباله استفاده شده است. در نهایت با استفاده از داده‌های واقعی در صنعت ساخت کاربرد روش پیشنهادی نشان داده شده است.

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