



and very high heat transfer rate in a short period of time [4], the Fourier's law cannot correctly predict the thermal behavior of the material.

In the mid-twentieth century, some scientists such as Morse and Feschbach [5], Cattaneo [6], and Vernotte [7], in different studies, achieved a new form of heat conduction equation to account for the effect of time lag between the heat flux vector and the resulting temperature gradient vector. This time lag is called the relaxation time,  $\tau$ . Thus, a classical heat wave model was established which is known as the Cattaneo-Vernotte heat conduction model. This modification in the Fourier's model transformed the energy conservation equation, which assumed a parabolic heat conduction equation, into a hyperbolic wave equation. The hyperbolic heat conduction model shows the effects of heat transfer phenomena at small time scales on conventional spatial scales. Joseph and Preziosi [8] and Ozisik and Tzou [9] reviewed the thermal relaxation phenomenon in the thermal wave propagation theory.

Several researchers have recently utilized the non-Fourier Cattaneo-Vernotte heat conduction model. Bargmann and Favata [10] used the model to analyze the laser heating in polycrystals. Sasmal and Mishra [11] studied the non-Fourier heat transfer in the combined analysis of heat conduction and thermal radiation in a two-dimensional square enclosure. Rahbari et al. [12] studied the non-Fourier heat conduction in a finite body with insulated boundaries. Zhao et al. [13] conducted a non-Fourier analytical study on a spherical body under surface thermal disturbances. Mishra and Sahai [14] investigated the non-Fourier heat conduction and thermal radiation in a spherical shell. In an analytical study, Fong and Lam [15] studied the phenomenon of thermal waves in a thin film. There are many other theoretical studies which are suitably established in the field of heat transfer [16-24].

In most studies that have been conducted on non-Fourier heat conduction, due to constant thermal properties the resulting equations were linearized, and the nonlinear study of such problems is rare in the literature but as we know, the behavior of materials is inherently nonlinear in nature, and the study of nonlinear problems mentioned in some cases is very important. On the other hand, in most studies, the heat source has been assumed constant, while in some cases, such as nuclear or chemical reactions, heat generation in the system is temperature-dependent.

In this study, the non-Fourier heat conduction problem in a semi-infinite body was studied. The Cattaneo-Vernotte thermal wave model was used. The thermal conductivity coefficient was assumed temperature-dependent which resulted in a non-linear equation. The heat source was also considered temperature-dependent. The numerical Mac-Cormack predictor-corrector method was used to solve the

equations. The investigation of the effects of a variable thermal conductivity and a variable heat source in a non-Fourier heat conduction problem is the novelty and originality of this study.

## 2. MATHEMATICAL MODELING

Consider a 1-D semi-infinite body. Energy equation assuming the presence of a heat source is as follows:

$$\rho c \frac{\partial T(x,t)}{\partial t} + \frac{\partial q(x,t)}{\partial x} - g = 0 \quad (1)$$

where  $\rho$  is the density of the body,  $c$  is specific heat of the body,  $T(x,t)$  is temperature function and  $q(x,t)$  is the function of heat flux,  $g$  is the heat source, and  $x$  and  $t$  are spatial and temporal variables, respectively. The conduction heat flux constitutive equation governing the problem based on the heat wave model [6, 7] can be written as follows:

$$\tau \frac{\partial q(x,t)}{\partial t} + q(x,t) + k \frac{\partial T(x,t)}{\partial x} = 0 \quad (2)$$

where  $\tau$  is the thermal relaxation time and  $k$  is thermal conductivity. A variation in thermal conductivity is considered as a linear function of temperature, as follows:

$$k = k_0 [1 + \gamma(T - T_0)] \quad (3)$$

The heat source can be also modeled as a linear function of temperature as the following equation:

$$g = g_0 [1 + \beta(T - T_0)] \quad (4)$$

The following dimensionless parameters are introduced:

$$\begin{aligned} FO &= \frac{\alpha_0 t}{2L}, \tilde{x} = \frac{x}{2L}, Ve^2 = \frac{\alpha_0 \tau_0}{L^2}, \\ \eta &= \frac{2g_0 L^2}{T_0 k_0}, \alpha = \frac{\eta \beta}{2}, \tilde{T}(\tilde{x}, FO) = \frac{T(x,t) - T_0}{T_0}, \\ \tilde{q}(\tilde{x}, FO) &= \frac{\alpha_0 q(x,t)}{T_0 k_0 c_0}. \end{aligned} \quad (5)$$

Thus the dimensionless form of Equations (1) and (2) are as follows:

$$\frac{\partial \tilde{T}(\tilde{x}, FO)}{\partial FO} + \frac{1}{Ve} \frac{\partial \tilde{q}(\tilde{x}, FO)}{\partial \tilde{x}} - \eta [1 + \beta \tilde{T}(\tilde{x}, FO)] = 0 \quad (6)$$

$$Ve^2 \frac{\partial \tilde{q}(\tilde{x}, FO)}{\partial FO} + 2\tilde{q}(\tilde{x}, FO) + Ve [1 + \gamma \tilde{T}(\tilde{x}, FO)] \frac{\partial \tilde{T}(\tilde{x}, FO)}{\partial \tilde{x}} = 0 \quad (7)$$

The initial and boundary conditions in the dimensionless form are also as follows:

$$\begin{aligned} \tilde{T}(\tilde{x}, 0) &= 0, \tilde{q}(\tilde{x}, 0) = 0, \tilde{q}(\infty, FO) = 0, \\ \tilde{T}(0, FO) &= \tilde{T}_p u(FO). \end{aligned} \quad (8)$$

### 3. NUMERICAL SCHEME

The Mac-Cormack predictor-corrector numerical method is used to solve the equations. First, Equations (6) and (7) should be written in vector form, as follows:

$$\frac{\partial \mathbf{E}}{\partial FO} + \frac{\partial \mathbf{F}}{\partial \tilde{x}} + \mathbf{H} = 0 \quad (9)$$

where,

$$\mathbf{E} = \begin{bmatrix} \tilde{T}(\tilde{x}, FO) \\ Ve^2 \tilde{q}(\tilde{x}, FO) \end{bmatrix} \quad (10)$$

$$\mathbf{F} = \begin{bmatrix} \frac{1}{Ve} \tilde{q}(\tilde{x}, FO) \\ Ve \tilde{T}(\tilde{x}, FO)(1 + 0.5\gamma \tilde{T}(\tilde{x}, FO)) \end{bmatrix} \quad (11)$$

$$\mathbf{H} = \begin{bmatrix} -\eta(1 + \beta \tilde{T}(\tilde{x}, FO)) \\ 2\tilde{q}(\tilde{x}, FO) \end{bmatrix} \quad (12)$$

Then, by imposing Mac-Cormack scheme to above equations, finite difference formulations are resulted [25]:

Predictor:

$$\tilde{\mathbf{E}}_i^{n+1} = \mathbf{E}_i^n - \frac{\Delta FO}{\Delta \tilde{x}} [\mathbf{F}_{i+1}^n - \mathbf{F}_i^n] - \Delta FO \mathbf{H}_i^n \quad (13)$$

Corrector:

$$\mathbf{E}_i^{n+1} = \frac{1}{2} [\mathbf{E}_i^n + \tilde{\mathbf{E}}_i^{n+1} - \frac{\Delta FO}{\Delta \tilde{x}} (\tilde{\mathbf{F}}_i^{n+1} - \tilde{\mathbf{F}}_{i-1}^{n+1}) - \Delta FO \tilde{\mathbf{H}}_i^{n+1}] \quad (14)$$

In above equations, subscript  $i$  indicates the grid point in spatial domain, superscript  $n$  denotes the time level in time domain, tilde symbol indicates the predicted value in  $n+1$  time level,  $\Delta \tilde{x}$  and  $\Delta FO$  are spatial and time steps, respectively. The forward finite differencing is used for predictor formulation and the backward differencing is used for corrector formulation.

### 4. RESULTS AND DISCUSSION

To evaluate the accuracy of the results obtained from the numerical method, the results were compared to Lewandowska and Malinowski [26], which is a linear analytical study (Figure 1). As can be seen in Figure 1, the employed numerical method was highly accurate. The acceptable agreement between the two graphs also confirms the accuracy of the numerical method. Quantitatively, there is a 0.85% average error for  $\alpha=0.05$  and 0.53% average error for  $\alpha=0.005$  between present study and analytical solution of Lewandowska and Malinowski [26]. The variations of various parameters and the resulting effects are discussed in the next sections.

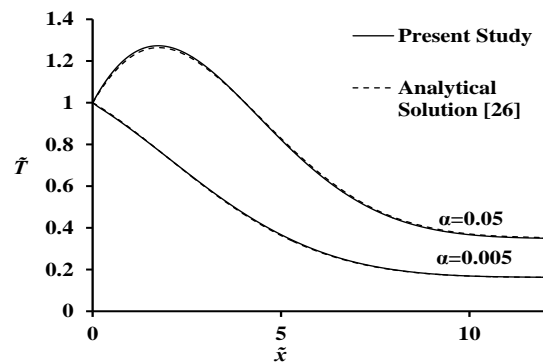


Figure 1. The comparison between present numerical solution and an analytical solution [26]

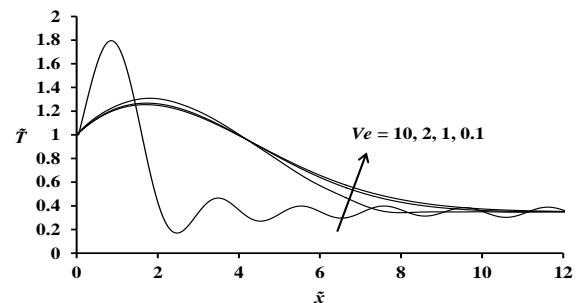
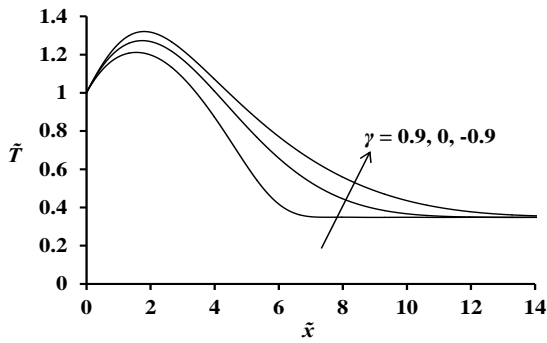
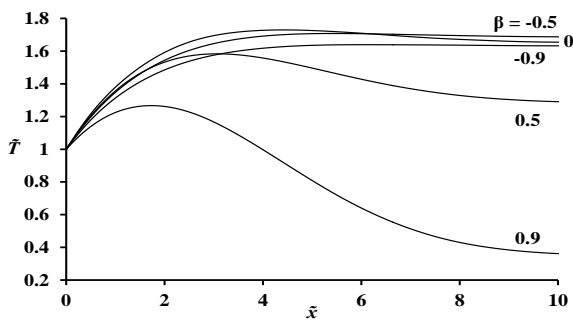


Figure 2. The effects of variations in Vernotte number on the temperature profile ( $\gamma=0.1$ ,  $\alpha=0.05$ )

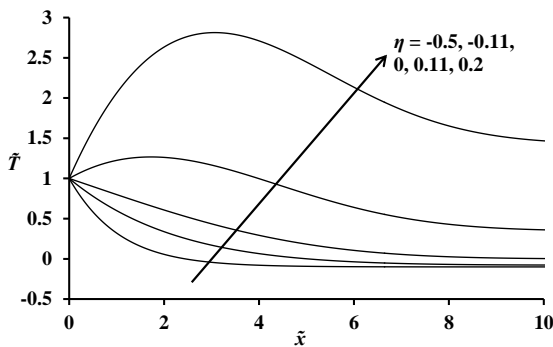
**4. 1. Variations in the Vernotte Number** Figure 2 shows the effect of variations in the Vernotte number on the temperature profile. In the early parts of the body, at a certain point, the temperature increased with increasing the Vernotte number. An increased Vernotte number increased the difference between the diagrams. By moving forward in the body, an increasing trend was observed in all diagrams, and the diagrams reached to a maximum. This increase is due to the temperature-dependency of the heat source. In fact, an increased temperature increased the energy amount of the heat source, which in turn, increased the temperature. This mutual feedback continued. However, energy dissipaters, such as heat resistance of the material prevent this trend from reaching an infinite temperature, and finally, at one point, i.e., the maximum point, the rising trend of temperature stops, and the temperature would then decrease. The declining trend of graphs became steeper and caused the three diagrams with lower Vernotte number coincide in one point (around  $\tilde{x}=4.2$ ). The declining trends continued after this point, and finally, all the curves converged to  $\tilde{T}=0.35$ , which shows that the balance between temperature and heat source had been established.



**Figure 3.** The effects of variations in  $\gamma$  on the temperature profile ( $Ve=1, \alpha=0.05$ )



**Figure 4.** The effects of variations in  $\beta$  on the temperature profile ( $Ve=1, \eta=0.11, \gamma=0.1$ )



**Figure 5.** The effects of variations in  $\eta$  on the temperature profile ( $Ve=1, \beta=0.9, \gamma=0.1$ )

Another noteworthy issue in the diagrams is the oscillatory behavior of the diagrams by increasing the Vernotte number. This behavior is especially evident at the endpoint of the diagram corresponding to  $Ve=2$  as well as the diagram corresponding to  $Ve=10$ . In fact, by increasing the Vernotte number, diagrams had a steeper slope in rises and declines and became more damped. The oscillatory behavior is a characteristic of the non-Fourier heat transfer, and higher oscillatory behavior indicates the greater deviation of the material thermal behavior from the Fourier state. Increase in Vernotte

number increased the maximum of the diagram which also occurred earlier. These behaviors can be justified considering the nature of the Vernotte number ( $Ve = \sqrt{\alpha_0 \tau_0} / L$ ). In fact, the materials with higher relaxation times, and consequently larger Vernotte numbers, show more evident non-Fourier behavior. This can be found in reference [27].

**4. 2. Effect of Thermal Conductivity Variations with Temperature**

Effect of thermal conductivity variations with temperature was also of interest. Figure 3 shows the effect of thermal conductivity coefficient ( $\gamma$ ) variations with temperature on the temperature profile. As can be seen in Figure 3, an increased  $\gamma$  decreased the temperature at a given point. An increased  $\gamma$  caused the maximum temperature occur at an earlier point. A smaller  $\gamma$  resulted in a more uniform and slower temperature variation and caused the temperature decay to the final temperature at a wider range. Figure 3 represents the importance of considering the thermal conductivity as a variable and indicates how much this parameter is affected by the temperature variations.

**4. 3. Effect of Variation in Heat Source Coefficient with Temperature**

Figure 4 shows the effect of variation in coefficient of heat source ( $\beta$ ) with temperature on the temperature profile. In negative values of  $\beta$ , variation in this parameter did not significantly affect the temperature, and in fact, the effect of negative slope of variation in heat source with temperature was negligible on the temperature profile. However, in the positive values of  $\beta$ , variation in this parameter significantly affected the temperature and reduced the temperature at a given point. The reduction was particularly significant at endpoints, and constant endpoint temperatures were different for different values of  $\beta$ . The maximum temperature occurred when the heat source was assumed constant and independent of temperature ( $\beta=0$ ). In fact, this figure shows that a constant heat source is acceptable only if the assumption is absolutely true and not an approximation to merely simplify the problem, because otherwise, the results would have significant error.

**4. 4. Effect of Variations in the Heat Source Magnitude**

Finally, it is noteworthy to look at the effect of variations in the dimensionless reference heat source magnitude ( $\eta$ ) on the temperature profile. Figure 5 shows these variations. Figure 5 shows that, as expected, the temperature increased with increase in  $\eta$ . In fact, similar to the previous figure, the variations in the negative values of these parameters had much less effect on the temperature profiles compared to the positive values. The maximum temperature occurred at positive values of  $\eta$  and moved forward with increase in

$\eta$ . Variations in  $\eta$  changed the final value of the constant temperature.

## 5. CONCLUSION

Non-Fourier heat transfer problem in a semi-infinite body was examined. The Cattaneo-Vernotte thermal wave model was employed. Thermal conductivity was assumed temperature-dependent which resulted in a non-linear equation. The heat source was also assumed temperature-dependent. The Mac-Cormack predictor-corrector numerical method was utilized to solve the equations. In summary, the following results were obtained:

1. An increased Vernotte number resulted in more significant non-Fourier effects and a more oscillatory behavior in the temperature profile.
2. The importance of considering the thermal conductivity temperature-dependence was proved. It was shown that increase in the thermal conductivity coefficient ( $\gamma$ ) decreases the temperature at a given point.
3. It was concluded that the changes in heat source coefficient ( $\beta$ ) significantly affects the temperature, and their increased values reduce the temperature at a given point.
4. The assumption of a constant thermal conductivity and constant heat source may result in a large error in the results. This issue should be carefully addressed.

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## Investigation of the Effects of Non-linear and Non-homogeneous Non-Fourier Heat Conduction Equations on Temperature Distribution in a Semi-infinite Body

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### P A P E R I N F O

چکیده

#### Paper history:

Received 03 July 2015

Received in revised form 31 August 2015

Accepted 24 December 2015

#### Keywords:

Heat Conduction

Mac-Cormack Numerical Method

Non-Fourier

Non-Homogeneous Equation

Non-Linear Equation

در این مقاله مسأله‌ی انتقال حرارت غیر فوری‌ای در یک جسم نیمه بی‌نهایت مورد بررسی قرار گرفت. برای تحلیل حرارتی مسئله از مدل هدایت حرارتی غیر فوری‌ای موج حرارتی استفاده شد. ضریب هدایت حرارتی متغیر با دما فرض شد و بدین ترتیب معادله‌ی غیر خطی بدست آمد. علاوه بر آن منبع تولید حرارت نیز متغیر با دما در نظر گرفته شد که معادله‌ای غیر همگن بدست آمد. برای حل معادلات، روش عددی پیشگو-اصلاحگر مک کورمک به کار گرفته شد. نتیجه گرفته شد که تحلیل غیر خطی مسائل انتقال حرارت غیر فوری‌ای اهمیت زیادی دارند و همچنین متغیر فرض کردن منبع حرارتی با دما، تغییرات زیادی را در نتایج نسبت به حالتی که منبع تولید حرارت ثابت فرض شود، به وجود می‌آورد.

doi:10.5829/idosi.ije.2015.28.12c.14