IJE TRANSACTIONS B: Applications Vol. 28, No. 11, (November 2015) 1624-1633

International Journal of Engineering

Journal Homepage: www.ije.ir

Investigating Zone Pricing in a Location-Routing Problem Using a Variable Neighborhood Search Algorithm

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PAPER INFO

Paper history:
Received 25 April 2015
Received in revised form 04 October 2015
Accepted 16 October 2015

Keywords:
Pricing, Zone Pricing
Location-Routing Problem
Variable Neighborhood Search

ABSTRACT

In this paper, we assume a firm tries to determine the optimal price, vehicle route and location of the depot in each zone to maximise its profit. Therefore, in this paper zone pricing is studied which contributes to the literature of location-routing problems (LRP). Zone pricing is one of the most important pricing policies that are prevalently used by many companies. The proposed problem is very applicable in the product distribution, such as fruit. The problem is formulated by two models consisting of a node and flow based model. The resulting nonlinear mixed integer models are approximated by a piecewise linearization method and the performance of them is compared. To cope with real-world cases, a variable neighborhood search (VNS) algorithm is developed and implemented in some instances. Three different combinations of local search are defined and the performance of them is compared with each other and two proposed models. The results of the computational study confirm that the suggested algorithm solves large instances efficiently compared to the proposed mathematical models. Moreover, the results show that the flow based model uses less computational time in comparison with the node based model.

***doi**: 10.5829/idosi.ije.2015.28.11b.10*

1. INTRODUCTION

Nowadays, not only the quality of supplying demands and services is a key factor for customers, but, the price of products is also another factor which affects the customer satisfaction. Companies always pursue a strategy that helps them to have an efficient movement of goods or workers. This issue helps them to receive more profit in their market. Christian [1] stated that the distribution costs account for approximately 10% of the firms' incomes and for more than 45% of the total logistics expenses.

Firms in spatial markets may implement a wide verity of pricing policies. Aras et al. [2] have used the pricing issue in the VRP. They integrated the concept of uniform delivered pricing and the selective multi-depot VRP. As another example, Lederer and Thisse [3] proposed a competitive situation in which, two firms try to optimize the location of their facilities and price of their products in order to maximize their profit.

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The market environment allows the firm to implement a wide variety of pricing policies, depending on the shipping costs passed to the customers such as delivery, milling, discriminatory and zone pricing. The first one implies that the same price is charged at the firm's door to all customers, regardless of their location; thus, the customers are incurred full of shipping costs. The second one means that the firm bears some or all of the shipping costs and the same delivered price is charged to all clients. A prevalent pricing policy, named zone pricing, is examined in this article. Taking into account this policy, simultaneously, several delivered prices for some predetermined zones should be determined. An important situation arises in which zones are given a priori corresponding to countries, natural areas or the economic regions. In U.K., the London Brick Company uses this pricing policy to determine identical prices within zones placed at the center of the plant. Another application of zone pricing is found in the domestic fuel and cement industries [4, 5] .

Zone pricing is a pricing policy with some advantages. The main advantage of zone pricing is diminishing the firm accounting costs. Another advantage is capturing high level of

Please cite this article as: M. Setak, M. Sadeghi-Dastaki , H. Karimi, Investigating Zone Pricing in A Location-Routing Problem Using A Variable Neighborhood Search Algorithm, International Journal of Engineering (IJE), TRANSACTIONS B: Applications Vol. 28, No. 11, (November 2015) 1624-1633

the gain; it would be made under perfect discriminatory pricing. Last, zone pricing can be a good pricing policy in the competitive environment. Incumbents tend to deter entry of competitors to the extent. They can react by decreasing prices in the zones of encroachment without changing prices in the other zones.

LRP is used frequently in many industries such as bill delivery [6] and Telecom network design [7] and so on. Readers in the field of LRP, can follow a valuable paper by Setak et al. [8, 9]. Decisions made in the LRPs influence on the price of products, so, this issue stimulated us to study about integrating pricing decisions and LRP. This issue is prevalently used in the companies that dispense vegetables, fruit, woolen garments, textile products, leather and shoes. When firms try to optimize the location of facilities, vehicle routing and product price simultaneously, they can improve the level of their profit and customer satisfaction in the market. In this problem, there is a firm that distributes goods to customers and needs to establish some depots in its vast market. The firm divided the market into some zones according to its policies. In the other hand, it decides to have a depot in each zone. Then, it should send goods with a vehicle to depots and after that, the firm should determine the route of the vehicle. Besides the depot location and routing, the price should be adopted in each zone separately. It is required to optimize its profit by applying a single depot vehicle routing problem in each zone. We assume a quadratic demand function in each zone. We suggest two models based on nodes and commodity flow. The objective function of them is mixed integer nonlinear programming, then we approximate and linearize it with the piecewise linearization method. If the price of products in each zone is equal to zero, the problem will clearly reduce to the LRP, which was proven to be NP-hard [10]. To handle the real-world cases, a heuristic VNS algorithm is proposed with four operators in its local search for solving the problem.

The remainder of the paper is structured as follows. The literature of pricing problems with the location, routing and LRPs is provided in section 2. Section 3 describes the model formulations and provides two formulations consisting of a node based and flow based model. Section 4 extends a VNS algorithm for our problem in solving large-scale problems and presents the description of the proposed heuristic solution procedure. Computational studies are reported in section 5. Section 6 presents some concluding remarks and directions for further research.

2. LITERATURE REVIEW

In this section, first, we concisely narrate the literature of pricing policies in location and VRP area. Then, we state how much researchers attend to integrate pricing decision and LRP.

In the location area, many researchers pursue the optimizing of the network by making decisions about prices of products and location of facilities simultaneously. Beckmann and Ingene [11] studied spatial monopoly and oligopoly market that had been overlooked in the literature on delivered pricing in location. Hansen et al. [12] worked on determining the uniform delivered pricing for a geographical system of demand functions, in order to maximize the firm profit. Beckmann and Ingene [11] studied spatial monopoly and oligopoly market that seems to be overlooked in the literature on delivered pricing in location. Hansen et al. [12] worked on determining the uniform delivered pricing for a geographical system of demand functions. Another pricing policy that entered to the location theory is discriminatory pricing. Hurter and Lederer [13] established a model that each firm has a production function. It sets discriminatory prices and locates in the plane. Afterwards, Kats [14] coped with a problem of finding location-price equilibrium in a market with a constraint on the number of various discriminatory prices. Hansen et al. [15] proposed a location problem under discriminatory pricing to maximize the firm profits.

One of the notable pricing policy is mill pricing that excludes the cost of transporting the commodities from the point of sale. In this area, Beckmann and Ingene [11] and Hansen et al. [15] discussed the situation with mill pricing in the cited article. In a geographical space, Dasci and Laporte [16] analysed a monopolist location and pricing decisions, willing to open several stores. Moreover, Diakova and Kochetov [17] addressed the problem of determining the optimal facility location and price that facilities can charge the different mill prices. In addition, Luer-Villagra and Marianov [18] have formulated a hub location and pricing problem and proposed a genetic algorithm. Zone pricing has formed another part of the literature of pricing policies for location problems. It consists of simultaneous decision making about several delivered prices, along with the zones in which, the prices should be applied. As a remarkable work, Hansen et al. [19] studied another situation with zone pricing in the cited article. Then, Hansen et al. [5] proposed a model and algorithm to determine optimal facility locations, tariff-zones, market areas and prices to maximize the profit of the firm under zone pricing.

Finally, in the VRP, Aras et al. [2] investigated the reverse logistics problem in which, a firm wishes to collect cores from its dealers. They proposed two flow-based and node-based models for their problem.

We reviewed the relative literature of pricing decisions in the above mentioned areas. To the best of our knowledge, researchers in the LRP have paid little attention to the integration of pricing decisions (specially zone pricing) into the LRP. So, we work on integrating zone pricing into the LRP. Moreover, interested readers in pricing issues are referred to an applicable paper by Tofigh and Mahmoudi [20]. In the next section, we propose two mathematical models.

3. MATHEMATICAL MODELS

In this research, we examined the problem that a firm dispenses the product to customers. Moreover, it needs to establish some depots in its market. The firm divided the market into some zones according to its policies. Actually, it wishes to have a depot in each zone. Then, it should send goods with a vehicle to depots and after that, the firm should decide on the vehicle route. Besides the depot location and routing, the price of products should be determined in each zone separately. It needs to optimize its profit by applying a single depot VRP in each zone. We assume a quadratic demand function in each zone. Moreover, we propose a flow-based and node-based model for the problem in the following.

Before we present two formulations of the problem, we list the same sets, parameters and variables utilized in both models. We have K zones and we follow a strategy to assign customer i_k to depots in each zone. We have M_k potential location for constructing a depot in each zone k . So the sets, parameters and variables in these models are defined as follows:

Sets: $k \in K$ (set of zones), $i_k \in I_k$ (set of customers), $m_k \in M_k$ (set of potential locations).

Parameters: $C_{i_k j_k}$ (travel cost between node i_k and j_k), G_{m_k} (initial cost for constructing depot at nod i_k), C_{0m_k} (travel cost between firm and depot m_k), $f_k(d_k)$ (demand function at zone k), d_k (represents all of independent variables such as price.)

Variables: W_k (unit price of products in zone k), $X_{i_k j_k}$ (the binary variable is used to show that the customer j_k is visited after i_k , if $X_{i_k j_k} = 1$.), Y_{m_k} (the binary variable is used to show that the potential location m_k is selected as depot for zone k, if $Y_{m_k} = 1$.).

3. 1. Node-Based Model

This model (LRP with zone pricing (LRPZP1)) contains a special variable U_{i_k} in addition to previous. The variable U_{i_k} helps us in constructing subtour elimination constraint according to the well-known MTZ constraint [21]. The first is given as follows:

$$\max \sum_{k} f_k(d_k) W_k - \sum_{i_k, j_k, i_k \neq j} C_{i_k j_k} X_{i_k j_k} - \sum_{m_k} (G_{m_k} + C_{0m_k}) Y_{m_k} \quad (1)$$

$$\sum_{i_k \neq j} X_{i_k j_k} = 1 \quad \forall j_k \quad (2)$$

$$\sum_{i_k \neq j} X_{i_k j_k} = 1 \quad \forall i_k \quad (3)$$

$$\sum_{j_k \neq i_k} X_{i_k j_k} = \sum_{j_k \neq i_k} X_{j_k i_k} \quad \forall i_k \quad (4)$$

$$\sum_{m_k} Y_{m_k} = 1 \quad \forall m_k \quad (5)$$

$$U_{i_k} - U_{j_k} + N_k X_{i_k j_k} \leq N_k - 1 \quad \forall i_k, j_k \quad i_k \neq j_k \quad i_k, j_k \neq 0 \ (node\ of\ depot) \quad (6)$$

$$X_{i_k j_k} = 0 \text{ or } 1 \quad (7)$$

$$U_{i_k} \geq 0 \quad (8)$$

$$Y_{m_k} = 0 \text{ or } 1 \quad (9)$$

$$W_k \geq 0 \quad (10)$$

The objective function is shown as (1). It gives the profit of the firm which should be maximized. Constraints (2) and (3) represent the concept of visiting a node only one time. Constraint (4) ensures that each node should be used with one input and output edges. Each zone should employ a depot, as demonstrated by constraint (5). Constraints (6) are subtour elimination constraints based on MTZ. Constraints (7)-(10) represent the types of variables.

The objective function includes two parts. The first represents the total income based on the demand function and the second one describes the total costs consisting of fixed and routing costs. There are different forms of non-negative demand function such as linear, quadratic and exponential. The above mentioned demand functions are given in (11)-(13). Moreover, Θ in exponential demand function, represents customers' sensitivity to price.

$$f_k(d_k) = C_k - A_k W_k \quad ,A_k > 1 \quad (11)$$

$$f_k(d_k) = C_k - A_k W^2{}_k \quad ,A_k > 1 \quad (12)$$

$$f_k(d_k) = \exp(-\Theta W_k) \quad (13)$$

The demand function creates nonlinearity in the objective function. Therefore, the problem is mixed integer nonlinear programming. In the light of this nonlinearity, we try to approximate the objective function by a piecewise linearization method. Use of this method is required to add some sets, parameters, variables and constraints to model. New inputs are defined as follows.

Sets: $q_k \in Q_k$ (set for each interval). Parameters: $Coeffs_{q_k}$ (coefficient for interval q_k of zone k), $Const_{q_k}$ (constant value for interval q_k of zone k), $Lower_{q_k}$ (lower bound of the interval q_k of zone k), $Upper_{q_k}$ (upper bound of the interval q_k of zone k). Variables: Y'_{q_k} (the binary variable is used to show that the interval q_k is selected at zone k, if $Y'_{q_k} = 1$.) And we should substitute the variable W_k by W_{kq_k} .

In this method, we divide the domain of nonlinear function to some intervals. Each interval has an upper and a lower bound. In each interval, we calculate the slope and intercept of the related linear piece. By defining above mentioned parameters, variables and assumptions, the new objective function and constraints are approximated by (14)-(18).

$$\max \sum_{k}\Big(\sum_{q_k}(coeff_{q_k} W_{kq_k} + const_{q_k} Y'_{q_k})\Big) - \sum_{i_k}\sum_{j_k} C_{i_k j_k} X_{i_k j_k} - \sum_{m_k}(G_{m_k} + C_{0m_k})Y_{m_k} \quad (14)$$

$$\sum_{q_k} Y'_{q_k} = 1 \qquad \forall k \quad (15)$$

$$W_{kq_k} \le upper(q_k) Y'_{q_k} \qquad \forall k, q_k \quad (16)$$

$$W_{kq_k} \ge lower(q_k) Y'_{q_k} \qquad \forall k, q_k \quad (17)$$

$$Y'_{q_k} = 0 \ \text{ or } \ 1 \quad (18)$$

3. 2. Flow-Based Model An alternative formulation can be developed for LRP with zone pricing (LRPZP2) used in writing commodity flow constraint that Wong [22] for the first time used a_k (node of depot in each zone) to a node u in the zone k. The new formulation is given as follows:

$$\max \sum_{k} f_k(d_k) W_k - \sum_{i_k}\sum_{j_k} C_{i_k j_k} X_{i_k j_k} - \sum_{m_k}(G_{m_k} + C_{0m_k})Y_{m_k} \quad (19)$$

(2), (3),(5), (7),(9), (10)

$$F^{u}_{i_k j_k} \le f_k(d_k) X_{i_k j_k} \qquad \forall i_k, j_k,\ i_k \ne j_k,\ \forall u \ne a_k \quad (20)$$

$$\sum_{j_k \ne a_k} F^{u}_{a_k j_k} - \sum_{j_k \ne a_k} F^{u}_{j_k a_k} = 1 \qquad \forall k,\ \forall u \ne a_k \quad (21)$$

$$\sum_{j_k \ne i_k} F^{u}_{i_k j_k} - \sum_{j_k \ne i_k} F^{u}_{j_k i_k} = 0 \qquad \forall k, i_k, u \ne a_k,\ i_k \ne u \quad (22)$$

$$\sum_{j_k \ne u} F^{u}_{u j_k} - \sum_{j_k \ne u} F^{u}_{j_k u} = -1 \qquad \forall k,\ \forall u \ne a_k \quad (23)$$

$$F_{i_k j_k} \ge 0 \quad (24)$$

The objective function (19) indicates the profit of the firm should be maximized. Constraint (20) shows maximum flow between any two nodes. Constraints (21)-(23) are the flow conservation constraints. Constraint (24) represents the types of variables that are available in the model.

It is quite clear, when we put the demand function, either linear or nonlinear, into the constraint (20), this constraint converts to a nonlinear constraint. This nonlinearity can be approximated by changing the constraint (20) to two constraints (25) and (26). In the constraint (26), the value of M is set at maximum possible demand in each zone.

$$F^{u}_{i_k j_k} \le f_k(d_k) \qquad \forall i_k j_k,\ i_k \ne j_k,\ \forall u \ne a_k \quad (25)$$

$$F^{u}_{i_k j_k} \le M X_{i_k j_k} \qquad \forall i_k j_k,\ i_k \ne j_k,\ \forall u \ne a_k \quad (26)$$

When we use the linear demand function of the above corrected model, constraint (20) will be the nonlinear demand function. Therefore, the previous approximation method is used.

To make a relation between selecting appropriate depot and price in each zone, we suppose the delivered price is related to shipping cost from the firm to depots. Therefore, the demand function is according to (27),

$$f_k(d_k) = C_k - A_k\Big(W_k + \sum_{m_k}(\alpha_k C_{0m_k})Y_{m_k}\Big)^2 \qquad A_k > 1 \quad (27)$$

where, the coefficient α_k is used for distributing shipping cost on all of the products. We assume that α_k is the inverse of maximum possible demand in each zone. Then, the delivered price in each zone is $W_k + \sum_{m_k}(C_{0m_k}\alpha_k)Y_{m_k})$.

As we know, the first term of objective function in (1) and (19) is the multiplication of demand function and price. Constraint (28) demonstrates this multiplication.

$$\sum_{k} f_k(d_k) W_k = \sum_{k}\Big(C_k W_k - A_k W_k^{3} - A_k W_k \sum_{m_k}(C_{0m_k}\alpha_k)^2 Y^2_{m_k} - 2A_k W^2_k \sum_{m_k}(C_{0m_k}\alpha_k)Y_{m_k}\Big) \quad (28)$$

In (28), $Y^2_{m_k}$ is a quadratic term. We know Y_{m_k} is a binary variable. Therefore, without any loss of generality, it is true that we consider $Y_{m_k} = Y^2_{m_k}$. Hence, constraint (29) is concluded.

$$\sum_{k} f_k(d_k) W_k = \sum_{k} \Big(C_k W_k - A_k W_k^{3} - A_k W_k \sum_{m_k}(C_{0m_k}\alpha_k)^2 Y_{m_k} - 2A_k W_k^{2} \sum_{m_k}(C_{0m_k}\alpha_k)Y_{m_k}\Big) \quad (29)$$

Proposition 1. Constraints (30)-(30) are the same as (29).
Proof. Without any loss of generality, we assume that $r_{m_k} = Y_{m_k} W_k$. Subsequently, $r^2_{m_k} = Y_{m_k} W^2_k$. Now we can rewrite constraint (29) as (30)-(33):

$$\sum_{k} f_k(d_k) W_k = \sum_{k} C_k W_k - A_k W_k^{3} - A_k \sum_{m_k}(C_{0m_k}\alpha_k)^2 r_{m_k} - 2A_k \sum_{m_k}(C_{0m_k}\alpha_k)\, r^2_{m_k} \quad (30)$$

$$r_{m_k} \le M'Y_{m_k} \qquad \forall m_k \qquad (31)$$

$$W_k \le r_{m_k} + M'(1-Y_{m_k}) \qquad \forall k, m_k \qquad (32)$$

$$r_{m_k} \ge 0 \qquad \forall m_k \qquad (33)$$

The maximum possible value for price in each zone can be an appropriate value for M'. We know Y_{m_k} is a binary variable, then, we have two cases.

Case 1: The coefficient of r_{m_k} and $r^2{}_{m_k}$ is negative in (30) and the objective function should be maximized. In this case, when $Y_{m_k}=1$, constraint (31) changes to $r_{m_k} \le M$ and $W_k \le r_{m_k}$ is the result of constraint (32). Therefore, according to maximizing the profit, the variable r_{m_k} will be W_k.

Case 2: When the variable $Y_{m_k}=0$, after simplification, the constraints (31) and (32) are equivalent to $r_{m_k} \le 0$ and $W_k - M' \le r_{m_k}$, respectively. In conclusion, r_{m_k} will be zero. □

Proposition 2. Constraints (30)-(33) are approximated by (34).

$$\sum_k f_k(d_k)W_k \approx \sum_k \sum_{m_k} \sum_{q_k} Y_{m_k}(Slope_{q_k} W_{kq_k} + Intercept_{q_k} Y'_{q_k}) \qquad (34)$$

In this approximation the variables and W_k are changed to W_{kq_k}.

Proof. In the light of the previous proof, we also have two cases.

Case 1: $r_{m_k} = W_k$: In this case, Equation (34) is equivalent to (35).

$$\sum_k f_k(d_k)W_k = \sum_k C_k W_k - A_k W_k{}^3 - A_k \sum_{m_k}(C_{0m_k}\alpha_k)^2 W_k - 2A_k \sum_{m_k}(C_{0m_k}\alpha_k) W_k^2 \qquad (35)$$

Case 2: $r_{m_k}=0$: Here, W_k is zero. Therefore, constraint (30) is the same as (32).

$$\sum_k f_k(d_k)W_k = 0 \qquad (36)$$

The results of the cases (i.e., (35) and (36)) show that we encounter with a Boolean space. Therefore, we need a variable to select the cases. The best variable to handle this situation is Y_{m_k}. Now, the piecewise linearization method is used rely on constraints (15) - (18) and (37).

$$\sum_k f_k(d_k)W_k \approx \sum_k \sum_{m_k} \sum_{q_k} Y_{m_k}(coeff_{q_k} W_{kq_k} + const_{q_k} Y'_{q_k}) \qquad (37)$$

Proposition 3. Constraints (38) - (47) make correctly linearize (37).

$$\sum_k f_k(d_k)W_k \approx \sum_k \sum_{m_k} (\sum_{q_k} (coeff_{q_k,m_k} r_{m_k q_k} + const_{q_k,m_k} t_{m_k q_k})) \qquad (38)$$

$$2t_{m_k q_k} \le Y'_{q_k} + Y_{m_k} \qquad \forall m_k, q_k \qquad (39)$$

$$\sum_{m_k} \sum_{q_k} t_{m_k q_k} = 1 \qquad \forall k \qquad (40)$$

$$W_{kq_k} \ge r_{m_k q_k} \qquad \forall k, m_k, q_k \qquad (41)$$

$$r_{m_k q_k} \le M'Y'_{q_k} \qquad \forall m_k, q_k \qquad (42)$$

$$r_{m_k q_k} \le M'Y_{m_k} \qquad \forall m_k, q_k \qquad (43)$$

$$W_{kq_k} \le r_{m_k q_k} + M'(2 - Y'_{q_k} - Y_{m_k}) \qquad \forall m_k, q_k \qquad (44)$$

$$r_{m_k q_k} \ge 0 \qquad \forall m_k, q_k \qquad (45)$$

$$t_{m_k q_k} = 0 \text{ or } 1 \qquad \forall m_k, q_k \qquad (46)$$

$$W_{kq_k} \ge 0 \qquad \forall k, q_k \qquad (47)$$

Proof. Equation (37) is a nonlinear equation. We linearize it by defining two variables $t_{m_k q_k}$ and $r_{m_k q_k}$ in which $t_{m_k q_k}$ is binary and equivalent to $Y_{m_k} Y'_{q_k}$ and $r_{m_k q_k}$ is nonnegative and equal to $Y_{m_k} W_{kq_k}$. In $t_{m_k q_k}$ four cases can occur for each m_k, q_k.

Case 1. $Y_{m_k}=0, Y'_{q_k}=0$: In this case, according to constraint (39) and (46), $t_{m_k q_k}$ will be zero.

Case 2. $Y_{m_k}=0, Y'_{q_k}=1$: The same as *case 1*.

Case 3. $Y_{m_k}=1, Y'_{q_k}=0$: The same as *case 1* and *case 2*.

Case 4. $Y_{m_k}=1, Y'_{q_k}=1$: In this case, according to constraints (39) and (46), $t_{m_k q_k}$ will be one.

Same as previous, for $r_{m_k q_k}$ four cases can occur for each m_k, q_k.

Case 1. $Y_{m_k}=0, Y'_{q_k}=0$: In this case, the constraints (42) and (43) change to $r_{m_k q_k} \le 0$ and constraint (44) becomes $W_{kq_k} \le r_{m_k q_k} + 2M'$, therefore, the variable $r_{m_k q_k}$ will be zero.

Case 2. $Y_{m_k} = 0, Y'_{q_k} = 1$: Here, constraints (42) and (43) convert to $r_{m_k q_k} \le M'$ and $r_{m_k q_k} \le 0$, respectively, and constraints (44) becomes $W_{kq_k} \le r_{m_k q_k} + M'$, therefore, the variable $r_{m_k q_k}$ becomes zero.

Case 3. $Y_{m_k} = 1, Y'_{q_k} = 0$: In this case, constraints (42) and (43) change to $r_{m_k q_k} \le 0$ and $r_{m_k q_k} \le M'$, respectively and constraints (44) also become $W_{kq_k} \le r_{m_k q_k} + M'$, so, the variable $r_{m_k q_k}$ becomes zero.

Case 4. $Y_{m_k} = 1, Y'_{q_k} = 1$: In this case, constraints (42) and (43) change to $r_{m_k q_k} \le M'$ and constraints (44) also become $W_{kq_k} \le r_{m_k q_k}$, and according to constraint (41), the variable $r_{m_k q_k}$ becomes one.

In addition, in the constraint (20), we have the same nonlinearity that can be approximated like the objective function.

4. THE SOLUTION METHOD

Variable neighbourhood search (VNS) algorithm is a heuristic method for solving combinatorial and global optimization problems, which was primarily introduced by Mladenović and Hansen [23]. A variety of valuable applications of VNS can be found in Melián and Mladenovic [24].

4. 1. Initialization: Generating Initial Solution

In the proposed algorithm, we need to generate initial solutions for the depot, route(s) and price in each zone. We apply a random permutation in determining preliminary depot. In our model, the potential locations for a depot in each zone are inputs. At this time, a random permutation of node $1, 2, \ldots, N_k$ is generated and used one by one to compare with the vector of potential locations.

Afterward, the first element of this vector is chosen as an initial depot for the zone k. Moreover, this algorithm generates initial route(s) for each zone by Clarke and Wright (C&W) method which is the most widely known heuristic for VRP [25]. Another important factor which should be determined is W_k. Therefore, with regards to the total revenue (R) of the firm in both models which is $R = \sum_k f_k(d_k)\,W_k$, we calculate the gradient of R for finding the best W_k. Hence, if ∇R be zero, then, W_k^* is obtained.

4. 2. Local Search

We propose the local search of our algorithm based on four operators. These operators improve the vehicle route in each zone. On the other hand, we can reduce the route cost of vehicle in each zone. Consequently, we improve the total firm profit by applying these operators. Four mentioned operators are 1-1 Move, 1-Exchange, 2-Echange and 2-Opt, which are defined in Table 1.

In this article, we consider three different types of orders, for the operators in the local search. In the first type (I), the algorithm generates a random number between 1 and 4. If the value of the random number is one, 1-1 Move should be operated. If the value of the random number is two, the algorithm operates 2-Opt. When the random number produced by the algorithm is three, 1-Exchange should be performed, and otherwise 2-Exchange should be operated. However, in the second type (II), the number and order of operators are given in generating random numbers. When the generated number is set at one, just 1-Exchange should be operated. For the other numbers, 1-1 Move, 2-Opt and 2-exchange are added, respectively. Finally the third one (III) is consisted of 1-1 Move, 2-Opt, 1-Exchange and 2-Exchange, respectively.

The VNS works to improve the objective function by finding optimal price and depot in each zone after passing from one of the procedures. Now, the algorithm selects the depot in each zone by generating a random number between 1 and the number of potential locations. The generated random number specifies which potential location should be chosen as a depot. Another part of the algorithm in the space of the objective function is finding the optimal price in each zone. The routes, prices, the cost of constructing depot and the shipment cost of products from the firm to the depot in each zone are determined. Then, the algorithm tries to reach the point that has more value in constraint (38). The price is selected as optimal price, if constraint (38) for this price has the highest value.

The objective function obtained by each iteration of the local search should be compared with the best known objective value. Therefore, if the objective function is improved, the solution should be updated. Moreover, the maximum number of iterations (we set it at 150) of the local search should be checked at the end of each iteration.

4. 3. Displaying The Solutions

When the termination condition is satisfied in the previous step, the algorithm is finished and the solution should be displayed in this step.

5. COMPUTATIONAL STUDY

All experiments are conducted on a PC equipped with an Intel Core i7 processor paced at 2.1 GHz, 6 GB of RAM and Windows operating system. The mixed integer linear programs are solved using CPLEX 12.2 and MATLAB ver.2012a software. The computational experiments are carried out on a set of randomly generated instances of the LRP. We consider instances having 49 nodes (consumers). A node is related to the firm. We assume the coordinate of the firm is $(0,0)$, and consumers are distributed in the plane. The distances between the firm and other nodes are calculated by Euclidean distance. The demand function in each zone is different from others and each zone has a special demand function itself. The parameters of the problems are:

$$C_1 = \alpha_1 = 100, C_2 = \alpha_2 = 200, C_3 = \alpha_3 = 300, A_1 = A_2 = A_3 = 2, K = 3,$$
$$N_1 = 15, N_2 = 15, N_3 = 19, m_1 = 1,2,..,5, m_2 = 16,17,...,20, m_3 = 31,32,...,37$$

5. 1. Results In Table 2 and Table 3, we report the results to provide a basis for comparing the accuracy and efficiency of the two models and the proposed algorithm. The exact solutions of each model calculated by CPLEX are shown in columns "Exact solution". Number of used nodes in the branch and bound tree is available in column "Node". Columns "Time" represents the solution time of each approach. After running LRPZP1 in CPLEX software, CPLEX cannot find the exact solution for some instances. Moreover, CPLEX cannot find the exact solution in one hour (3600s). However, LRPZP2 finds the exact solution in all of the instances in a reasonable time. Objective function in all of the instances except the above mentioned instances, is the same. The column "RG" shows the relative gap between the exact solution and the solution of VNS algorithm. More analysis and some statistical tests are shown in the next subsection.

5. 2. Discussion and Statistical Analysis The statistic is widely employed in the field of research. In fact, it is almost impossible to come to an informed deduction in any part of the research without statistics. In this research, the results from statistical hypothesis tests (i.e., paired student's T-tests), show LRPZP2 uses less time for obtaining an exact solution versus LRPZP1 with a 0.05 significance level. The results of the hypothesis test are presented in Table 4. In addition, hypothesis test proves the used node for the branch and bound tree in LRPZP2 is less than LRPZP1. These results show the comparative superiority of LRPZP2 over LRPZP1 in performance.

The proposed VNS algorithm shows a good performance in approximating the objective functions. The mean of RG for all instances in the VNS (type I), the VNS (type II) and the VNS (type III) are 0.02628, 0.01681 and 0.01935, respectively.

TABLE 1. Description of operators

Operator	Definition
1-1 Move	For two selected nodes, the first node is omitted from its current position and is placed after the second node.
1-Exchange	For two selected nodes, they swap their positions.
2-Exchange	For two selected nodes, the first node and its successor exchange their positions with the second node and its successor.
2-Opt	For two selected nodes, the two arcs connecting them with their successors are omitted, the nodes are linked, their successors are linked, and finally the chain between the successor of the first node and the second node is reversed.

TABLE 2. Results of running two models

Instance	LRPZP1			LRPZP2		
	Exact solution	**Node**	**Time (s)**	**Exact solution**	**Node**	**Time (s)**
1	684.30	1018	7.97	684.30	4453	396.55
2	1174.99	1516	5.92	1174.99	1208	165.92
3	1293.30	572	2.70	1293.30	4882	336.41
4	916.98	810359	1000.02*	916.98	4203	310.13
5	604.87	658355	1000.02*	604.87	3677	623.63
6	930.20	756747	1000.02*	930.20	2379	239.68
7	464.65	491	3.03	464.65	1819	187.64
8	1073.61	535691	1000.02*	1074.63	3722	307.54
9	1178.23	3680	16.74	1178.23	1983	242.50
10	671.00	115333	178.76	671.00	6711	545.47
11	299.00	491	3.94	299.00	1200	425.53
12	879.09	8282	26.03	879.09	1634	251.16
13	945.22	170447	287.40	945.22	1378	236.72
14	1315.01	282091	1000.06	1315.01	4964	559.11
15	834.02	468685	701.29	834.02	5740	378.98
16	578.79	22524	44.08	578.79	2763	224.44
17	1246.12	746398	1000.02*	1246.12	3016	290.10
18	470.21	765691	1000.03*	470.21	2364	245.47
19	1242.47	873430	1000.02*	1242.47	2691	231.01
20	932.95	809465	1000.03*	932.95	4546	367.96
21	1563.03	791583	1000.03*	1566.72	4616	449.41
22	817.97	561279	1000.03*	817.97	2824	265.41
23	610.20	703696	1000.02*	610.20	2551	340.49
24	1569.78	52113	108.62	1569.78	2169	238.67
25	1172.68	668819	1000.02*	1172.68	2638	289.33
26	448.60	3194	14.63	448.60	5642	378.90
27	947.81	687059	1000.02*	947.81	4055	308.69
28	727.32	241583	890.84	727.32	2351	479.76
29	1316.92	455612	1000.04*	1317.14	5285	1001.25
30	880.96	537	12.47	880.96	3670	610.66

* CPLEX cannot find exact solution for these instances and we run them for one hour (3600s)

These values show a slight relative gap in finding solutions. Also, these values and results from hypothesis tests confirm the comparative superiority of the VNS (Type II) over the VNS (Type III) and the VNS (Type I), and the superiority of the VNS (Type III) over the VNS (Type I). Therefore, it is clear when the decision maker (DM) pays attention to the RG, the VNS (Type II) is comparatively better than VNS (Type III) and VNS (Type I) and the VNS (Type III) is comparatively better than VNS (Type I).

TABLE 3. Results of running VNS

Instance	VNS (Type I)		VNS (Type II)		VNS (Type III)	
	RG	Time (s)	RG	Time (s)	RG	Time (s)
1	0.007	5.39	0.017	4.17	0.019	6.30
2	0.018	5.58	0.017	4.35	0.032	6.53
3	0.005	5.46	0.004	4.37	0.003	5.28
4	0.004	5.32	0.002	4.38	0.004	5.31
5	0.048	3.98	0.024	4.44	0.031	5.38
6	0.016	3.99	0.005	4.54	0.000	5.38
7	0.027	4.04	0.027	4.48	0.027	4.49
8	0.021	4.19	0.009	4.48	0.023	5.41
9	0.010	4.05	0.011	4.34	0.010	5.38
10	0.095	4.17	0.022	4.29	0.016	5.65
11	0.079	4.15	0.079	4.20	0.101	5.57
12	0.010	4.19	0.010	4.35	0.010	5.44
13	0.032	4.15	0.020	4.20	0.012	5.51
14	0.010	4.13	0.010	4.29	0.010	5.46
15	0.018	4.11	0.004	4.30	0.011	5.47
16	0.013	4.10	0.013	4.22	0.050	5.45
17	0.013	4.04	0.008	4.28	0.008	5.48
18	0.069	4.04	0.066	4.26	0.066	5.42
19	0.037	4.20	0.015	4.32	0.020	5.48
20	0.038	4.35	0.011	4.23	0.002	5.46
21	0.015	4.47	0.010	4.29	0.010	5.74
22	0.015	4.21	0.007	4.47	0.007	5.71
23	0.030	4.36	0.019	6.41	0.028	5.63
24	0.012	4.41	0.027	5.80	0.012	5.56
25	0.024	4.33	0.024	4.47	0.018	5.64
26	0.042	4.38	0.037	5.10	0.032	5.59
27	0.030	4.30	0.000	4.48	0.004	5.39
28	0.035	4.23	0.001	4.51	0.011	5.72
29	0.011	4.22	0.001	4.69	0.002	5.83
30	0.005	4.17	0.005	4.63	0.005	5.87
Mean	0.026		0.017		0.019	

Moreover, the results confirmed when the solution time is the main factor for DM, the best alternatives are VNS (Type II) and VNS (Type I) between all proposed models and approaches. Another advantage of our algorithm is using less time compared to LRPZP1 and LRPZP2 and hypothesis tests accept this claim at the significance level 0.05.

TABLE 4. Results of hypothesis tests

Hypothesis test	P-value
H_0: Solution time of LRPZP2 is greater than or equal to solution time of LRPZP1	0.019
H_0: Used nodes for B&B in LRPZP2 is greater than or equal to LRPZP1	0.000
H_0: The solution time of the VNS (Type I) is less than or equal to the solution time of LRPZP1	1.000
H_0: The solution time of the VNS (Type I) is less than or equal to the solution time of LRPZP2	1.000
H_0: The solution time of the VNS (Type II) is less than or equal to the solution time of LRPZP1	1.000
H_0: The solution time of the VNS (Type II) is less than or equal to the solution time of LRPZP2	1.000
H_0: The solution time of the VNS (Type III) is less than or equal to the solution time of LRPZP1	1.000
H_0: The solution time of the VNS (Type III) is less than or equal to the solution time of LRPZP2	1.000
H_0: The solution time of the VNS (Type II) is equal to the solution time of the VNS (Type I)	0.108
H_0: The solution time of the VNS (Type II) is less than or equal to the solution time of the VNS (Type III)	1.000
H_0: The solution time of the VNS (Type I) is less than or equal to the solution time of the VNS (Type III)	1.000
H_0: The RG of the VNS (Type II) is less than or equal the RG of the VNS (Type I)	0.998
H_0: The RG of the VNS (Type II) is less than or equal the RG of the VNS (Type III)	0.911
H_0: The RG of the VNS (Type I) is less than or equal the RG of the VNS (Type III)	0.031

6. CONCLUSION

This paper extends the literature of the LRP by providing two models of LRPZP. In this problem, we have a firm that tries to supply its market to maximize its profit. The firm tries to make decisions about the following three issues: (i) which strategy can be good for routing of vehicle in each zone? (ii) what price should be selected for demands in each zone? (iii) which potential location can be the best alternative for constructing a depot in each zone? In this article, we present two formulations consisting of node-based and flow-based model. In addition, a heuristic VNS algorithm for solving large-scale problem is proposed. Some test instances are solved by two models and the VNS algorithm. Then, some hypothesis tests are carried out. Two models are compared based on solution time and the used node in branch and the bound tree method. The results of the tests show the comparative superiority of LRPZP2 over LRPZP1. Other consequences of test instances are related to the comparison between models and the VNS algorithm. Results of hypothesis tests show the comparative superiority of the VNS algorithm over two models based on solution time. We suggest three types of combinations for the local search of the algorithm.

The second type gets the best performance between all of them. In this type, order of operators is indicated by generating random integer numbers between one and four.

If the generated number is set at one, 1-Exchange should be operated. In case of two, 1-1 Move is run after 1-Exchange. In the case of three, 2-Opt is also performed. Finally, in the case of four, 2-Exchange should be applied after the previous operators. The performance of this heuristic in terms of both accuracy and efficiency is considered to be quite promising, according to our computational results obtained on 30 randomly generated instances. As a future research direction, we intend to focus on LRPZP in a competitive situation. The firm can be considered as an entrant that wants to enter a big market in the presence of incumbents. It should make a decision about price, vehicle route and selecting depots. Another research direction is to examine our problem by taking into account the quality-dependent price of demands, since, another key factor for the customers is the level of feature and quality.

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Investigating Zone Pricing in a Location-Routing Problem Using a Variable Neighborhood Search Algorithm

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P A P E R I N F O

Paper history:
Received 25 April 2015
Received in revised form 04 October 2015
Accepted 16 October 2015

Keywords:
Pricing, Zone Pricing
Location-Routing Problem
Variable Neighborhood Search

چکیده

در این مقاله، شرکتی در نظر گرفته شده که به دنبال تعیین بهینه قیمت، مسیر وسیله نقلیه و مکان دپو در هر ناحیه، با هدف بیشینه سازی سود می‌باشد؛ بنابراین این مقاله، موضوع قیمت‌گذاری منطقه‌ای را وارد ادبیات مسائل مکان‌یابی–مسیریابی کرده است. قیمت‌گذاری منطقه ای یکی از مهمترین سیاست‌های قیمت‌گذاری است که توسط بسیاری از شرکت‌ها مورد استفاده قرار می‌گیرد. مساله پیشنهادی در توزیع محصولاتی مثل میوه کاربرد فراوان دارد. مساله توسط دو مدل مبتنی بر گره و جریان فرمولبندی شده است. مدل برنامه‌ریزی عدد صحیح غیرخطی بدست آمده، توسط یک روش قطعه‌قطعه‌خطی تقریب زده شد و عملکرد آنها مورد مقایسه قرار گرفته است. به منظور سازگاری با دنیای واقعی، یک الگوریتم جستجوی همسایگی متغیر توسعه یافت و برای تعدادی مساله اجرا گردید. سه ترکیب مختلف جستجوی محلی معرفی شد و عملکرد آنها با هم و نیز با دو مدل پیشنهادی مورد مقایسه قرار گرفت. نتایج حاصل از محاسبات کارایی الگوریتم پیشنهادی در حل مسائل بزرگ به جای مدل‌های پیشنهادی را تایید می‌نماید. به علاوه نتایج نشان می‌دهند که مدل مبتنی بر جریان زمان محاسباتی کمتری را به نسبت مدل مبتنی بر گره استفاده می‌نماید.

doi*: 10.5829/idosi.ije.2015.28.11b.10*