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Economic Order Quantity for Deteriorating Items with Non Decreasing Demand and Shortages Under Inflation and Time Discounting

R. P. Tripathi

Department of Mathematics, Graphic Era University, Dehradun (UK) India

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ABSTRACT

Some products like green vegetables, volatile liquids and others deteriorate continuously due to evaporation, spoilage etc. In this study, an inventory model is developed for deteriorating items with linearly time dependent demand rate under inflation and time discounting over a finite planning horizon. Shortages are allowed and linearly time dependent. Mathematical model is presented for the proposed model. We show that total profit is concave with respect fraction of scheduling period *k*. The results are discussed with the help of numerical example. A sensitivity analysis of the optimal solution with respect to the key parameters is also discussed.

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1. INTRODUCTION

Economic order quantity (EOQ) model is generally used for finding the optimal order quantity in order to maximize the total profit. The EOQ model assumes that the present order for a given item is received into inventory at one scheduled time. Many research papers appeared and published in the national/ international journals considered that demand rate is constant. However, in actual practice demand rate for items is not always constant; it may be time- dependent, exponential time- dependent or stock- dependent. In this paper, we consider demand rate in linearly time- dependent under time dependent shortages and inflation. Large number of researchers has published their work considering time – dependent demand. Silver and Meal [1] presented EOQ model for the case of a varying demand. Donaldon [2] established inventory model with a linearly time-dependent demand. Teng et al.[3] developed an EOQ model under trade credit financing with increasing demand. In this direction, many researchers like Silver [4], Mitra et al. [5], Ritchie [6] made valuable contribution. Tripathi and Mishra [7] presented EOQ model for linearly time dependent demand and exponential time- dependent holding cost. Tripathi [8] established an inventory model for time varying demand and constant demand. Tripathi and Mishra [9] presented the problem of determining the retailer's optimal price and optimal total profit when supplier permits delay in payments for an order of a product.

Most of the items in the universe deteriorate overtime. Thus, the loss due to deterioration cannot be ignored. Chang et al. [10] established an EOQ model for deteriorating items under supplier credits linked to order quantity. Chung and Huang [11] presented an EPQ (Economic Production Quantity) model for retailer where the supplier offers a permissible delay in payments. Teng et al. [12] established EOQ model for deteriorating item under trade credits. Balkhi and Benkherouf [13] developed an inventory model for deteriorating items with stock- dependent and time – varying demand rates over a finite planning horizon. Many related research papers can be found in Giri et al. [14], Sarker et al. [15], Chung et al. [16], Liao et al. [17], Aggarwal and Jaggi [18] , Jamal et al. [19], Huang and Hsu [20], Ouyang et al. [21], Hung [22] , Teng and Chang [23].

It is believed that the inflation does not affect in the study of inventory policy, but this assumption may not

*Corresponding Author's Email: tripathi_rp0231@rediffmail.com (R.P.Tripathi)

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be true in any type of business transaction. At present, many counties are suffering large scale of inflation. Hence, the effect of inflation cannot be discarded in the study of inventory policy. Buzacott [24] developed an EOQ model with inflation subject to different types of pricing policies. Jaggi et al. [25] presented the optimal inventory replenishment policy of deteriorating item under inflationary conditions using discounted cash flow (DCF) approach over a finite planning horizon. Wee and Law [26] developed the problem with finite replenishment rate of deteriorating items taking account of time value of money. Chang [27] presented an inventory model for deteriorating items with inflation under a situation in which the supplier provides the purchase or a permissible delay of payments if the purchaser orders a large quantity. Sarker et al. [28] dealt with an economic manufacturing quantity (EMQ) model for the selling price and the time- dependent pattern in an imperfect production process. Some researchers developed inventory models in which shortages are allowed. Hou [29] derived an inventory model for deteriorating items with stock- dependent consumption rate and shortages under inflation and time discounting over a finite planning horizon. Basu and Sinha [30] presented a general inventory model with due consideration to factors of time dependent partial backlogging and time dependent deterioration. Jaggi et al. [31] developed an inventory model for imperfect quality items under permissible delay in payments where shortages are allowed and fully backorders parallel to the screening process out of the items which are of perfect quality. Ghiami et al. [32] investigated a two- echelon supply chain model for deteriorating inventory in which the retailer's warehouse has a limited capacity. Taleizadeh and Nematollahi [33] investigated the effects of time value of money and inflation on the optimal ordering policy in an inventory control system. Some research work done by Yang et al. [34], Wee et al. [35], Bhunia et al. [36], and Ouyang and Chang [37], Yhmadi et al. [38], Tripathi and Uniyal [39], Modarres and Taimury [40], Rabbani and Manavizadeh [41] in this direction.

The rest of the research paper is organized as follows. In section 2, we provide the assumption and notations. In section 3, we formulate mathematical model. The optimal solution is provided in section 4. In section 5, numerical examples are discussed. The sensitivity analysis is provided in section 6. Conclusion and future research directions are provided in the last section 7.

2. ASSUMPTIONS AND NOTATION

The model is developed with the help of following assumptions:

1. The demand rate in linearly time- dependent and continuous function of time
2. The lead time is negligible and replenishment rate in infinite
3. Shortages are allowed and completely backlogged
4. The deterioration rate is constant
5. Instantaneous cash flow is considered for product transactions

In addition, the following notations are used throughout the manuscript in developing the model:

T replenishment cycle
H planning horizon
m the number of replenishment during the planning horizon
k fraction of scheduling period ($0 < k < 1$)
t_j the total time elapse up to and including the jth replenishment cycle (j = 1 ,2, ------m) where $t_0 = 0$ and $t_1 = T$ and $t_m = H$
t_j' the time for jth inventory level (i.e. jth replenishment cycle) drops to zero (j = 1,2,-- m)
t_j-t_j' shortage occurs during this period ($j = 1, 2$---m)
Q the lot size fir 2nd, 3rd ----- nth cycles
Q_1 the maximum inventory level
Q_2 the maximum shortage level
Θ the deterioration rate ($0 < \Theta < 1$)
A the cost of per replenishment, &/order
c the per unit cost of the item, &/unit
h the per unit inventory holding cost per unit time, $/unit/unit time
s the per unit shortage cost per unit time, $/unit/unit time
f the discount rate
i the inflation rate
$r = f - i$ the net discount rate of inflation (constant)
m* optimal number of replenishment during the planning horizon
k* optimal fraction of scheduling period
SR the sales revenue
OC ordering cost
SC shortage cost
HC holding cost
MC material cost
TP_1 present value of total profit
TP (m, k) present value of total profit over a finite planning horizon H

3. MATHEMATICAL FORMULATION

We divide the planning horizon H into m equal parts of length $T = H/m$. Thus, the reorder times during the planning horizon H are $t_j = jT'$ ($j = 0, 1, 2,$ ------m). The demand for positive inventory and negative inventory are same and equal to linearly time dependent. For period of no shortage in each interval [$jT', (j + 1) T'$] is a fraction of the period kT $(0 < k < 1)$. At time $t_j' = (k + j-1)T$, $(j = 1, 2,----m)$ before they are backordered.

At starting point $T_0' = 0$, the first replenishment lot size is Q_1. The inventory level decreases during the time interval $[0,T_1]$ due to time dependent demand and deterioration until it becomes to zero at $t = T_1$. Shortages occur during the interval $[T_1, T']$ and are accumulated until $t = T'$, before they are backordered. Hence, the inventory level at any time't' can be represented by the following differential equations:

$$\frac{dI(t)}{dt} + \theta I(t) = -(a + b.t), \quad 0 \le t \le T_1 \tag{1}$$

$$\frac{dI(t)}{dt} = -(a + b.t), \quad T_1 \le t \le T \tag{2}$$

The solutions of (1) and (2) (with the condition $I(T_1) = 0$ are given by:

$$I(t) = \frac{1}{\theta}\left\{\left(a - \frac{b}{\theta}\right)\left(e^{\theta(T_1 - t)} - 1\right) + b\left(T_1 e^{\theta(T_1 - t)} - t\right)\right\},\ 0 \le t \le T_1 \tag{3}$$

$$I(t) = -(t - T_1)\left\{a + \frac{b}{2}(T + T_1)\right\},\ T_1 \le t \le T \tag{4}$$

Thus, the maximum inventory level and maximum shortage quantity during the first replenishment cycle are:

$$Q_1 = \frac{1}{\theta}\left\{\left(a - \frac{b}{\theta}\right)\left(e^{\theta kH/m} - 1\right) + \frac{bkH}{m} e^{\theta kH/m}\right\} \tag{5}$$

$$Q_2 = \frac{(1-k)H}{m}\left\{a + \frac{b(1+k)H}{2m}\right\} \tag{6}$$

The present value of sales revenue during $[0,T]$ in the first replenishment cycle is given by:

$$SR = p\int_0^T a + b.t)e^{-rt}dt = \frac{p}{r}\left\{\left(a + \frac{b}{r}\right)\left(1 - e^{-rT}\right) - bTe^{-rT}\right\}$$
$$= \frac{\mathrm{p}}{\mathrm{r}}\left\{\left(a + \frac{b}{r}\right)\left(1 - e^{-rH/m}\right) - \frac{bH}{m} e^{-rH/m}\right\} \tag{7}$$

The present value of ordering cost in the first replenishment cycle is given by $OC = A$ (8)

The present value of holding cost during the time interval $[0,T_1]$ in the first replenishment cycle is:

$$HC = h\int_0^{T_1} I(t)e^{-rt}dt =$$
$$\frac{h}{\theta}\left[\left(a - \frac{b}{\theta} + \frac{bkH}{m}\right)\left(\frac{e^{\theta kH/m} - e^{-rkH/m}}{\theta + r}\right) + \frac{b}{r}\left(\frac{kH}{m}e^{-rkH/m} + \frac{e^{-rkH/m} - 1}{r}\right)\right] \tag{9}$$

The present value of shortage cost during the first replenishment cycle is (the shortage occurs during the time interval $[T_1, T'_1]$)

$$SC = s\int_{T_1}^{T'_1} -I(t)e^{-rt}dt = \frac{s}{r}\left\{a + \frac{b(1+k)H}{2m}\right\}$$
$$\left\{\frac{(1-k)H}{m} + \frac{e^{-rH/m} - e^{-rkH/m}}{r}\right\} \tag{10}$$

The replenishment is finished at $t = 0$ and T, the items are consumed by time dependent demand and deterioration during $[0\ ,T_1]$. The present value of material cost during the first replenishment cycle is:

$$MC = cQ_1 + ce^{-rT}\int_{T_1}^{T}(a + b.t)dt = cQ_1 + ce^{-rT}(T - T_1)\left\{a + \frac{b}{2}(T + T_1)\right\}$$
$$= \frac{\mathrm{c}}{\theta}\left\{\left(a - \frac{b}{\theta}\right)\left(e^{\theta kH/m} - 1\right) + \frac{bkH}{m}e^{\theta kH/m}\right\} + \tag{11}$$
$$\frac{ce^{-rH/m}(1-k)H}{m}\left\{a + \frac{b(1+k)H}{2m}\right\}$$

Therefore, the present value of total profit of inventory system during the first replenishment cycle is given by:

$$TP_1 = SR - OC - HC - SC - MC \tag{12}$$

In this process, we have considered m cycles during the planning horizon. It is also assumed that inventory started at maximum level and ends at zero, an extra replenishment at $T_m' = H$ is required to follow the backorders of the best cycle in the planning horizon. Thus, $(m+1)$ replenishments are required in the planning horizon H. Therefore, the first replenishment lot size is Q_1, the 2^{nd} , 3^{rd}, ... m^{th} replenishment lot size is
$Q = Q_1 + Q_2$, where

$$Q_2 = \int_{T_1}^{T}(a + b.t)dt \tag{13}$$

And the last or $(m+1)^{th}$ replenishment lot size is:

$$Q_2 = (T - T_1)\left\{a + \frac{b}{2}(T + T_1)\right\} = \frac{(1-k)H}{m}\left\{a + \frac{b(1+k)H}{2m}\right\} \tag{14}$$

Thus, the present value of total profit of the inventory system over a finite planning horizon H is:

$$TP(m,k) = \sum_{j=0}^{m-1} TP_1 e^{-r_j T} + Ae^{-rH} = TP_1\left(\frac{1 - e^{-rH}}{1 - e^{-rH/m}}\right) + Ae^{-rH} \tag{15}$$

where, $T = H/m$ and TP_1 is given in (12).

$$TP(m,k) =$$
$$\left[\frac{p}{r}\left\{\left(a + \frac{b}{r}\right)\left(1 - e^{-rH/m}\right) - \frac{bH}{m}e^{-rH/m}\right\} - A\left\{1 - \left(\frac{1 - e^{-rH/m}}{e^{rH} - 1}\right)\right\} - \right.$$
$$\frac{h}{\theta}\left\{\left(a - \frac{b}{\theta} + \frac{bkH}{m}\right)\left(\frac{e^{\theta kH/m} - e^{-rkH/m}}{\theta + r}\right) + \frac{b}{r}\left(\frac{kH}{m}e^{-rkH/m} + \frac{e^{-rkH/m} - 1}{r}\right)\right\}$$
$$-\frac{s}{r}\left\{a + \frac{b(1+k)H}{2m}\right\}\left\{\frac{(1-k)H}{m} + \frac{e^{-rH/m} - e^{-rkH/m}}{r}\right\} \tag{16}$$
$$-\frac{c}{\theta}\left\{\left(a - \frac{b}{\theta}\right)\left(e^{\theta kH/m} - 1\right) + \frac{bkH}{m}e^{\theta kH/m}\right\} -$$
$$\left.\left\{a + \frac{b(1+k)H}{2m}\right\}\right]\left(\frac{1 - e^{-rH}}{1 - e^{-rH/m}}\right)$$

4. DETERMINATION OF OPTIMAL SOLUTION

The present value of total profit *TP (m, k)* is a function of two variables *m* and *k* where *m* is discrete variable and *k* is a continuous variable. The optimal value of *TP (m, k)* is obtained by differentiating Equation (16) with respect to *k*. The necessary and sufficient condition for *TP (m, k)* to maximize is $\frac{d(TP(m,k))}{dk}=0$, for which $\frac{d^2(TP(m,k))}{dk^2}<0$. Differentiating Equation (16) with respect to *k* and putting to zero, we get

$$\frac{h}{\theta}\left\{\left(a-\frac{b}{\theta}+\frac{bkH}{m}\right)\left(\frac{\theta e^{\theta kH/m}+re^{-rkH/m}}{\theta+r}\right)+\frac{b(e^{\theta kH/m}-e^{-rkH/m})}{\theta+r}-\frac{bkH}{m}e^{-rkH/m}\right\}$$
$$+\frac{s}{r}\left[\frac{b}{2}\left\{\frac{(1-k)H}{m}+\frac{e^{-rH/m}-e^{-rkH/m}}{r}\right\}+\left\{a+\frac{b(1+k)H}{2m}\right\}\left(e^{-rkH/m}-1\right)\right]+ \tag{17}$$
$$c\left(a+\frac{bkH}{m}\right)\left(e^{\theta kH/m}-e^{-rH/m}\right)=0$$

$$\frac{d^2(TP(m,k))}{dk^2}=\frac{H^2}{m^2}\left[-\frac{h}{\theta}\times\right.$$
$$\left\{\left(a-\frac{b}{\theta}+\frac{bkH}{m}\right)\left(\frac{\theta^2e^{\theta kH/m}-r^2e^{-rkH/m}}{\theta+r}\right)+2b\left(\frac{\theta^2e^{\theta kH/m}-r^2e^{-rkH/m}}{\theta+r}\right)-\right.$$
$$\left.b\left(1-\frac{rkH}{m}\right)e^{-rkH/m}\right\}+\frac{s}{\theta}\left\{b\left(1-e^{-rkH/m}\right)+r\left(a+\frac{bH+bkH}{2m}\right)e^{-rkH/m}\right\}- \tag{18}$$
$$\left.c\left\{b\left(1+\frac{\theta kH}{m}\right)e^{\theta kH/m}+a\theta e^{\theta kH/m}-be^{-rH/m}\right\}\right]\left(\frac{1-e^{-rH}}{1-e^{-rH/m}}\right)<0$$

It is difficult to handle Equation (17) to find the exact value of k. The second order approximation is used for exponential terms i.e.
$e^{\theta kH/m}\approx 1+\theta kH/m+(\theta kH)^2/2m^2$ etc. we get

$$\frac{h}{\theta}\left\{\left(a-\frac{b}{\theta}\right)+\left(a+\frac{bkH}{m}\right)(\theta-r)\frac{kH}{m}+\frac{brkH}{\theta m}+\left(a-\frac{b}{\theta}+\frac{bkH}{m}\right)\left(\theta^2-\theta r+r^2\right)\frac{k^2H^2}{2m^2}\right.$$
$$\left.+\frac{bk^2H^2}{2m^2}\left(\theta+r-\frac{r^2}{m}\right)\right\}+\frac{sH}{m}\left[\frac{bH\left(1-k^2\right)}{4m}-k\left\{a+\frac{b(1+k)H}{2m}\right\}\left(1-\frac{rkH}{2m}\right)\right]+ \tag{19}$$
$$\frac{c\ (\theta k+r)}{m}\left(a+\frac{bkH}{m}\right)\left(1+\frac{(\theta k-r)k^2H}{2m}\right)=0$$

Accordingly the total profit *TP(m, k)* and order quantity *Q* is given by:

$$TP(m,k)\approx\left[\frac{pH}{m}\left\{a\left(1-\frac{rH}{2m}\right)+\frac{bH}{2m}\left(1-\frac{rH}{m}\right)\right\}-A\left\{1-\frac{1}{m}+\frac{rH}{2}\left(1+\frac{1}{m^2}\right)\right\}\left(1+\frac{rH}{2}\right)^{-1}-\right.$$
$$\frac{hkH}{\theta m}\left\{\left(a-\frac{b}{\theta}+\frac{bkH}{m}\right)\left(1+\frac{(\theta-r)kH}{2m}\right)-\frac{bkH}{2m}\left(1-\frac{rkH}{m}\right)\right\}-$$
$$\frac{sH^2(1-k^2)}{2m^2}\left\{a+\frac{b(1+k)H}{2m}\right\}-\frac{ckH}{m}\left\{a\left(1+\frac{\theta kH}{2m}\right)+\frac{bkH}{2m}\left(1+\frac{\theta kH}{m}\right)\right\} \tag{20}$$
$$\left.-\frac{cH\left(1-\frac{rH}{m}+\frac{r^2H^2}{2m^2}\right)}{m}\left\{a+\frac{b(1+k)H}{2m}\right\}\right]\frac{m\left(1-\frac{rH}{2}\right)}{\left(1-\frac{rH}{2m}\right)}$$

And

$$Q\approx\frac{\mathrm{H}}{\mathrm{m}}\left\{a\left(1+\frac{\theta k^2H}{2m}\right)+\frac{bH}{2m}\left(1+\frac{\theta k^3H}{m}\right)\right\} \tag{21}$$

5. NUMERICAL EXAMPLE

An example is provided to explain the validity of theoretical results discussed in section 4 with the following date:

Let us consider the parameter values *a = 600, b = 10, Ɵ = 0.2, H = 10, r = 0.15, h = 2, c = 50,* s = 30, in appropriate units. We see that the number of replenishments (from Table 1) *m = 9*, the total profit *TP(m,k)* becomes maximum. Thus, the optimal values of m and k are *m* = 9* and *k* = 0.327693*, respectively, and the maximum total profit *TP(m*,k*) = $* 173918.00. We have $T = H/m^* = 1.111$ year, $T_1^* = k^*T^* = 0.3641$ year and Q* = 680.842 units.

6. SENSITIVITY ANALYSIS

The sensitivity analysis is performed with the variation of various key parameters using the same data as in the above numerical example at optimal value of m = m* = 9. On varying one parameter at a time keeping other parameters same.

All the observations computed in Table 2 can be summed as follows:

(1).The increase of parameter '*b*' results slight increase in order quantity (*Q*) and increase in total profit *TP(m,k).* That is, change in '*b*' leads slight positive change in *Q* and positive change in *TP(m,k).*

(2). When deterioration rate (*Ɵ*) increases, the ordering cost (*Q*) decreases and total profit TP(m,k) increases. That is, change in deterioration rate will lead negative change in *Q* and positive change in *TP(m,k).*

(3). When holding cost (*h*) increases, order quantity (*Q*) increases and total profit *TP(m,k)* decreases. That is, change in holding cost (h) will lead positive change in *Q* and negative change in *TP(m,k).*

(4). When purchase cost (c) increases, order quantity *Q* slightly increases and total profit *TP(m,k)* slightly decreases. That is, change in *c* leads slight positive change in *Q* and negative change in *TP(m,k).*

(5). When shortage cost (s) increases, order quantity *Q* slightly increases and total profit *TP(m,k)* decreases. That is, the change in s will lead slight positive change in *Q* and negative change in *TP(m,k).*

TABLE 1. Optimal solution

m	*k*	*T*	*Q*	*TP(m*,k*)*
2	0.11313	5.00000	3144.38	35134.50
3	0.138906	3.33333	2068.52	122376.00
4	0.168337	2.50006	1541.95	150571.00
5	0.199229	2.0006	1229.59	16306 2.00
6	0.23085	1.66667	1022.83	169263.00
7	0.262898	1.4285	875.863	172349.00
8	0.295209	1.25000	766.033	173677.00
9	0.327693*	1.11100*	680.842*	173918.00*
10	0.360304	1.00000	612.836	173443.00
11	0.393008	0.90909	557.291	172470.00
12	0.425781	0.83333	511.071	171139.00
13	0.458610	0.76923	472.008	169541.00
14	0.491483	0.71428	438.560	167738.00
15	0.524392	0.66667	409.598	165776.00
16	0.557332	0.62500	384.276	163687.00
17	0.590298	0.58824	361.947	161495.00
18	0.623287	0.55556	342.112	159218.00
19	0.656297	0.52632	324.375	156872.00
20	0.689324	0.50000	308.418	154467.00

m* = 9 optimal solution

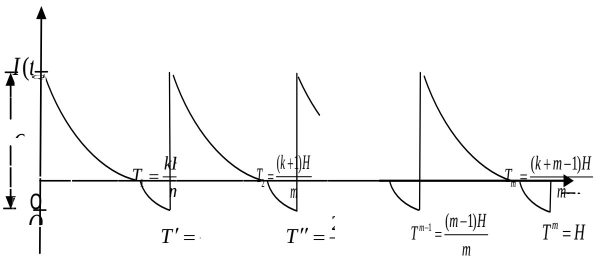


Figure 1. Graph between time and inventory

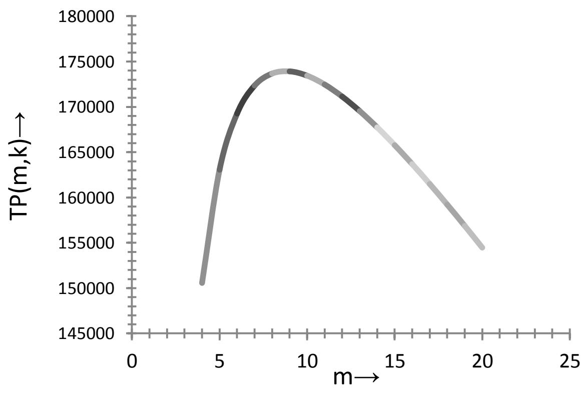


Figure 2. Figure between *m* vs. *TP(m,k)*

TABLE 2. Effect of changing the parameters '*b'*, deterioration rate '*Θ'*, holding cost *'h'*, purchase cost *'c'* and shortage cost *'s'* at optimal $m = m^* = 9$.

b	*k*	*Q*	*TP(m*,k*)*
11	0.325142	681.34	174311.00
12	0.322599	681.838	174703.00
13	0.320063	682.338	175095.00
14	0.317535	682.839	175486.00
15	0.315014	683.341	175877.00
Θ	***k***	***Q***	***TP(m*,k*)***
0.05	0.827391	685.711	117673.00
0.10	0.547441	684.052	150707.00
0.15	0.407320	682.126	165792.00
0.25	0.276795	679.970	178942.00
0.30	0.241595	679.354	182337.00
h	***k***	***Q***	***TP(m*,k*)***
2.5	0.406936	685.198	165623.00
3.0	0.488586	690.682	156714.00
3.5	0.572934	697.413	147152.00
4.0	0.660326	705.533	136889.00
4.5	0.751189	715.220	125871.00
5.0	0.846050	726.693	114030.00
c	***k***	***Q***	***TP(m*,k*)***
55	0.331686	681.039	163733.00
60	0.335711	681.240	153474.00
65	0.339770	681.445	143140.00
70	0.343861	681.654	132730.00
100	0.369141	683.002	68613.50
32	0.305807	679.806	174151.00
35	0.278043	678.596	174011.00
37	0.262220	677.958	173659.00
40	0.246530	677.362	172414.00
45	0.213826	676.240	170829.00

7. CONCLUSIONS

In this study, we have developed *EOQ* model for deteriorating items with shortages under inflation considering demand as a function of time. We have provided mathematical formulation of the problem discussed above and have given an optimal solution. We have used second order approximation for exponential terms to obtain the closed form optimal solution. We have also shown that total profit *TP (m,k)* is concave function with respect to '*k*'. Next, sensitivity analysis has been discussed for optimal solution to changes in the values of various parameters. Here are some managerial phenomena that we obtain from the sensitivity analysis:

- If parameter 'b' increases, total profit increases, total profit *TP (m,k)* increases
- If deterioration rate (*Θ*) increases, total profit *TP (m,k)* increases
- If the holding cost increases , total profit *TP (m,k)* decreases
- In purchase cost (c) increases, total profit *TP (m,k)* decreases
- If shortage cost (s) increases , total profit *TP (m,k)* decreases

It is observed that the total profit *TP (m,k)* increases for 'b' and deterioration rate (*Θ)* and decreases for holding cost (h), purchase cost (c) and shortage cost (s).

The model proposed in this paper can be extended in several ways. We may extend the model for two parameters weibull distribution deteriorating items. We could also consider demand rate as stock- dependent, function of selling price as well as quadratic time varying.

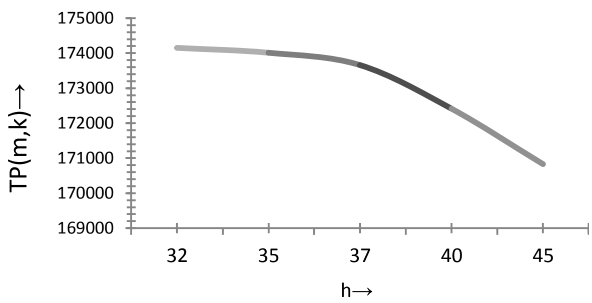
8. REFERENCES

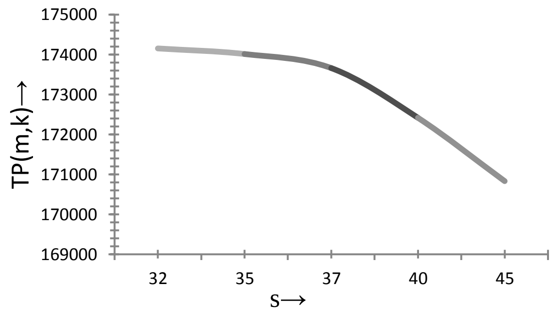
1. Silver, E.A. and Meal, H.C., "A simple modification of the eoq for the case of a varying demand rate", ***Production and Inventory Management,*** Vol. 10, No. 4, (1969), 52-65.
2. Donaldson, W., "Inventory replenishment policy for a linear trend in demand--an analytical solution", ***Operational Research Quarterly,*** (1977), 663-670.
3. Teng, J.-T., Min, J. and Pan, Q., "Economic order quantity model with trade credit financing for non-decreasing demand", ***Omega***, Vol. 40, No. 3, (2012), 328-335.
4. Silver, E.A., "A simple inventory replenishment decision rule for a linear trend in demand", ***Journal of the Operational Research Society,*** (1979), 71-75.
5. Mitra, A., Cox, J.F. and Jesse Jr, R.R., "A note on determining order quantities with a linear trend in demand", ***Journal of the Operational Research Society,*** (1984), 141-144.
6. Ritchie, E., "Practical inventory replenishment policies for a linear trend in demand followed by a period of steady demand", ***Journal of the Operational Research Society,*** (1980), 605-613.
7. Tripathi, R.P. and Mishra, S.M., "Eoq model with linear time-dependent demand and different holding cost function",***International Journal of Mathematics in Operational Research Quarterly,*** (2015) 47-64.
8. Tripathi, R., "Inventory model with different demand rate and different holding cost", ***International Journal of Industrial Engineering Computations,*** Vol. 4, No. 3, (2013), 437-446.
9. Tripathi, R.P. and Misra, S.S., "Optimal pricing and lot-sizing policy with time - dependent demand rate under trade credits", ***International Journal of Mathematics and Computer Application Research,*** Vol. 2, No. 3, (2013), 106-120.
10. Chang, C.-T., Ouyang, L.-Y. and Teng, J.-T., "An eoq model for deteriorating items under supplier credits linked to ordering quantity",***Applied Mathematical Modelling,*** Vol. 27, No. 12, (2003), 983-996.
11. Chung, K.-J. and Huang, Y.-F., "The optimal cycle time for epq inventory model under permissible delay in payments", ***International Journal of Production Economics,*** Vol. 84, No. 3, (2003), 307-318.
12. Teng, J.-T., Chang, C.-T. and Goyal, S.K., "Optimal pricing and ordering policy under permissible delay in payments", ***International Journal of Production Economics,*** Vol. 97, No. 2, (2005), 121-129.
13. Balkhi, Z.T. and Benkherouf, L., "On an inventory model for deteriorating items with stock dependent and time-varying demand rates", ***Computers & Operations Research,*** Vol. 31, No. 2, (2004), 223-240.
14. Giri, B., Pal, S., Goswami, A. and Chaudhuri, K., "An inventory model for deteriorating items with stock-dependent demand rate", ***European Journal of Operational Research,*** Vol. 95, No. 3, (1996), 604-610.
15. Sarker, B.R., Mukherjee, S. and Balan, C.V., "An order-level lot size inventory model with inventory-level dependent demand and deterioration", ***International Journal of Production Economics,*** Vol. 48, No. 3, (1997), 227-236.
16. Chung, K.-J., Chu, P. and Lan, S.-P., "A note on eoq models for deteriorating items under stock dependent selling rate", ***European Journal of Operational Research,*** Vol. 124, No. 3, (2000), 550-559.
17. Liao, H.-C., Tsai, C.-H. and Su, C.-T., "An inventory model with deteriorating items under inflation when a delay in payment is permissible", ***International Journal of Production Economics,*** Vol. 63, No. 2, (2000), 207-214.
18. Aggarwal, S. and Jaggi, C., "Ordering policies of deteriorating items under permissible delay in payments", ***Journal of the Operational Research Society,*** (1995), 658-662.
19. Jamal, A., Sarker, B.R. and Wang, S., "Optimal payment time for a retailer under permitted delay of payment by the wholesaler", ***International Journal of Production Economics,*** Vol. 66, No. 1, (2000), 59-66.
20. Huang, Y.-F. and Hsu, K.-H., "An eoq model under retailer partial trade credit policy in supply chain", ***International Journal of Production Economics,*** Vol. 112, No. 2, (2008), 655-664.
21. Ouyang, L.-Y., Teng, J.-T. and Chen, L.-H., "Optimal ordering policy for deteriorating items with partial backlogging under permissible delay in payments", ***Journal of Global Optimization,*** Vol. 34, No. 2, (2006), 245-271.
22. Huang, Y.-F., "Optimal retailer's replenishment policy for the epq model under the supplier's trade credit policy", ***Production Planning & Control,*** Vol. 15, No. 1, (2004), 27-33.
23. Teng, J.-T. and Chang, C.-T., "Optimal manufacturer's replenishment policies in the epq model under two levels of trade credit policy",***European Journal of Operational Research,*** Vol. 195, No. 2, (2009), 358-363.
24. Buzacott, J., "Economic order quantities with inflation", ***Operational Research Quarterly,*** (1975), 553-558.
25. Jaggi, C.K., Aggarwal, K. and Goel, S., "Optimal order policy for deteriorating items with inflation induced demand", ***International Journal of Production Economics,*** Vol. 103, No. 2, (2006), 707-714.
26. Wee, H.-M. and Law, S.-T., "Economic production lot size for deteriorating items taking account of the time-value of money", ***Computers & Operations Research,*** Vol. 26, No. 6, (1999), 545-558.
27. Chang, C.-T., "An eoq model with deteriorating items under inflation when supplier credits linked to order quantity",

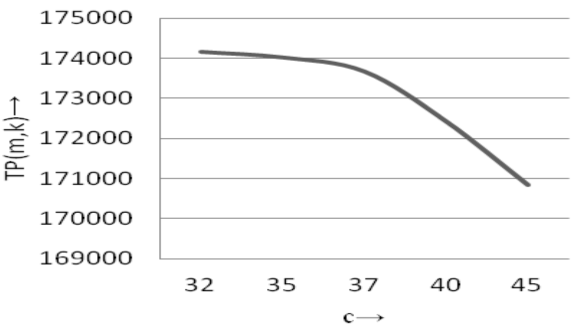
***International Journal of Production Economics*,** Vol. 88, No. 3, (2004), 307-316.

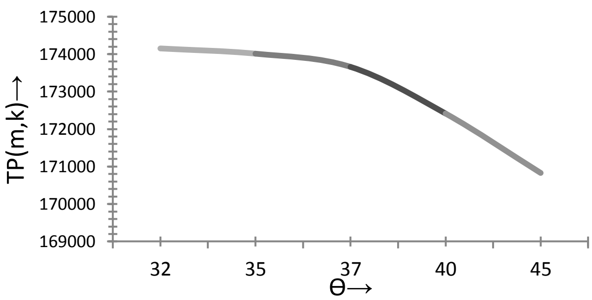
28. Sarkar, B., Mandal, P. and Sarkar, S., "An emq model with price and time dependent demand under the effect of reliability and inflation",***Applied Mathematics and Computation*,** Vol. 231, (2014), 414-421.
29. Hou, K.-L., "An inventory model for deteriorating items with stock-dependent consumption rate and shortages under inflation and time discounting", ***European Journal of Operational Research*,** Vol. 168, No. 2, (2006), 463-474.
30. Basu, M., Sinha, S., Kalyani, N. and Bengal, W., "An inflationary inventory model with time dependent demand with weibull distribution deterioration and partial backlogging under permissible delay in payments", ***Control and Cybernetics*,** Vol. 36, No. 1, (2007), 203.
31. Jaggi, C.K., Goel, S.K. and Mittal, M., "Credit financing in economic ordering policies for defective items with allowable shortages", ***Applied Mathematics and Computation*,** Vol. 219, No. 10, (2013), 5268-5282.
32. Ghiami, Y., Williams, T. and Wu, Y., "A two-echelon inventory model for a deteriorating item with stock-dependent demand, partial backlogging and capacity constraints", ***European Journal of Operational Research*,** Vol. 231, No. 3, (2013), 587-597.
33. Taleizadeh, A.A. and Nematollahi, M., "An inventory control problem for deteriorating items with back-ordering and financial considerations", ***Applied Mathematical Modelling*,** Vol. 38, No. 1, (2014), 93-109.
34. Yang, H.-L., Teng, J.-T. and Chern, M.-S., "An inventory model under inflation for deteriorating items with stock-dependent consumption rate and partial backlogging shortages", ***International Journal of Production Economics*,** Vol. 123, No. 1, (2010), 8-19.
35. Wee, H.-M., Huang, Y.-D., Wang, W.-T. and Cheng, Y.-L., "An epq model with partial backorders considering two backordering costs", ***Applied Mathematics and Computation*,** Vol. 232, (2014), 898-907.
36. Bhunia, A., Jaggi, C.K., Sharma, A. and Sharma, R., "A two-warehouse inventory model for deteriorating items under permissible delay in payment with partial backlogging", ***Applied Mathematics and Computation*,** Vol. 232, (2014), 1125-1137.
37. Ouyang, L.-Y. and Chang, C.-T., "Optimal production lot with imperfect production process under permissible delay in payments and complete backlogging",***International Journal of Production Economics*,** Vol. 144, No. 2, (2013), 610-617.
38. Yazdi, A.A. and Honarvar, M., "A two stage stochastic programming model of the price decision problem in the dual-channel closed-loop supply chain", ***International Journal of Engineering-Transactions B: Applications*,** Vol. 28, No. 5, (2015), 738-746.
39. Tripathi, R.P. and Uniyal, A.K., "Eoq model with cash flow oriented and quantity", ***International Journal of Engineering*,** Vol. 27, No. 7, (2014), 1107-1112.
40. Modarres, M. and Taimury, E., "Optimal solution in a constrained distribution system",***International Journal of Engineering Transactions A*,** Vol. 15, (2002), 179-190.
41. Rabbani, M., Manavizadeh, N. and Balali, S., "A stochastic model for indirect condition monitoring using proportional covariate model", ***International Journal of Engineering Transactions A Basics*,** Vol. 21, No. 1, (2008), 45-56.

APPENDIX









Figures between h, s,c,Θ and TP(m,k) (from sensitivity analysis)

Economic Order Quantity for Deteriorating Items with Non Decreasing Demand and Shortages Under Inflation and Time Discounting

R. P. Tripathi

Department of Mathematics, Graphic Era University, Dehradun (UK) India

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چکیده

برخی از محصولات مانند سبزیجات سبز، مایعات فرار و محصولات دیگر به علت تبخیر، فساد و غیره به طور مداوم رو به زوال می روند. در این مطالعه، یک مدل جدید برای اقلام فاسدشدنی که سرعت تقاضای آن تحت تورم و زمان تنزیلی در یک افق برنامه ریزی محدود به طور خطی به زمان وابسته است، توسعه یافته است. کمبودها مجاز بوده و به طور خطی به زمان وابسته است. مدل ریاضی برای مدل پیشنهادی ارائه شده است. ما نشان می دهیم که سود کلی با توجه به کسری از دوره زمان بندی k تقعر یافته است. نتایج به دست آمده با کمک مثال عددی مورد بحث قرار گرفته است. یک تحلیل حساس از راه حل بهینه با توجه به پارامترهای کلیدی نیز بحث شده است.

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