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TECHNICAL NOTE

A Numerical Analysis for the Effect of Slip Velocity and Stenosis Shape on Non-newtonian Flow of Blood

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ABSTRACT

The aim of this paper is to study the effect of slip velocity and shape of stenosis on non-Newtonian flow of blood through a stenosed arterial segment. Blood is modeled as Bingham-Plastic fluid in a uniform circular tube with a radially non-symmetric stenosis. The problem is investigated by a joint effort of analytical and numerical techniques. The influence of stenosis shape parameter, slip velocity, stenosis height and yield stress on blood flow through a stenosed artery has been examined. The variations of wall shear stress, resistance to flow, volumetric flow rate and axial velocity with stenosis shape parameter, yield stress and slip velocity have been shown graphically. It is noticed that axial velocity and volumetric flow rate was increased with slip but was decreased with yield stress. This information of blood could be useful in the development of new diagnosis tools for many diseases.

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1. INTRODUCTION

In developed and developing countries, one of the major health hazards is atherosclerosis, which refers to the narrowing of the arterial lumen. It means that the inner open space of an artery, due to sub-endothelial build-up of fatty or lipid material rich in cholesterol and proliferation of the connective tissues are believed to be the factors that accelerate the growth of the disease. This may lead to hypertension, myocardial infarction etc. This abnormal and unnatural growth in the arterial wall thickness is called stenosis which disturbs the flow of blood appreciably. The fluid mechanical study of blood flow in artery bears some important aspects due to the engineering interest as well as the feasible medical applications. A lot of investigations are performed for the prevention and cure of atherosclerosis which results in better understanding the nature of this type of disease.

M.M. Lih [1] and Mishra et al. [2] found that whole blood is predominantly a suspension of erythrocytes in plasma and behaves as a non-Newtonian fluid at low shear rates in micro-vessels. Published literature also reveals that in the stenotic region, blood exhibits low shear rate. Dwyer et al. [3] have emphasized that the formation of intravascular plaques and the impingement of ligaments and spurs on the blood vessel wall are some of the major factors for the initiation and development of this vascular disease. Chakravarty et al. [4] developed mathematical model of non-linear two-dimensional blood flow in tapered arteries in the presence of stenosis taking the vascular wall deformability to be elastic and the flowing blood contained in is treated to be Newtonian. Mishra et al. [5] studied bell shaped geometry to develop a mathematical model for studying the non-Newtonian flow of blood through a stenosed arterial segment. Utilizing the Herschel-Bulkley fluid model, Jain et al. [6] examined the effect of mild stenosis on blood flow, in an irregular axisymmetric artery with oscillating pressure gradient. Venkateshwarlu and Rao [7] have discussed numerical

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solution of unsteady blood flow through indented tube. Dash et al. [8] studied the pulsatile as well as steady flow pattern in a narrow catheterized artery taking blood as non-Newtonian fluid (Casson fluid) and estimated the increase in frictional resistance due to catheterization in a narrow artery. Dash et al. [9], Jayaraman et al. [10] discussed the changes in various flow characteristics in a catheterized curved artery with or without stenosis. Daripa et al. [11] have analyzed the numerical study of pulsatile blood flow in an eccentric catheterized artery, using a fast algorithm and in considering blood as to behave like a Newtonian fluid. Sankar [12] has studied a two-fluid model for the pulsatile flow of blood in a stenosed artery, by considering the core layer as a Casson fluid (non-Newtonian fluid) and the peripheral layer as a Newtonian fluid.

Johnston et al. [13] investigated a mathematical model to study the wall shear stress in four different human right coronary arteries using non-Newtonian blood model, as well as the usual Newtonian model of blood viscosity. Tu et al. [14] studied pulsatile flow of blood through arterial stenosis and used finite element simulation to obtain the expression for various flow characteristics. Tandon et al. [15] developed a model for blood flow through a stenotic tube. In all of the above studies the shape of stenosis was considered to be symmetrical about the axis as well as radius of the flow cylinder. Shit et al. [16] explored the effect of externally imposed body acceleration and magnetic field on peristaltic flow of blood through a stenosed arterial segment. A mathematical model for magneto-hydrodynamics (MHD) blood flow in a stenosed artery under porous medium is developed by Jain et al. [17], considering the cosine shaped geometry of the stenosis. Singh et al. [18] formulated a mathematical model to study the effects of shape parameter and stenosis length on the resistance to flow and wall shear stress under stenotic conditions by considering, laminar, steady, one dimensional, non-Newtonian and fully developed flow of blood through axially symmetric but radially non-symmetric stenosed artery. Assuming blood as non-Newtonian fluid (Casson fluid) and artery as circular tube, Bali et al. [19] investigated the response of external applied magnetic field on the flow of blood through a multiple stenosed artery. Nanda et al. [20] investigated a mathematical model for analyzing flow characteristics through a multiple stenosed narrow artery. The radially non-symmetric stenosis has been analysed by Singh et al. [21], Srivastava [22] Sanyal and Maji [23], Haldar [24]. They studied the effect of stenosis shape parameter on different flow characteristics employing traditional no slip conditions at the constricted wall.

A number of investigators like Brunn [25], and Chaturani et al. [26] have suggested the likely presence of slip (a velocity discontinuity) at the flow boundaries (or in their immediate neighbourhood). In view of the possible presence of a red cell slip at the wall, Mishra et al. [27] employed the momentum integral technique to investigate the problem of blood flow through a stenosed vessel taking the velocity slip condition at the arterial wall. Mishra et al. [28] developed a mathematical model for studying, analytically, blood flow through a stenosed arterial segment by taking into account the slip velocity at the wall of the artery. Mallik et al. [29] studied blood flow through an atherosclerotic artery with slip velocity at wall. A power law fluid model of the blood has been utilized in this study to account for the presence of red cells (erythrocytes) in plasma. Owing to the fact that due to permeability of the vessel wall, consideration of the no-slip condition at the wall may not be valid, the present study on blood flow in stenosed arteries is carried out to investigate the effects of stenosis shape parameter on resistance to flow, wall shear stress, volumetric flow rate and axial velocity with stenosis size, stenosis length and radial distance modeling blood as Bingham-Plastic fluid and introducing a velocity slip condition at the wall of the artery which makes the present model more general.

2. FORMULATION OF THE PROBLEM

In the present study, we have considered an arterial segment having stenosis which is symmetrical about the axis but non- symmetrical with respect to radial co-ordinates. The geometry of the stenosis may be written as:

$$\frac{R(z)}{R_0} = 1 - A\left[L_0^{m-1}(z-d)-(z-d)^m\right]; \quad d \le z \le d + L_0$$
$$= 1; otherwise \tag{1}$$

where, R_0 is the radius of the artery (assumed to be a rigid circular tube) outside the stenosis, $R(z)$ is the radius of the stenosed portion of the arterial segment, L_0 is the length of the stenosis, d indicates its location, m is a parameter determining the shape of stenosis and is referred to as stenosis shape parameter($m \ge 2$). Radially symmetric stenosis occurs when $m = 2$, and a parameter A is given by:

$$A = \frac{\delta}{R_0 L_0^{m-1}} \frac{m^{m/(m-1)}}{m-1} \tag{2}$$

where, δ denotes the maximum height of stenosis at $z = d + L_0 / m^{1/(m-1)}$. The ratio of the stenosis height to the radius of the normal artery is much less than unity ($\delta / R_0 \ll 1$). The schematic diagram of the flow is given by the Figure 1. Blood is assumed to flow in steady, laminar and fully developed manner. The constitutive equation in one dimensional form for Bingham-Plastic fluid with the shearing stress τ, is given by:

$$-\frac{du}{dr} = f(\tau) = \frac{\tau - \tau_0}{k}; \tau \geq \tau_0$$
$$= 0; \tau < \tau_0 \quad (3)$$

where, u stands for the axial velocity of blood, τ_0 the yield stress and k the viscosity coefficient of blood.
The equation governing the flow of blood is taken (Mishra, et al. [7]) in the form:

$$-\frac{dp}{dz} = \frac{1}{r}\frac{d(r\tau)}{dr} \quad (4)$$

in which, p stands for the pressure at any point, ($-dp/dz$) is called the pressure gradient and (z,r) are co-ordinates with z measured along the axis and r measured normal to the axis of the artery.
The boundary conditions for the problem stated above may be listed as:

$u = u_s$ at $r = R(z)$ (slip velocity condition) (5)

τ is finite at $r = 0$ (regularity condition) (6)

3. ANALYTICAL SOLUTION OF THE PROBLEM

As per the published literature and available physiological data, blood flow in the neighborhood of the vessel wall can be considered as Newtonian, if the shear rate of blood is high enough. However, the shear rate is very small towards the center of the artery (circular tube), the non-Newtonian behaviour of blood is more evident (cf. Mishra et al. [23]).
Integrating Equation (4) and using the regularity condition (6), we get:

$$\tau = -\frac{r}{2}\frac{dp}{dz} \quad (7)$$

The corresponding yield stress in the core region is given by:

$$\tau_0 = -\frac{r_0}{2}\frac{dp}{dz} \quad (8)$$

where, r_0 is the radius of the core region.
The expression for the wall shear stress τ_R may be written as:

$$\tau_R = -\frac{R}{2}\frac{dp}{dz} \quad (9)$$

where, $R=R(z)$. From Equation (7), it is clear that the shearing stress τ is proportional to the radial distance r. Blood will flow only if the shearing stress is more than the yield stress (i.e. if $\tau \geq \tau_0$ or $r \geq r_0$). And if the shearing stress is smaller than the yield stress (i.e. if $\tau < \tau_0$ or $r < r_0$), then blood will not flow. Therefore, it is apparent for the region $0 \leq r < r_0$, the equation of flow is:

$$\frac{du(r)}{dr} = 0, \quad r < r_0$$

which gives on integration, $u(r) = u_0 = constant$

where, u_0 is the velocity of blood in the core region.
For the region $r_0 \leq r \leq R(z)$, using Equations (7) and (8) in Equation (3) we get the velocity function $u(r)$ as:

$$\frac{du}{dr} = \frac{P}{2k}(r - r_0) \quad (10)$$

where:

$$P = P(z) = -\frac{dp}{dz}$$

represents the pressure gradient.
Using the slip velocity condition (5) for integrating Equation (10) between the limits r and $R(z)$ we get:

$$\int_r^R du = \frac{P}{2k}\int_r^R (r - r_0)dr$$

This yields:

$$u(r) = u_s + \frac{P}{4k}\left[(R - r_0)^2 - (r - r_0)^2\right], r_0 \leq r \leq R(z) \quad (11)$$

The expression for velocity of blood in the core region when $r = r_0$ may be written as:

$$u_0 = u_s + \frac{P}{4k}(R - r_0)^2 \quad (12)$$

The volumetric flow rate Q can be defined as:

$$Q = \int_0^R 2\pi r u(r)du = \int_0^{r_0} 2\pi r u(r)du + \int_{r_0}^R 2\pi r u(r)du \quad (13)$$

Using Equations (11) and (12) in (13), we get the expression for the volumetric flow rate Q as:

$$Q = \pi R^2 u_s + \frac{\pi P R^4}{8k}\left(1 + \frac{2}{3}\alpha + \frac{1}{3}\alpha^2\right)(1 - \alpha)^2 \quad (14)$$

where $\alpha = \frac{r_0}{R} = \frac{\tau_0}{\tau_R}$

when $\tau_0 / \tau_R \ll 1$ Equation (14) reduces to:

$$Q = \pi R^2 u_s + \frac{\pi P R^4}{8k}\left(1 - \frac{4}{3}\alpha\right) \quad (15)$$

From Equation (15), pressure gradient is written as follows:

$$-\frac{dp}{dz} = \frac{8k}{\pi R^4}\left(Q - \pi R^2 u_s\right) + \frac{8}{3}\frac{\tau_0}{R} \quad (16)$$

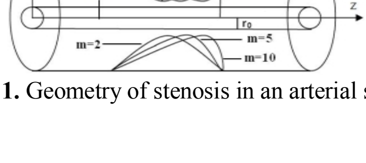

Figure 1. Geometry of stenosis in an arterial segment

Integrating Equation (16) along the length of the artery, if $P = P_1$ at $z = 0$ and $P = P_2$ at $z = L$, we obtain:

$$P_1 - P_2 = \frac{8kQ}{\pi R_0^4}\int_0^L \frac{dz}{(R/R_0)^4} - \frac{8ku_s}{R_0^2}\int_0^L \frac{dz}{(R/R_0)^2} + \frac{8}{3}\frac{\tau_0}{R_0}\int_0^L \frac{dz}{R/R_0} \tag{17}$$

where, R/R_0 can be obtained from Equation (1)

Thus, the resistance to flow λ_R defined by:

$$\lambda_R = \frac{P_1 - P_2}{Q} \tag{18}$$

which may be expressed as:

$$\lambda_R = \frac{8k}{\pi R_0^4}\left[L - L_0 + \int_d^{d+L_0}\frac{dz}{(R/R_0)^4}\right] - \frac{8ku_s}{QR_0^2}\left[L - L_0 + \int_d^{d+L_0}\frac{dz}{(R/R_0)^2}\right] + \frac{8}{3}\frac{\tau_0}{QR_0}\left[L - L_0 + \int_d^{d+L_0}\frac{dz}{R/R_0}\right] \tag{19}$$

Outside the stenosis (when $R = R_0$), the resistance to flow λ_N is given by:

$$\lambda_N = 8\left(\frac{k}{\pi R_0^4} + \frac{1}{3}\frac{\tau_0}{QR_0}\right)L \tag{20}$$

A non-dimensional expression for the resistance to flow may be put as:

$$\lambda = \frac{3kQ.I_1 - 3\pi k u_s R_0^2.I_2 + \pi R_0^3\tau_0.I_3}{3kQ + \pi R_0^3\tau_0} \tag{21}$$

where:

$$I_1 = 1 - \frac{L_0}{L} + \frac{1}{L}\int_d^{d+L_0}\frac{dz}{(R/R_0)^4},\quad I_2 = 1 - \frac{L_0}{L} + \frac{1}{L}\int_d^{d+L_0}\frac{dz}{(R/R_0)^2},$$

$$I_3 = 1 - \frac{L_0}{L} + \frac{1}{L}\int_d^{d+L_0}\frac{dz}{R/R_0}.$$

The wall shear stress obtained from (9) and (16) is given by:

$$\tau_R = \frac{4k}{\pi R_0^3}\frac{Q - \pi R_0^2 (R/R_0)^2 u_s}{(R/R_0)^3} + \frac{4}{3}\tau_0 \tag{22}$$

In the absence of any constriction (when $R = R_0$), the expression for wall shear stress, τ_N reads:

$$\tau_N = \frac{4kQ}{\pi R_0^3} + \frac{4}{3}\tau_0 \tag{23}$$

The expression for wall shear stress in a non-dimensional form may be given as:

$$\tau = \frac{3k\left[Q - \pi R_0^2 (R/R_0)^2 u_s\right] + \pi R_0^3 (R/R_0)^3 \tau_0}{(R/R_0)^3[3kQ + \pi R_0^3\tau_0]} \tag{24}$$

4. NUMERICAL COMPUTATION AND DISCUSSION OF RESULTS

The main aim of the present study was to quantify the influence of the stenosis shape parameter and the slip velocity at the arterial wall on the various flow characteristics. The effects of stenosis height, yield stress, axial distance and radial distance on wall shear stress, resistance to flow, axial velocity of blood and volumetric flow rate have also been observed and shown graphically. The analytical expressions for wall shear stress, resistance to flow, axial velocity of blood and volumetric flow rate derived in the previous section were executed by using MATLAB 7.8.

The computations were carried out for different values of stenosis height δ/R_0 = 0.2 (mild stenosis), 0.4, 0.6 (moderate stenosis), 0.8 (severe stenosis), slip velocity (u_s = 0, 0.1 and 0.2), yield stress (τ_0 = 0 and 0.1) and stenosis shape parameter (m = 2, 3,…, 11). The non-dimensional wall shear stress, resistance to flow, axial velocity of blood and volumetric flow rate are obtained in the Equations (25), (22), (11) and (15), respectively. The Bingham-Plastic fluid model reduces to Power-Law fluid when τ_0 = 0 and the radially non-symmetric stenosis becomes symmetric when m = 2. In Figuers 2 and 3, the variation in the axial velocity of blood with radial distance has been shown. From these figures, it is found that the axial velocity increases as stenosis shape parameter m and slip velocity u_s increase and the velocity decreases with increasing radial distance and the yield stress τ_0.

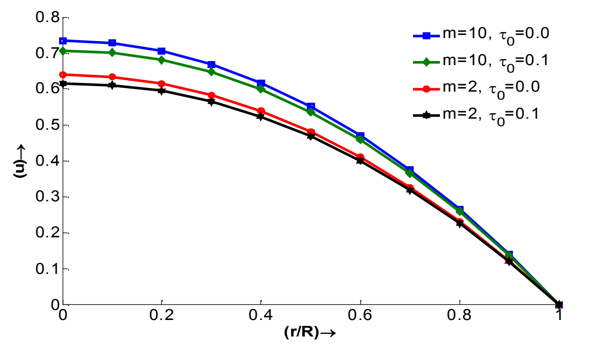

Figure 2. Variation of axial velocity along radial distance for different values of stenosis shape parameter (m) and yield stress (τ_0)

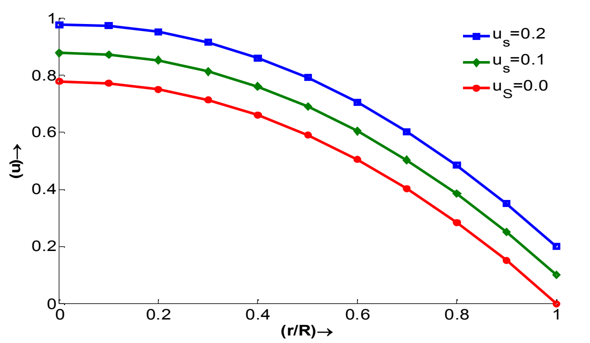

Figure 3. Variation of axial velocity along radial distance for different values of stenosis slip velocity (u_s)

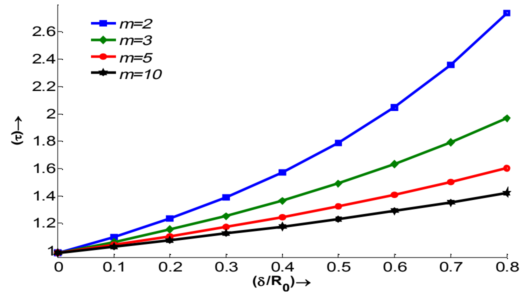

Figure 4. Variation of wall shear stress with stenosis height for different values of shape parameter (*m*)

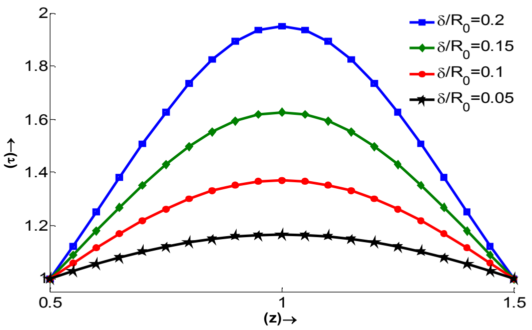

Figure 5. Variation of wall shear stress with axial distance for different values of stenosis height (δ/R_0)

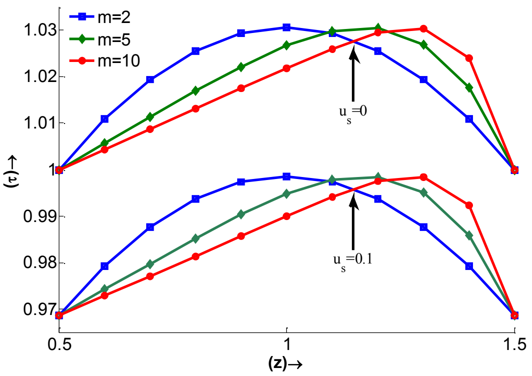

Figure 6. Variation of wall shear stress with axial distance for different values of shape parameter (*m*) and slip velocity (u_s)

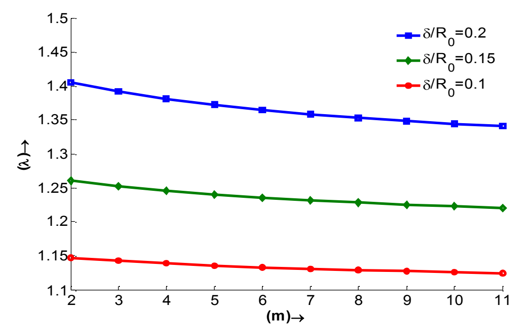

Figure 7. Graph of resistance to flow with stenosis shape parameter for different values of stenosis height (δ/R_0)

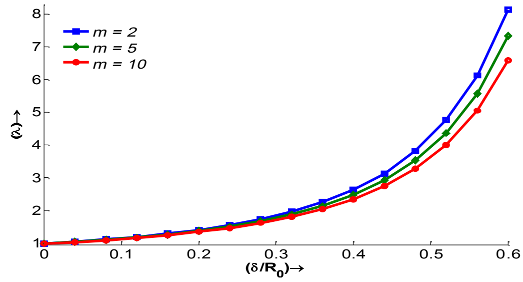

Figure 8. Graph of resistance to flow with stenosis height for different values of stenosis shape parameter (*m*)

These results are consistent to the observations of Singh et al. [21] which validate our work. Figure 4 consists of the variation of wall shear stress, τ with stenosis height (δ/R_0) and shape parameter (*m*). It is evident that the wall shear stress increases in the stenotic region as stenosis height increases. This result is similar to that of Mishra et al. [6]. The wall shear stress attains maximum value in case of radially symmetric stenosis (m = 2) and starts diminishing as stenosis losing its symmetry i.e. stenosis shape parameter *m* becoming larger. By the inspection of the Figures 5 and 6, it is revealed that the wall shear stress has the minimum value at the extremities of stenosis, then it starts increasing with stenosis height along the axial distance of the artery and attains the maximum value at the stenosis throats and it goes on decreasing to the minimum value. Figure 6 represents the variation of the wall shear stress with stenosis shape parameter.

The wall shear stress shows same variations for the equidistant values of *z* from the extremities of the constriction when m = 2 which indicates the radially non-symmetric stenosis becomes symmetric when m = 2. It can also be seen that if slip velocity u_s increases then the wall shear stress decreases. The influence of stenosis shape parameter and stenosis size on the impedance (resistance to flow) has been revealed in Figures 7 and 8. It can be found easily that the flow resistance increases as the size of constriction increases but decreases with increasing shape parameter although the decrease is comparatively small. The variation in the volumetric flow rate of blood along the axial length of the artery for different values of the stenosis shape parameter *m*, slip velocity u_s, stenosis height δ/R_0 and yield stress τ_0 is illustrated in the Figures 9 and 10. The effect of stenosis size and the shape parameter on the volumetric flow rate of blood for different values of the slip velocity u_s and yield stress τ_0 is shown in the Figures 11 and 12. It can be noted that the volumetric flow rate decreases with the increase in stenosis height and the yield stress and that the flow rate increases with an increase in slip velocity and stenosis shape parameter. One can further observe that the flow rate takes the minimum value at the throats of stenosis irrespective of the size of stenosis.

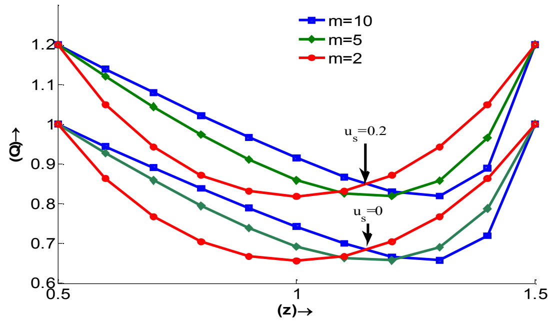

Figure 9. Variation of volumetric flow rate v/s axial distance for different values of stenosis shape parameter (m) and slip velocity (u_s)

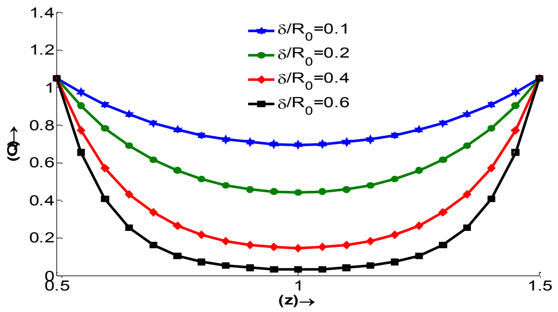

Figure 10. Variation of volumetric flow rate with axial length for different values of stenosis height (δ/R_0) and yield stress (τ_0)

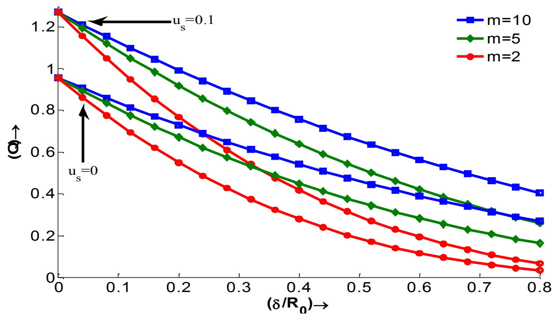

Figure 11. Variation of volumetric flow rate with stenosis height for different values of stenosis shape parameter (m) and slip velocity (u_s)

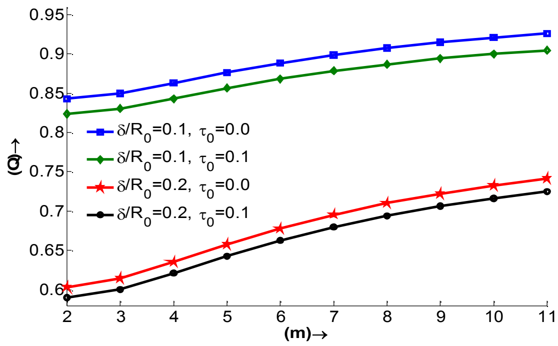

Figure 12. Variation of volumetric flow rate with stenosis shape parameter for different values of stenosis height (δ/R_0) and yield stress (τ_0)

5. CONCLUDING REMARKS

In the present work, we have investigated the influence of slip velocity and stenosis shape parameter on the different flow characteristics of blood behaving as a non-Newtonian fluid theoretically. Resistance to flow and wall shear stress increases as stenosis grows and stenosis shape parameter increases axial velocity and the rate of flow of blood. These increases are however, small due to non-Newtonian behaviour of the blood. It is observed that increase in shape parameter increases the wall shear stress in the upstream of the throat but decreases in the downstream. From the computational results it can be concluded that although slip velocity does not have any considerable effect on the flow resistance, but it has a significant role to play in reducing wall shear stress. Also, the volumetric flow rate is enhanced when the slip velocity increases. It appears that the non-Newtonian behaviour of the blood is helpful in the functioning of diseased arterial circulation.

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A Numerical Analysis for the Effect of Slip Velocity and Stenosis Shape on Non-Newtonian Flow of Blood

TECHNICAL NOTE

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چکیده

هدف از این مقاله بررسی اثر سرعت لغزش و شکل تنگی در جریان غیر نیوتنی از خون از طریق بخش شریانی تنگ است. خون به عنوان سیال بینگهام پلاستیک در یک لوله دایره ای یکنواخت با تنگی شعاعی غیر متقارن مدل شده است. مسئله با یک تلاش مشترک از تکنیک های تحلیلی و عددی بررسی شده است. تاثیر پارامتر شکل تنگی، سرعت لغزش، تنگی ارتفاع و عملکرد استرس در جریان خون از میان عروق تنگ مورد بررسی قرار گرفته است. تغییرات تنش برشی دیوار، مقاومت در برابر جریان، دبی حجمی و سرعت محوری با پارامتر شکل تنگی، عملکرد استرس و سرعت لغزش به صورت گرافیکی نشان داده شده است. این نتیجه حاصل شد که سرعت محوری و سرعت جریان حجمی با لغزش افزایش اما با استرس عملکرد کاهش یافت. این اطلاعات از خون می تواند در توسعه ابزار تشخیص جدید برای بسیاری از بیماری ها مفید باشد.

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