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A Simple Method for Modeling Open Cracked Beam

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ABSTRACT

A simple method is proposed to model the open cracked beam structures. In this method, crack is modeled as a beam element. Hence cracked beam can be assumed to be a beam with stepped cross sections, and problem of determining natural frequency and mode shape of cracked beam, can be solved as determining these characteristics for a beam with different lengths and cross sections. With this work, it is not necessary to model crack as lumped flexibility model in according to fracture mechanics and related sciences to obtain crack stiffness, and this spring model of crack can be used for further analysis.

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1. INTRODUCTION

The presence of a crack in structural members decreases their stiffness and natural frequencies. We can group different models of cracked structure in three basic categories [1]: equivalent reduction in local stiffness (smeared crack models), lumped flexibility models, and continuous cracked bar and beam models. A smeared crack model usually consists of a finite element model of the structure in which the damage is represented by an equivalent reduction in the stiffness of a particular element or group of the elements. Better representations of the effect of a crack over a region may be achieved by developing special finite elements, as reported by Haisty and Springer [2] and Gounaris and Dimaroginas [3]. Ibrahim [4] proposes an elasto-plastic finite element capable of accounting for the plastic deformation at the crack tip. Lumped flexibility models are based on sub-structuring concept. The undamaged portions of the structure are modeled using standard techniques such as FEM, component mode synthesis, or partial differential equations, and the crack is represented by lumped springs or by a compliance matrix, usually derived from the expression for the strain energy release rate or stress intensity factor. The first investigations are attributed to Kirmser [5]. They represented the effect of a notch by equivalent forces and moments at the location of the geometrical discontinuity. Dimarogonas [6] proposed the derivation of compliance constants from fracture mechanics and used it for vibration analysis. Christides and Barr initially developed models for transverse vibrations of a symmetric, double-edge cracked Euler-Bernoulli beam [7] and for the torsional vibrations of a cracked bar [8]. Both models rely on the characterization of the stress concentration due to the crack by means of a decay function. The mixed variational theorem used is an extension of the Hu-Washizu stationary principle[9], and it is now referred in the literature as the Hu-Washizu-Barr [10] principle. Christides and Barr's model was improved and extended in following years by Shen ad Pierre [11-13], first by introducing an alternative estimation of the decay parameter from 2D finite element models and later by applying the same ideas to develop a model for beams with a single edge breathing crack. Chondors and Dimarogonas [14-17] used a similar variational approach, but they derived the so-called crack functions from energy considerations and fracture mechanics concepts.

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Studies on nonlinear behavior of cracked beam using perturbation models have been proposed, e.g., by Gudmundson [18], Tsyfanskii [19], Plakhtienko and Yasinskii [20] and Ballo [21]. A review on the vibration analysis for a damage occurrence of a cantilever beam is given by Jassim et al. [22]. AL-Shudeifat used finite element modeling approach to study asymmetric cracked rotor [23]. Dynamic behavior of single-cracked beams considering the effects of axial stiffness, and shear deformations is carried out by Gomes and Almeida [24]. The damage is modeled using a rotational spring that simulates the crack based on fracture mechanics theory. Caddemi and Morassi investigate mathematical modeling and exact solutions of multi-cracked Euler–Bernoulli beams [25]. Dixit and Hodges, gave a general formulation termed the "Unified Framework", which yielded *n*th-order expressions governing mode shapes and natural frequencies for damaged elastic structures such as rods, beams, plates and shells of any shape [26]. Rakideh et al. presented identification of crack location in beams with different boundary conditions based on neural network method [27].

The most used method for modeling vibration characteristics of cracked beam is the lumped flexibility model. In this model, the cracked section is considered as a torsional (rotational) spring. The stiffness of this spring is obtained from fracture mechanics, in which boundary conditions, location, opening (or length) of the crack is not important. Here a method is presented for modeling of crack, in which all of these parameters are considered in modeling of crack. In the proposed method, crack is modeled as a beam element. Hence a cracked beam can be assumed to be a beam with stepped cross sections, and problem of determining natural frequency and mode shape of cracked beam, can be solved as determining these characteristics for a beam with different cross sections. With this work, it is not necessary to model crack in according to fracture mechanics. With this modeling method, position, width and height of cracks are directly included in modeling. With putting aside fracture mechanics from analysis, modeling of crack will be simple, and analytical solution of crack problem will be possible.

2. MODELING OF CRACKED BEAM

Figure 1 (a) shows a beam of length L and thickness h, containing an edge crack of depth h_c located at a distances x_1 and x_2 from the left end, and the middle point of the crack is $(x_1+x_2)/2$ which is assigned as eL, Young's modulus of elasticity E and mass density ρ are assumed to be constant.

3. LUMPED MODELING OF CRACK

The lumped flexibility form for modeling cracked beam is simply shown in Figure 1 (b). In this simple model, the crack is modeled by massless spring with rotational (torsional) stiffness of K_C. The effects of discontinuities in axial and transverse displacements are considered to be negligible compared with those of discontinuity in bending slope. The cracked beam is divided into two sub-beams connected by the rotational spring with stiffness of K_C at the cracked section whose bending stiffness can be given as:

$$K_C = \frac{1}{G} \tag{1}$$

where, G is the flexibility due to the crack and can be derived as[28]:

$$\frac{\left(1-\nu^2\right)K_1^2}{E(a)} = \frac{M^2}{2}\frac{dG}{da} \tag{2}$$

M, is the bending moment at the cracked section; K_1 is the stress intensity factor (SIF) under mode I bending load; and E is Young's modulus at the crack tip. The magnitude of SIF can be obtained from the data given in the literature [29] through Lagrange interpolation technique as:

$$K_1 = \frac{6M\sqrt{\pi h\eta}}{bh^2}F(\eta), \quad \eta = \frac{a}{h}\ (\eta \le 0.7) \tag{3}$$

where, $\eta \le 0.7$ implies that crack depth ratio changes from 0.0 to 0.7, and:

$$\begin{aligned} F(\eta) &= 0.6384\eta^2 - 1.035\eta^3 + 3.7201\eta^4 \\ &-5.1773\eta^5 + 7.553\eta^6 - 7.332\eta^7 + 2.4909\eta^8 \end{aligned} \tag{4}$$

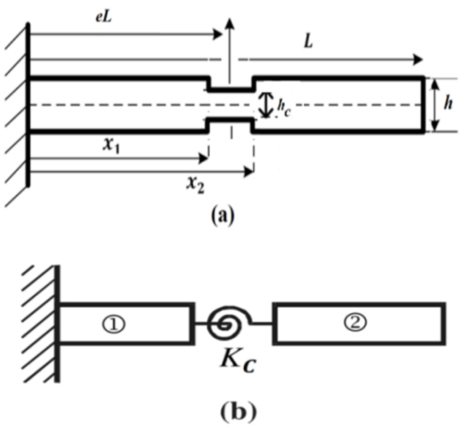

Figure 1. (a) Beam with single crack and (b) lumped flexibility modeling of cracked beam.

The expression of $F(\eta)$ for other values of Young's modulus ratios can also be obtained by using Lagrange interpolation formula. Substituting Equation (4) into Equation (3) and then into Equation (2) and integrating the results give:

$$G=\int_0^{\eta}\frac{72\pi\left(1-v^2\right)\eta F^2(\eta)}{E(\eta h)h^2}d\eta \tag{5}$$

From Eqsuations (5) and (1), the bending stiffness of the cracked section required for lumped flexibility modeling can be determined.

4. CRACKED EULER-BERNOULLI BEAM WITH LUMPED FLEXIBILITY MODELS

Consider an elastic cracked Euler-Bernoulli beam of length L, uniform cross-section area A and moment of inertia I, with a crack at middle position of eL as shown in Figure 1. Differential equations of motion and boundary conditions on two segments of the beam are as follows:

$$\left(EIw''\right)''+\rho A\ddot{w}=0,\ 0<x<eL,\ eL<x<L \tag{6}$$

$$EIw''\delta w'\big|_{(0,L)}=0,\ EIw'''\delta w\big|_{(0,L)}=0 \tag{7}$$

where, $w(x,t)$ is the beam transverse deflection and ρ mass density per unit length, A cross section area of beam, E Young's module of elasticity and I is area moment of inertial. Also, at crack location:

$$\begin{aligned}&w\big|_{eL^+}=w\big|_{eL^-}=w(eL),\ EIw''\big|_{eL^-}^{eL^+}=0\\&EIw'''\big|_{eL^-}^{eL^+}=0,\ EIw''\big|_{eL^-}^{eL^+}=K_c\,w'\big|_{eL^-}^{eL^+}\end{aligned} \tag{8}$$

Following dimensionless parameter are defined:

$$\xi=\frac{x}{L},\ \tau=\sqrt{\frac{EI}{\rho AL^4}}t,\ \eta_i=\frac{w_i}{L},\ (i=1,2) \tag{9}$$

By substituting dimensionless parameter from Equation (9), in Equations (6) and (7), assuming harmonic solution as $\eta_i(\tau,\xi)=Ae^{\lambda_i\xi}e^{i\bar{\omega}\tau}$, the related characteristic equation will be obtained with eigenvalues of $\lambda_{1,2}^2=\pm\bar{\omega}=\pm\beta^2$, $(i=1,2)$. Then mode shape of beam has following form [30]:

$$\begin{aligned}\mathrm{X}_{1,2}(\xi)&=C_{1,5}\cosh\beta\xi+C_{2,6}\sinh\beta\xi\\&+C_{3,7}\cos\beta\xi+C_{4,8}\sin\beta\xi\end{aligned} \tag{10}$$

Values of β_i and the ratios of coefficients of mode shapes of C_i/C_1, can be obtained from applying boundary and compatibility conditions at crack location.

5. CRACKED TIMOSHENKO BEAM WITH LUMPED FLEXIBILITY MODELS

Differential equations of motion and boundary conditions for transverse deflection w and slope of ψ for beam in according to Timoshenko beam theory is as follows [30]:
For $0\le x<eL$ and $eL<x\le L$:

$$\begin{aligned}&\rho A\ddot{w}-kAG\left(w''-\psi'\right)=0,\\&\rho I\ddot{\psi}-EI\psi''-kAG\left(w'-\psi\right)=0\end{aligned} \tag{11}$$

and boundary conditions are:

$$EI\psi'\delta\psi\big|_{0,L}=0,\ kAG\left(w'-\psi\right)\delta w\big|_{0,L}=0 \tag{12}$$

and at crack location:

$$\begin{aligned}&w\left(t,eL^+\right)=w\left(t,eL^-\right),\ \psi'\big|_{eL^-}^{eL^+}=\frac{K_c}{EI}w'\big|_{eL^-}^{eL^+}\\&kAG\left(w'-\psi\right)\big|^{eL^+}=kAG\left(w'-\psi\right)\big|^{eL^-}\end{aligned} \tag{13}$$

where, $w(x,t)$ is transverse deflection, and $\psi(x,t)$ is the slope of the deflection curve, G indicates shear modulus of elasticity. k represents the shear correction factor, which is assumed to be $5/6$. Following dimensionless quantities are introduced:

$$\begin{aligned}&\xi=\frac{x}{L},\ \vartheta=\frac{kG}{E},\ r=\frac{AL^2}{I},\ s=\vartheta r,\tau=\sqrt{\frac{EI}{\rho AL^4}}t\\&\eta_i=\frac{w_i}{L}\end{aligned} \tag{14}$$

With assuming a harmonic solution in following form $\eta_i(\tau,\xi)=\mathrm{X}_i(\xi)e^{i\bar{\omega}\tau}$ and $\psi_i(\tau,\xi)=\Psi_i(\xi)e^{i\bar{\omega}\tau}$ and obtaining $\Psi_i(\xi)$ in terms of $\mathrm{X}_i(\xi)$, we have [30]:

$$\frac{d^4\mathrm{X}_i}{d\xi^4}+\bar{\omega}^2\left(\frac{1}{r}+\frac{1}{s}\right)\frac{d^2\mathrm{X}_i}{d\xi^2}+\bar{\omega}^2\left(\frac{\bar{\omega}^2}{sr}-1\right)\mathrm{X}_i=0 \tag{15}$$

in the form of $\mathrm{X}_i=A_ie^{\lambda\xi}$, the characteristic equation for determining of mode shape eigenvalue of λ and mode shape will be obtained as:

$$\lambda_{1,2}=\sqrt{\bar{\omega}\sqrt{\frac{\bar{\omega}^2}{4}\left(\frac{1}{r}-\frac{1}{s}\right)^2+1}\mp\frac{\bar{\omega}^2}{2}\left(\frac{1}{r}+\frac{1}{s}\right)} \tag{16}$$

$$\begin{aligned}\mathrm{X}_{1,2}(\xi)&=C_{1,5}\cosh\lambda_1\xi+C_{2,6}\sinh\lambda_1\xi\\&+C_{3,7}\cos\lambda_2\xi+C_{4,8}\sin\lambda_2\xi,\\ \Psi_{1,2}(\xi)&=\left(\frac{\bar{\omega}^2}{s\lambda_1}+\lambda_1\right)\left(C_{1,5}\sinh\lambda_1\xi+C_{2,6}\cosh\lambda_1\xi\right)\\&+\left(\frac{\bar{\omega}^2}{s\lambda_2}-\lambda_2\right)\left(C_{3,7}\sin\lambda_2\xi-C_{4,8}\cos\lambda_2\xi\right)\end{aligned} \tag{17}$$

Accordingly the boundary conditions are:

$$\begin{aligned}&X_1(e)=X_2(e),\ X_2'(e)-X_1'(e)-\frac{1}{K_{cb}}\Psi_1'(e)=0\\&\Psi_2'(e)-\Psi_1'(e)=0,\\&X_2'(e)-X_1'(e)-\Psi_2(e)+\Psi_1(e)=0\end{aligned} \tag{18}$$

where, K_{cb} is given by following equation:

$$K_{cb}=\frac{L}{6h\pi F(\eta)} \tag{19}$$

6. CRACKED EULER-BERNOULLI BEAM WITH CRACKS AS STEPPED CHANGE IN BEAM'S CROSS SECTION

As pointed out in the introduction section, here crack is modeled as a beam element with different cross sections other than intact section of beam. Hence a cracked beam as shown in Figure 1 can be modeled as three interconnecting beams, with different areas, moment of inertias, heights and lengths. Then intact and cracked segments of beam has area moment of inertia of $I_i=1/12bh_i^3$ and $I_c=1/12b(h-h_c)^3$, respectively, where h is the thickness of the beam and h_c is the depth of the crack. The length of cracked beam segment is equal to its opening, i.e. x_2-x_1. The equation of motion for Euler-Bernoulli cracked beam is similar to Eqsuations (6) and (7), and related mode shape solution is similar to Equation (10). Then for a beam according to Figure 1 and with one crack, we have (from left end of beam):

$$\begin{aligned}&\frac{d^4X_i}{d\xi^4}-\theta_i^4X_i=0,\ m_i=\rho bh_i,\\&I_i=\frac{1}{12}bh_i^3,\ \theta_i^4=\frac{m_iL^4}{EI_i}\omega^2\\&\bar{W}_i(\xi)=C_{1,i}\cosh\theta_i\xi+C_{2,i}\sinh\theta_i\xi\\&+C_{3,i}\cos\theta_i\xi+C_{4,i}\sin\theta_i\xi,\ x_{i-1}\le x\le x_i\end{aligned} \tag{20}$$

For cracked section $h_i=h-h_c$. Since the value of ω (natural frequency) is high, for simplicity in numerical calculation it is better to select θ_i as unknown. The ratio of θ_i/θ_1 can be written as:

$$\frac{\theta_2}{\theta_1}=\sqrt{\frac{h_1}{h_2}},\ \frac{\theta_3}{\theta_2}=\sqrt{\frac{h_2}{h_3}} \tag{21}$$

and boundary and compatibility conditions at two ends of beam and location of cracks are:

$$\begin{aligned}&X_1''\delta X_1'\big|_0=0,\ X_3''\delta X_3'\big|_1,\ X_1'''\delta X_1\big|_0=0,\ X_3''\delta X_3\big|_1=0\\&X_1(\xi_1)=X_2(\xi_1),\ \ X_1'(\xi_1)=X_2'(\xi_1)\\&EI_1X_1''(\xi_1)=EI_2X_2''(\xi_1),\ EI_1X_1'''(\xi_1)=EI_2X_2'''(\xi_1)\\&X_2(\xi_2)=X_3(\xi_2),\ \ X_2'(\xi_2)=X_3'(\xi_2)\\&I_2X_2''(\xi_2)=I_3X''(\xi_2),\ I_2X_2'''(\xi_2)=I_3X_3'''(\xi_2)\end{aligned} \tag{22}$$

7. CRACKED TIMOSHENKO BEAM WITH CRACKS AS STEPPED CHANGE IN BEAM'S CROSS SECTION

Following the same procedure as described for Euler-Bernoulli beam, and with considering the crack as a beam with length equal to its opening and thickness equal to the thickness of the intact section minus depth of beam, following modeling of cracked beam in according to Timoshenko beam theory is given. The equation of motion for Timoshenko beam is given by Eqsuations (11) and (12), and related mode shape solution is also given by Equation (17). Then for a beam in according to Figure 1 and with one crack, we have: $(i=1,2,3,0\le\xi<\xi_1,\xi_1<\xi<\xi_2,\xi_2<\xi\le1)$:

$$\begin{aligned}&X_i(\xi)=C_{i,1}\cosh\lambda_{i,1}\xi+C_{i,2}\sinh\lambda_{i,1}\xi\\&+C_{i,3}\cos\lambda_{i,2}\xi+C_{i,4}\sin\lambda_{i,2}\xi\end{aligned}$$

$$\begin{aligned}&\Psi_i(\xi)=\left(\frac{\bar{\omega}^2}{s_1\lambda_{i,1}}+\lambda_{i,1}\right)\left[C_{i,1}\sinh\lambda_{i,1}\xi+C_{i,2}\cosh\lambda_{i,1}\xi\right]\\&+\left(\frac{\bar{\omega}^2}{s_1\lambda_{i,2}}-\lambda_{i,2}\right)\left[C_{i,3}\sin\lambda_{i,2}\xi-C_{i,4}\cos\lambda_{i,2}\xi\right]\end{aligned} \tag{23}$$

Boundary and compatibility conditions for Timoshenko-beam are:

$$\begin{aligned}&EI\Psi_1'\delta\Psi_1\big|_0=0,\ \ EI\Psi_3'\delta\Psi_3\big|_1=0\\&kAG(X_1'-\Psi_1)\delta X_1\big|_0=0,\ kAG(X_3'-\Psi_3)\delta X_3\big|_1=0\\&X_1(\xi_1)=X_2(\xi_1),\ \Psi_1(\xi_1)=\Psi_2(\xi_1)\\&EI_1\Psi_1'\big|_{\xi_1}=EI_2\Psi_2'\big|_{\xi_1},\\&kA_1G(X_1'-\Psi_1)\big|_{\xi_1}=kA_2G(X_2'-\Psi_2)\big|_{\xi_1}\\&X_2(\xi_2)=X_3(\xi_2),\ \Psi_2(\xi_2)=\Psi_3(\xi_2)\\&EI_2\Psi_2'\big|_{\xi_2}=EI_3\Psi_3'\big|_{\xi_2},\\&kA_2G(X_2'-\Psi_2)\big|_{\xi_2}=kA_3G(X_3'-\Psi_3)\big|_{\xi_2}\end{aligned} \tag{24}$$

It should be noted that formulation of the intact beam is similar to cracked beam, with putting aside the compatibility boundary conditions of cracked beam [30].

8. RESULTS AND DISCUSSIONS

Now after presenting the proposed crack modeling method and describing lumped flexibility modeling of crack, the results obtained from these two methods and comparison between them are given for Euler-Bernoulli and Timoshenko beam theories. Steel-high strength

4340 by the width, $b = 12.5\times10^{-3}$m has been utilized for comparing the results. The mechanical and geometrical parameters of cracked beam are: $L = 1\text{m}$, $h / L = 0.25$, $\rho = 7860\,\text{kg}/\text{m}^3$, $E = 210$ GPa . For the presented results acronyms of EB, TB, IB, PM and LM for Euler Bernoulli, Timoshenko beam, intact beam (without crack), present method and lumped method are selected, to simply referring the considered cases.

Table 1 shows predicted natural frequencies for cracked clamped-clamped (CC) beam for Euler-Bernoulli and Timoshenko theories, in according to the stepped beam and lumped flexibility methods for crack opening of $d_c / L = 0.004$. Natural frequencies of intact beam are shown in this table.

As seen from this table, there are very good conformity between these two results. From the results given in Table 1, it can be seen that natural frequency prediction in accordance to the lumped flexibility model presented in the literature [31] are lower than the results obtained from stepped change in cross section modeling of the beam, i.e. the amount of predicted stiffness reduction in according to lumped flexibility model is greater than stepped modeling of crack, although the difference between these two methods are very negligible. The accuracy of the presented modeling approach will be more clear in the following.

In Tables 2 and 3, three first natural frequencies of Euler-Bernoulli of clamped-free (CF) and clamped-simple (pined) (CS), are calculated for different values of crack' depth. The crack location is assumed to be $\eta = 0.5$ and the opening of crack is $d_c / L = 0.002$. Natural frequencies of intact beam (IB) are shown for comparison. Again as seen from these results, there are very good conformity between these two different modeling of cracks, and this show the effectiveness of the proposed modeling method of crack.

TABLE 1. Three first natural frequencies for clamped-clamped beam, different crack depths and $\eta = 0.5$, $d_c / L = 0.004$.

$\frac{h_c}{h}$	Intact beam [30]		Cracked beam			
			Lumped method [31]		Presented method	
	EB	TB	EB	TB	EB	TB
	22.3	22.27	22.32	22.23	22.3	22.2
0.1	61.6	61.06	61.67	61.06	61.6	61.06
	120.9	118.8	120.5	118.5	120.7	118.6
	22.37	22.27	22.20	22.11	22.33	22.23
0.2	61.6	61.06	61.67	61.06	61.6	61.06
	120	118	119	117	120	118.4

As expected with increasing depth of crack, natural frequencies of cracked beam are reduced, and with increasing or decreasing the rigidity of boundary conditions, natural frequencies are changed accordingly.

TABLE 2. Three first natural frequencies for CF beam, for crack location $\eta = 0.5$, crack opening $d_c / L = 0.002$ and various crack depths.

$\frac{h_c}{h}$	CF					
	Intact beam [30]		Cracked beam			
			Presented method		Lumped modeling [31]	
	EB	TB	EB	TB	EB	TB
	3.516	3.51	3.51	3.51	3.51	3.51
0.1	22.03	21.95	22.0	21.9	21.97	21.8
	61.69	61.19	61.69	61.19	61.69	61.19
	3.51	3.51	3.51	3.51	3.50	3.50
0.2	22.03	21.95	22.00	21.92	21.80	21.73
	61.69	61.19	61.6	61.1	61.6	61.1
	3.51	3.51	3.51	3.51	3.49	3.49
0.3	22.03	21.95	21.9	21.8	21.5	21.4
	61.69	61.19	61.6	61.1	61.6	61.1
	3.516	3.514	3.51	3.50	3.47	3.47
0.4	22.03	21.95	21.8	21.8	21.0	21.0
	61.69	61.19	61.6	61.1	61.6	61.1

TABLE 3. Three first natural frequencies for CS beam, for crack location $\eta = 0.5$, crack opening $d_c / L = 0.002$ and various crack depth

$\frac{h_c}{h}$	CS					
	Intact beam [30]		Cracked beam			
			Presented method		Lumped modeling [31]	
	EB	TB	EB	TB	EB	TB
	15.4	15.3	15.4	15.3	15.3	15.3
0.1	49.9	49.6	49.9	49.6	49.9	49.6
	104.2	102.8	104.1	102.8	104.0	102.6
	15.4	15.3	15.4	15.3	15.3	15.2
0.2	49.9	49.6	49.9	49.6	49.8	49.5
	104.2	102.8	104.1	102.7	103.3	102.0
	15.4	15.3	15.3	15.3	15.1	15.1
0.3	49.9	49.6	49.9	49.6	49.8	49.4
	104.2	102.8	103.9	102.6	102.2	100.9
	15.4	15.3	15.3	15.3	14.9	14.9
0.4	49.9	49.6	49.9	49.5	49.6	49.3
	104.2	102.8	103.6	102.3	100.6	99.3

In Table 4, the effect of varying crack opening on three first natural frequencies of CC beam for crack location of $\eta = 0.5$ and depth of $h_c/h = 0.1$ is shown. From this table, it can be seen that for lower frequencies, natural frequencies predicted in according to lumped flexibility model of Euler-Bernoulli beam is greater than stepped modeling. While in higher models, lumped flexibility model gives higher prediction of natural frequency in comparison to present modeling of crack.

The effect of crack opening on natural frequency of Euler-Bernoulli's beam in according to these two mentioned methods of crack modeling will be more clear, with Tables 5. and 6. In these figures, three first natural frequencies of beams with different boundary conditions with one crack at location of $\eta = 0.5$, depth of $h_c/h = 0.5$ and different amounts of crack openings are shown. As expected, in all cases with increasing the crack opening, the amounts of natural frequencies are reduced. Now the effects of varying crack location on natural frequencies of beam are investigated. Table 7 shows the effect of crack location on natural frequency of beam. As seen from these tables, the differences between these two presented methods with varying the location of crack is more apparent in this case, i.e. change in location of crack. The amounts of differences are dependent to boundary conditions, and the amount of proximity of crack to the nodal point of related intact mode shape. In Figure 2, mode shapes and their slopes for three first natural frequencies of cracked CC beam obtained from stepped modeling of crack are shown. As seen from this figure, mode shapes and their slops are continuous in cracked section of beam, while there are qualitative change in the shape of slope of mode shapes in cracked section of beam, i.e. $\partial^2 w/\partial x^2$ and $\partial^3 w/\partial x^3$ are discontinuous in the cracked domain.

TABLE 4. Three first natural frequencies for clamped-clamped beam (CC) for different crack openings, $\eta = 0.5$ and $h_c/h = 0.1$.

3	Intact beam [30]		Cracked beam			
			Lumped modeling [31]		Presented method	
	EB	TB	EB	TB	EB	TB
	22.3	22.2	22.3	22.2	22.3	22.2
	61.6	61.0	61.6	61.0	61.6	61.0
0.001	120.9	118.8	120.5	118.5	120.8	118.7
	61.6	61.0	61.6	61.0	61.6	61.0
	120.9	118.8	120.5	118.5	120.7	118.69
	22.3	22.2	22.3	22.2	22.3	22.2
0.005	61.6	61	61.6	61	61.6	61
	120	118	120	118	120	118
	22.3	22.2	22.3	22.2	22.3	22.2
0.01	61.6	61.0	61.6	61.0	61.6	61.0
	120	118	120	118	120	118

TABLE 5. Three first natural frequencies for CF beam with different crack openings, $\eta = 0.5$ and $h_c/h = 0.5$.

$\frac{d_c}{L}$	Intact beam [30]		Cracked beam			
			Presented method		Lumped method [31]	
	EB	TB	EB	TB	EB	TB
	3.51	3.51	3.5	3.5	3.45	3.45
0.001	22.0	21.9	21.8	21.8	20.4	20.3
	61.6	61.1	61.6	61.1	61.6	61.1
	3.51	3.51	3.50	3.50	3.45	3.45
0.002	22.03	21.9	21.7	21.6	20.4	20.3
	61.6	61.1	61.6	61.1	61.6	61.1
	3.51	3.51	3.5	3.4	3.45	3.45
0003	22.0	21.9	21.6	21.5	20.4	20.3
	61.6	61.1	61.6	61.1	61.6	61.1

TABLE 6. Three first natural frequencies for CS beam with different crack openings, $\eta = 0.5$ and $h_c/h = 0.5$.

$\frac{d_c}{L}$	Intact beam [30]		Cracked Beam			
			Presented method		Lumped Method [31]	
	EB	TB	EB	TB	EB	TB
	15.4	15.3	15.3	15.3	14.6	14.6
0.001	49.9	49.6	49.9	49.5	49.5	49.1
	104.2	102.8	103.6	102.3	98.5	97.2
	15.4	15.3	15.2	15.2	14.6	14.6
0.002	49.9	49.6	49.8	49.5	49.5	49.1
	104	102	103	101	98.5	97.2
	15.4	15.3	15.2	15.1	14.6	14.6
0.003	49.9	49.6	49.8	49.5	49.5	49.1
	104	102	102	101	98.5	97.2

Finally some results are presented to show the validity of the presented method with experimental or other available results given in literature. For this purpose a model with geometric and mechanical properties of $E = 207\,Gpa$, $\rho = 7860\,kg/m^3$, $\upsilon = 0.3$, $b = 12.7\,mm$, $h = 12.7\,mm$, $L = 400\,mm$, $\eta = 0.3$, and $d_0/L = 5$ are selected [32]. Experimental results for these parameters are given elsewhere [32]. The obtained results are shown in Table 8. As seen from this table, a good agreement between the results of the proposed method and the experimental model exists, and this shows the validity of the presented modeling of crack. The proposed modeling approach can be used to investigate the beam with different crack shapes, such as circular, elliptical and V shape cracks. Figure 3 shows a beam with V shape crack; there is no analytical solution for these problems, so to obtain the natural frequencies the Galerkin method should be used. Table 9 shows three first natural frequencies of Euler-Bernoulli cantilever cracked beam with V shape crack shown in Figure 3. For modeling crack, stepped beam modeling presented is used and beam is modeled in according to Euler-Bernoulli theory. Galerkin method is used for obtaining natural frequencies.

TABLE 7. Three first natural frequencies for CC, with different locations of crack, crack opening $d_c/L = 0.002$ and depth of $h_c/h = 0.5$.

Crack location	Intact beam [30]		Cracked beam			
			Presented method		Lumped modeling [31]	
	EB	TB	EB	TB	EB	TB
	22.3	22.2	22.2	22.1	21.5	21.4
0.1	61.6	61.0	61.5	60.9	61.2	60.6
	120	118	120	118	120	118
	22.37	22.2	22.3	22.2	22.3	22.2
0.2	61.6	61.0	61.4	60.8	60.3	59.7
	120	118	119	117	114	112
	22.3	22.2	22.2	22.1	21.5	21.4
0.4	61.6	61.0	61.3	60.7	59.7	59.2
	120	118	120	118	119	117

TABLE 8. Comparing results of the presented theory and experimental data of simply supported beam.

Natural frequency (Hz)	Proposed method	Experimental model in [32]
1^{st}	176.752	-
2^{nd}	702.085	700
3^{rd}	1615.467	1616
4^{th}	2852.20	2864

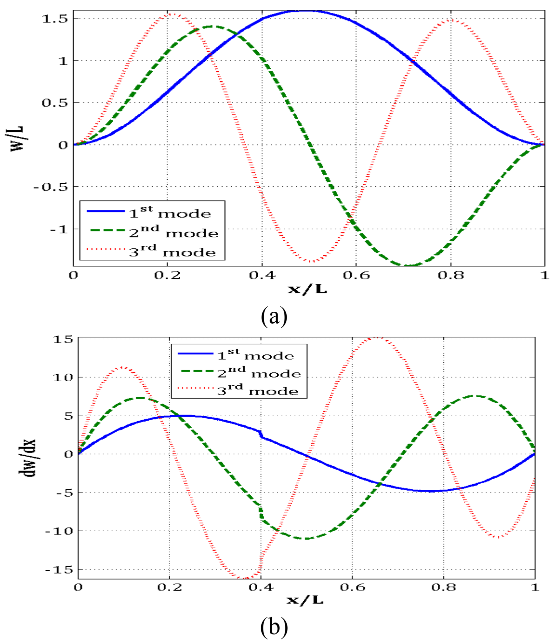

Figure 2. (a) Three first mode shapes and (b) slopes of cracked CC beam, for crack location $\eta = 0.4$.

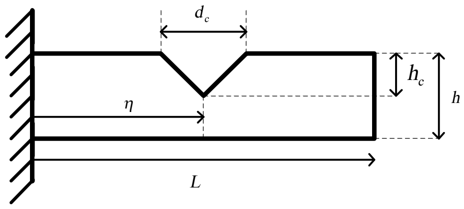

Figure 3. A cantilever cracked beam with V shape geometric.

TABLE 9. Three first natural frequencies for CF, with different crack openings and depth and crack location $\eta = 0.5$.

$\frac{h_c}{h}$	Intact beam	Cracked beam	
		Different crack openings	
		0.0025	0.00125
	3.5160	3.5158	3.5159
0.1	22.0344	22.0294	22.0319
	61.6972	61.6972	61.6972
	3.5160	3.5153	3.5156
0.2	22.0344	22.0163	22.0254
	61.6972	61.6971	61.6972
	3.5160	3.5147	3.5153
0.3	22.0344	21.9980	22.0162
	61.6972	61.6971	61.6972
	3.5160	3.5139	3.5150
0.4	22.0344	21.9768	22.0057
	61.6972	61.6970	61.6971
	3.5160	3.5130	3.5145
0.5	22.0344	21.9551	21.9950
	61.6972	61.6969	61.6971

As clear from obtained results, natural frequencies decrease with increasing crack depth and crack opening, and their values are less than intact beam.

9. CONCLUSION

There are different methods for modeling of cracks in the literature. In the current work a simple method is presented for modeling of crack. This method is based on modeling crack as a stepped change in cross section of beam. With this modeling approach, crack will be a beam element, with specific depth, area, area moment of inertia, and length, and similar to intact beam, its solution will be obtained analytically. Based on the proposed idea, beams with different number of cracks and boundary conditions are simulated and results are compared with well-known lumped flexibility modeling of crack, in which crack is modeled as massless rotational spring. The obtained results show good conformity of the stepped modeling of crack with results obtained from lumped flexibility model. The presented modeling approach is very simple, can simply be extended to structures with different shapes of cracks such as V or circular crack shapes that do not have analytical solutions.

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A Simple Method for Modeling Open Cracked Beam

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چکید

یک روش ساده برای مدل‌سازی ترک باز در ساختارهای به شکل تیر ارایه شده است. در این روش ترک به صورت یک المان تیر مدل‌سازی می‌شود. از این رو تیر به صورت یک تیر با تغییر پله‌ای در سطح مقطع در نظر گرفته می‌شود، و مساله تعیین فرکانس طبیعی و شکل تیر به صورت حل این مشخصه‌ها برای یک تیر با طول و سطح مقطع متفاوت قابل حل خواهد بود. با این کار نیاز به مدل‌سازی تیر به صورت تیر با انعطاف‌پذیری متمرکز بر طبق تئوری شکست و یا علوم مربوطه برای تعیین سفتی ترک و استفاده از این مدل فنر برای تحلیل‌های بعدی مربوطه نخواهد بود.

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