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Investigation of Thermoelastic Damping in the Longitudinal Vibration of a Micro Beam

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ABSTRACT

In the design of high Quality factor (Q) micro or nano beam resonators, different dissipation mechanisms may have damaging effects on the quality factor. One of the major dissipation mechanisms is the thermoelastic damping (TED) that needs an accurate consideration for prediction. In this paper, TED of the longitudinal vibration of a homogeneous micro beam with both ends clamped have been investigated. A Galerkin method has been used to analyze TED for the first mode of vibration of the micro beam. Then the quality factor and longitudinal vibrations frequency are obtained. Changing of quality factor versus geometrical properties and ambient temperature for different materials are plotted.

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1. INTRODUCTION

These days Microelectromechanical systems (MEMS) and Nanoelectromechanical systems (NEMS) are playing an important role in science and engineering applications. Lots of benefits make MEMS and NEMS attractive for commercial activities. Micro pumps, ink jet printer heads, micro sensors and airbag accelerometers are small number of examples of devices that MEMS have replaced successfully [1].

Micro mechanical resonators are one of important applications of MEMS. Micro-beams and micro-plates are employed widely in micro resonators and they are used a lot in applications such as radio frequency (RF) filters, sensors, charge detectors and gyrometers [1]. In many usages, such as resonant sensors and RF-MEMS filters where increasing the sensitivity and accuracy of devices is needed, obtaining high amounts of quality factor is an essential issue [2].

Thermoelastic damping (TED) has been displayed lately to be a major source of inherent damping in MEMS [2]. For the first time Zener [3, 4] found that TED plays an important role in resonator's dissipations. Lifshitz and Roukes [5] obtained an analytical solution for the quality factor of micro-beams and studied its size-dependency. Landau and Lifshitz [6] presented an exact expression for damping coefficient of thermoelastic vibration. Evoy et al. [7] and Duwel et al. [8] experimentally shown that TED is a major source of damping in MEMS and NEMS. Nayfeh and Younis [2] and De and Aluru [9] studied one dimensional parabolic model of heat conduction ignoring longitudinal direction; therefore, their equations of motion and heat transfer were one side coupled. Guo and Rogerson [10] studied two dimensional parabolic (TDP) model of heat conduction in the presence of TED. Sun et al. [11] investigated two dimensional hyperbolic heat conduction model when the TED exists with one relaxation time in micro-beam resonators, but the influence of these assumptions in quality factor (Q) of TED (QTED) is undetermined. Rezazadeh et al. [1] studied the effects of applying TDP heat conduction model and one dimensional hyperbolic (ODH) heat conduction model with one relaxation time on the

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QTED for a micro-beam resonator separately. They demonstrated the size-dependency of QTED for various values of thicknesses and lengths and behavior of QTED when the applied bias DC voltage is near the pull-in voltage and they compared QTED with Q of air damping. Khanchehgardan et al. [12] investigated thermo-elastic damping in nano-beam resonators based on nonlocal theory of elasticity and the Euler-Bernoulli beam assumptions. Rozshart [13] indicated experimentally that TED decreases the Q of devices in micro scale. JafarSadeghi-Pournaki et al. [14] investigated the pull-in phenomenon of functionally graded (FG) capacitive nanocantilevers subjected to an electrostatic force and thermal moment due to an applied voltage and thermal shock considering the intermolecular force within the framework of nonlocal elasticity theory to account for the small scale effect. Choi et al. [15] used the model order reduction for a finite element formulation based on the weak form of fully coupled thermoelastic problems. Vahdat and Rezazadeh [16] investigated the effects of residual and axial stresses on TED in capacitive micro-beam resonators and governed coupled thermoelastic equations by applying 2D non-Fourier heat conduction model. They used a Galerkin based finite element formulation to analyze TED for the first mode of vibration with both ends clamped. Yasumura et al. [17] presented the dependence of thermo-mechanical dissipation on cantilever material, geometry, and surface treatments for arrays of silicon–nitride, polysilicon, and single-crystal silico,n also they have studied thermo-mechanical noise effects on the Q. Rezazadeh et al. [18] obtained analytical expressions for Q by applying modified couple stress theory (MCST) for plane stress and plane strain conditions of gold and nickel micro-beam resonators.

Investigations about longitudinal vibrations are few in comparison with transversal vibrations, and these vibrations are quite different [19]. For example, the natural frequencies in transversal vibration are much lower than those of longitudinal vibration [19]; and probably can achieve high Q, thus, the longitudinal vibration of beams is studied. In-plain vibration can occur in transversal vibration of sandwich panels. The Kantorovich–Krylov method was employed by Wang and Wereley to study that [20]. Also longitudinal vibrations occurs when the work piece material is e.g. piezoelectric or magnetostrictive; these type of material strain when an electrical and magnetically field is applied across them and by fluctuating these fields work piece vibrates [21, 22]. Shah-mohammadi-Azar et al. [23] presented the mechanical analysis of a fixed-fixed nano-beam that is sandwiched with two piezoelectric layers based on nonlocal theory of elasticity. Gorman presented an accurate analytical solution for free in-plain vibration (FIV) of completely free rectangular plate and lately for the fully clamped plate by method of superposition [19, 24]. He also used the superposition method to analyze FIV of rectangular plates with elastic support normal to the boundaries [25]. Bardell et al. reached the in-plain frequencies for simply supported, clamped and free plates using the Rayleigh-Ritz method [26]. Kobayashi et al. [27] investigated the in-plain vibration of a rectangular plate with point support and the Ritz method was employed to solve it. Seok et al. [28] performed an analysis of the free in-plain vibration by means of a variational approximation procedure for a cantilever rectangular plate. In-plain free vibration of rectangular plates with in-plain elastic support and completely free was examined by Gutierrez and Laura, they employed an extension of the method used by Mikhlin to achieve the lowest frequency [29]. Exact analytical analysis of free in-plain vibrations with a pair of opposite simply supported boundaries was done by Xing and Liu [30]. Du et al. [31] studied in-plain vibration of plates with classical and uniform elastically restrained edges by developing an analytical Fourier series method. Singh and Muhammad [32], Woodcock et al. [33] and Farag and Pan [34] used the Ritz energy method to study the in-plain vibration of plates. Dozio [35] developed the Ritz method using a set of trigonometric functions to obtain in-plain vibration of rectangular plates with arbitrary non-uniform elastic edge restraints. Andrianov et al. [36] studied free in-plain vibration of rectangular plates using homotopy perturbation approach. Liu and Xing [37] used separation of variable method to study free in-plain vibrations of isotropic and orthotropic rectangular plates. Hyde et al. [38] investigated FIV of rectangular plates through Ritz discretization of the Rayleigh quotient.

Microelectromechanical actuators are used a lot in different systems because of their advantages, such as, low energy consumption, low cost, favorable scaling property, low driving power, relative ease of fabrication, large deflection capacity and etc. [39].

According to our knowledge, TED of the longitudinal vibration of micro-beams is not studied. Therefore, in this paper TED in longitudinal vibration of micro-beams are studied and the Galerkin method is employed to solve it. Some obtained results in special conditions are verified by comparing them with exact solution of free longitudinal vibration of micro-beams.

2. MODEL DESCRIPTION AND PROBLEM FORMULATION

An isotropic thermoelastic micro mechanical beam with both ends clamped initially at a uniform temperature T_0 is studied. A Cartesian coordinate system is employed for the micro-beam, as shown in Figure 1. The origin of the coordinates is placed at the left end of the micro-beam. L, h and b are length, thickness and width of the beam, respectively.

2. 1. Stress and Strain Fields A general strain field results from both mechanical and thermal effects [40, 41]:

$$e_{ij} = e_{ij}^{(M)} + e_{ij}^{(T)} \tag{1}$$

To construct a general three-dimensional constitutive law for linear elastic materials, we assume that each stress component is linearly related to each strain component [40]:

$$\sigma_{ij} = C_{ijkl} e_{kl} \tag{2}$$

where C_{ijkl} is a fourth-order elasticity tensor and its components include all the parameters necessary to characterize the material.
It can be shown that the most general form that satisfies this isotropy condition is given by [40]:

$$C_{ijkl} = \alpha \delta_{ij}\delta_{kl} + \beta \delta_{ik}\delta_{jl} + \gamma \delta_{il}\delta_{jk} \tag{3}$$

where α , β , and γ are arbitrary constants and δ is the Kronecker delta. Using the general form of Equation (3) in the stress-strain relation Equation (2) gives [40]:

$$\sigma_{ij} = \lambda e_{kk}\delta_{ij} + 2\mu e_{ij} \tag{4}$$

in which λ and μ are Lame′′s constant and shear modulus, respectively. Equation (4) can be written out as [40]:

$$e_{ij}^{(M)} = \frac{1+\nu}{E}\sigma_{ij} - \frac{\nu}{E}\sigma_{kk}\delta_{ij} \tag{5}$$

The thermal strains in an unrestrained solid can be written in the linear constitutive form [40, 41]:

$$e_{ij}^{(T)} = \alpha(T - T_0)\delta_{ij} \tag{6}$$

in which α , σ_{ij} , ν and E are the thermal expansion coefficient, stress tensor, Poisson's ratio and Young's modulus, respectively. Combining Equation (5) and Equation (6), gives:

$$e_{ij} = \frac{1+\nu}{E}\sigma_{ij} - \frac{\nu}{E}\sigma_{kk}\delta_{ij} + \alpha(T - T_0)\delta_{ij} \tag{7}$$

The corresponding results for the stress in terms of strain can be written as:

$$\sigma_{ij} = \lambda e_{kk}\delta_{ij} + 2\mu e_{ij} - (3\lambda + 2\mu)\alpha(T - T_0)\delta_{ij} \tag{8}$$

Rewriting Equation (8) in terms of E and ν concludes:

$$\sigma_{ij} = \frac{E}{1+\nu}e_{ij} + \left(\frac{\nu}{1+\nu}\right)\left(\frac{E}{1-2\nu}\right)e_{kk}\delta_{ij} - \left(\frac{E}{1-2\nu}\right)\alpha(T - T_0)\delta_{ij} \tag{9}$$

Figure 1. Schematic view of micro-beam with both ends clamped

2. 2. Equation of Motion Equation of motion [40] is:

$$\sigma_{ij,j} + \rho b_i = \rho a_i \tag{10}$$

where ρ is the mass density, b is a body force and a is an acceleration. Rewriting Equation (10) in terms of displacements gives [40]:

$$\begin{aligned} &\lambda u_{k,ki} + \mu(u_{i,jj} + u_{j,ij}) - (3\lambda + 2\mu)\alpha(T - T_0)_{,i} \\ &+\rho b_i = \rho \ddot{u}_i \end{aligned} \tag{11}$$

By neglecting body forces Equation (11) simplifies to:

$$\begin{aligned} &\lambda u_{k,ki} + \mu(u_{i,jj} + u_{j,ij}) \\ &-(3\lambda + 2\mu)\alpha(T - T_0)_{,i} = \rho \ddot{u}_i \end{aligned} \tag{12}$$

2. 3. Heat Equation Heat conduction equation is:

$$kT_{,ii} = \rho c\dot{T} + (3\lambda + 2\mu)\alpha T_0 \dot{e}_{ii} - \rho h \tag{13}$$

where k and c are the thermal conductivity and the specific heat at a constant pressure, respectively. Rewriting Equation (13) in terms of displacement with no sources ($h = 0$), gives:

$$kT_{,ii} = \rho c\dot{T} + (3\lambda + 2\mu)\alpha T_0 \dot{u}_{i,i} - \rho h \tag{14}$$

So, Equations (12) and (14) are coupled. Simplifying and writing Equations (12) and (14) in dimensionless forms gives:

$$\frac{\partial^2 \hat{u}}{\partial \hat{x}^2} - B_1\frac{\partial \hat{\theta}}{\partial \hat{x}} = \frac{\partial^2 \hat{u}}{\partial \hat{t}^2} \tag{15}$$

$$\frac{\partial^2 \hat{\theta}}{\partial \hat{x}^2} = B_2\frac{\partial \hat{\theta}}{\partial \hat{t}} + B_3\frac{\partial^2 \hat{u}}{\partial \hat{t}\partial \hat{x}} \tag{16}$$

where

$$\theta = T - T_0 \quad B_1 = \frac{\alpha T_0(1+\nu)}{1-2\nu} \quad B_2 = \frac{cl}{k}\sqrt{\frac{\rho E}{(1+\nu)}} \quad B_3 = \frac{\alpha l}{k}\sqrt{\frac{E^3}{\rho(1+\nu)}} \tag{17}$$

and the dimensionless parameters in Equations (15) and (16) are defined as:

$$\hat{u} = \frac{u}{l},\ \hat{x} = \frac{x}{l},\ \hat{\theta} = \frac{\theta}{T_0},\ \hat{t} = \frac{t}{t_0},\ t_0^2 = \frac{\rho l^2(1+\nu)}{E} \tag{18}$$

2. 4. Solving the Governing Equations The Galerkin method is applied to solve Equations (15) and

(16). Thereby, it can be approximated in terms of linear combinations of finite number of suitable shape functions with time dependent coefficients:

$$\hat{u}(\hat{x},\hat{t})=\sum_{n=1}^{N}\phi_n(\hat{x})a_n(\hat{t}) \quad \hat{\theta}(\hat{x},\hat{t})=\sum_{m=1}^{M}\psi_m(\hat{x})b_m(\hat{t}) \tag{19}$$

Suitable shape functions are chosen according to the Galerkin method as follows:

$$\phi_n(\hat{x})=\sin(n\pi\hat{x}) \quad , \quad \psi_m(\hat{x})=\sin(m\pi\hat{x}) \tag{20}$$

these shape functions satisfy the both end clamped boundary conditions of our problem and that is sufficient according to the Galerkin method [42].
Solutions for first term of displacement and second term of thermo are expanded, in which $a_1(\hat{t})$ and $b_2(\hat{t})$ are considered as follow:

$$a_1(\hat{t})=\alpha_1 e^{s\hat{t}} \quad b_2(\hat{t})=\beta_2 e^{s\hat{t}} \tag{21}$$

where Equation (21) is one of the general and common solutions for $a_1(\hat{t})$ and $b_2(\hat{t})$ [42]. Finally the simple form is derived:

$$\begin{bmatrix} -\frac{\pi^2}{2}-\frac{s^2}{2} & \frac{4}{3}B_1 \\ -\frac{4}{3}B_3 s & -2\pi^2-\frac{1}{2}B_2 s \end{bmatrix}\begin{Bmatrix}\alpha_1\\ \beta_2\end{Bmatrix}=\begin{Bmatrix}0\\0\end{Bmatrix} \tag{22}$$

Hence, the natural frequencies of the system can be obtained by solving the following equation:

$$\det\begin{bmatrix} -\frac{\pi^2}{2}-\frac{s^2}{2} & \frac{4}{3}B_1 \\ -\frac{4}{3}B_3 s & -2\pi^2-\frac{1}{2}B_2 s \end{bmatrix}=0 \tag{23}$$

3. NUMERICAL RESULTS

The following micro-beams in Table 1 are investigated to compare the effects of length, thickness, ambient temperature and material on Q [18, 43] .

According to the complex frequency approach, quality factor of thermo-elastic damping (Q_{TED}) can be achieved as [1, 5]:

$$Q_{TED}=\frac{1}{2\zeta}\cong\frac{1}{2}\left|\frac{\mathrm{Re}(\omega)}{\mathrm{Im}(\omega)}\right| \tag{24}$$

The coefficient of linear thermal expansion assumes to be constant in this study [1].

To appear the effect of thermo on longitudinal vibrations, odd modes for displacement and even modes for thermo shall be chosen and vice versa and displacement counters (n , i) shall be equal to each other and thermal counters (m , j) shall be also equal to each other.

Table 2 shows the evaluated complex frequency results for the first fifth modes of displacement and first second thermal modes. It shows that the frequency increases by increasing the displacement's modes as it is expected and there is relation between imaginary part of natural frequency and mode numbers of displacement ($n\pi$, $n=1,2,...$). Negative real part is appeared because of damping mechanism (thermo-elastic damping) to show that responses of the system will damp during the time.

The results of Q are shown as the same as many references such as [1, 2] to show the variations of Q clearly.

Transversal vibrations have been investigated in the literature [11] and these vibrations are quite different in comparison with longitudinal vibrations and the solution methods are not similar.

For micro-beams with mentioned properties in Table 1, the numerically obtained values of Q_{TED} are illustrated for first mode of displacement and second thermal mode in Figure 2 for different values of length at constant ambient temperature ($T_0=300\ k$). As it shows, by increasing the length of micro-beams the quality factor is increased, but it is more important and much greater for SiC and Si than others.

TABLE 1. Material properties of micro-beams [18, 43]

Parameters	Unit	Si	SiC	Poly Silicon	Gold	Nickel
Young's modulus (E)	Gpa	169	400	160	79	210
Poisson's ratio (ν)	---	0.28	0.185	0.22	0.44	0.31
Thermal conductivity (k)	$\frac{w}{mk}$	150	70	148	318	92
Density (ρ)	$\frac{kg}{m^3}$	2300	3200	2330	19320	8900
Specific heat at constant volume (Cv)	$\frac{j}{kgk}$	695	938	107	129	438
Coefficient of linear thermal expansion (α)× 10^{-6}	k^{-1}	2.6	3	4.7	14.21	13

TABLE 2. Complex frequency of Silicon carbide for first fifth mode of displacement and first second mode of thermo

n	m	i	j	Complex frequency (ω)
1	2	1	2	- 0.0000001092 ± 3.142358i
2	1	2	1	- 0.0000000068 ± 6.283568i
3	2	3	2	- 0.0000000393 ± 9.4256052i
4	1	4	1	- 0.0000000003 ± 12.566401i
5	2	5	2	- 0.0000000022 ± 15.708041i

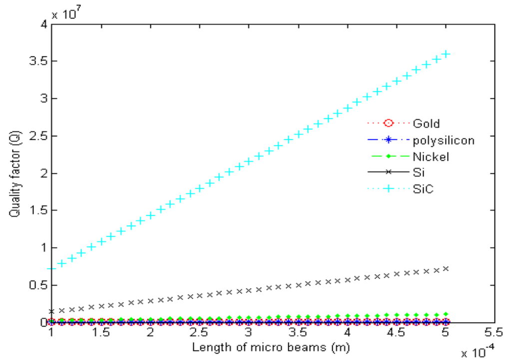

Figure 2. QTED versus length of micro-beams at $T_0 = 300\ k$ for SiC, Si, Nickel, Polysilicon and Gold

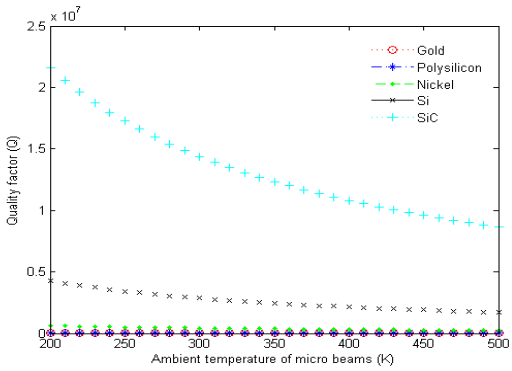

Figure 3. Effect of various ambient temperatures on QTED with $L = 200\ \mu m$ for SiC, Si, Nickel, Polysilicon and Gold

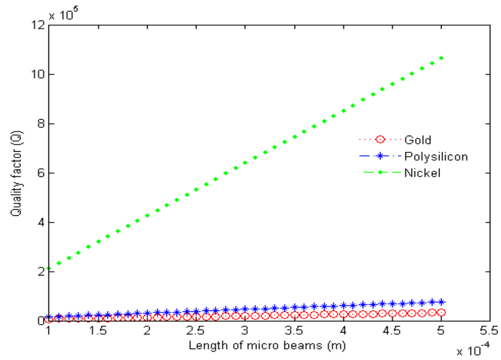

Figure 4. QTED versus length of micro-beams at $T_0 = 300\ k$ for Nickel, Polysilicon and Gold

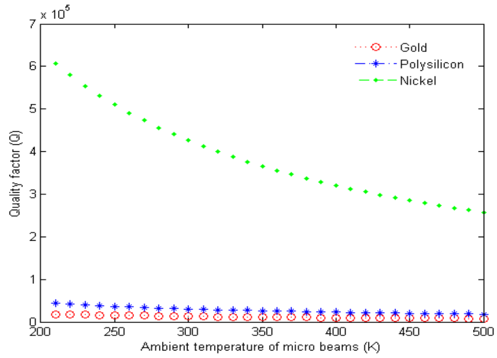

Figure 5. Effect of various ambient temperatures on QTED with $L = 200\ \mu m$ for Nickel, Polysilicon and Gold

TABLE 3. Differences between Q_{TED} in longitudinal and transversal vibrations for silicon micro-beam with L= 200 µm

			T_0 = 300 K	T_0 = 400 K
Q	in this paper	$\times 10^6$	2.861	2.146
Q	[3, 4]	$\times 10^6$	≈ 0.38	≈ 0.291
Q	[5]	$\times 10^6$	≈ 0.39	≈ 0.294
Q	[16]	$\times 10^6$	≈ 0.40	≈ 0.303
Q	[43]	$\times 10^6$	≈ 0.47	≈ 0.350

The obtained values of quality factor with respect to variations of ambient temperature with constant length ($L = 200\ \mu m$) are shown in Figure 3 for all mentioned materials in Table 1, for first mode of displacement and second thermal mode. It shows that by increasing the ambient temperature QTED is decreased.

As the variation rate of the Gold, Polysilicon and Nickel is much lower than the Si and SiCarbide because of their mechanical properties, so Figure 4 and Figure 5 are illustrated to show their variations versus length and ambient temperature much better.

Table 3 shows the differences between the Q_{TED} in longitudinal vibrations in this paper and Q_{TED} in transversal vibrations that represented in the literature [3-5, 16, 43] for same material, length and temperature. By comparing the natural frequencies of this paper with natural frequency of free longitudinal vibrations by ignoring thermo effects [44] a good verification is derived.

4. CONCLUSION

This paper presents thermoelastic damping in longitudinal vibrations of micro-beams. The problem is solved by Galerkin method. The result shows that increment of the ambient temperature of micro-beam decreases the Q_{TED} and increment of the length of micro-beam increases the quality factor. The results show that the quality factor for longitudinal vibration is higher than that of the transverse one. A good verification is derived for the natural frequency of this work in comparison with free vibrations without thermo effects. The data presented in this paper provide useful information for other researchers that work in this field.

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Investigation of Thermoelastic Damping in the Longitudinal Vibration of a Micro Beam

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چکیده

در طراحی فاکتور کیفیت بالا، مکانیزم های اتلافی میکرو و نانو تیرها می توانند تاثیر منفی روی فاکتور کیفیت داشته باشند. یکی از مکانیزم های اتلافی مهم ترموالاستیک دمپینگ می باشد که برای پیش بینی آن نیازمند مطالعات دقیقی هستیم. در این مقاله به بررسی ترموالاستیک دمپینگ ارتعاشات طولی در یک میکرو تیر همگن دو سر گیر دار می پردازیم. برای تحلیل ترموالاستیک دمپینگ در مود ارتعاشی اول میکرو تیر، روش گلرکین مورد استفاده قرار گرفته است. سپس فاکتور کیفیت و فرکانس ارتعاشات طولی بدست آمده است. نحوه تغییرات فاکتور کیفیت نسبت به ابعاد و دمای محیط برای جنس های مختلف ترسیم شده است.

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