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A Balancing and Ranking Method based on Hesitant Fuzzy Sets for Solving Decision-making Problems under Uncertainty

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ABSTRACT

The purpose of this paper is to extend a new balancing and ranking method to handle uncertainty for a multiple attribute analysis under a hesitant fuzzy environment. The presented hesitant fuzzy balancing and ranking (HF-BR) method does not require attributes' weights through the process of multiple attribute decision making (MADM) under hesitant conditions. For the rating of possible alternatives, firstly, they are defined as hesitant fuzzy terms and then converted into hesitant fuzzy sets. Second, an outranking matrix indicates that a possible alternative overcomes the other alternatives regarding to each chosen attribute. Third, the outranking matrix is triangularized which means that we prepare provisional order of possible alternatives or implicit preordering under hesitant conditions. Eventually, the empirical order of alternatives goes through variant operations of balancing and screening that needs continuous application of a balancing axiom to the advantages–disadvantages table. It links incompatible attributes with pair-wise comparisons of the possible alternatives for the multiple attribute analysis. Finally, we present an application example for the supplier selection to show the applicability and feasibility of the proposed HF-BR method in the hesitant fuzzy setting.

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1. INTRODUCTION

Since fuzzy set theory was introduced by Zadeh [1], It has widely used in uncertain situations for solving the problems. These fields can include management [2], artificial intelligence [3], pattern recognition [4] and decision making [5-8]. Decision making is a process that is described as final outcome of decision problems and helps decision makers (DMs) for the selection of suitable alternative or a set of alternatives. In reality, researchers often focus on decision-making problems in uncertain and imprecise situations. The multiple attribute decision making (MADM) has created an efficient frame for the comparison respecting to the assessment of multiple incompatible attributes. In classical evaluation, the MADM is based on crisp approach, but in fuzzy multiple attribute decision making (FMADM) we usually estimate the performance values by using fuzzy terms. However, in real-world applications, the objects can be regarded as hesitant and uncertain values because the DMs' preferences are vague/ hesitant. Thus, the attributes of decision-making problems in some situations can be expressed by fuzzy values [9, 10], such as fuzzy interval-valued [11-14], intuitionistic fuzzy values [15-17], linguistic variables [18, 19], and hesitant fuzzy elements (HFEs) [20-23]. In this respect, Mousavi et al. [9] proposed a hierarchical multi-attribute group decision-making approach under a fuzzy environment for evaluating and ranking the new product ideas. Vahdani and Zandieh [19] solved their fuzzy MCDM problem with linguistic variables which were described as triangular fuzzy numbers. Mousavi et al. [13] considered their decision-making problems under an uncertain environment with interval-valued fuzzy numbers with linguistic variables.

In addition, in some complex situations, the DMs for the margin of error, decreasing the uncertainty and risks

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want to assign their judgments by several membership degrees for an element under a set. Thus, hesitant fuzzy set (HFS) has been first introduced by Torra and Narukawa [24] and Torra [25] as a very useful tool for handling the situations. Torra and Narukawa [24] and Torra [25] have discussed about relationship between the intuitionistic fuzzy set and HFS, and they have showed that the intuitionistic fuzzy set was obtained with envelope of the HFS. For more information about the HFS, Rodriguez et al. [26] presented an overview on HFSs by preparing an obvious perspective on various concepts and tools that were related to this fuzzy set. In this respect, the HFS could be very effective tool in order to avoid this issue. Thus, each attribute can be defined as the HFS and expressed in terms of the DMs' preferences. Also, the characteristic of HFSs has caused more functional for modeling of hesitancy to define the membership degree of an element. The HFS has been received much attention, and it has been successfully implemented in various decision-making fields [27-31]. Moreover, using different aggregation operators can lead to different selections; it is pointed out that Torra [32] discussed about the element selection for type of assessments.

Xia and Xu [29] developed some hesitant fuzzy aggregation operators and described the relationships among them. They also used the properties for solving MADM problems. In this respect, Zhu et al. [28] developed the geometric bonferroni mean, the geonmetric mean, and the normalization method under the hesitant fuzzy environment. In addition, they defined the hesitant fuzzy choquet geometric bonferroni mean and the hesitant fuzzy geometric bonferroni mean. Xu and Xia [33] proposed a distance measure for HFSs and discussed about their applications and relations of them. They suggested an idea based on similarity and distance measures for the MADM problem. Xu and Xia [34] reported a detailed study on distance and correlation measures for HFSs and then discussed about their applications. Xia et al. [30] focused on some other aggregation operators for HFSs and used them on group decision making. Wei [31] developed some models in several priority levels for hesitant fuzzy MADM problems and for hesitant fuzzy information. They extended some prioritized aggregation operators. The HFSs are regarded as a very helpful and effective tool to deal with hesitant and uncertain situations because DMs could express his/her ideas exactly and perfectly. In addition, when the DMs assign same membership values to an element, two membership degrees should be emerged to only once [31]. Also, when we specify that the exact value of attributes is impossible or difficult, the HFS is very helpful tool to deal with this situation. Thus, in this paper we employ hesitant fuzzy information to solve our decision problems.

In this paper, we introduce a new method based on the HFS with balancing and ranking, namely HF-BR, to solve MADM problems under uncertainty. The presented HF-BR method outranks the alternatives versus attributes using a four-stage algorithm. The motivation for using these sets on decision-making method is that; sometimes the DMs define some different membership of an element regarding the rating of alternative versus several conflicting attributes. It is difficult that the membership of an element put into a set and in some situations. These difficulties are caused by a hesitation between a few variant values. Main core of the algorithm is based on the balancing which means that the provisional ordering is further assessed by utilizing the balancing operations regarding to advantages–disadvantages table. For the ordering of pairs for the possible alternatives, this assists in constructing the strict superiority relations. For example, two DMs discuss the membership of x into A, one assign 0.2 and the other 0.5. Hence, the doubt on the possible values is someway limited. In this paper, we review the definition of these sets on the MADM. As mentioned before, the HFSs permit us to have several membership values for a single element x in the reference set X. The empirical order of possible alternatives goes through variant operations of balancing and screening that need continuous application of a balancing axiom to the advantages–disadvantages table. The balancing approach is different from the classical MADM methods regarding the function of prior weights to the conflicting criteria. Furthermore, it prepares a combination of the balancing for the relative advantages and disadvantages of pairs for possible alternatives while respecting to the importance of the attributes or factors concurrently. The balancing problem consists of the comparison for two possible alternatives that are regarded as separate binary decision-making problems versus a set of advantages and disadvantages. These representative comparisons provide the balancing problem properly.

The structure of the paper is as follows; in section 2, some basic concepts and operations are reviewed. In section 3, the proposed HF-BR method under HFSs is illustrated. In section 4, the proposed method is applied to an application example in order to show the verification of the proposed method. Finally, some conclusions and suggestions have been presented in section 5.

2. PRELIMINARIES

In this section, we briefly review some basic notions and operations of the HFSs.

Definition 1. Let X be a universe of discourse, then we define a HFS, E on X in terms of a function $h_E(x)$ as if when we apply to X returns a proper subset of [0, 1]

[29]. Also, we explain the HFS by a mathematical symbol:

$$E = \{< x, h_E(\mathrm{x}) > | \mathrm{x} \in \mathrm{X}\} \tag{1}$$

where, $h_E(\mathrm{x})$ is defined as some possible membership degrees of an element; in other words, this is a set of some values in [0, 1]. Also, for convenience Xia and Xu [29] named $h = h_E(\mathrm{x})$ as hesitant fuzzy element (HFE), and the set of all HFEs is H.

*Definition 2.*Torra and Narukawa [24] purposed the following basic operations for hesitantfuzzy sets, Let *h*, h_1, and h_2 be HFS, then proposed operations are as follows:

- Lower bound

$$h^-(x) = \min\ h(x) \tag{2}$$

- Upper bound

$$h^+(x) = \max\ h(x) \tag{3}$$

- α-upper bound

$$h_\alpha^+(x) = \{h \in h(x) \mid h \geq \alpha\} \tag{4}$$

- α-lower bound

$$h_\alpha^-(x) = \{h \in h(x) \mid h \leq \alpha\} \tag{5}$$

- Complement

$$h^c(x) = \cup_{\gamma \in h(x)} \{1 - \gamma\} \tag{6}$$

- Union

$$(h_1 \cup h_2)(x) = \{h \in (h_1(x) \cup h_2(x)) \mid h \geq \max(h_1^-, h_2^-)\}$$

Equivalently:

$$\tilde{h}_1 \cup \tilde{h}_2 = \cup_{\gamma_1 \in \tilde{h}_1, \gamma_2 \in \tilde{h}_2} \max\{\gamma_1, \gamma_2\} \tag{7}$$

- Intersection

$$(h_1 \cap h_2)(x) = \{h \in (h_1(x) \cap h_2(x)) \mid h \leq \min(h_1^+, h_2^+)\}$$

Equivalently:

$$\tilde{h}_1 \cap \tilde{h}_2 = \cup_{\gamma_1 \in \tilde{h}_1, \gamma_2 \in \tilde{h}_2} \min\{\gamma_1, \gamma_2\} \tag{8}$$

*Definition 3.*Consider a fixed set X, an intuitionistic fuzzy set (IFS), E on X is demonstrated as $E = (\langle x_i, \mu_E(x_i), v_E(x_i)\rangle)$ for $x_i \in X$.

According to each element x_i, $\mu_E(\mathrm{x}_i)$ is an membership degree and $v_E(\mathrm{x}_i)$ is an non-membership degree under the terms of $0 \leq \mu_E(x_i) + v_E(x_i) \leq 1$ for $x_i \in X$ [15, 35, 36]. For convenience, Xu [37] called $(\mu_E(\mathrm{x}_i), v_E(\mathrm{x}_i))$ as intuitionistic fuzzy value (IFV) and the set of all IFVs is V.

*Definition 4.*Let *h* be a hesitant fuzzy set, we define the intuitionistic fuzzy sets $A_{env}(\mathrm{h})$ with the envelope of h as $(\mu(\mathrm{x}) = \mathrm{h}^-, v(\mathrm{x}) = 1 - \mathrm{h}^+)$, according to $h^- = \min\{\gamma \mid \gamma \in \mathrm{h}\}$ and $h^+ = \max\{\gamma \mid \gamma \in \mathrm{h}\}$. Torra and Narukawa [24] described the relationship between HFS and IFS as follows:

$$A_{env}(h^c) = (A_{env}(h))^c \tag{9}$$

$$A_{env}(h_1 \cup h_2) = A_{env}(h_1) \cup A_{env}(h_2) \tag{10}$$

$$A_{env}(h_1 \cap h_2) = A_{env}(h_1) \cap A_{env}(h_2) \tag{11}$$

*Definition 5.*According to relationship between the HFE and IFV, Xia and Xu [29] described some new operations on the HFE as below:

$$\tilde{h}_1 \oplus \tilde{h}_2 = \cup_{\gamma_1 \in \tilde{h}_1, \gamma_2 \in \tilde{h}_2} \{\gamma_1 + \gamma_2 - \gamma_1 . \gamma_2\} \tag{12}$$

$$\tilde{h}_1 \otimes \tilde{h}_2 = \cup_{\gamma_1 \in \tilde{h}_1, \gamma_2 \in \tilde{h}_2} \{\gamma_1 . \gamma_2\} \tag{13}$$

$$h^\lambda = \cup_{\gamma \in h} \{\gamma^\lambda\} \tag{14}$$

$$\lambda h = \cup_{\gamma \in h} \{1 - (1 - \gamma)^\lambda\} \tag{15}$$

Definition 6. Liao and Xu [38] proposed the subtraction and division operations of HFS based on the relationship between the HFS and IFV and subtraction and division operations of the IFS as below:

$$h_1 - h_2 = \cup_{\gamma_1 \in h_1, \gamma_2 \in h_2} \begin{Bmatrix} \frac{\gamma_1 - \gamma_2}{1 - \gamma_2} & if\, \gamma_1 \geq \gamma_2 \ and\ \gamma_2 \neq 1; \\ 0 & otherwise \end{Bmatrix} \tag{16}$$

$$h_1 / h_2 = \cup_{\gamma_1 \in h_1, \gamma_2 \in h_2} \begin{Bmatrix} \frac{\gamma_1}{\gamma_2} & if\, \gamma_1 \leq \gamma_2 \ and\ \gamma_2 \neq 0; \\ 1 & otherwise \end{Bmatrix} \tag{17}$$

Definition 7. Consider m possible alternatives as $A_1, A_2, ..., A_m$ and decision makers can choose n criteria as $C_1, C_2, ..., C_n$. x_{ij} is the membership degree A_i with attention to criterion C_j and it is not determined exactly, only we know $x_{ij} \in [x_{ij}^l, x_{ij}^u]$. The normalized hesitant fuzzy values (η_{ij}^l and η_{ij}^u) for $i = 1, ..., m$ and $j = 1, ..., n$ can be calculated based on [39] as follows:

$$\eta_{ij}^l = \frac{x_{ij}^l}{\sqrt{\sum_{i=1}^m [(x_{ij}^l)^2 + (x_{ij}^u)^2]}} \tag{18}$$

$$\eta_{ij}^{u} = \frac{x_{ij}^{u}}{\sqrt{\sum_{i=1}^{m}[(x_{ij}^{l})^2 + (x_{ij}^{u})^2]}} \quad (19)$$

where, the interval $[\eta_{ij}^{l}, \eta_{ij}^{u}]$ is normalized from $[x_{ij}^{l}, x_{ij}^{u}]$.

3. PROPOSED NEW HESITANT FUZZY BALANCING AND RANKING METHOD

In this section, we present a new balancing and ranking of decision-making process for the multi-attributes analysis with hesitant fuzzy setting, namely HF-BR. Hesitant fuzzy terms are used for the ratings of possible alternatives by the DMs under incompatible attributes. Hence, the proposed hesitant fuzzy stepwise ordering method denotes a transitive overall to provide the order of a finite set of possible alternatives. Notably that all ranking methods have advantages and disadvantages; therefore, when a decision-making method improves in some property, it usually looses another one. The proposed method does not have the defect of other classical fuzzy MADM methods from a viewpoint of weights for conflicting attributes. It means that lack of information or shortage of information to ascertain the weights of effective attributes to come up with the outranking of alternatives can effect on the final ranking. In this regard, the proposed method does not require the attributes' weights through the process of the MADM under hesitant conditions. Strassert and Prato [40] first introduced the balancing and ranking method for solving a decision-making problem. The hesitant fuzzy MADM problem is solved by using a four-step method, called the HF-BR method in this paper. First, we evaluate the performance of possible alternatives via hesitant fuzzy terms which are described as hesitant fuzzy sets. Second, we show the frequency of each possible alternative that is dominated to all other alternatives according to each attribute. Third, for achieving an implicit provisional order or pre-ordering of possible alternatives, the outranking matrix is triangularised. Fourth, we obtain advantages–disadvantages table that combines the attribute with the pair-wise comparisons of possible alternatives.

As we review the related literature and by considering advantages of the HFS tool, we point out that the DMs could express their opinions and assign several membership degrees for each alternative with respect to the selected attributes. In this paper, the DMs utilize hesitant fuzzy terms for the rating of performance values. The concept of hesitant fuzzy terms is very helpful in ill-defined and hesitant situations. These hesitant fuzzy terms can be transformed into interval-valued hesitant fuzzy sets (IVHFSs) as provided in Table 1. The IVHFSs have been introduced by Chen et al. [41]. Notebly, there are similar recommended tables for coventional decision-making process in the literature [42-45].

3. 1. Data Table, Outranking Matrix and Provisional Order of Possible Alternatives

To implement a new version of MADM with hesitant fuzzy setting, namely HF-BR method under uncertainty, the main steps are described as follows:

1. The implementation of possible alternatives is defined by hesitant fuzzy terms which are represented as hesitant fuzzy sets. Then, hesitant fuzzy sets are normalized.
2. An outranking matrix is defined to show the frequency of alternative that dominates among all other alternatives against each attribute.
3. To determine a provisional order of possible alternatives or implicit pre-ordering, the outranking matrix can become triangularized.
4. The provisional order of the possible alternatives is achieved by several operations of the balancing and screening. It also needs sequential application of the balancing principle to be defined as advantages–disadvantages table that incorporates the attributes with the pair-wise comparisons of possible alternatives under a hesitant fuzzy environment.

3. 2. Advantages-disadvantages Table

The advantages–disadvantages table is defined with the pair-wise comparison of possible alternatives. The head row of the table consists of the votes for the outranking matrix. In fact, the number of advantages should equal to the number of positive votes. In addition, the number of disadvantages should equal to the number of negative votes. The head row consists of all possible pairs of possible alternatives. If we have m alternatives, the maximum number of pairs is $z = \frac{m(m-1)}{2}$.

The pair-wise comparisons are created with respect to quantities, i.e., on a cardinal scale. For example, S_1 has comparative advantage related to S_4 since S_4 is inferiorto S_1 with respect to the first attribute (C_1).

TABLE 1. Linguistic variables expressed by the HFS.

Linguistic terms	Interval-valued hesitant fuzzy set
Very high (VH)	[0.8,0.9]
High (H)	[0.7,0.8]
Moderately high (MH)	[0.6,0.7]
Fair (F)	[0.5,0.6]
Moderately low (ML)	[0.4,0.5]
Low (L)	[0.25,0.4]
Very low (VL)	[0.1,0.25]

As a result, it is denominated as ${}_{1/4}A_1$. The table involves the votes of outranking matrix explained how the quasi votes are divided by attributes or equivalently, the attribute relies on advantages and disadvantages.

3. 3. Triangularization of the Outranking Matrix Triangularization of the outranking matrix is defined to specify a new order of the possible alternatives. The triangular matrix out of a set of $P = j!$ orders, reorders the j possible alternatives in the matrix of the final order, the sum of the values above the main diagonal is a maximum. In the triangular matrix, it will be only zero below the main diagonal, a situation which is mentioned as the total order structure. Generally, the order of possible alternatives, mentioned by the outranking matrix, is not the final entire order of alternatives. The degree for the linearity in a triangularized matrix can be calculated by λ as follows:

$$\lambda = \frac{\sum_{j<k} r_{jk}}{\sum_{j\neq k} r_{jk}}, \qquad 0.5 \le \lambda \le 1 \tag{18}$$

λ represents how much an order of possible alternatives digresses from the ideal case of $\lambda = 1$, which denotes a strong linear order, say, $A-C$, in which the transmissibility situation uses (if $A > B$ and $B > C$, then $A > C$). In the worst case, there is not a linear order, and $\lambda = 0.5$, but a cycle, say, $A > B > C > A$, and contrariwise [19, 46]. This assists the ordering of pairs of the possible alternatives in establishing strict superiority relations.

3. 4. Balancing Problem Each comparison of two possible alternatives in advantages–disadvantages table is illustrated a separate binary decision problem. It is called a balancing problem that includes the comparison of two possible alternatives by regarding a set of advantages and disadvantages. The binary problem is solved with attention to the advantages and disadvantages of possible alternatives. They are further reordered. By taking into account the overall ordering of possible alternatives, a final solution is achieved when this conversion is completed. Providing a maximum number of transitivity implications triangularization is the principal objective, when the $(m-1)$ pairs of possible alternatives alongside and above the diagonal are determined. For instance, if the pair-wise comparisons alongside and above the diagonal, S_1/S_2, S_2/S_3, S_3/S_4 and S_4/S_5, are determined, six remaining pair-wise comparisons S_1/S_3, S_1/S_4, S_1/S_5, S_2/S_4, S_2/S_5 and S_3/S_5 are reported. Such implicative comparisons are provided as below:

$S_1 > S_2$ and $S_2 > S_3 \rightarrow S_1 > S_3$
$S_1 > S_3$ and $S_3 > S_4 \rightarrow S_1 > S_4$
$S_1 > S_4$ and $S_4 > S_5 \rightarrow S_1 > S_5$
$S_2 > S_3$ and $S_3 > S_4 \rightarrow S_2 > S_4$
$S_2 > S_4$ and $S_4 > S_4 \rightarrow S_2 > S_5$
$S_3 > S_4$ and $S_4 > S_5 \rightarrow S_3 > S_5$

These implicative comparisons can comfort the balancing problems. In the best case, explained above, where all pair-wise comparisons alongside and above the diagonal (S_1/S_2, S_2/S_3, S_3/S_4 and S_4/S_5) are encompassed which skip out four balancing problems solved previously.

3. 5. Role of Judgment The balancing approach outperforms the classical MADM methods for allocating the prior weights to the incompatible attributes. In addition, it allows a combination of the balancing of the respective advantages and disadvantages of pairs for the possible alternatives while considering the variant significance of the attributes. Regarding the advantages-disadvantages table works at the factual level due to comprising each pair between the possible alternatives, no other qualitative relations are defined compared with the factual relations.

3. 6. Final Ordering of the Possible Alternatives The final ordering of the possible alternatives which are incompatible with the superiority relations is obtained by the sequential elimination from the complete counting of orders. In our deciosion problem, the number of possible orders will be $p = j!$.

4. APPLICATION EXAMPLE

In this section, an application example is presented from the recent literature [19] to illustrate the proposed HF-BR method for decision-making problems under the hesitant fuzzy environment. In this application example, 5 possible alternatives or suppliers are compared against 5 incompatible attributes that are described as follows:

1) Profitability of supplier (c_1);
2) Relationship closeness (c_2);
3) Technological capability (c_3);
4) Conformance quality (c_4); and
5) Conflict resolution (c_5).

4. 1. Data Table and Outranking Matrix The DMs or experts use the hesitant fuzzy terms, defined in Table 1, to appraise the ratings of possible alternatives against each selected attribute for the decision-making problem. The ratings of the five possible alternatives by the DMs regarding to the selected attributes are reported in Table 2. The hesitant fuzzy appraisement, explained in Table 2, is transformed into hesitant fuzzy sets to construct the hesitant fuzzy sets decision matrix. These results are reported in Table 3. Then, the hesitant fuzzy normalized decision matrix is established by regarding definition 7 and Eqs. (18)-(19). The related results have been given in Table 4.

4. 2. Advantages-disadvantages Table In our application example, $z=10$. The pair-wise comparisons are created respecting to the quantities, i.e., on a cardinal scale. Table 5 including the votes of outranking matrix explains how the quasi votes are split by the attributes or equivalently, the attribute depends on advantages and disadvantages. Also, the advantages and disadvantages are defined as A_i and D_i $(i=1,2,...,n)$.

TABLE 2. Ratings of five possible alternatives by DMs against the selected attributes

Attributes	Feature	Alternatives				
		S_1	S_2	S_3	S_4	S_5
C_1	Profitability of supplier	MH	ML	F	L	H
C_2	Relationship Closeness	H	F	MH	ML	L
C_3	Technological Capability	F	MH	H	L	ML
C_4	Conformance Quality	VH	H	MH	ML	F
C_5	Conflict resolution	MH	ML	F	H	L

MH moderately high, *ML* moderately low, *F* fair, *L* low, *H* high, *VH* very high.

TABLE 3. Hesitant fuzzy sets decision matrix of five possible alternatives.

Attributes	Features	Alternatives				
		S_1	S_2	S_3	S_4	S_5
C_1	Profitability of supplier	[0.6,0.7]	[0.4,0.5]	[0.5,0.6]	[0.25,0.4]	[0.7,0.8]
C_2	Relationship Closeness	[0.7,0.8]	[0.5,0.6]	[0.6,0.7]	[0.4,0.5]	[0.25,0.4]
C_3	Technological Capability	[0.5,0.6]	[0.6,0.7]	[0.7,0.8]	[0.25,0.4]	[0.4,0.5]
C_4	Conformance Quality	[0.8,0.9]	[0.7,0.8]	[0.6,0.7]	[0.4,0.5]	[0.5,0.6]
C_5	Conflict resolution	[0.6,0.7]	[0.4,0.5]	[0.5,0.6]	[0.7,0.8]	[0.25,0.4]

TABLE 4. Hesitant fuzzy normalized decision matrix

Attributes	Alternatives				
	S_1	S_2	S_3	S_4	S_5
C_1	[0.271,0.316]	[0.216,0.270]	[0.248,0.298]	[0.161,0.258]	[0.434,0.496]
C_2	[0.316,0.361]	[0.270,0.324]	[0.298,0.348]	[0.258,0.323]	[0.155,0.248]
C_3	[0.226,0.271]	[0.324,0.379]	[0.348,0.397]	[0.161,0.258]	[0.248,0.310]
C_4	[0.361,0.407]	[0.379,0.433]	[0.298,0.348]	[0.258,0.323]	[0.310,0.372]
C_5	[0.271,0.316]	[0.216,0.270]	[0.248,0.298]	[0.452,0.517]	[0.155,0.248]

TABLE 5. Advantages–disadvantages table for ten pairs of potential alternatives and selected attributes

Attributes	C_1	C_2	C_3	C_4	C_5	$\sum A_i$	$\sum D_i$
S_1/S_2	$_{1/2}A_1$	$_{1/2}A_2$	$_{1/2}D_3$	$_{1/2}A_4$	$_{1/2}A_5$	4	1
S_1/S_3	$_{1/3}A_1$	$_{1/3}A_2$	$_{1/3}D_3$	$_{1/3}A_4$	$_{1/3}A_5$	4	1
S_1/S_4	$_{1/4}A_1$	$_{1/4}A_2$	$_{1/4}A_3$	$_{1/4}A_4$	$_{1/4}D_5$	4	1
S_1/S_5	$_{1/5}D_1$	$_{1/5}A_2$	$_{1/5}A_3$	$_{1/5}A_4$	$_{1/5}A_5$	4	1
S_2/S_3	$_{2/3}D_1$	$_{2/3}D_2$	$_{2/3}D_3$	$_{2/3}A_4$	$_{2/3}D_5$	1	4
S_2/S_4	$_{2/4}A_1$	$_{2/4}A_2$	$_{2/4}A_3$	$_{2/4}A_4$	$_{2/4}D_5$	4	1
S_2/S_5	$_{2/5}D_1$	$_{2/5}A_2$	$_{2/5}A_3$	$_{2/5}A_4$	$_{2/5}A_5$	4	1
S_3/S_4	$_{3/4}A_1$	$_{3/4}A_2$	$_{3/4}A_3$	$_{3/4}A_4$	$_{3/4}D_5$	4	1
S_3/S_5	$_{3/5}D_1$	$_{3/5}A_2$	$_{3/5}A_3$	$_{3/5}A_4$	$_{3/5}A_5$	4	1
S_4/S_5	$_{4/5}D_1$	$_{4/5}A_2$	$_{4/5}D_3$	$_{4/5}D_4$	$_{4/5}A_5$	2	3

4. 3. Triangularization of the Outranking Matrix

The triangularization of the outranking matrix is executed in order to achieve a new order of the possible alternatives. The consequent triangular outranking matrix is demonstrated as R^T that is explained in Table 6. The triangular matrix reorders the j alternatives systematically so that, out of a set of $P=j!$ orders (in our application example $P=5!=120$), in the matrix of the final order the sum of the values above the main diagonal is a maximum. The linearity degree of the matrix defined in Table 7 is 0.78. The performance orders of the five possible alternatives versus each selected attribute based on Table 4 are described as follows:

$C_1 : S_5 > S_1 > S_3 > S_2 > S_4$
$C_2 : S_1 > S_3 > S_2 > S_4 > S_5$
$C_3 : S_3 > S_2 > S_1 > S_5 > S_4$
$C_4 : S_1 > S_2 > S_3 > S_5 > S_4$
$C_5 : S_4 > S_1 > S_3 > S_2 > S_5$

4. 4. Balancing of the Problem

The balancing problem contains the comparison of two possible alternatives with according to a set of advantages and disadvantages. As we provide in the first column of Table 6, S_1 / S_2 mentioned a separate binary decision making problem including four advantages and one disadvantage. This denotes that S_1 have an advantage compared to S_2. Next, the binary problem is solved with according to the advantages and disadvantages of possible alternatives, and they are further reordered. The triangular outranking matrix define in Table 7 and represent the following provisional ordering of the possible alternatives as: $S_1 > S_3 > S_2 > S_5 > S_4$. Thus, the corresponding comparisons are represented as follows:

$S_1 > S_2$ and $S_2 > S_3 \rightarrow S_1 > S_3$
$S_1 > S_3$ and $S_3 > S_4 \rightarrow S_1 > S_4$
$S_1 > S_4$ and $S_4 > S_5 \rightarrow S_1 > S_5$
$S_2 > S_3$ and $S_3 > S_4 \rightarrow S_2 > S_4$
$S_2 > S_4$ and $S_4 > S_4 \rightarrow S_2 > S_5$
$S_3 > S_4$ and $S_4 > S_5 \rightarrow S_3 > S_5$

These indicative comparisons show the prior balancing problems. In the best status explained above, where all pair-wise comparisons alongside and above the diagonal ($S_1/S_2, S_2/S_3, S_3/S_4$ and S_4/S_5) are approved, skipped out four balancing problems, and then solved as demonstrated in Table 7.

4. 5. Role of Judgment

In our application example, the final order of the possible alternatives for the decision-making problem is achieved by respecting to 10 balancing problems, specified in Table 8.

4. 6. Final Ordering of the Possible Alternatives

In the application example, the number of possible orders is $p = j!$ and j is 5, then $P = 5! = 120$. According to 120 orders, 60 orders having S_1 before S_2 are omitted as S_1 explained a strict superiority over S_2.

TABLE 6. Outranking matrix (R)

Alternatives	S_1	S_2	S_3	S_4	S_5
S_1	-	4	4	4	4
S_2	1	-	1	4	4
S_3	1	4	-	4	4
S_4	1	1	1	-	2
S_5	1	1	1	3	-

TABLE 7. Triangular outranking matrix (R^T)

Alternatives	S_1	S_3	S_2	S_5	S_4
S_1	-	4	4	4	4
S_3	1	-	4	4	4
S_2	1	1	-	4	4
S_5	1	1	1	-	3
S_4	1	1	1	2	-

TABLE 8. Final triangular outranking matrix and final order of five potential alternatives

Alternatives	S_1	S_3	S_2	S_5	S_4
Triangular outranking matrix					
S_1	-	4	4	4	4
S_3	1	-	4	4	4
S_2	1	1	-	4	4
S_5	1	1	1	-	3
S_4	1	1	1	2	-
First provisional triangular outranking matrix					
S_1	-	5	4	4	4
S_3	0	-	4	4	4
S_2	1	1	-	4	4
S_5	1	1	1	-	3
S_4	1	1	1	2	-
Second provisional triangular outranking matrix					
S_1	-	5	5	4	4
S_3	0	-	5	4	4
S_2	0	0	-	4	4
S_5	1	1	1	-	3
S_4	1	1	1	2	-
Third triangular outranking matrix					
S_1	-	5	5	5	4
S_3	0	-	5	5	4
S_2	0	0	-	5	4
S_5	0	0	0	-	3
S_4	1	1	1	2	-
Final triangular outranking matrix					
S_1	-	5	5	5	5
S_3	0	-	5	5	5
S_2	0	0	-	5	5
S_5	0	0	0	-	5
S_4	0	0	0	0	-

If pair-wise comparisons alongside and above the diagonal, S_1 / S_3, S_1 / S_4, S_1 / S_5 and S_2 / S_4 are as supposed, then a stepwise decrease of the residual 60 orders becomes possible. Eventually, the overall order of the possible alternatives based on their performances is $S_1 > S_3 > S_2 > S_5 > S_4$.

5. CONCLUDING REMARKS

This paper proposed a new hesitant fuzzy balancing and ranking (HF-BR) method for decision-making process with the hesitant fuzzy sets (HFSs) to solve the decision-making problems under imprecise and uncertain situations. The proposed HF-BR method can help the experts or decision makers (DMs) to evaluate the possible alternatives versus multiple incompatible attributes in the real-life engineering and management fields. The HF-BR can deal with hesitant conditions. Since specified weights of the selected incompatible attributes is a difficult and required time task, the HF-BR method, without weights of the attributes can be used to solve the complex decision-making problems. The procedure has outranked the possible alternatives with respect to the attributes that utilized four steps decision-making process for the multi-attributes analysis. First, the performance of possible alternatives has been evaluated by using hesitant fuzzy terms which have been expressed as the HFSs. Then, the HFSs have been normalized. Second, an outranking matrix has been defined, mentioned the frequency with which one possible alternative dominated all other possible alternatives respecting to each selected attribute. Third, the outranking matrix has been triangularized to represent an implicit provisional order or pre-ordering of the possible alternatives. Fourth, the provisional order of possible alternatives has been reported by different operations of balancing and screening. Finally, an application example has been proved and validated the process of proposed hesitant fuzzy decision-making. The main advantage of the HF-BR method is that in the proposed method there is no requirement for determining the weight of the attributes. Also, it utilizes hesitant fuzzy terms convertible to the HFSs for evaluating possible alternatives and selected attributes, considering the weights of attributes in other MADM methods which highly effected on the ranking result of alternatives. Afterwards, taking account of the HFSs in the proposed HF-BR method appropriately demonstrates the imprecise or hesitant information. These HFSs are more capable than classical fuzzy methods that help the DMs to confirm that the recommended hesitant statement is adequately obvious in the conditions. For future research, developing a new compromise ranking is suggested to enhance the decision-making process for the chosen problems under hesitant environments.

6. REFERENCES

1. Zadeh, L.A., "Fuzzy sets", ***Information and control***, Vol. 8, (1965), 338-353.
2. Doria, S., "Characterization of a coherent upper conditional prevision as the Choquet integral with respect to its associated Hausdorff outer measure", ***Annals of Operations Research***, Vol. 195, (2012), 33-48.
3. Greco, S., Matarazzo, B., and Giove, S., "The Choquet integral with respect to a level dependent capacity", Fuzzy sets and Systems, Vol. 175, (2011), 1-35.
4. Demirel, T., Demirel, N.C., and Kahraman, C., "Multi-criteria warehouse location selection using Choquet integral", Expert Systems with Applications, Vol. 37, (2010), 3943-3952.
5. Qin, J. and Liu, X., "Study on Interval Intuitionistic Fuzzy Multi-Attribute Group Decision Making Method based on Choquet Integral", ***Procedia Computer Science***, Vol. 17, (2013), 465-472.
6. Bonetti, A., Bortot, S., Fedrizzi, M., Marques Pereira, R. A., and Molinari, A. , "Modelling group processes and effort estimation in project management using the Choquet integral: An MCDM approach", Expert Systems with Applications, Vol. 39, No. 18, (2012), 13366-13375.
7. Samantra, C., Datta, S., and SankarMahapatra, S., "Risk Assessment in IT Outsourcing using Fuzzy Decision-Making Approach: An Indian Perspective", Expert Systems with Applications, Vol. 41, (2014), 4010–4022.
8. Wang, J.Q., et al., "Multi-criteria outranking approach with hesitant fuzzy sets", ***OR Spectrum***, Vol. 36, (2014), 1001-1019.
9. Mousavi, S.M., Torabi, S.A., and Tavakkoli-Moghaddam, R., "A hierarchical group decision-making approach for new product selection in a fuzzy environment". ***Arabian Journal for Science and Engineering***, Vol. 38, No. 11, (2013), 3233-3248.
10. Mousavi, S.M., Jolai, F., and Tavakkoli-Moghaddam, R., "A fuzzy stochastic multi-attribute group decision-making approach for selection problems", ***Group Decision and Negotiation***, Vol. 22, No. 2, (2013), 207-233.
11. Vahdani, B., Tavakkoli-Moghaddam, R., Mousavi, S. M., and Ghodratnama, A. "Soft computing based on new interval-valued fuzzy modified multi-criteria decision-making method", ***Applied Soft Computing***, Vol. 13, No. 1, (2013), 165-172.
12. Cao, Q.-w. and Wu, J., "The extended COWG operators and their application to multiple attributive group decision making problems with interval numbers", ***Applied Mathematical Modelling,*** Vol. 35, No. 5, (2011), 2075-2086.
13. Vahdani, B., Mousavi, S.M., Tavakkoli-Moghaddam, R., and Hashemi, H., "A new design of the elimination and choice translating reality method for multi-criteria group decision-making in an intuitionistic fuzzy environment". ***Applied Mathematical Modelling***, Vol. 37, No. 4, (2013), 1781-1799.
14. Yue, Z., "An extended TOPSIS for determining weights of decision makers with interval numbers", Knowledge-Based Systems, Vol. 24, (2011), 146-153.
15. Atanassov, K.T., "Intuitionistic fuzzy sets", ***Fuzzy sets and Systems***, Vol. 20, No. 1, (1986), 87-96.
16. Hashemi, H., Bazargan, J., and Mousavi, S.M., "A Compromise Ratio Method with an Application to Water Resources Management: An Intuitionistic Fuzzy Set", ***Water Resources Management***, Vol. 27, (2013), 2029-2051.

17. Xu, Z., "Intuitionistic preference relations and their application in group decision making", ***Information Sciences***, Vol. 177, (2007), 2363-2379.

18. Parreiras, R. O., Ekel, P. Y., Martini, J. S. C., and Palhares, R. M., "A flexible consensus scheme for multicriteria group decision making under linguistic assessments", ***Information Sciences***, Vol. 180, No. 7, (2010), 1075-1089.

19. Vahdani, B., and Zandieh, M., "Selecting suppliers using a new fuzzy multiple criteria decision model: the fuzzy balancing and ranking method",***International Journal of Production Research,*** Vol. 48, (2010), 5307-5326.

20. Yu, D., Wu, Y., and Zhou, W., "Multi-Criteria Decision Making Based on Choquet Integral Under Hesitant Fuzzy Environment", ***Journal of Computational Information Systems,*** Vol. 7, (2011), 4506-4513.

21. Zhang, N. and Wei, G., "Extension of VIKOR method for decision making problem based on hesitant fuzzy set". ***Applied Mathematical Modelling,*** Vol. 37, (2013), 4938–4947.

22. Liao, H. and Xu, Z., "A VIKOR-based method for hesitant fuzzy multi-criteria decision making", ***Fuzzy Optimization and Decision Making***, Vol. 12, (2013), 373-392.

23. Xu, Z. and Zhang, X., "Hesitant fuzzy multi-attribute decision making based on TOPSIS with incomplete weight information", Knowledge-Based Systems, Vol. 52, (2013), 53-64.

24. Torra, V. and Narukawa, Y. On hesitant fuzzy sets and decision. FUZZ-IEEE. IEEE International Conference on Fuzzy Systems,(2009)

25. Torra, V., "Hesitant fuzzy sets", ***International Journal of Intelligent Systems***, Vol. 25, No. 6, (2010), 529-539.

26. Rodríguez, R. M., Martínez, L., Torra, V., Xu, Z. S., and Herrera, F., Hesitant Fuzzy Sets: State of the Art and Future Directions, ***International Journal of Intelligent Systems***, Vol. 29, No. 6, (2014), 495-524.

27. Rodriguez, R.M., Martinez, L., and Herrera, F., "Hesitant fuzzy linguistic term sets for decision making", ***IEEE Transactions on Fuzzy Systems***, Vol. 20, (2012), 109-119.

28. Zhu, B., Xu, Z., and Xia, M., "Hesitant fuzzy geometric Bonferroni means",***Information Sciences,*** Vol. 205, (2012), 72-85.

29. Xia, M. and Xu Z., "Hesitant fuzzy information aggregation in decision making", ***International Journal of Approximate Reasoning***, Vol. 52, (2011), 395-407.

30. Xia, M., Xu, Z., and Chen, N., "Some hesitant fuzzy aggregation operators with their application in group decision making", ***Group Decision and Negotiation,*** Vol. 22, No. 2, (2013), 259-279.

31. Wei, G., Hesitant fuzzy prioritized operators and their application to multiple attribute decision making. Knowledge-Based Systems, Vol. 31, (2012),176-182.

32. Torra, V., "Element selection for intuitionistic fuzzy sets", ***International Journal of Knowledge Engineering and Soft Data Paradigms***, Vol. 2, No. 2, (2010), 160-168.

33. Xu, Z. and Xia, M., "Distance and similarity measures for hesitant fuzzy sets", *Information Sciences*, Vol. 181, No. 11, (2011), 2128-2138.

34. Xu, Z. and Xia, M., "On distance and correlation measures of hesitant fuzzy information", ***International Journal of Intelligent Systems***, Vol. 26, No. 5, (2011), 410-425.

35. Atanassov, K.T., "More on intuitionistic fuzzy sets", ***Fuzzy sets and Systems***, Vol. 33, (1989), 37-45.

36. Atanassov, K.T., "Two theorems for intuitionistic fuzzy sets", ***Fuzzy sets and Systems***, Vol. 110, No. 2, (2000), 267-269.

37. Xu, Z., "Intuitionistic fuzzy aggregation operators", ***IEEE Transactions on Fuzzy Systems,*** Vol. 15, No. (6), (2007), 1179-1187.

38. Liao, H. and Xu, Z., "Subtraction and division operations over hesitant fuzzy sets", ***Journal of Intelligent and Fuzzy Systems,*** Vol. 27, (2014), 65-72.

39. Jahanshahloo, G.R., Hosseinzadeh Lotfi, F., and Davoodi, A., "Extension of TOPSIS for decision-making problems with interval data: Interval efficiency", ***Mathematical and Computer Modelling***, Vol. 49, (2009), 1137-1142.

40. Strassert, G. and Prato, T., "Selecting farming systems using a new multiple criteria decision model: the balancing and ranking method", ***Ecological Economics***, Vol. 40, No. 2, (2002), 269-277.

41. Chen, N., Xu Z., and Xia M., "Interval-valued hesitant preference relations and their applications to group decision making", ***Knowledge-Based Systems***, Vol. 37, (2013), 528-540.

42. Tavakkoli-Moghaddam, R., Mousavi, S.M., and Heydar, M., "An integrated AHP-VIKOR methodology for plant location selection", ***International Journal of Engineering Transactions B: Applications***, Vol. 24, No. 2, (2011), 127-137.

43. Mousavi, S.M., Makui, A., Raissi, S., and Mojtahedi, S.M.H., "A multi-criteria decision-making approach with interval numbers for evaluating project risk responses", ***International Journal of Engineering Transactions B: Applications***, Vol. 25, No. 2, (2012), 117-124.

44. Kaboli, A., Aryanezhad, M.B., and Shahanaghi, K., Tavakkoli-Moghaddam, R., "A holistic approach based on mcdm for solving location problems", ***International Journal of Engineering Transactions A: Basics***, Vol. 20, No. 3, (2007), 251-262.

45. Vahdani, B., Salimi, M., and Mousavi, S.M., "A new compromise decision-making model based on TOPSIS and VIKOR for solving multi-objective large-scale programming problems with a block angular structure under uncertainty", ***International Journal of Engineering Transactions B: Applications***, Vol. 27, No. 11, (2014), 1673-1680.

46. Żak, J., "The methodology of multiple-criteria decision making in the optimization of an urban transportation system: case study of Poznań City in Poland", ***International Transactions in Operational Research***, Vol. 6, No. 6, (1999), 571-590.

A Balancing and Ranking Method based on Hesitant Fuzzy Sets for Solving Decision-Making Problems under Uncertainty

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چکیده

هدف از این مقاله، توسعه‌ی یک روش جدید متعادل‌سازی و رتبه بندی برای تجزیه و تحلیل چند معیاره در محیط فازی تردیدی است. روش ارائه شده نیازمند وزن معیارها در فرآیند تصمیم‌گیری چند معیاره در محیط تردیدی نیست. برای ارزیابی گزینه های ممکن، ابتدا آنها با متغیرهای زبانی تعریف می شوند و سپس به مجموعه‌ی فازی تردیدی تبدیل می-گردند. همچنین با تشکیل ماتریس برتری نشان داده می شود که کدام گزینه بر دیگر گزینه ها با توجه به معیارها برتری دارد. سپس ماتریس برتری مثلثی می‌گردد؛ این بدان معناست که گزینه های ممکن به طور موقت مرتب شده و یا به عبارت دیگر به طور تلویحی در شرایط تردیدی از قبل مرتب‌سازی می شوند. در نهایت، این رتبه بندی تجربی گزینه ها باید در عملیات‌های متفاوت متعادل‌سازی و غربال‌گری روش مذکور قرار گیرد که از اینرو نیازمند یک برنامه‌ی مستمر از اصل متعادل‌سازی بنام جدول مزایا–معایب است. این جدول، با مقایسه ی دوبه‌دو گزینه‌های ممکن با توجه به معیارها، میزان برتری آنها را می‌سنجد. در آخر با استفاده از یک مطالعه‌ی کاربردی در زمینه‌ی انتخاب تامین‌کنندگان، کاربردپذیری و امکانپذیر بودن روش پیشنهادی در شرایط فازی تردیدی نشان داده می شود

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