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Outsourcing through Three-dimensional Competition

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ABSTRACT

In this paper, we study an outsourced supply chain consisting of one buyer and two suppliers in which the buyer outsources manufacturing of a physical product to two competing suppliers. The suppliers compete for the buyers' demands share, and the buyer allocates the demands to the competing suppliers based on three-dimensional allocation functions. We consider two certain types of allocation functions which depend on price, service level and product quality level. They include the exponential allocation function and the Cobb-Douglas allocation function. A three-stage game-theoretic framework is presented to derive the equilibrium values. Since the problem lacks a closed-form solution, numerical studies are conducted over a wide range of some key parameters.

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1. INTRODUCTION

A supply chain consists of suppliers, manufacturers, distributors, retailers and end customers, cooperating to meet the customers' needs. Over the past several decades, outsourcing of inputs has become a major business phenomenon in industries and has played an essential role in supply chains [1-3]. Outsourcing can be stated as "the allocation of business activities from a source internal to a source outside of the organization" [4]. The main aim of outsourcing is to achieve lower costs and higher operational efficiency rates. In fact, nowadays, in order to make supply chains more effective, outsourcing cannot be ignored [3]. In decision making about outsourcing, buyers often consider a multiple sourcing strategy. In other words, the buyers encounter competing suppliers. In general, competition, as a useful mechanism, can improve the suppliers' performance from the buyer's viewpoint.

In this paper, we consider a dual-sourcing problem faced by a buyer who commits to outsource the manufacturing of a given product to two selected suppliers. In our proposed model, the buyer allocates demand to the suppliers using a three-dimensional allocation function. For this problem, we try to determine the behavior of the competing suppliers.

This paper is organized as follows. The next section presents a brief review of the related literature and underlines our contribution. Section 3 presents the model description including the model assumptions and formulation. Section 4 introduces a three-stage game-theoretic framework. Section 5 presents some computational results and a sensitivity analysis with respect to some parameters. Finally, section 6 concludes the paper and makes suggestions for future research.

2. LITERATURE REVIEW AND CONTRIBUTION

Stream of investigations treated various issues of outsourcing in competitive environments. The papers in this field can be divided into two categories. Some of them focus on the aspects of outsourcing decision making, while some others try to take into account the competition among suppliers for the demand share of a single buyer which outsources a production input. The main focus of the first-category study is on assessing the effectiveness of outsourcing and deciding about the rate of outsourcing. Cachon and harker [5], Dube et al. [6], Chang et al. [7], Ni et al. [8], Kumar et al. [9], McIvor [10] and Bae et al. [11] have focused on effectiveness of

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outsourcing or outsourcing decision making. The manner in which the demand is allocated to the competing suppliers can be taken as exogenous or endogenous. In exogenous manner, the allocation parameters are taken as fixed and given, while in endogenous manner the allocation parameters are taken as decision variables. Companies are increasingly utilizing a range of criteria to evaluate the performance of their suppliers [12]. Some of the most commonly used criteria in the literature are as follows: price, service level (fill rate), lead time, product quality, supplier's reliability, production capacity and inventory level. Therefore, we can classify the second-category investigations in terms of exogenous or endogenous parameters based on which suppliers compete. Next, we categorize the studies with endogenous parameters based on the number of considered parameters.

Suppliers' competition based on one or more exogenous parameters for demand share has been studied in a stream of research in the literature. Moorthy [13], Banker et al. [14], Hall and Porteus [15], Tsay and Agrawal [16], Gans [17], Chayet and Hopp [18], Boyaci and Gallego [19], Allon and Federgruen [20], Matsubayashi [21], Ozer and Raz [22], Lu et al. [23], Hafezolkotob and makui [24] and Ahmadvand et al. [25] have taken exogenous parameters into consideration.

In Gilbert and Weng [26], Ha et al. [27], Cachon and Zhang [28], Benjaffar et al. [2], Ching et al. [29], Elahi et al. [30] and Elahi [12], the authors have studied firms' competition based on a single endogenous parameter.

As far as the present authors' search through the literature shows there is only one work that considers an allocation policy with more than one endogenous parameter. Jin and Ryan [3] considered a problem faced by a single buyer who must decide how to allocate a demand to two make-to-stock suppliers based on both suppliers' prices and service levels (fill rates). In fact, in their model, the suppliers face a two-dimensional strategy space. The buyer utilizes an exponential allocation function that characterizes the relative importance of the price versus the service. In their model, the buyer's objective is to minimize his total cost, and the suppliers' objective is to maximize their profits. In the real world, companies are interested in using multi-criteria to evaluate the performance of their suppliers. In other words, buyers buy a product or service based on not only its price but also its supplier's service level, its quality, etc. For example, price and quality competition is very common in the broadband Internet market in Japan [21]. As another example, Royal Philips Electronics puts 15% weight on the price among other service and innovation-related criteria [12]. However, some companies place a lower weight on some criteria as compared to other ones, though ignoring these criteria does not seem logical. As noted in Jin and Ryan [3], the analysis of a setting with a multi-dimensional strategy space is more sophisticated than other settings with single-dimensional strategy space.

In this paper, we will extend the model of Jin and Ryan [3] by considering a three-dimensional strategy space and propose a game-theoretic framework that has the ability to be generalized to a strategy space with more dimensions. In our model, the buyer allocates demands to the competing suppliers based on the price and the service level as well as the product quality. We will try to account for the buyer's trade-off among these three criteria by using two certain allocation functions: 1) the exponential allocation function, and 2) the Cobb-Douglas allocation function.

3. MODEL DESCRIPTION

We examine a supply chain consisting of one buyer and two suppliers in which the buyer outsources manufacturing of a physical product to two suppliers. The suppliers operate as make-to-stock and compete for the buyer's share of demand based on price, service level and product quality. In a make-to-stock environment, the typical measures of service level consist of fill rate, expected order delay and the probability that the order delay does not exceed a quoted lead time [2, 12, 30]. In this paper, in consistence with Benjaffar et al. [2], Elahi et al. [30], Jin and Ryan [3] and Elahi [12], we assume that the service level is measured by fill rate that is determined by the supplier's base-stock level. In addition, we use the mean of the quality characteristic of interest as a proxy for product quality. Here, we consider the specification limits for the quality characteristic of interest, and the suppliers try to determine the optimum process mean. It should be noted that when a product does not fulfill at least one of the specification limits, the item is reprocessed or scrapped and sold at a discount price. Hence, the process mean may be set higher to reduce the costs incurred due to producing defective products. On the other hand, an increase in the process mean causes an increase in the production cost [31].

In order to minimize his average cost, the buyer must use an allocation policy to allocate a fraction of the demand to his suppliers. In this paper, we focus on two certain allocation functions: the exponential and the Cobb-Douglas allocation functions that have been widely used in marketing and operation management [32]. The exponential allocation function can be represented as:

$$\eta_i = \frac{e^{s_i - \alpha p_i + \beta \mu_i}}{\sum_{j=1}^{2} e^{s_j - \alpha p_j + \beta \mu_j}} \tag{1}$$

where α and β can be interpreted as the relative importance of price and quality vs. service level, respectively. In fact, η_i specifies the fraction of the demand that is allocated to supplier i based on his price (p_i), service level (s_i) and product quality (μ_i). The Cobb-Douglas allocation function can be written as:

$$\eta_i = \frac{s_i^{\gamma} p_i^{-\alpha} \mu_i^{\beta}}{\sum_{j=1}^{2} s_i^{\gamma} p_i^{-\alpha} \mu_i^{\beta}} \tag{2}$$

where γ, α and β denote the absolute elasticity of supplier i 's score function with respect to its own service level, price and quality level, respectively.

The notations (parameters and decision variables), assumptions and mathematical formulation are as follows:

3. 1. Parameters

f_0 : Variable production cost per unit for each supplier

ε_q : Quality related variable cost for each supplier

ε_s : Service related variable cost for each supplier

ε : Performance inspection cost per unit for each supplier

λ : The rate of Poisson demand process at the buyer

μ_p: The rate of exponentially distributed production times at each supplier

ρ : The utilization rate for each supplier ($\rho = \frac{\lambda}{\mu_p}$)

L : The lower specification limit for the product of each supplier

x_0 : The target value of product quality level

h : The holding cost per unit

b : The backorder cost per unit

3. 2. Decision Variables

s_i : The service level provided by supplier i , $i = 1,2$

p_i : The price offered by supplier i , $i = 1,2$

μ_i : The unknown mean of the normal quality characteristic of supplier i 's product (X_i), $X_i \equiv N[\mu_i, \sigma_i^2]$, where σ_i is the known standard deviation of X_i , $i = 1,2$

3. 3. Assumptions

Suppliers are homogeneous with identical cost structures.

Suppliers operate in a make-to-stock environment.

Suppliers have perfect and complete information on each others' price, service level and quality level.

Capacity and process mean adjustment are inexpensive.

Collusion is not considered between the suppliers.

There is a lower specification limit for the quality characteristic of interest.

Only the buyer bears responsibility for the backorder cost and deviation cost from the target value of the product quality level.

Surrogate variable is not considered for the quality characteristic of interest.

Products with $X_i < L$ are not sold. The scrap cost for the non-conformance product is assumed to be zero.

3. 4. Model Formulation

This section describes the supplier's and buyer's objectives and introduces the profit functions of each of them.

3. 4. 1. Supplier's Problem

The supplier's objective is to determine the price, service level and quality level based on a given allocation policy to maximize her profit function. The choice of the policy is subject to the behavior of the competing supplier. The supplier's profit function can be determined by the following equation:

Supplier's profit=total revenue - total production cost - total capacity cost - total quality cost - total performance inspection cost - total expected holding cost

The total expected holding cost is expressed as $\text{h.(expected inventory)}$. Using the approaches of Benjaffar et al. [2], Ching et al. [29], Jin and Ryan [3] and Elahi [12] and modifying them based on our assumptions, we may express the expected inventory mathematically as:

$$E(I_i) = \frac{\ln(1-s_i)}{\ln(\rho_i')} - \frac{\rho_i'}{1-\rho_i'} s_i \tag{3}$$

where $\rho_i' = \frac{\lambda}{\mu_p(1-\phi(\frac{L-\mu_i}{\sigma_i}))} = \rho \times \frac{1}{1-\phi(\frac{L-\mu_i}{\sigma_i})}$, $\phi(.)$ and $1-\phi(\frac{L-\mu_i}{\sigma_i})$ are the cumulative distribution function of the standard normal variable and the probability of producing non-defective products, respectively. Therefore, the supplier's profit function can now be expressed as:

$$\begin{aligned} \pi_i &= \eta_i \lambda (p_i (1-\phi(\frac{L-\mu_i}{\sigma_i})) - f_0 - \frac{\varepsilon_s}{\rho} - \varepsilon_q \mu_i - \varepsilon_i) \\ &-h.(\frac{\ln(1-s_i)}{\ln\rho - \ln(1-\phi(\frac{L-\mu_i}{\sigma_i}))} - \frac{\rho}{1-\rho-\phi(\frac{L-\mu_i}{\sigma_i})} s_i) \end{aligned} \tag{4}$$

3. 4. 2. Buyer's Problem

The buyer's objective is to choose the allocation functions' parameters and the fraction of the demand that must be allocated to each supplier based on the chosen parameters to minimize his long-term expected cost. The buyer's cost function can be determined by the following equation:

Buyer's cost function= total purchasing cost + total expected backorder cost +total expected quality loss

The total expected backorder cost is expressed as: $b.(\text{expected backorder})$. The approaches of Benjaffar et al. [2], Ching et al. [29], Jin and Ryan [3] and Elahi [12] have also been used to find the expected backorder. The expected backorder incurred by supplier i can be written as:

$$E(B_i)=\frac{\rho}{1-\rho-\phi(\frac{L-\mu_i}{\sigma_i})}(1-s_i) \tag{5}$$

In this paper, in line with Chen and Kao [33], Chen and Koo [34] and Chen and Lu [35], we use the adopted Taguchi's quadratic quality loss function for the buyer's quality loss cost. Taguchi's quadratic quality loss function has been widely used in quality control literature [35]. Quality loss per unit ($Loss(X)$) can be presented as $Loss(X)=k(X-x_0)^2$, where k and x_0 are the quality loss coefficient and target value of product quality level, respectively. Therefore, the expected quality loss per unit is given by:

$$E(Loss(X_i))=\int_{-\infty}^{+\infty}k(X_i-x_0)^2 f(x)dx=k(\sigma_i^2+(\mu_i-x_0)^2) \tag{6}$$

where $f(x)$ is the normal density function. Hence, the buyer's cost function can be stated mathematically as:

$$C=\sum_{i=1}^{2}\eta_i\lambda p_i+\sum_{i=1}^{2}b\frac{\rho}{1-\rho-\phi(\frac{L-\mu_i}{\sigma_i})}(1-s_i)+\sum_{i=1}^{2}k\eta_i\lambda(\sigma_i^2+(\mu_i-x_0)^2) \tag{7}$$

4. COMPETITION EQUILIBRIUM

In the present study, we develop a three-stage sequential game-theoretic model, in which the suppliers decide on the quality level in the first stage, the suppliers choose their service level in the second stage, and the suppliers make the pricing decisions in the third stage. We can summarize the proposed game stages as follows:

Stage 1: each supplier decides on the quality level to be provided (μ_i)

Stage 2: each supplier decides on the service level to be provided (s_i)

Stage 3: each supplier decides on the price to be offered (p_i)

Equilibrium analysis based on the exponential allocation function similar to Bae et al. [11], we begin from stage 3 to find the Subgame Nash Equilibrium. In other words, firstly, we assume that the quality level and the service level are given, and each supplier competes for a demand share by choosing a price that maximizes her profit.

It should be noted that Jin and Ryan [3] proved the existence of a unique and symmetric equilibrium for the game with a two-dimensional strategy space. It is straightforward to prove that there exists a unique and symmetric equilibrium for the proposed model by applying a similar approach; therefore, we save the proof by taking it for granted. Substituting η_i in (1) into π_i in (4) gives the profit function in terms of $p_i, s_i, \mu_i, p_j, s_j, \mu_j$ as follows:

$$\begin{aligned}\pi_i(p_i,s_i,\mu_i,p_j,s_j,\mu_j)&=\frac{e^{s_i-\alpha p_i+\beta\mu_i}}{\sum_{j=1}^{2}e^{s_j-\alpha p_j+\beta\mu_j}}\times\\ &\lambda(p_i(1-\phi(\frac{L-\mu_i}{\sigma_i}))-f_0-\frac{\varepsilon_s}{\rho}-\varepsilon_q\mu_i-\varepsilon_i)\\ &-h.(\frac{\ln(1-s_i)}{\ln\rho-\ln(1-\phi(\frac{L-\mu_i}{\sigma_i}))}-\frac{\rho}{1-\rho-\phi(\frac{L-\mu_i}{\sigma_i})}s_i)\end{aligned} \tag{8}$$

By applying the first-order condition to $\pi_i(p_i,s_i,\mu_i,p_j,s_j,\mu_j)$ with respect to p_i and letting $p_i=p_j=p$, $s_i=s_j=s$ and $\mu_i=\mu_j=\mu$, given the quality level and service level, the unique p that maximizes the profit function is determined as in the following: $\partial\pi_i/\partial p_i=0$. The first-order condition yields

$$p=\frac{2(1-\phi(\frac{L-\mu}{\sigma}))+\alpha(f_0+\frac{\varepsilon_s}{\rho}+\varepsilon_q\mu+\varepsilon)}{\alpha(1-\phi(\frac{L-\mu}{\sigma}))} \tag{9}$$

At the second stage, each supplier decides upon the service level given the quality level. Using the first-order condition on (8) by differentiating with respect to s_i and letting $p_i=p_j=p$, $s_i=s_j=s$ and $\mu_i=\mu_j=\mu$, the following is obtained:

$$\begin{aligned}\frac{\partial\pi_i}{\partial s_i}&=\frac{1}{4}\lambda(p(1-\phi(\frac{L-\mu}{\sigma})-f_0-\frac{\varepsilon_s}{\rho}-\varepsilon_q\mu-\varepsilon)\\ &+h.[(\frac{1}{1-s})(\frac{1}{\ln\rho-\ln(1-\phi(\frac{L-\mu}{\sigma}))})+\frac{\rho}{1-\rho-\phi(\frac{L-\mu}{\sigma})}]=0\end{aligned} \tag{10}$$

Substituting P in (9) into (10) and solving the first-order condition, the optimal service level as a function of quality level is obtained as follows:

$$s=1+\frac{2h\alpha(1-\rho-\phi(\frac{L-\mu}{\sigma}))}{\lambda(1-\phi(\frac{L-\mu}{\sigma}))(1-\rho-\phi(\frac{L-\mu}{\sigma}))+2h\alpha\rho}\times\frac{1}{\ln\rho-\ln(1-\phi(\frac{L-\mu}{\sigma}))} \tag{11}$$

It should be noted that the condition $\rho<1-\phi(\frac{L-\mu}{\sigma})$ is needed to ensure that the service level is smaller than 1.At the first stage, individual suppliers choose the quality level simultaneously. Applying the first-order condition in (8) with respect to μ_i and substituting $p_i=p_j=p$, $s_i=s_j=s$ and $\mu_i=\mu_j=\mu$, the following is obtained:

$$\frac{\partial \pi_i}{\partial \mu_i} = \frac{\beta}{4}\lambda(p(1-\phi(\frac{L-\mu}{\sigma})) - f_0 - \frac{\varepsilon_s}{\rho} - \varepsilon_q \mu - \varepsilon) + \frac{1}{2}\lambda[\frac{p}{\sigma}\varphi(\frac{L-\mu}{\sigma}) - \varepsilon_q] + h[\frac{\varphi(\frac{L-\mu}{\sigma})(-\ln(1-s))}{\sigma(1-\phi(\frac{L-\mu}{\sigma}))(\ln\rho - \ln(1-\phi(\frac{L-\mu}{\sigma})))^2} - \frac{\rho\varphi(\frac{L-\mu}{\sigma})s}{\sigma(1-\rho-\phi(\frac{L-\mu}{\sigma}))^2}] = 0 \quad (12)$$

where $\varphi(.)$ is the standard normal density function. By incorporating P in (9) and $s$in (11) into (12), we can write (12) in terms of only one decision variable,μ. Solving the resulting Equation (12) for μ gives the equilibrium value of the quality level provided by each supplier. Hence, the equilibrium values of the price and the service level can be derived by substituting the determined μ into (9) and (11), respectively. Due to the nonlinear terms involved in the resulting Equation (12), ($\varphi(.)$ and $\phi(.)$), it is difficult to provide a closed-form expression forμ. Therefore, the equilibrium values of the proposed model can be obtained by using a numerical search. It is to be note that in order to ensure the concavity of the profit function, the second-order condition should be checked at the equilibrium point. Hence, we need to show that the Hessian matrix is negative definite. In other words, we need to show that the leading principal minors of the Hessian matrix alternate in sign. The leading principal minor is determinant of the leading principal sub-matrix obtained by deleting the last $n-k$ rows and columns of a $n \times n$ matrix, where $k = 0,1,...,n-1$. The Hessian matrix of the profit function and the leading principal minors are given as follows:

$$H = \begin{pmatrix} \frac{\partial^2 \pi_i}{\partial p_i^2} & \frac{\partial^2 \pi_i}{\partial p_i \partial s_i} & \frac{\partial^2 \pi_i}{\partial p_i \partial \mu_i} \\ \frac{\partial^2 \pi_i}{\partial s_i \partial p_i} & \frac{\partial^2 \pi_i}{\partial s_i^2} & \frac{\partial^2 \pi_i}{\partial s_i \partial \mu_i} \\ \frac{\partial^2 \pi_i}{\partial \mu_i \partial p_i} & \frac{\partial^2 \pi_i}{\partial \mu_i \partial s_i} & \frac{\partial^2 \pi_i}{\partial \mu_i^2} \end{pmatrix} \quad A_1 = \left|\frac{\partial^2 \pi_i}{\partial p_i^2}\right|$$

$$A_2 = \begin{vmatrix} \frac{\partial^2 \pi_i}{\partial p_i^2} & \frac{\partial^2 \pi_i}{\partial p_i \partial s_i} \\ \frac{\partial^2 \pi_i}{\partial s_i \partial p_i} & \frac{\partial^2 \pi_i}{\partial s_i^2} \end{vmatrix} \quad A_3 = \begin{vmatrix} \frac{\partial^2 \pi_i}{\partial p_i^2} & \frac{\partial^2 \pi_i}{\partial p_i \partial s_i} & \frac{\partial^2 \pi_i}{\partial p_i \partial \mu_i} \\ \frac{\partial^2 \pi_i}{\partial s_i \partial p_i} & \frac{\partial^2 \pi_i}{\partial s_i^2} & \frac{\partial^2 \pi_i}{\partial s_i \partial \mu_i} \\ \frac{\partial^2 \pi_i}{\partial \mu_i \partial p_i} & \frac{\partial^2 \pi_i}{\partial \mu_i \partial s_i} & \frac{\partial^2 \pi_i}{\partial \mu_i^2} \end{vmatrix} \quad (13)$$

where the second-order partial derivatives should be derived from the first-order condition Equations. If $A_1 < 0, A_2 > 0$ and $A_3 < 0$, then H is negative definite. Equilibrium analysis based on the cobb-dougl as allocation function by substituting η_i in (2) into π_i in (4), the supplier's profit function can be expressed in terms of $p_i, s_i, \mu_i, p_j, s_j, \mu_j$ as follows:

$$\pi_i(p_i, s_i, \mu_i, p_j, s_j, \mu_j) = \frac{s_i^\gamma p_i^{-\alpha} \mu_i^\beta}{\sum_{j=1}^{2} s_i^\gamma p_i^{-\alpha} \mu_i^\beta}\lambda(p_i(1-\phi(\frac{L-\mu_i}{\sigma_i})) - f_0 - \frac{\varepsilon_s}{\rho} - \varepsilon_q \mu_i - \varepsilon_i) - h.(\frac{\ln(1-s_i)}{\ln\rho - \ln(1-\phi(\frac{L-\mu_i}{\sigma_i}))} - \frac{\rho}{1-\rho-\phi(\frac{L-\mu_i}{\sigma_i})}s_i) \quad (14)$$

Similar to the previous case, we begin from stage 3. In this stage, equating the first derivative of π_i in (14) with respect to p_i zero and letting $p_i = p_j = p$, $s_i = s_j = s$ and $\mu_i = \mu_j = \mu$ yields:

$$p = \frac{\alpha(f_0 + \frac{\varepsilon_s}{\rho} + \varepsilon_q \mu + \varepsilon)}{(\alpha - 2)(1 - \phi(\frac{L-\mu}{\sigma}))} \quad (15)$$

In order to ensure that $p > 0$, we require $\alpha > 2$. Incorporating P in (15) into π_i in (14) and applying the first-order condition given the $p_i = p_j = p$, $s_i = s_j = s$ and $\mu_i = \mu_j = \mu$, the following is given:

$$\frac{\partial \pi_i}{\partial s_i} = \frac{2\gamma\lambda}{\alpha-2}(f_0 + \frac{\varepsilon_s}{\rho} + \varepsilon_q \mu + \varepsilon) + \frac{4h}{\ln\rho - \ln(1-\phi(\frac{L-\mu}{\sigma}))}\frac{s}{1-s} + \frac{4h\rho}{1-\rho-\phi(\frac{L-\mu}{\sigma})}s = 0 \quad (16)$$

Now, defining

$$A = \frac{2\gamma\lambda}{\alpha-2}(f_0 + \frac{\varepsilon_s}{\rho} + \varepsilon_q \mu + \varepsilon),$$

$$B = \frac{4h}{\ln(1-\phi(\frac{L-\mu}{\sigma})) - \ln\rho} \text{ and } C = \frac{4h\rho}{1-\rho-\phi(\frac{L-\mu}{\sigma})},$$

we obtain the supplier's equilibrium service level in terms of A, B and C as follows:

$$s = \frac{-A - B + C + \sqrt{A^2 + B^2 + C^2 + 2AB + 2AC - 2BC}}{2C} \quad (17)$$

Finally, at stage 1 of the proposed game, differentiating π_i in (14) with respect to μ_i and letting $p_i = p_j = p$, $s_i = s_j = s$ and $\mu_i = \mu_j = \mu$, we have:

$$\frac{\partial \pi_i}{\partial \mu_i} = \frac{\beta}{4\mu}\lambda(p(1-\phi(\frac{L-\mu}{\sigma})) - f_0 - \frac{\varepsilon_s}{\rho} - \varepsilon_q \mu - \varepsilon) + \frac{1}{2}\lambda(\frac{p}{\sigma}\varphi(\frac{L-\mu}{\sigma}) - \varepsilon_q) - h(\frac{\ln(1-s)\varphi(\frac{L-\mu}{\sigma})}{\sigma(1-\phi(\frac{L-\mu}{\sigma}))(\ln\rho - \ln(1-\phi(\frac{L-\mu}{\sigma})))^2} + \frac{\rho\varphi(\frac{L-\mu}{\sigma})}{\sigma(1-\rho-\phi(\frac{L-\mu}{\sigma}))^2}s) = 0 \quad (18)$$

Substituting Pin (15) and s in (17) into (18), we can derive equilibrium quality level. Because of nonlinear items in (18), it is difficult to derive a unique solution analytically and to find a closed-form solution for μ. Therefore, this can be solved by a numerical search as well. In this case, in order to ensure that the profit function is concave, the second-order condition should be satisfied. Hence, it is sufficient to show that the Hessian matrix is negative definite. The Hessian matrix of the profit function and the leading principal minors are also given by (13), where the second-order partial derivatives should be derived from the first-order condition Equations.

5. COMPUTATIONAL RESULTS

As already mentioned, the problem lacks explicit expression for the supplier's quality level. Therefore, in this section, a numerical study with a sensitivity analysis of some key parameters is carried out to illustrate the behavior of the proposed model.

5. 1. Numerical Study We assume that the input parameters are set as:
$\alpha=7$, $\beta=2$, $\gamma=1$, $\lambda=1000$, $\sigma=1.5$, $\rho=0.8$, $L=40$, $f_0=5$, $\varepsilon_s=2$, $\varepsilon_q=0.3$, $\varepsilon=1$, $h=2$, $b=10$, $k=10$ $x_0=43$.
Under the given parameters, the equilibrium values of the decision variables as well as the optimal supplier's profit and the optimal buyer's cost are depicted in Table 1.The results in Table 1 show that under the considered condition, both sides gain more profit by applying the Cobb-Douglas allocation function in the model. As can be seen in Table 1, in competition with the Cobb-Douglas allocation function, the suppliers offer higher prices, higher service levels and lower quality levels than in competition with the exponential allocation function.

5. 2. Sensitivity Analysis In this section, the effect of parameters *α*, *β* and *ρ* on the equilibrium values of *P*, *s*and μ will be investigated by performing a sensitivity analysis. The results of the sensitivity analysis are summarized with respect to *α* in Table 2. It is to be noted that we only allow *α* to vary from 5 to 20 and fix the other parameters similar to those in the previous sub-section.

Table 2 indicates that the equilibrium values of *P*, *s*and *μ* decrease in *α* for both types of competition (games with the exponential allocation function and the Cobb-Douglas allocation function). As shown in Table 2, the difference between the equilibrium prices of the two competition modes decreases as the relative importance of the price vs. service level *α* increases. This is similar to the impact of *α* on the difference between the equilibrium quality levels and in contrast with the impact of *α* on the difference between the equilibrium service levels.

TABLE 1. Equilibrium values of the decision variables (with $\alpha/\gamma=7,\ \beta/\gamma=2$)

	p^*	s^* (%)	μ^*	π^*	C^*	A_1	A_2	A_3
Exponential Allocation Function	22.353	88.7	45.205	130.359	93491.3	-1749.5	1229806.3	-204864261.6
Cobb-Douglas Allocation Function	30.611	99.4	44.44	4327.823	73848.8	-35802380	77271067944	-666676409515143000

TABLE 2. Sensitivity analysis with respect to *α*

α	5		7		9		15		20	
	E*.	Co**.	E.	Co.	E.	Co.	E.	Co.	E.	Co.
p^*	22.519	36.865	22.353	30.611	22.081	28.041	21.917	25.122	21.871	24.184
s^* (%)	91.6	99.6	88.7	99.4	85.8	99.3	77.9	98.7	72.3	98.3
μ^*	45.38	45.38	45.21	44.44	44.41	44.17	44.00	43.91	43.9	43.84
π^*	184.99	7330.341	130.353	4327.823	110.232	3070.827	58.983	1635.172	43.931	1173.267
C^*	101914	116248.1	93491.3	73848.8	64485	64232	54553.6	55997	52516.8	53743

*E.: Exponential Allocation Function **Co.: Cobb-Douglas Allocation Function

TABLE 3. Sensitivity analysis with respect to *β*

β	2		4		6		8		10	
	E*.	Co**.	E.	Co.	E.	Co.	E.	Co.	E.	Co.
p^*	22.353	30.611	23.99	33.5	23.877	32.5	24.016	34.036	23.95	33.99
s^* (%)	88.7	99.4	88.7	99.5	88.7	99.5	88.7	99.5	88.7	99.5
μ^*	45.21	44.44	50.67	51.43	50.305	49.045	50.77	52.7	50.53	52.615
π^*	130.4	4327.823	130.4	4746.612	130.4	4603.8	130.4	4822.97	130.4	4817.58
C^*	1284472	1688792	158614	248246	141046	190601	166179	512388	150507	488410

*E.: Exponential Allocation Function **Co.: Cobb-Douglas Allocation Function

TABLE 4. Sensitivity analysis with respect to ρ

ρ	0.5		0.6		0.7		0.8		0.9	
	E*.	Co**.	E.	Co.	E.	Co.	E.	Co.	E.	Co.
p^*	23.461	32.743	23.191	31.794	22.714	31.117	22.353	30.611	22.075	30.222
s^* (%)	96.1	99.8	94.7	99.7	92.6	99.6	88.7	99.4	78.7	98.7
μ^*	45.235	45.53	45.22	44.48	45.22	44.45	45.21	44.44	45.205	44.44
π^*	135.404	4655.415	134.132	4515.11	132.518	4411.601	130.353	4327.823	127.574	4244.051
C^*	96315	78653	94976	76347	94501	74787	93491.3	73848.8	93233	73460

*E.: Exponential Allocation Function **Co.: Cobb-Douglas Allocation Function

Table 3 shows the results of the sensitivity analysis of the equilibrium values with respect to β. We vary the value of β from 2 to 10 and let $\alpha=7$, $\gamma=1$, $\lambda=1000$, $\sigma=1.5$, $\rho=0.8$, $L=40$, $f_0=5$, $\varepsilon_s=2$, $\varepsilon_q=0.3$, $\varepsilon=1$, $h=2$, $b=20$, $k=50$ and $x_0=50$.

As showed by Table 3, an increase in β has no significant impact on the equilibrium service level. We also find that there are no clear trends in the equilibrium price and the equilibrium quality level. However, an interesting insight is that the trend of the price level is similar to that of the quality level. In other words, the buyer pays a higher price for a higher-quality product received, and vice versa. This implies that the buyer makes a trade-off between the higher price paid and the higher quality received.

The results of the sensitivity analysis of the equilibrium values with respect to ρ are shown in Table 4. In this case, the input parameters are given as: $\alpha=7$, $\beta=2$, $\gamma=1$, $\lambda=1000$, $\sigma=1.5$, $L=40$, $f_0=5$, $\varepsilon_s=2$, $\varepsilon_q=0.3$, $\varepsilon=1$, $h=2$, $b=10$, $k=10$ and $x_0=43$.

As Table 4 shows, once the utilization rate ρ increases, none of the equilibrium values increase. In other words, the buyer makes a trade-off between the price and the service level, as ρ increases.

6. CONCLUSION

In recent years, outsourcing of inputs has become a major phenomenon in industries. On the other hand, in decision making about outsourcing, the splitting of an order among multiple suppliers, namely multi-sourcing, is one of the sourcing strategies that has recently been regarded. In this setting, the buyer encounters competing suppliers. Therefore, understanding the behavior of the buyer and suppliers is an important issue.

This paper is a study of an outsourced supply chain consisting of one buyer and two suppliers in which the buyer outsources manufacturing of a physical product to two competing suppliers. The proposed model postulates a three-dimensional strategy space in which the buyer allocates his demands to the competing suppliers based on the price, the service level and the product quality level. In this paper, two certain types of allocation function, namely the exponential allocation function and the Cobb-Douglas allocation function have been used. A three-stage game-theoretic framework is presented to derive the equilibrium values. Since the problem does not have a closed-form solution, numerical studies are carried out over a wide range of some key parameters. The numerical results show that the equilibrium values of price, service level and quality level tend to decrease, as α increases for both competition modes. Service level is not sensitive to β. The numerical results also indicate that as β increases, the buyer makes a trade-off between the higher price paid and the higher quality received. He also makes a trade-off between the higher price paid and the higher service level received as ρ increases. In this case, future research is suggested to be done in some directions. It is interesting to consider the competition at the two levels of a supply chain where the buyers compete for the market share and the suppliers compete for the buyers' demand share. Another extension is to consider heterogeneous suppliers in cost structures and utilization rates. Further extension would be to relax some of the assumptions considered in the paper. For example, a setting may be considered in which the buyer and the suppliers bear responsibility for the back order cost.

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Outsourcing through Three-dimensional Competition

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چکیده

در این مقاله، یک زنجیره تأمین شامل یک خریدار و دو تأمین‌کننده در نظر گرفته می‌شود. در این زنجیره تأمین، خریدار ساخت یک قطعه یا محصول فیزیکی را به دو تأمین‌کننده رقیب برون‌سپاری می‌نماید. تأمین‌کنندگان برای کسب سهم بیشتری از تقاضای خریدار به رقابت می‌پردازند. خریدار تقاضای خود را بر پایه یک تابع تخصیص سه بعدی به تأمین-کنندگان رقیب تخصیص می‌دهد. در این مقاله، دو تابع تخصیص نمایی و کاب- داگلاس در نظر گرفته می‌شود که هر یک تابعی از قیمت، سطح خدمت و سطح کیفیت محصول می‌باشد. برای یافتن جوابهای تعادلی مدل، یک چارچوب نظریه بازی سه مرحله‌ای ارائه می‌شود. از آنجا که مسأله فاقد یک جواب تحلیلی و فرم بسته است، مطالعات عددی و تحلیل حساسیت بر روی برخی از پارامترهای کلیدی مدل انجام خواهد شد.

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