IJE TRANSACTIONS A: Basics Vol. 27, No. 7, (July 2014) 1107-1112

International Journal of Engineering

Journal Homepage: www.ije.ir

TECHNICAL NOTE

EOQ Model with Cash Flow Oriented and Quantity Dependent Under Trade Credits

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PAPER INFO

Paper history:
Received 14 June 2013
Received in revised form 27 September 2013
Accepted in 06 March 2014

Keywords:
Inventory
Cash Flow
Inflation
Liquid Forming
Quantity Dependent Demand
Permissible Delay

ABSTRACT

Inventory models in which the demand rate depended on the stock are based on the common real- life observation that greater product availability tends to stimulate more sales. In this study, we develop an inventory model to determine an optimal ordering policy for quantity dependent demand rate and time dependent holding cost items with delay in payments permitted by the supplier under inflation and time discounting. Mathematical models have been derived under two situations. These situations are: Case I: cycle time greater than or equal to permissible delay period and Case II: cycle time less than permissible delay period. Finally, numerical example is given to illustrate the proposed model.

doi: *10.5829/idosi.ije.2014.27.07a.12*

1. INTRODUCTION

In the classical EOQ model, it was assumed that the supplier is paid for the items as soon as the items are received. However, in daily life the supplier frequently offers its customers a permissible delay in payments to attract new customers who consider it to be a type of price reduction. The retailer can sell the goods to accumulate revenue and earn interest before the end of trade credit period. But if the payment is delayed beyond this period (credit period) a higher interest will be charged. Thus the trade credit is an important source of financing for intermediate purchasers of goods and services and plays a large role in our economy.

Over the last two decades several researchers have studied inventory models of deteriorating items such as medicines, blood banks, green vegetables, fashion goods and volatile liquids. etc. Goyal [1] developed an EOQ model under conditions of permissible delay in payments. He ignored the difference between the selling price and the purchase cost, and concluded that the economic replenishment interval and order quantity generally increases marginally under the permissible delay in payments. Ghare and Schrader [2] developed a model for an exponentially decaying inventory. Covert and Philip [3] then extended the Ghare and Schrader's constant deteriorating rate to a two parameter Weibull distribution. Shah and Jaiswal [4] and Aggrawal [5] presented and re-established an order level inventory model with a constant rate of deterioration, respectively. Dave and Patel [6] considered an inventory model for deteriorating items with time proportional demand when shortages were prohibited. Tripathi and Misra [7] developed a model credit financial in economic ordering policies of non-deteriorating items with time dependent demand rate considering three cases. Tripathi and Kumar [8, 9] also developed model credit financing in economic ordering policies of time dependent deteriorating items. A cash flow oriented EOQ model with deteriorating items under permissible delay in payments is discussed by Hou and Lin [10] considering time dependent demand rate for non-deteriorating items. Many related articles could be found by Heng et al. [11], Sarkar et al. [12] and Balkhi and Benkherouf [13].

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Please cite this article as: R. P.Tripathi, A. Kumar Uniyal, EOQ Model with Cash Flow Oriented and Quantity Dependent Under Trade Credits. International Journal of Engineering (IJE) Transactions A: Basics, Vol. 27, No. 7, (2014): 1107-1112

Many countries have high annual inflation rate. Inflation also influences demand of certain product. As inflation increases, the value of money goes down which erodes the future of saving and forces one for more current spending. These spending are on deteriorating and luxury products that give rise to demand of these products. Most of the countries have suffered from large – scale inflation and sharp decline in the purchasing power of money last several decades. As a result while determining the optimal ordering policy, the effects of inflation and time value of money cannot be ignored. The pioneer research in this direction was done by Buzacott [14], who developed and EOQ model inflation subject to different types of pricing policies. Other related articles can be found in Misra [15] and Ray and Choudhary [16] and Liao et al. [17] , Singh et al. [18] developed a model two ware house inventory model for deteriorating items with shortages under inflation and time value of money. Effects of variable inflationary conditions on an inventory model with inflation- proportional demand rate were discussed by Mirzazadeh [19]. Jain et al. [20] presented an economic production quantity (EPQ) model for deteriorating items with stock- dependent demand and shortages. Torabi and Jenabi [21] studied the problem of lot sizing scheduling and delivery of several items in a two stage supply chain over a finite planning horizon. Wee and Widyadana [22] developed integrated single vendor single – buyer inventory model with multiple deliveries. Saha et al. [23] developed multi- item economic order quantity (EOQ) inventory models for breakable units with stock- dependent demand under imprecise constraints. Liao and Huang [24] presented an inventory model for optimizing the replenishment cycle time for a single deteriorating item under a permissible delay in payments and constraints on warehouse capacity. Sarkar [25] presented an EOQ model for finite replenishment rate with different discount rates on purchasing costs where demand and deterioration rate are both time dependent. Chung and Liao [26] discussed the optimal order quantity of the EOQ model that is not only dependent on the inventory policy but also on firm credit policy. Soni and Shah [27] developed mathematical models to formulate optimal ordering policies for retailer when demand is partially consistent and partially dependent on the stock and supplier offers progressive credit period to settle the account. Mishra and Jain [28] developed maintainability policy for deteriorating system with inspection and common cause failure.

In classical EOQ model, the demand rate is considered constant but independent of stock status. For consumer goods, the consumption rate may be influenced by the stock levels that is the consumption rate may go up or down with the on – hand stock level. Hou [29] derived an inventory model for deteriorating items with stock- dependent consumption rate and shortages under inflation and time discounting over a finite planning horizon. Alfares [30] considered the inventory policy for an item with a stock – level dependent demand rate and a storage- time dependent holding cost. Ray and Chaudhuri [31] developed a finite time horizon deterministic economic order quantity inventory model with shortages, where the demand rate at any instant depends on the on- hand inventory at that instant. Urban [32] presented an inventory models incorporating financing agreements with both suppliers and customers. Uttayakumar and Geetha [33] investigated an instantaneous replenishment inventory model for deteriorating items under cost minimization, considering stock- dependent consumption rate

This study develops an inventory model for stock dependent demand rate when a delay in payments is permissible. Shortages are not allowed and the effect of inflation rate and delay in payments are discussed. Mathematical models are also derived under two circumstances i.e. case I and case II. In addition, expressions for inventory systems total cost are derived for the above two cases. The main objective of this paper is to determine optimal (minimum) total present value of the cost over the time horizon H.

This paper is organized as follows. In section 2, we introduce the notations and assumptions used in the model. In section 3, the model formulation are proposed for analysis. Numerical example is explained in section 4. Finally, conclusion and future research directives are given in the last section 5.

2. NOTATION AND ASSUMPTION

The following notations are used through the manuscript:

H	Length of planning horizon
T	Replenishment cycle time
N	Number of replenishment during the planning horizon, n = H/T
Q	Order quantity, units per cycle
D	Demand rate per unit time, units/unit time
A_0	Ordering cost at time zero, \$/order
c	Per unit cost of the item
h	Inventory holding cost per unit per unit time excluding interest charges, \$.unit/unit time
r	Discount rate represent the time value of money
f	Inflation rate
k	The net discount rate of inflation, k = r-f
I_e	The interest earned per \$ per unit time

I_c The interest charged per \$ in stocks per unit time by the supplies, $I_c > I_e$

m The permissible delay in settling the account.

C_1 The present value of total replenishment cost

C_2 The present value of total purchasing cost

A The present value of total holding cost over the time horizon H

i_p The present value of the interest payable during the first replenishment cycle

I_p The present value of the total interest payable over the time horizon H.

I(t) Inventory level at time t

E Interest earned during the first replenishment cycle.

E_1 The present value of the total interest earned over the time horizon H

$Z_1(n)$ The total present value of the costs over the time horizon H. (for case I)

$Z_2(n)$ The total present value of the costs over the time horizon (for case II)

In addition, the following assumptions are used throughout the paper.

1. The lead time is zero
2. The demand rate is quantity dependent, i.e. $D = D\{I(t)\} = \alpha\,\{I(t)\}^{\beta}$, $\alpha > 0,\ 0 < \beta < 1$.
3. Holding cost is a function of time i.e. h = h(t) = h t
4. The model under consideration is inflation dependent
5. A single is considered
6. Shortages are not allowed

3. MATHEMATICAL MODEL

The inventory level I(t) at time t during the first replenishment cycle is depleted by the effect of demand only. Thus, the variation of inventory at time t with respect to't' is governed by the following differential equation.

$$\frac{dI(t)}{dt} = \alpha\,\{I(t)\}^{\beta},\ 0 < t < T = H/n \tag{1}$$

The solution of (1) is given by

$$I(t) = \{\alpha\ (1-\beta)\ (T - t)\}^{\frac{1}{(1-\beta)}} \tag{2}$$

The order quantity Q is given by

$$I(0) = Q = \{\alpha(1-\beta)T\}^{\frac{1}{(1-\beta)}} \tag{3}$$

The present value of the total replenishment cost is given by

$$C_1 = A_o\left(\frac{1-e^{-kH}}{1-e^{-kT}}\right);\ T = H/n \tag{4}$$

The present value of total purchasing costs is given by

$$C_2 = c\{\alpha(1-\beta)T\}^{\frac{1}{1-\beta}}\left(\frac{1-e^{-kH}}{1-e^{-kT}}\right) \tag{5}$$

The present value of total purchasing cost is given by

$$A = \left(\frac{(1-\beta)^2}{(2-\beta)(3-2\beta)}\right)\left\{T^{\left(\frac{3-2\beta}{1-\beta}\right)} - \left(\frac{1-\beta}{4-3\beta}\right)2kT^{\left(\frac{4-3\beta}{1-\beta}\right)} + \frac{3(1-\beta)^2 3k^2}{(4-3\beta)(5-4\beta)}T^{\left(\frac{5-4\beta}{1-\beta}\right)}\right\}\left(\frac{1-e^{-kH}}{1-e^{-kT}}\right) \tag{6}$$

Case I : *m* ≤ *T* = *H*/*n*

The present value of the interest payable during the first replenishment cycle is given by

$$i_p = cI_c\,\{\alpha(1-\beta)\}^{\frac{1}{1-\beta}}\left(\frac{1-\beta}{2-\beta}\right).\left\{\left(1 - km + \frac{k^2m^2}{2}\right)(T-m)^{\frac{(2-\beta)}{(1-\beta)}} + \left(\frac{1-\beta}{3-2\beta}\right)k(km-1)(T-m)^{\frac{(3-2\beta)}{(1-\beta)}} - \left(\frac{(1-\beta)^2}{(3-2\beta)(4-3\beta)}\right)(T-m)^{\frac{(4-3\beta)}{(1-\beta)}}\right\} \tag{7}$$

The present value of total interest payable over the time horizon *H*

$$I_p = i_p\left(\frac{1-e^{-kH}}{1-e^{-kT}}\right) \tag{8}$$

The present value of the interest earned over the time horizon *H*

$$E = cI_e\,\{\alpha(1-\beta)\}^{\frac{1}{1-\beta}}\left(\frac{1-\beta}{2-\beta}\right)T^{\frac{(2-\beta)}{(1-\beta)}}.\left[1 + \left(\frac{1-\beta}{3-2\beta}\right)\left\{-2kT + \left(\frac{1-\beta}{4-3\beta}\right)T^2\right\}\right] \tag{9}$$

The present value of the total interest earned over the time horizon *H* is

$$E_1 = E\left(\frac{1-e^{-kH}}{1-e^{-kT}}\right) \tag{10}$$

Thus, the present value of the total present value of the costs over the time horizon *H* is

$$Z_1(n) = C_1 + C_2 + A + I_p - E_1 \tag{11}$$

Case II : $m > T = H/n$

The interest earned in the first cycle is the sum of interest earned during the time period $[0, T]$ and the interest earned from the costs invested during the time period (T, m) is given by

$$E_2 = E + cI_e(m - T)e^{-kT}\{\alpha(1-\beta)\}^{\frac{1}{(1-\beta)}} \tag{12}$$

Thus, the present value of the total interest earned over the time horizon H is given by

$$E_3 = \left[E + cI_e(m-T)e^{-kT}\{\alpha(1-\beta)T\}^{\frac{1}{1-\beta}}\right]\left(\frac{1-e^{-kH}}{1-e^{-kT}}\right) \tag{13}$$

The total present value of the costs,

$Z_2(n)$ is given by $Z_2(n) = C_1 + C_2 + A - E_3$ (14)

At $m = T = H/n$, we obtain $Z_1(n) = Z_2(n)$, we have

$$Z(n) = \begin{cases} Z_1(n) \text{ if } T = H/n \geq m \\ Z_2(n) \text{ if } T = H/n < m \end{cases}$$

where, $Z_1(n)$ and $Z_2(n)$ is given in Equations (11) and (14).

Differentiating (11) and (14) two time partially with respect to 'n' we obtain

$$\frac{\partial^2 Z_1(n)}{\partial n^2} > 0 \text{ ,and } \frac{\partial^2 Z_2(n)}{\partial n^2} > 0$$

Which shows that $Z_1(n)$ and $Z_2(n)$ are convex function 'n'.

4. NUMERICAL EXAMPLE

An example is devised here to illustrate the results of the general model in this study with the following data: α = 500, β = 0.2, A = \$80/order, the holding cost excluding interest charges, h = \$2.4/unit/year, the per unit cost, c = \$15 unit, the constant rate of deterioration, θ = 0.15, the net discount rate of inflation, k = 0.12 /\$/ year, the interest charged per \$ in stocks per year by the supplier, I_c = \$0.18/\$/year, the interest earned per \$ per year, I_e = \$0.16/\$/year and the planning horizon, H = 5 years. The permissible delay in settling account, m = 60 days = 60/360 = 0.166666 years (assume 360 days per year).The computational results is shown in the Table 1 with the variation of total present value of cost with respect to order number 'n'. We find the Case II is optimal option in credit policy. From the case, the minimum total present value of costs is found when the number of replenishment, n is 40. With 40 replenishments, the optimal cycle time T is 0.15 year, the optimal order quantity, Q = 166.989 units and the optimal total present value of costs, Z(n) = \$79225.2. From the above data, the following figures is drawn between order 'n' vs. Total cost (Figure 1(a)); and cycle time 'T' vs total cost (Figure 1(b)). The above curves are convex which proves the validity of the theoretical results that total cost Z(n) is convex function of order number 'n'.

TABLE 1. The numerical results "variation of TVC with the variation of order number 'n' keeping all parameters same as in numerical example".

Case	n	T	'Q'	TVC
I	30	0.200000	239.256	80198.9
	31	0.193548	229.647	80071.1
	32	0.187500	220.712	79965.0
	33	0.181818	212.384	79879.0
	34	0.176471	204.605	79810.7
	35	0.171429	197.324	79761.5
	36	0.166666	190.495	79728.0
II	37	0.162162	184.082	79294.3
	38	0.157895	178.047	79242.9
	39	0.153846	172.359	79233.3
	40*	**0.150000**	**166.989**	**79225.2**
	41	0.146341	161.913	79231.7
	42	0.142857	157.105	79251.1
	43	0.139535	152.556	79282.8
	44	0.136363	148.235	79326.1

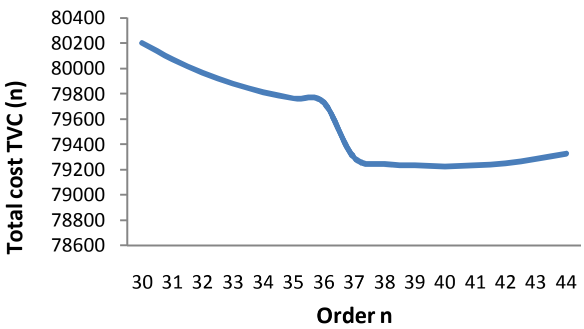

Figure 1(a). Total cost vs. order 'n'.

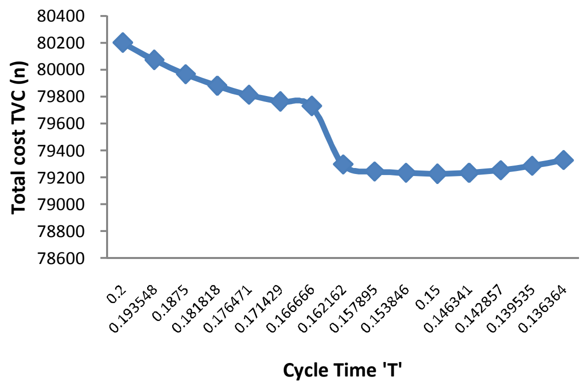

Figure 1(b). Total cost vs. cycle time 'T'.

5. CONCLUSION AND FUTURE RESEARCH

This study presents inventory model for stock dependent demand rate under stock dependent demand rate under permissible delay in payments, shortages are not allowed and the effect of inflation rate, stock dependent demand rate and permissible delay in payments are discussed as well. Numerical example shows that the total relevant cost is minimum at replenishment number n = 40 and the minimum total cost is Z(n) = $79225.2.

The proposed model can be extended for time dependent deteriorating items and time dependent demand rate. We can extend this paper by considering probabilistic demand, the demand which depend upon the current stock and inflation dependent demand rate.

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PAPER INFO

Paper history:
Received 14 June 2013
Received in revised form 27 September 2013
Accepted in 06 March 2014

Keywords:
Inventory
Cash Flow
Inflation
Liquid Forming
Quantity Dependent Demand
Permissible Delay

چکیده

مدل های فهرست که در آن ها سرعت تقاضا به موجودی وابسته است، بر پایه مشاهدات زندگی واقعی بنا شده است که موجودیت بیشتر محصول تمایل دارد که فروش بیشتری را نیز تحریک کند. در این مطالعه، ما یک مدل فهرست را برای تعیین سیاست بهینه مرتب سازی برای سرعت تقاضای وابسته به مقدار و برگزاری اقلام هزینه وابسته به زمان همراه با تاخیر در پرداخت که توسط تهیه کننده تحت تورم و کاهش زمان اجازه داده می شود، توسعه داده ایم. مدل های ریاضی بر پایه دو موقعیت ایجاد شده اند. این دو موقعیت مورد اول: زمان چرخه ای بیشتر یا برابر با مدت تاخیر اجازه داده شده و مورد دوم: زمان چرخه ای کمتر از مدت اجازه داده شده است. در پایان، مثال های عددی برای نمایش دادن مدل پیشنهادی ارائه شده است.

doi: *10.5829/idosi.ije.2014.27.07a.12*