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RESEARCH NOTE

Analytical Analysis of the Dual-phase-lag Heat Transfer Equation in a Finite Slab with Periodic Surface Heat Flux

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ABSTRACT

This work uses the dual-phase-lag (DPL) model of heat conduction to demonstrate the effect of temperature gradient relaxation time on the result of non-Fourier hyperbolic conduction in a finite slab subjected to a periodic thermal disturbance. DPL model combines the wave features of hyperbolic conduction with a diffusion-like feature of the evidence not captured by the hyperbolic case. For the first time, the analytical solution of DPL model of heat conduction equation is obtained adopting Laplace transform method and inversion theorem. The temperature profiles at the front and rear surfaces of the slab are calculated for various temperature gradient relaxation times. The phase and amplitude difference between the front and the rear surface are calculated numerically as a function of the temperature gradient relaxation time, which have been reported previously as a function of heat flux relaxation time. The results demonstrate that increasing the temperature gradient relaxation time leads to the lower phase difference and upper amplitude difference between the temperature responses of the front and rear surfaces.

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1. INTRODUCTION

The non-Fourier dual-phase-lag (DPL) model of heat conduction is used to extend the results of non-Fourier conduction in the study of Tang and Araki [1] with a finite slab subjected to a periodic thermal disturbance that was reported with hyperbolic conduction. Since the DPL model has been shown to give good agreement with experiments across a wide range of length and time scales for engineering materials [2, 3], it provides a more comprehensive treatment of the non-Fourier heat conduction compared to the interpretation [1] where hyperbolic conduction model is applied.

Applying heat sources such as laser and microwave with extremely short duration or very high frequency has numerous applications for practical engineering problems such as surface melting of metal [4] and sintering of ceramics [5]. In these situations, the Fourier's law of heat conduction fails to describe the heat transfer process and the non-Fourier effect becomes significant [6-9]. Thus describing the heat conduction process in such conditions with non-Fourier models (especially by DPL model since it is supported experimentally [2]) is essential.

Analytical solution of DPL model of heat conduction equation is more complicated in compare with the hyperbolic case and to the best of authors' knowledge there is not any closed form solution of this model in the literature. Consequently, the motivation for the work here is to extend the analysis of the previous work[1] that is aimed at eventually developing better tools to predict transient temperature in finite slab.

In theory, the Fourier's heat conduction equation leads to the solutions exhibiting infinite propagation speed of thermal signals. In order to eliminate this paradox, two different kinds of models are usually adopted in the related literatures. One is the macroscopic thermal wave model, postulated by Cattaneo[10] and Vernotte[11]:

$$q=-\lambda\nabla T-\tau_q\frac{\partial}{\partial t}q \qquad (1)$$

where, q is the heat flux, T is the temperature, λ is the thermal conductivity and τ_q is the heat flux relaxation time. Although the non-Fourier thermal wave model can remedy the physically unreasonable heat penetration speed, in some cases introduces unusual behaviors [12],

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physically impossible solutions like negative thermal energies [13, 14], violates the second law of thermodynamics by yielding negative values for entropy [15, 16], does not consider the relaxation times between electrons and the atomic lattice [9] and when it comes to the importance of microstructural interaction effects, is not capable to describe the fast transient heat transfer process [17]. To provide a macroscopic description free from the aforementioned defects, Tzou [9] developed a dual-phase-lag (DPL) model by generalizing the Fourier's law and supported it experimentally [2].

The non-Fourier dual-phase-lag (DPL) model is deduced by adding two time constants, namely the phase-lags of temperature gradient and heat flux, in the Fourier heat flux equation to provide a macroscopic description that can capture the microscopic interaction effects in heat transfer process and suggest some behaviors of heat conduction, while neither the macroscopic thermal wave model nor the Fourier thermal diffusion model can do [9]:

$$q = -\lambda \nabla T - \lambda \tau_T \frac{\partial}{\partial t} \nabla T - \tau_q \frac{\partial q}{\partial t} \tag{2}$$

where, τ_T is the temperature gradient relaxation time. Equation (2) and the conservation equation of energy lead to the following equation to describe the lagging behavior:

$$a \nabla^2 T + a \tau_T \frac{\partial}{\partial t} \nabla^2 T = \frac{\partial T}{\partial t} + \tau_q \frac{\partial^2 T}{\partial t^2} \tag{3}$$

where, a is the thermal diffusivity and $\sqrt{a/\tau_q}$ is the propagation speed of temperature wave. In addition to its application in the ultrafast pulsed-laser heating, the DPL heat conduction equation arises in describing and predicting phenomena such as temperature pulses propagation in superfluid liquid helium, non-homogeneous lagging response in porous media, thermal lagging in amorphous materials, and the effects of material defects and thermomechanical coupling [18].

Since the DPL model developed, the analysis of it has been a concern for various kinds of mediums and dimensions under different conditions. Tang and Araki [18] introduced a generalized macroscopic model and solved it analytically using green functions in treating the transient heat conduction problems in finite rigid slabs irradiated by short pulse lasers. They illustrated wavy, wavelike, and diffusive behavior predicted by the model. Antaki[19] considered various thermal lagging behaviors in a semi-infinite slab under step surface heat flux. In addition,Antaki[20]employed the DPL heat conduction model, which is based on the well-known two phase lags concept [9], to interpret the non-Fourier heat conduction phenomena in processed meats against thermal wave model which was used by Mitra et al. [21]. The spatial behaviour of solutions of some problems for the DPL heat equation on a semi-infinite cylinder is studied by Horgan and Quintanilla [22]. Liu [23] analyzed the DPL thermal behaviors in two-layered thin films with an interface thermal resistance by considering the radiation boundary condition. Wang et al. [24] developed two solution structure theorems for the DPL equation under linear boundary conditions. Ang[25] considered the numerical solution of a two-dimensional thermal problem governed by a third-order partial differential equation (DPL model) derived from a non-Fourier heat flux model which may account for thermal waves and/or microscopic effects. Zhang and Zhao [26] derived a two-dimensional governing microscale heat transport equation (DPL model) and proposed an unconditionally stable finite difference scheme to discretize the governing equation. They used a preconditioned conjugate gradient method to solve the resulting sparse linear systems. Han et al. [27] numerically analyzed a two-dimensional lagging thermal behavior under short-pulse-laser heating on surface in both rectangular coordinate and axially symmetric systems. Ghazanfarian and Shomali[28] investigated the numerical simulation of non-Fourier transient heat transfer in a two-dimensional sub-100 nm metal-oxide-semiconductor field-effect transistor (MOSFET). They introduced the dual-phase-lag (DPL) model with a specific normalization procedure for the modeling of nanoscale heat transport. They concluded that the combination of the DPL model with mixed-type temperature boundary condition is able to predict the heat flux and temperature distribution obtained from the Boltzmann transport equation (BTE) more accurate than the ballistic-diffusive equations (BDE). Lee et al. [29] applied the DPL heat transfer model and solved it using an efficient numerical scheme involving the hybrid application of the Laplace transform and control volume methods in conjunction with hyperbolic shape functions to investigate the transient heat transfer in a thin metal film exposed to short-pulse laser heating. They showed that the phase lag of the heat flux tends to induce thermal waves with sharp wave-fronts separating heated and unheated zones in the metal film, while the phase lag of the temperature gradient destroys the waveforms and increases the thermally disturbed zone.Hu and Chen [30] studied the transient temperature distribution in a cracked half-plane under temperature impact loading using the DPL heat conduction model. More recently, Lam [31] provided a unified solution of parabolic and hyperbolic heat conduction models in a one-dimensional thin film subjected to a time-varying and spatially-decaying laser energy source incident on both surfaces.

All previous works adopting the DPL model for a constant or pulse heat flux or a sudden temperature change on a slab. In addition, the heat transfer analysis with periodic heat flux on finite slab has only been studied by Fourier and hyperbolic thermal wave equations [1]. Due to the particular form of the relationship between q and T in DPL model, the boundary condition in the case of time-dependent one is more complicated in comparison with the hyperbolic thermal wave model and it makes more difficult procedure to obtain analytical solution for DPL model.

This paper considers transient heat conduction in a finite slab exposed to a periodic heat flux by applying the DPL heat conduction model. For the first time, the analytical solution of the DPL heat conduction equation is derived using Laplace transform method and inversion theorem. Calculations are performed to exhibit the wavelike (hyperbolic) behavior by comparing it with the diffusive ones and the influence of temperature gradient relaxation time on the hyperbolic conduction characteristics is investigated that has not been yetreported. The temperature profiles at the front and rear surfaces are calculated for different values of $\tau_T\omega$, which have been presented for different values of $\tau_q\omega$ previously [1]. The non-Fourier effects are discussed by comparing the phase and amplitude between the front and rear surfaces.

2. ANALYSIS

Consider a slab as a finite medium with the thickness of L and insulated boundaries where one-dimensional heat conduction and constant thermal properties prevail. The medium is initially in equilibrium at temperature $T(x,0) = 0$, from time $t = 0$ the external surface at $x = 0$ is exposed to a periodic heat flux with the amplitude q_0 and the frequency ω. In this situation, the general DPL heat conduction Equation (3), and the boundary and the initial conditions in the non-dimensional form are:

$$\left(1+\Gamma\frac{\partial}{\partial Fo}\right)\frac{\partial^2 V}{\partial X^2}=\frac{\partial V}{\partial Fo}+\Lambda\frac{\partial^2 V}{\partial Fo^2} \tag{4}$$

$$-Fo_1\left(1+\Gamma\frac{\partial}{\partial Fo}\right)\frac{\partial V(0,Fo)}{\partial X}=Q(0,Fo)+\Lambda\frac{\partial Q(0,Fo)}{\partial Fo},\quad Q(0,Fo)=\cos\left(\frac{Fo}{Fo_1}\right) \tag{5a}$$

$$\frac{\partial V(1,Fo)}{\partial X}=0 \tag{5b}$$

$$V(X,0)=0,\quad \frac{\partial V(X,0)}{\partial Fo}=0,\quad Q(X,0)=0 \tag{6}$$

The governing equations are converted into non-dimensional equations using the following parameters:

$$V(X,Fo)=\frac{T(x,t)}{q_0/\rho c\omega L},\quad Q(X,Fo)=\frac{q(x,t)}{q_0},\quad Fo_1=\frac{a}{\omega L^2},\quad X=\frac{x}{L},\quad Fo=\frac{at}{L^2},\quad Ve^2=\Lambda=\frac{a\tau_q}{L^2},\quad \Gamma=\frac{a\tau_T}{L^2} \tag{7}$$

where ρ is the slab density, a is the slab thermal diffusivity, x and t are the coordinate and time variables, X is the dimensionless coordinate variable and V and $Q$are the dimensionless temperature distribution and heat flux density of the slab, respectively. Fo is the Fourier number (dimensionless time), Ve is the Vernotte number Fo_1 is the dimensionless incident heat flux frequency and $1/Ve=L/\sqrt{(a\tau_q)}$ denotes the dimensionless speed of propagation of temperature wave. Here, $X$is the dimensionless coordinate and Γ and Λ are the dimensionless temperature gradient and heat flux relaxation times, respectively.

If the Laplace transformation is applied to Equation (4) and (5), by taking into account the initial conditions (6), the following subsidiary equation and the BCs are obtained as:

$$\frac{d^2\bar{V}}{dX^2}-B\bar{V}=0,\quad B=\frac{s+\Lambda s^2}{1+\Gamma s} \tag{8}$$

$$-Fo_1(1+\Gamma s)\frac{d\bar{V}(0;s)}{d\bar{X}}=(1+\Lambda s)\bar{Q}(0;s),\quad \bar{Q}(0;s)=\frac{Fo_1^2 s}{1+(Fo_1 s)^2} \tag{9a}$$

$$\frac{d\bar{V}(1;s)}{dX}=0 \tag{9b}$$

where$s$ is the Laplace domain variable and B is a simple fractional function in Laplace domain. Equation (8) is solved with respect to the conditions of Equation (9),

$$\bar{V}(X;s)=\frac{Fo_1}{1+(Fo_1 s)^2}\frac{\sqrt{B}\cosh\left[\sqrt{B}(1-X)\right]}{\sinh\sqrt{B}} \tag{10}$$

In Equation (10) $\bar{V}(X;s)$ is the slab temperature field in Laplace domain. A similar problem in a finite medium is studied by Tzou[32] but with dirichlet boundary condition. For evaluating the inverse Laplace transform of DPL model he pointed out that:"*The branch points resulting from the mixed-derivative term in DPL governing equation (Equation (21) [32]) prevent me to obtaining an analytical inversion; therefore, I invoke the numerical algorithm developed previously [33] for the purpose of inversion*". In this study a complete analytical inversion of Equation (10) is provided and checked by applying the procedure of inversion on the temperature field obtained by Tzou [32] in the Laplace domain. Equation (10) can be expressed as:

$$\bar{V}(X;s)=\frac{Fo_1}{1+(Fo_1 s)^2}\times\left\{\frac{1+\frac{B(1-X)^2}{2!}+\frac{B^2(1-X)^4}{4!}+\cdots}{1+\frac{B}{3!}+\frac{B^2}{5!}+\cdots}\right\} \tag{11}$$

According to the definition of the inverse Laplace transformation [34] and the residue theorem (Bromwich contour integration), the original function of $\bar{V}(X;s)$ can be expressed as:

$$V(X,Fo)=\int_{\gamma-i\infty}^{\gamma+i\infty} e^{sFo}\bar{V}(X;s)\,ds = \sum_{n=1}^{M} residual\left[e^{sFo}\bar{V}(X;s),s_n\right] \tag{12}$$

where, $s_1,s_2,\ldots,s_n$ are all poles of $\bar{V}(X;s)$ and γ is the Bromwich line which all the above poles are in the left side of it on the complex plain. Letting the denominator of $\bar{V}(X;s)$ be zero, all of the poles can be obtained as follows:

$$1+(Fo_1 s)^2=0 \Rightarrow s_1=i/Fo_1\,,\quad s_2=-i/Fo_1 \tag{13a}$$

$$\sinh\sqrt{B}=0\Rightarrow \quad s_{1n},s_{2n}=\frac{-\left(1+\Gamma\mu_n^2\right)\pm\sqrt{\left(1+\Gamma\mu_n^2\right)^2-4\Lambda\mu_n^2}}{2\Lambda} \quad \mu_n=\frac{2n-1}{2}\pi,\quad n=1,2,3,\ldots \tag{13b}$$

In Equation (13) i is the imaginary unit of complex plain and μ is the eigenvalues of problem. Since all poles are of the first order, the residues are [34]:

$$\mathrm{Re}\left[e^{sFo}\bar{V}(X;s),s_1\right]=\frac{\sqrt{B\left(\frac{i}{Fo_1}\right)}\cosh\left[\sqrt{B\left(\frac{i}{Fo_1}\right)}(1-X)\right]}{2i\sinh\left[\sqrt{B\left(\frac{i}{Fo_1}\right)}\right]}e^{i\frac{Fo}{Fo_1}} \tag{14a}$$

$$\mathrm{Re}\left[e^{sFo}\bar{V}(X;s),s_2\right]=\frac{\sqrt{B\left(\frac{-i}{Fo_1}\right)}\cosh\left[\sqrt{B\left(\frac{-i}{Fo_1}\right)}(1-X)\right]}{-2i\sinh\left[\sqrt{B\left(\frac{-i}{Fo_1}\right)}\right]}e^{-i\frac{Fo}{Fo_1}} \tag{14b}$$

$$\sum_{m=1}^{2}\sum_{n=1}^{\infty}\mathrm{Re}\left[e^{sFo}\bar{V}(X;s),s_{mn}\right]=\sum_{m=1}^{2}\sum_{n=1}^{\infty}e^{s_{mn}Fo}\times \frac{Fo_1}{1+(Fo_1 s_{mn})^2}\frac{-2\mu_n^2(1+\Gamma s_{mn})\cos\left[\mu_n(1-X)\right]}{\left(2\Lambda s_{mn}+1+\Gamma\mu_n^2\right)\cos\mu_n} \tag{14c}$$

where,

$$B\left(\frac{i}{Fo_1}\right)=\frac{iFo_1-\Lambda}{Fo_1(i\Gamma+Fo_1)},\ B\left(\frac{-i}{Fo_1}\right)=\frac{iFo_1+\Lambda}{Fo_1(i\Gamma-Fo_1)} \tag{14d}$$

In Equation (14) denotes the residue. From Equation (14), the inverse transformation of $\bar{V}(X;s)$ is obtained as follows:

$$V(X,Fo)=\mathrm{Re}\left[e^{sFo}\bar{V}(X;s),s_1\right]+\mathrm{Re}\left[e^{sFo}\bar{V}(X;s),s_2\right] +\sum_{m=1}^{2}\sum_{n=1}^{\infty}\mathrm{Re}\left[e^{sFo}\bar{V}(X;s),s_{mn}\right] \tag{14e}$$

In other words, the dimensionless temperature response of the slab can be express as:

$$V(X,Fo)=\sqrt{\frac{iFo_1-\Lambda}{Fo_1(i\Gamma+Fo_1)}}e^{i\frac{Fo}{Fo_1}}\times\frac{\cosh\left[\sqrt{\frac{iFo_1-\Lambda}{Fo_1(i\Gamma+Fo_1)}}(1-X)\right]}{2i\sinh\left[\sqrt{\frac{iFo_1-\Lambda}{Fo_1(i\Gamma+Fo_1)}}\right]} +\sqrt{\frac{iFo_1+\Lambda}{Fo_1(i\Gamma-Fo_1)}}e^{-i\frac{Fo}{Fo_1}}\times\frac{\cosh\left[\sqrt{\frac{iFo_1+\Lambda}{Fo_1(i\Gamma-Fo_1)}}(1-X)\right]}{-2i\sinh\left[\sqrt{\frac{iFo_1+\Lambda}{Fo_1(i\Gamma-Fo_1)}}\right]} +\sum_{m=1}^{2}\sum_{n=1}^{\infty}\frac{Fo_1}{1+(Fo_1 s_{mn})^2}\times\frac{-2\mu_n^2(1+\Gamma s_{mn})\cos\left[\mu_n(1-X)\right]}{\left(2\Lambda s_{mn}+1+\Gamma\mu_n^2\right)\cos\mu_n}e^{s_{mn}Fo} \tag{15}$$

3. RESULTS AND DISCUSSION

By applying Equation (15), the numerical computation is performed in order to display the temperature profile arising from a periodic surface heat source at $x = 0$ on a finite slab. The temperature response during an oscillatory surface thermal disturbance with the period $Fo_1 = 0.25$ at the rear surface ($X= 1$) for $Ve = 0.8$ and different values for Γ is shownin Figure 1a.For checking the reasonability of the solution, we consider two limit situations of above solution according to the Lam's [31] study. One is $\tau_T\rightarrow 0$, i.e. the DPL solution should go back to the hyperbolic thermal wave solution and another limit situation of the above solution is $\tau_T\rightarrow \tau_q$, i.e. the DPL solution should go back to the parabolic Fourier solution [32]. As observers in this figure, when these limit situations are applied in the solution of DPL model, the solution coincides with the hyperbolic thermal wave and parabolic Fourier solutions.

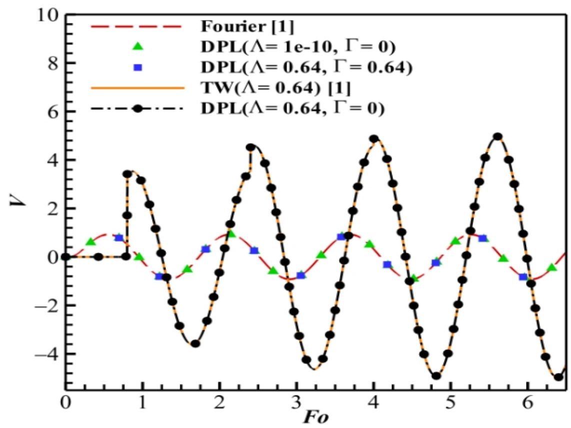

Figure 1a. Accuracy of the analytical procedure by applying suitable limit situations on the solution of the DPL based on the Lam [31] validation procedure.

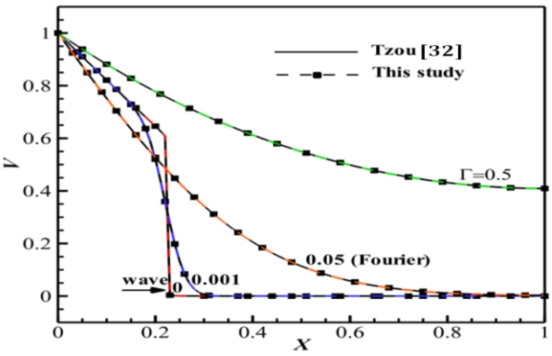

Figure 1b. Comparison between the results of the analytical and numerical inversion procedures are used in this and Tzou's [32] study, respectively when Fo = 0.05 and Λ= 0.05.

As mentioned in Sec. 2, the accuracy of the analytical inversion procedure is also checked by comparing it with the numerical one which is used by Tzou [32]. This comparison is conducted when both analytical and numerical inversion procedures are applied on the transformed temperature field in the Tzou's[32] study. The results of this process are shown in Figure 1b, where the consistency of them approved conducted analytical inversion once more.

The temperature response at both surfaces are shown in Figures 2a and 2b, respectively for a periodic surface heat flux with the period Fo_1= 0.25 and various values of $\Gamma/Fo_1 = \tau_T\omega$. A similar result with hyperbolic model for various values of $Ve^2/Fo_1 = \Lambda/Fo_1 = \tau_q\omega$ is presented by Tang and Araki [1]. They concluded that, at both surfaces, when $Ve \leq$ 0.07, the difference between the results from the hyperbolic and the parabolic equation is below 1%. In other words, the inequality $Ve \leq$ 0.07 represents the condition of the occurrence of difference in temperature response between the Fourier and the non-Fourier solution. In this study, by gradual increase of $\tau_T\omega$ in DPL model, between the range of parabolic and hyperbolic models (i.e. $0 \leq \tau_T\omega \leq 2.56$) when Ve= 0.8, the ability of the DPL model in covering the situations between the parabolic and hyperbolic models is illustrated at both front and rear surfaces in Figures 2a and 2b, respectively. The condition $\tau_T\omega$ = 0, 0.004, 0.4, 1.2 and 2.56 corresponds to Γ = 0, 0.001, 0.1, 0.3 and 0.64. The values Γ = 0 and 0.64 are related to the hyperbolic and parabolic models, respectively.

Tang and Araki [1] presented a discussion about the phase and amplitude difference between the front and rear surfaces of the periodic temperature response for the parabolic and hyperbolic models and concluded that this discussion is significant only for small values of Vernotte number, because increasing Vemakes the temperature response deviate from a normal periodic curve (see Figure 1 in [1]). According to the results of DPL model (like Figures 2a and 2b), it is not a comprehensive conclusion because when τ_T exists, the deviation from a normal periodic curve does not occur and a discussion about the phase and amplitude difference between the front and rear surfaces maybe important in high valuesof Vernotte number. In this regard, the results for phase and amplitude difference between the front and the rear surface temperature responses vs. $\Gamma^{1/2}$ number are presented in order to illustrate the effect of temperature gradient relaxation time on these parameters. The phase and amplitude difference between the front and the rear surface vs. $\Gamma^{1/2}$ number for a given period Fo_1= 0.25 and Ve = 0.8 are shown in Figures 3a and 3b, respectively. The vertical coordinates of these figures respectively are $\Delta\varphi = (\varphi_F - \varphi_R)/2\pi Fo_1$ and A_F-A_R. The phase difference φ_F-φ_R is determined by the time difference of the first minimum point in temperature response between the front and rear surfaces and $A_F$and A_R are the amplitude of the first minimum point of the front and rear surface, respectively [1]. The results indicate that the temperature gradient relaxation time has an important effect on the phase and amplitude difference between front and rear surfaces. As value of $\Gamma^{1/2}$ is increased, the phase and amplitude difference of the surfaces decrease and increase, respectively.

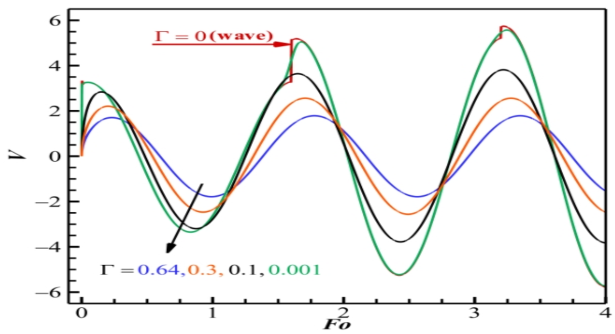

Figure 2a. Effect of $\tau_T\omega$ on the temperature response at the front surface (Fo_1=0.25)

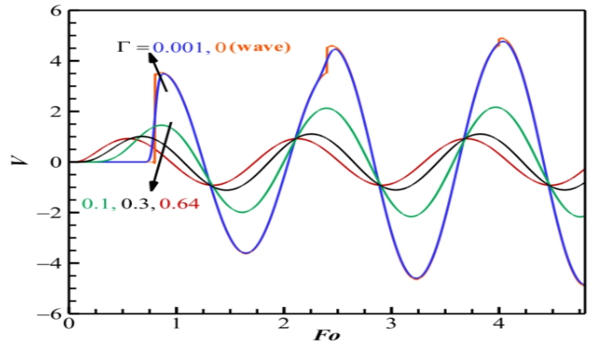

Figure 2b. Effect of $\tau_T\omega$ on the temperature response at the rear surface (Fo_1= 0.25)

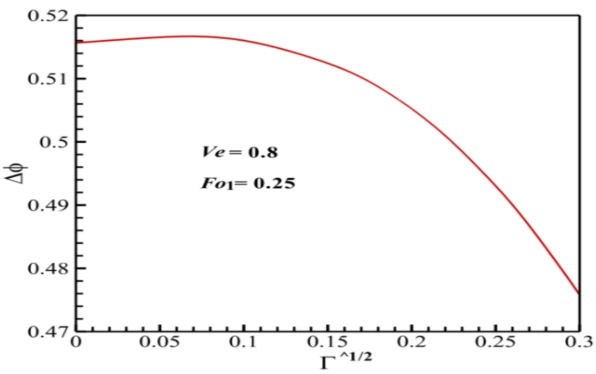

Figure 3a. Phase difference between the front and the rear surface as a function of number $\Gamma^{1/2}$

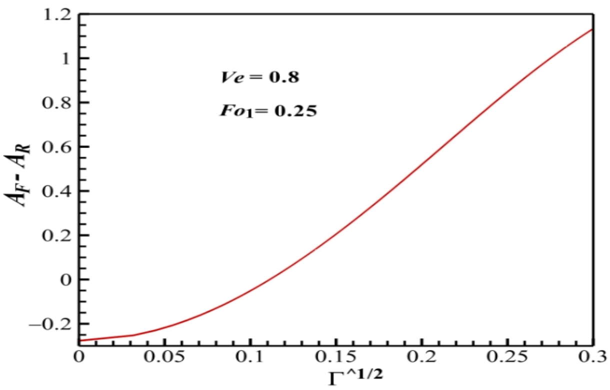

Figure 3b. Amplitude difference between the front and the rear surface as a function of number $\Gamma^{1/2}$

5. CONCLUSION

For the first time, the exact analytical solution of the dual-phase-lag (DPL) heat conduction equation under the condition of periodic thermal disturbance is derived. The existence of interpretations for the measurements with hyperbolic heat conduction equation in a finite slab subjected to a periodic thermal disturbance, suggests that additional study is needed to help clarify the nature of conduction using the dual-phase-lag (DPL) model in such slab. Corresponding to the actual situation that a finite rigid slab under periodic heating exists, various behaviors of heat conduction (the wavelike and diffusion), are obtained by adjusting the relaxation parameters in the DPL heat conduction equation. The temperature responses calculated here are compared with two previous related results in literature and the calculations show good agreement with them. For the first time, the effect of temperature gradient relaxation time on the phase and amplitude difference between the front and rear surfaces are computed and the results indicate that the phase and amplitude difference of the surfaces decrease and increase, respectively as the temperature gradient is increased.

6. REFERENCES

1. Tang, D. and Araki, N., "Non-fourier heat conduction in a finite medium under periodic surface thermal disturbance", ***International Journal of Heat and Mass Transfer***, Vol. 39, No. 8, (1996), 1585-1590.
2. Tzou, D.Y., "Experimental support for the lagging behavior in heat propagation", ***Journal of Thermophysics and Heat Transfer***, Vol. 9, No. 4, (1995), 686-693.
3. Tzou, D. and Chiu, K., "Temperature-dependent thermal lagging in ultrafast laser heating", ***International Journal of Heat and Mass Transfer***, Vol. 44, No. 9, (2001), 1725-1734.
4. Appleton, B. and Celler, G., "Laser and electron beam interactions with solids, Cambridge Univ Press, (1982).
5. Sutton, W.H., Brooks, M.H. and Chabinsky, I., Microwave processing of materials., Pittsburgh, PA (USA); Materials Research Society.(1988)
6. Prokhorov, A.M., Konov, V., Ursu, I. and Mihailescu, I.N., "Laser heating of metals, Hilger Bristol, (1990).
7. Joseph, D.D. and Preziosi, L., "Heat waves", ***Reviews of Modern Physics***, Vol. 61, No. 1, (1989), 41.
8. Ozisik, M. and Tzou, D., "On the wave theory in heat conduction", ***Journal of Heat Transfer***, Vol. 116, No. 3, (1994), 526-535.
9. Tzou, D.Y., "Macro-to micro-scale heat transfer: The lagging behavior, CRC Press, (1996).
10. Cattaneo, C., "Sur une forme de lequation de la chaleur eliminant le paradoxe dune propagation instantanee", ***Comptes Rendus Hebdomadaires Des Seances De L Academie Des Sciences***, Vol. 247, No. 4, (1958), 431-433.
11. Vernotte, P., "Les paradoxes de la theorie continue del'equation de la chaleur", ***Comptes Rendus Hebdomadaires Des Seances De L Academie Des Sciences***, Vol. 246, No., (1958), 3154-3155.
12. Taitel, Y., "On the parabolic, hyperbolic and discrete formulation of the heat conduction equation", ***International Journal of Heat and Mass Transfer***, Vol. 15, No. 2, (1972), 369-371.
13. Bai, C. and Lavine, A., "On hyperbolic heat conduction and the second law of thermodynamics", ***Journal of Heat Transfer***, Vol. 117, No. 2, (1995), 256-263.
14. Körner, C. and Bergmann, H., "The physical defects of the hyperbolic heat conduction equation", ***Applied Physics A***, Vol. 67, No. 4, (1998), 397-401.
15. Al-Nimr, M. and Naji, M., "On the phase-lag effect on the non-equilibrium entropy production", ***Microscale Thermophysical Engineering***, Vol. 4, No. 4, (2000), 231-243.
16. Ahmadikia, H. and Rismanian, M., "Analytical solution of non-fourier heat conduction problem on a fin under periodic boundary conditions", ***Journal of mechanical science and technology***, Vol. 25, No. 11, (2011), 2919-2926.
17. Brorson, S., Fujimoto, J. and Ippen, E., "Femtosecond electronic heat-transport dynamics in thin gold films", ***Physical Review Letters***, Vol. 59, (1987), 1962-1965.
18. Tang, D. and Araki, N., "Wavy, wavelike, diffusive thermal responses of finite rigid slabs to high-speed heating of laser-pulses", ***International Journal of Heat and Mass Transfer***, Vol. 42, No. 5, (1999), 855-860.
19. Antaki, P.J., "Solution for non-fourier dual phase lag heat conduction in a semiinfinite slab with surface heat flux",

International Journal of Heat and Mass Transfer, Vol. 41, No. 14, (1998), 2253-2258.

20. Antaki, P.J., "New interpretation of non-fourier heat conduction in processed meat", ***Journal of Heat Transfer***, Vol. 127, No. 2, (2005), 189-193.

21. Mitra, K., Kumar, S., Vedevarz, A. and Moallemi, M., "Experimental evidence of hyperbolic heat conduction in processed meat", ***Journal of Heat Transfer***, Vol. 117, No. 3, (1995), 568-573.

22. Horgan, C. and Quintanilla, R., "Spatial behaviour of solutions of the dual-phase-lag heat equation", ***Mathematical Methods in the Applied Sciences***, Vol. 28, No. 1, (2005), 43-57.

23. Liu, K.-C., "Analysis of dual-phase-lag thermal behaviour in layered films with temperature-dependent interface thermal resistance", ***Journal of Physics D: Applied Physics***, Vol. 38, No. 19, (2005), 3722.

24. Wang, L., Xu, M. and Zhou, X., "Well-posedness and solution structure of dual-phase-lagging heat conduction", ***International Journal of Heat and Mass Transfer***, Vol. 44, No. 9, (2001), 1659-1669.

25. Ang, W.-T., "A laplace transformation dual-reciprocity boundary element method for a class of two-dimensional microscale thermal problems", ***Engineering Computations***, Vol. 19, No. 4, (2002), 467-478.

26. Zhang, J. and Zhao, J.J., "Unconditionally stable finite difference scheme and iterative solution of 2d microscale heat transport equation", ***Journal of Computational Physics***, Vol. 170, No. 1, (2001), 261-275.

27. Han, P., Tang, D. and Zhou, L., "Numerical analysis of two-dimensional lagging thermal behavior under short-pulse-laser heating on surface", ***International Journal of Engineering Science***, Vol. 44, No. 20, (2006), 1510-1519.

28. Ghazanfarian, J. and Shomali, Z., "Investigation of dual-phase-lag heat conduction model in a nanoscale metal-oxide-semiconductor field-effect transistor", ***International Journal of Heat and Mass Transfer***, Vol. 55, No. 21, (2012), 6231-6237.

29. Lee, H.-L., Chen, W.-L., Chang, W.-J., Wei, E.-J. and Yang, Y.-C., "Analysis of dual-phase-lag heat conduction in short-pulse laser heating of metals with a hybrid method", ***Applied Thermal Engineering***, Vol. 52, No. 2, (2013), 275-283.

30. Hu, K. and Chen, Z., "Transient heat conduction analysis of a cracked half-plane using dual-phase-lag theory", ***International Journal of Heat and Mass Transfer***, Vol. 62, (2013), 445-451.

31. Lam, T.T., "A unified solution of several heat conduction models", ***International Journal of Heat and Mass Transfer***, Vol. 56, No. 1, (2013), 653-666.

32. Tzou, D., "A unified field approach for heat conduction from macro-to micro-scales", ***Journal of Heat Transfer***, Vol. 117, No. 1, (1995), 8-16.

33. Tzou, D., Ozisik, M. and Chiffelle, R., "The lattice temperature in the microscopic two-step model", ***Journal of Heat Transfer***, Vol. 116, No. 4, (1994), 1034-1038.

34. Arpaci, V.S., "Conduction heat transfer, Addison-Wesley Reading, (1966).

Analytical Analysis of the Dual-phase-lag Heat Transfer Equation in a Finite Slab with Periodic Surface Heat Flux

RESEARCH NOTE

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چکیده

در این مطالعه از مدل تأخیر فاز دوگانه معادله هدایت گرما برای تعیین اثر زمان آسایش حرارتی گرادیان دما روی نتایج هدایت گرمای غیر فوریه هذلولوی در یک ورق محدودی که در معرض اغتشاش گرمایی نوسانی قرار گرفته، استفاده شده است. مدل تأخیر فاز دوگانه ویژگی‌های موج گونه‌ای و پخش گونه‌ای هدایت گرما را ترکیب می‌کند که مدل هذلولوی قادر به این کار نیست. برای اولین بار حل تحلیلی مدل تأخیر فاز دوگانه معادله هدایت گرما با استفاده از روش تبدیل لاپلاس و تئوری وارونه سازی به دست آمده است. پروفیل‌های توزیع دما در سطح و پشت ورق برای زمان‌های مختلف آسایش حرارتی گرادیان دما به دست آمده‌اند. اختلاف فاز و دامنه بین پاسخ‌های دمایی سطح و پشت به عنوان تابعی از زمان آسایش حرارتی گرادیان دما که قبلاً به عنوان تابعی از زمان آسایش حرارتی شار گرما ارائه شده‌اند، به صورت عددی محاسبه شده‌اند. نتایج نشان می‌دهند که افزایش زمان آسایش حرارتی گرادیان دما باعث کاهش اختلاف فاز و افزایش اختلاف دامنه بین پاسخ‌های دمایی سطح و پشت می‌شود..

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