International Journal of Engineering

Journal Homepage: www.ije.ir

Mechanical Response of a Piezoelectrically Sandwiched Nano-beam Based on the Nonlocal Theory

A. Shah-Mohammadi-Azar, A. Khanchehgardan, G. Rezazadeh *, R. Shabani

Department of Mechanical Engineering, University of Urmia, Urmia, Iran

PAPER INFO

Paper history:
Received 03 May 2013
Received in revised form 11 June 2013
Accepted 20 June 2013

Keywords:
NEMS
Nonlocal Elasticity
Electric Potential
Piezoelectric Layer
Piezoelectric Actuation
Electrostatic Actuation

ABSTRACT

This article deals with the mechanical analysis of a fixed-fixed nano-beam based on nonlocal theory of elasticity. The nano-beam is sandwiched with two piezoelectric layers through its upper and lower surfaces. The electromechanical coupled equations governing the problem are derived based on nonlocal theory of elasticity considering Euler-Bernoulli beam assumptions. Also, nonlocal piezoelectricity is according to Maxwell's electrostatic equations. The piezoelectric layers are subjected to a voltage to tune the stiffness of the nano-beam. The equations are solved through step by step linearization method and Galerkin's weighted residual method. The results are compared with those of the local model. The effect of piezoelectric voltages on the non-locality of the model is investigated as well.

doi: *10.5829/idosi.ije.2013.26.12c.12*

1. INTRODUCTION

Nano-beams/tubes are vastly considered in the nano-electromechanical systems (NEMS); that is why studying the mechanical behavior of the tiny media has become more attractive during the last decade. For example, Shabani et al. [1] studied static and dynamic response of carbon nanotube nanotube based nano-tweezers. Jafari et al. [2] validated the shell theory for modeling the radial breathing mode of a single-walled carbon nanotube. Mohammadimehr et al. [3] investigated postbuckling equilibrium path of a long thin-walled cylindrical shell (Single-Walled Carbon Nanotube) under axial compression using energy method. Shafiei et al. [4] studied the effect of composition on properties of Cu-Ag nanocomposites synthesized by heat treatment. Scientists have found out that mechanical behavior in tiny structures may be strongly influenced by phonon dispersion and atomic interaction according to lattice dynamics; these length dependent properties of materials are considered in length scale constant. According to, classical theory of elasticity could not be exacted. Eringen attended stress at a point of a medium dependent on the whole points of the medium [5, 6]. Furthermore, the stress gradient should be considered. Many works have been focused on studying the mechanical behavior of the nano-beams/tubes based on the non-local theory.

Firstly, Eringen [7] proposed the general theory of non-local piezoelectricity. The problems for a crack or two cracks considering non-local theory in elastic or piezoelectric materials were investigated by Zhou and Jia [8] and Zhou et al. [9]. Ke et al. [10] investigated the nonlinear vibration of the piezoelectric nano-beams based on the nonlocal theory and Timoshenko beam theory, where the piezoelectric nano-beam is subjected to an applied voltage and a uniform temperature change. Ghorbanpour Arani et al. [11] studied electro-thermo nonlinear vibration of a piezo-polymeric rectangular micro plate made from polyvinylidene fluoride (PVDF) reinforced by zigzag double walled boron nitride nanotubes (DWBNNTs).

In spite of the few studies on nonlocal piezoelectric medium, there is a large number of the works on sandwiched and FG piezo beams. Rezazadeh et al. used piezoelectric actuation to control the pull-in voltage of a fixed–fixed and cantilever MEM (micro electro

*Corresponding Author Email: g.reazadeh@urmia.ac.ir (G. Rezazadeh)

mechanical) actuators [12]. Azizi et al. [13] modeled a FG Fixed-Fixed beam, the composite varying through the thickness from piezoelectric base to silicon. Also, Azizi et al. [14] studied parametric excitation of a piezoelectrically actuated system near Hopf bifurcation. Saeedi-Vahdat et al. [15] probed thermoelastic damping (TED) in a tunable micro-beam resonator sandwiched with a pair of piezoelectric layers. Ahmadian et al. [16] studied piezoelectric and functionally graded materials in designing electrostatically actuated micro switches.

In these achievements, the authors have employed linear electric potential distribution through the piezoelectric layer according to Crawley proposed model [17]. Quek and Wang [18] proposed electric potential distribution as the combination of half-cosine and linear distribution.

According to this model Pietrzakowski [19] investigated the effect of electric potential distribution on the piezoelectrically vibration control of composite Kirchhoff plate, and Shah-Mohammadi-Azar et al. [20] investigated the effect of electric potential distribution on electromechanical behavior of Micro-beam with piezoelectric and electrostatic actuation. Ke et al. [10] used combination of half-cosine and linear distribution of electric potential distribution to derive governing equation of the electric displacement through the piezoelectric layer based on nonlocal elasticity theory. To the authors' knowledge, the nonlocal model of piezoelectrically sandwiched nano-beam has not been studied yet.

In this paper, the coupled electromechanical equations of a sandwiched nano-beam based on nonlocal elasticity theory are derived. The nano-beam in the case of Fixed-Fixed boundary conditions attended to be parallel with a Fixed substrate and be applied to electrostatic and Casimir pressures. The effect of non-locality on the nano-beam static stability and deflection due to applied pressures is studied.

2. PROBLEM FORMULATION

As shown in Figure 1a fixed-fixed nano-beam (NEM actuator), bonded with piezoelectric layers from the upper and lower sides is considered. Length L, thickness h, width b, gap from the fixed substrate g_o and piezoelectric layers with thickness h_p are the nano-beam's geometrical properties. The nano-beam is actuated with an electrostatic voltage V and is tuned by a piezoelectric voltages V_p.

Firstly, Eringen [5] expressed the dependence of stress field at a point to strains of the body whole points. According to this stress tensor at a point x can be expressed as:

$$\sigma = \int_V K\left(\left|x' - x\right|, \tau\right)\Omega(x')dx' \tag{1}$$

where $\Omega(x)$ is the classical, macroscopic stress tensor at point x, and K denotes the nonlocal modulus, $|x'-x|$ the Euclidean distance, and τ the dimensionless length scale (material constant) which depends on the internal and external properties like the Lattice spacing and wave length.

$$\tau = \frac{e_0 a}{L} \tag{2}$$

e_0 is the material constant, and a and L are the internal and external characteristic lengths, respectively. For stress tensor we have following form:

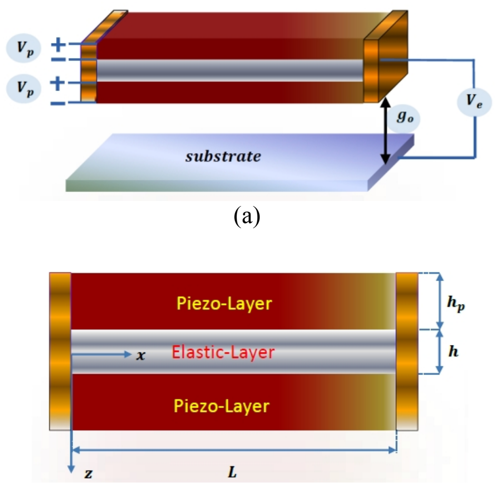
$$\Omega(x) = C(x):\varepsilon(x) \tag{3}$$

where $C(x)$ is the fourth order elasticity tensor and: denotes the 'double dot product'.

According to Eringen [7] expression coupled electromechanical nonlocal constitutive equations have the following form:

$$\begin{aligned} \sigma_{ij} - (e_0 a)^2 \nabla^2 \sigma_{ij} &= c_{ijkl}\varepsilon_{kl} - e_{kij}E_k \\ D_{ij} - (e_0 a)^2 \nabla^2 D_{ij} &= e_{ikl}\varepsilon_{kl} + \varepsilon_{ik}^{\varepsilon} E_k \end{aligned} \tag{4}$$

where E' is the Young's modulus and $\sigma(x)$ and $\varepsilon(x)$ are the normal stress and strain in the axial direction of the nonlocal nanotube. For limiting nano-scale $e_0 a \to 0$, the nonlocal effect can be neglected and the nonlocal stress σ approaches that of the corresponding classical stress $\sigma' = E'\varepsilon(x)$. Equation (3) in one-dimensional form can be reduced into Equations (5) and (6) [10]:



(a)

(b)

Figures 1. Schematic of the fixed- fixed NEM actuator with piezoelectric layers

$$\sigma_x - (e_0 a)^2 \frac{\partial^2 \sigma_x}{\partial x^2} = c_{11,b}\varepsilon_x - \bar{e}_{31} E_3 \tag{5}$$

$$\begin{aligned} D_1 - (e_0 a)^2 \frac{\partial^2 D_1}{\partial x^2} &= \bar{\varepsilon}_1^{\varepsilon} E_1 \\ D_3 - (e_0 a)^2 \frac{\partial^2 D_3}{\partial x^2} &= \bar{e}_{31}\varepsilon_x + \bar{\varepsilon}_3^{\varepsilon} E_3 \end{aligned} \tag{6}$$

where $\bar{c}_{11}, \bar{e}_{31}, \bar{\varepsilon}_3^{\varepsilon}$, $\bar{\varepsilon}_1^{\varepsilon}$ are indicated modified coefficients of piezoelectric material [21] which are presented in appendix 1, and for $b/h \geq 5$ the elastic layer effective modulus of elasticity can be approximated by plane stress as: $E = E'/(1-\nu^2)$.

The electric potential distribution in the actuator layers proposed like the form suggested by Quek and Wang [18], as the combination of a half-cosine and linear variation as following:

$$\Phi(x,z,t) = -\cos(\frac{\pi z_1}{h_p})\Phi_p(x,t) + \frac{z_1}{h_p}\Phi_a(t) \tag{7}$$

where z_1 is the local coordinate measured from the center of the piezoelectric layer center. The first term of Equation (7) shows direct piezoelectric layer effect, famed as eigen potential, induced due to mechanical strain of the beam and $\Phi_a = -V_p$ is the external applied potential difference along the piezoelectric layer thickness of the actuator layer. According to electrostatic Maxwell's electric field equation of the beam , electric field and electric potential are related to each other as:

$$\begin{aligned} \nabla \times E = 0 \Rightarrow E_1 &= -\frac{\partial \Phi(x,z)}{\partial x}, \\ E_3 &= -\frac{\partial \Phi(x,z)}{\partial z} \Rightarrow \\ E_1 &= \cos(\frac{\pi z_1}{h_p})\frac{\partial \Phi_p}{\partial x}, \\ E_3 &= -\frac{\pi}{h_p}\sin(\frac{\pi z_1}{h_p})\Phi_p + \frac{V_p}{h_p} \end{aligned} \tag{8}$$

On the other hand, the free charge density ρ_f inside the piezoelectric fixed-fixed nano-beam is zero. The electrostatic Maxwell's electric displacement equation is as following :

$$\nabla . D = \rho_f \ \ ,\rho_f = 0 \Rightarrow \frac{\partial D_3}{\partial z} + \frac{\partial D_1}{\partial x} = 0 \tag{9}$$

If we substitute E_1 and E_3 of Equation (8) and $\varepsilon_x = -z\partial^2 w / \partial x^2$ we will have:

$$\begin{aligned} & D_3 - \frac{\partial^2 D_3}{\partial x^2} = -\bar{e}_{31} z \frac{\partial^2 w}{\partial x^2} + \\ & \bar{\varepsilon}_3^{\varepsilon}(-\frac{\pi}{h_p}\sin(\frac{\pi z_1}{h_p})\Phi_p + \frac{V_p}{h_p}) \\ & D_1 - \frac{\partial^2 D_1}{\partial x^2} = \bar{\varepsilon}_1^{\varepsilon}(\cos(\frac{\pi z_1}{h_p})\frac{\partial \Phi_p}{\partial x}) \end{aligned} \tag{10}$$

By applying weighted integration over the electric displacements in Equation (10), we can obtain Equations (11) and (12) [10], also considering Equation (9) at last Equation (13) is obtainable.

$$\begin{aligned} & \int_{-\frac{h_p}{2}}^{\frac{h_p}{2}} (D_3 - \frac{\partial^2 D_3}{\partial x^2})(\frac{\pi}{h_p}\sin(\frac{\pi z_1}{h_p}))dz = \\ & \int_{-\frac{h_p}{2}}^{\frac{h_p}{2}} \left(\begin{aligned} & -\bar{e}_{31} z \frac{\partial^2 w}{\partial x^2} + \\ & \bar{\varepsilon}_3^{\varepsilon}(-\frac{\pi}{h_p}\sin(\frac{\pi z_1}{h_p})\Phi_p + \frac{V_p}{h_p}))(\frac{\pi}{h_p}\sin(\frac{\pi z_1}{h_p}) \end{aligned} \right) dz \end{aligned} \tag{11}$$

$$\begin{aligned} & \int_{-\frac{h_p}{2}}^{\frac{h_p}{2}} (D_1 - \frac{\partial^2 D_1}{\partial x^2})\cos(\frac{\pi z_1}{h_p})dz = \\ & \int_{-\frac{h_p}{2}}^{\frac{h_p}{2}} \bar{\varepsilon}_1^{\varepsilon} \frac{\partial \Phi_p}{\partial x}\cos(\frac{\pi z_1}{h_p})^2 dz \end{aligned} \tag{12}$$

$$-\frac{2\bar{e}_{31}h_p}{\pi}\frac{\partial^2 w}{\partial x^2} - \frac{\pi^2 \bar{\varepsilon}_3^{\varepsilon}}{2h_p}\Phi_p + \frac{h_p \bar{\varepsilon}_1^{\varepsilon}}{2}\frac{\partial^2 \Phi_p}{\partial x^2} = 0 \tag{13}$$

In order to obtain the governing equation of the lateral vibration of the nano-beam, the total moment at a given cross section of the nano-beam is:

$$M_{tot} = M_{piezo} + M_{elastic} \tag{14}$$

where $M_{elastic}$ relates to the elastic layer moment, and M_{piezo} relates to the bonded piezoelectric layers moment. And also based on the Euler-Bernoulli moment we have:

$$M = \int \sigma_x z dA \tag{15}$$

Considering Equation (5) and substituting E_1, E_3 and ε_x as told before the moment of piezoelectric and elastic layers will be as:

$$\begin{aligned} M_{piezo} &= \int_{-\frac{h}{2}-h_s}^{-\frac{h}{2}} \left((e_0 a)^2 \frac{\partial^2 \sigma_x}{\partial x^2} + \bar{c}_{11}\varepsilon_x - \bar{e}_{31}E_3 \right) zbdz \\ &+ \int_{\frac{h}{2}}^{\frac{h}{2}+h_s} \left((e_0 a)^2 \frac{\partial^2 \sigma_x}{\partial x^2} + \bar{c}_{11}\varepsilon_x - \bar{e}_{31}E_3 \right) zbdz \\ M_{elastic} &= \int_{-\frac{h}{2}}^{\frac{h}{2}} \left((e_0 a)^2 \frac{\partial^2 \sigma_x}{\partial x^2} + E_e \varepsilon_x \right) zbdz \end{aligned} \tag{16}$$

Referring to Equations (14) and (16):

$$M_{tot} = M_{piezo} + M_{elastic} = (e_0 a)_{eq}^2 \frac{\partial^2 M_{tot}}{\partial x^2} - (EI)_{eq} \frac{\partial^2 w}{\partial x^2} + \frac{4b\overline{e}_{31} h_p}{\pi} \Phi_p \tag{17}$$

where w is the deflection and $(e_0 a)_{eq}$ the equal nonlocal parameter assumed to be equal with each of the layers nonlocal parameter, and $(EI)_{eq}$ the equal stiffness rigidity as: $(EI)_{eq} = (EI)_{elastic} + (EI)_{piezo}$. Also based on the Euler-Bernoulli beam theory we have:

$$\frac{\partial^2 M_{tot}}{\partial x^2} + N_p \frac{\partial^2 w}{\partial x^2} + q(x,t) = (\rho bh)_{eq} \frac{\partial^2 w}{\partial t^2} \tag{18}$$

where, $(\rho bh)_{eq}$ is the equivalent mass of the nano-beam per unit length, and $q(x,t)$ a distributed force on the nano-beam. The stretching force N_p induced by applied piezoelectric voltages is [20]:

$$N_p = 2\overline{e}_{31} b \Phi_a(t) = -2\overline{e}_{31} b V_p \tag{19}$$

Considering Equation (17) and (19) we can obtain:

$$\begin{aligned} &((e_0 a)_{eq}^2 N_p + (EI)_{eq}) \frac{\partial^4 w}{\partial x^4} - \frac{4b\overline{e}_{31} h_p}{\pi} \frac{\partial^2 \Phi_p}{\partial x^2} \\ &((e_0 a)_{eq}^2 (\rho bh)_{eq}) \frac{\partial^4 w}{\partial x^2 \partial t^2} + (e_0 a)_{eq}^2 \frac{\partial^2 q(x,t)}{\partial x^2} \\ &+ (\rho bh)_{eq} \frac{\partial^2 w}{\partial t^2} - N_p \frac{\partial^2 w}{\partial x^2} = q(x,t) \end{aligned} \tag{20}$$

Applied pressures to the nano-beam are:

$$q = \frac{\varepsilon_o b V^2}{2(g_o - w)^2} + \frac{\hbar c b \pi^2}{240(g_o - w)^4} \tag{21}$$

where the first and second terms of Equation (32) refer to electrostatic [22], and Casimir pressure [23] respectively. $\hbar = 1.055 \times 10^{-34} Js$ is the Planck's constant, $C = 2.998 \times 10^8 ms^{-1}$ the speed of light, and $A = \pi^2 C \rho_1^2$ Hamaker constant which lies in the range of $(0.4-4) \times 10^{-19} J$ [24]. Considering Equations (20) and (21) the governing equation of the deflection of the nano-beam can be simplified as:

$$\begin{aligned} &((EI)_{eq} + (e_0 a)_{eq}^2 N_p) \frac{\partial^4 w}{\partial x^4} - (e_0 a)_{eq}^2 \rho bh \frac{\partial^4 w}{\partial x^2 \partial t^2} \\ &+ \rho bh \frac{\partial^2 w}{\partial t^2} - \frac{4b\overline{e}_{31} h_p}{\pi} \frac{\partial^2 \Phi_p}{\partial x^2} + (e_0 a)_{eq}^2 (\frac{\partial w}{\partial x})^2 \\ &(\frac{3\varepsilon_o b V^2}{(g_o - w)^4} + \frac{\hbar c b \pi^2}{12(g_o - w)^6}) + (e_0 a)_{eq}^2 \frac{\partial^2 w}{\partial x^2} \\ &(\frac{\varepsilon_o b V^2}{(g_o - w)^3} + \frac{\hbar c b \pi^2}{60(g_o - w)^5}) = \frac{\varepsilon_o b V^2}{2(g_o - w)^2} \\ &+ \frac{\hbar c b \pi^2}{240(g_o - w)^4} \end{aligned} \tag{22}$$

The accompanying boundary conditions for the fixed-fixed open circuit [25] sandwiched nano-beam are:

$$\begin{aligned} &\Phi_p(0) = 0, \quad \Phi_p(L) = 0, \\ &w(0) = 0, \quad w(L) = 0, \frac{\partial w}{\partial x}(0) = 0; \quad \frac{\partial w}{\partial x}(L) = 0 \end{aligned} \tag{23}$$

In order to make analysis easier, we prefer to non-dimensionlize the governing equations as following:

$$\frac{\partial^2 \widehat{w}}{\partial \xi^2} + \chi_1 \Phi_p + \chi_2 \frac{\partial^2 \Phi_p}{\partial \xi^2} = 0 \tag{24}$$

$$\begin{aligned} &(\kappa_1 + 1) \frac{\partial^4 \widehat{w}}{\partial \xi^4} + \kappa_2 \frac{\partial^4 \widehat{w}}{\partial \xi^2 \partial t^{*2}} + \frac{\partial^2 \widehat{w}}{\partial t^{*2}} + \kappa_3 \frac{\partial^2 \Phi_p}{\partial \xi^2} \\ &+ \kappa_4 \frac{\partial^2 \widehat{w}}{\partial \xi^2} + \frac{\partial^2 \widehat{w}}{\partial \xi^2} (\frac{\kappa_5 V^2}{(1-\widehat{w})^3} + \frac{\kappa_6}{(1-\widehat{w})^5}) + (\frac{\partial \widehat{w}}{\partial \xi})^2 \\ &(\frac{\kappa_7 V^2}{(1-\widehat{w})^4} + \frac{\kappa_8}{(1-\widehat{w})^6}) = \frac{\alpha V^2}{(1-\widehat{w})^2} + \frac{\beta}{(1-\widehat{w})^4} \end{aligned} \tag{25}$$

The new parameters are defined in appendix 2.

3. SOLUTION

In the static analysis, because of confronting with nonlinear force terms, gradual increment of the applied force is considered, which results in deflection and electric potential distribution in a slow trend. To this, according to the step by step linearization method (SSLM), governing equations are solved in sequent steps of force increment in linearized form keeping the first two terms of Taylor expansion. The value of the electrostatic pressure depends on both the voltage and gap where the value of Casimir pressure only depends on the gap.

In order to apply SSLM to Casimir pressure, we need a virtual parameter for gradually applying of this force. This can be possible by multiplying Casimir pressure to a virtual parameter (Υ) and changing the virtual parameter from zero to 1, step by step [26].

$$\begin{cases} V^{k+1} = V^k + \delta V \\ \Upsilon^{k+1} = \Upsilon^k + \delta\Upsilon \end{cases} \Rightarrow \begin{cases} w^{k+1} = w^k + \delta w = w^k + \Omega^k \\ \Phi_p^{k+1} = \Phi_p^k + \delta\Phi_p = \Phi_p^k + \varpi^k \end{cases} \tag{26}$$

Ω^k and ϖ^k are the deflection and electric potential increment in $(k)th$ step. In $(k)th$ step we have:

$$(1+\kappa_1)\frac{\partial^4 \widehat{w}^k}{\partial\xi^4}+\kappa_3\frac{\partial^2\Phi_p^i}{\partial\xi^2}+\kappa_4\frac{\partial^2\widehat{w}^k}{\partial\xi^2}+\frac{\partial^2\widehat{w}^k}{\partial\xi^2}$$
$$(\frac{\kappa_5(V^k)^2}{(1-\widehat{w}^k)^3}+\frac{\kappa_6}{(1-\widehat{w}^k)^5})+(\frac{\partial\widehat{w}^k}{\partial\xi})^2(\frac{\kappa_7(V^k)^2}{(1-\widehat{w}^k)^4}$$
$$+\frac{\kappa_8}{(1-\widehat{w}^k)^6})=\frac{\alpha(V^k)^2}{(1-\widehat{w}^k)^2}+\Upsilon^k\frac{\beta}{(1-\widehat{w}^k)^4} \tag{27}$$

$$\frac{\partial^2\widehat{w}^k}{\partial\xi^2}+\chi_1\Phi_p^k+\chi_2\frac{\partial^2\Phi_p^k}{\partial\xi^2}=0 \tag{28}$$

And in the case of $(k+1)th$ step, substituting Equation (26) into the statically governing equations and subtracting the terms indicating into $(k)th$ step we can obtain:

$$(1+\kappa_1)\frac{\partial^4\Omega^k}{\partial\xi^4}+\kappa_3\frac{\partial^2\varpi^k}{\partial\xi^2}+\kappa_4\frac{\partial^2\Omega^k}{\partial\xi^2}+\frac{\partial^2\widehat{w}^k}{\partial\xi^2}$$
$$((\frac{3\kappa_5(V^k)^2}{(1-\widehat{w}^k)^4}+\frac{5\kappa_6\Upsilon^k}{(1-\widehat{w}^k)^6})\Omega^k+\frac{2\kappa_5V^k\delta V}{(1-\widehat{w}^k)^3}$$
$$+\frac{\kappa_6\delta\Upsilon}{(1-\widehat{w}^k)^5})+\frac{\partial^2\Omega^k}{\partial\xi^2}(\frac{\kappa_5(V^k)^2}{(1-\widehat{w}^k)^3}+\frac{\kappa_6\Upsilon^k}{(1-\widehat{w}^k)^5}$$
$$+\frac{2\kappa_5V^k\delta V}{(1-\widehat{w}^k)^3}+\frac{\kappa_6\delta\Upsilon}{(1-\widehat{w}^k)^5})+(\frac{\partial\widehat{w}^k}{\partial\xi})^2((\frac{4\kappa_7(V^k)^2}{(1-\widehat{w}^k)^5}$$
$$+\frac{6\kappa_8\Upsilon^k}{(1-\widehat{w}^k)^7})\Omega^k+\frac{2\kappa_7V^k\delta V}{(1-\widehat{w}^k)^4}+\frac{\kappa_8\delta\Upsilon}{(1-\widehat{w}^k)^6})$$
$$+2\frac{\partial\widehat{w}^k}{\partial\xi}\frac{\partial\Omega^k}{\partial\xi}(\frac{\kappa_7(V^k)^2}{(1-\widehat{w}^k)^4}+\frac{\kappa_8\Upsilon^k}{(1-\widehat{w}^k)^6}$$
$$+\frac{2\kappa_7V^k\delta V}{(1-\widehat{w}^k)^4}+\frac{\kappa_8\delta\Upsilon}{(1-\widehat{w}^k)^6})=\frac{2\alpha V^k\delta V}{(1-\widehat{w}^k)^2}$$
$$+\frac{\beta\delta\Upsilon}{(1-\widehat{w}^k)^4}+(\frac{2\alpha(V^k)^2}{(1-\widehat{w}^k)^3}+\frac{4\beta\Upsilon^k}{(1-\widehat{w}^k)^5})\Omega^k \tag{29}$$

$$\frac{\partial^2\Omega^k}{\partial\xi^2}+\chi_1\varpi^k+\chi_2\frac{\partial^2\varpi^k}{\partial\xi^2}=0 \tag{30}$$

where ϖ^k and Ω^k are assumed as a combination of a complete set of n linearly independent shape functions [27] as:

$$\Omega^n\cong\sum_{k=1}^{n}a^k\mu^k(\xi),\quad \varpi^n\cong\sum_{k=1}^{n}b^k\beta^k(\xi) \tag{31}$$

By substituting Equation (31) into Equations (29) and (30), and employing Galerkin weighted residual method we can obtain:

$$\left[\boldsymbol{K^m}-\boldsymbol{K^e}\right]_{\boldsymbol{n}\times\boldsymbol{n}}\left[\boldsymbol{a}\right]_{\boldsymbol{n}\times 1}+\left[\boldsymbol{K^{pi1}}\right]_{\boldsymbol{n}\times\boldsymbol{n}}\left[\boldsymbol{b}\right]_{\boldsymbol{n}\times 1}=\left[\boldsymbol{F}\right]_{\boldsymbol{n}\times 1}$$
$$\left[\boldsymbol{K^a}\right]_{\boldsymbol{n}\times\boldsymbol{n}}\left[\boldsymbol{a}\right]_{\boldsymbol{n}\times 1}+\left[\boldsymbol{K^{pi2}}\right]_{\boldsymbol{n}\times\boldsymbol{n}}\left[\boldsymbol{b}\right]_{\boldsymbol{n}\times 1}=0 \tag{32}$$

where the elements of the matrices are presented in Appendix 3.

4. NUMERICAL RESULTS

4. 1. Validation of Results Using the foregoing numerical procedure and writing a code in Matlab software results for different cases are obtained and for some cases are compared with those, available in the literature.

To this end, we compared our results for the static pull-in voltage with those of the MEMCAD (3D simulation) [28] and FDM (Finite difference method) [12] for a fixed-fixed electrostatic MEM actuator. For this purpose, we considered properties of the MEM actuator with Young's modulus $169GPa$, thickness $3\mu m$, width $50\mu m$ and gap $1\mu m$ for two different Length values, and settled zero the Casimir pressure and piezoelectric layer thickness. The results and comparisons are illustrated in Table 1.

4. 2. Current Paper Results Here as a case study PZT-4 is considered as the piezoelectric layers, which are bonded to the upper and lower side of the silicon nano-beam with original mechanical and electrical properties [29], and geometrical dimensions as listed in Table 1.

In order to investigate the effect of non-locality, variation of the pull-in voltage versus the mid-point deflection of the fixed-fixed nano-beam considering different nonlocal parameters are shown in Figure 2. As shown in Figure 2, by increasing the nonlocal parameter, the pull-in voltage has a decreasing trend, where nonlocal parameter increases, the difference between the results of the classic theory and nonlocal theory is significant. Also in Figure 3 variation of the non-dimensional frequency versus the applied electrostatic voltage considering different nonlocal parameters are shown.

TABLE 1. The values of the pull-in voltages for the MEM actuator

	Obtained results*	MEM CAD [28]	FDM [12]	Δ_1 (%)	Δ_2 (%)
L=350 μm	20.2	20.3	20.2	0.49	0
L=250 μm	39.4	40.1	39.5	1.75	0.25

Similar to the results presented in Figure 2, by increasing the nonlocal parameter differences between the results of the classic theory and nonlocal theory increase. As it is clear from Figures 2 and 3, in the case of the fixed-fixed nano-beam, increasing the nonlocal parameter results in decrement of the pull-in voltage.

Deflection of the fixed-fixed nano-beam due to applied pressures for different nonlocal parameters are shown in Figure 4,. By increasing the effect of non-locality, lateral deflection of the nano-beam increases. In Figure 5, effect of non-locality for different length to thickness ratios is investigated. As shown in this figure, if the thickness of the nano-beam is comparable with its length, non-locality effect on the mechanical behavior (pull-in instability) of the nano-beam becomes more important. In the case of lower length to thickness ratios, especially when the nonlocal parameter increases, results of the local and nonlocal elasticity theories differ with each other significantly. On the other hand, for higher length to thickness ratios results of the nonlocal theory converge to those of the local theory and the effect of increasing the nonlocal parameter becomes trivial.

TABLE 2. Geometrical and material properties of the silicon nano-beam and Piezoelectric layers

	Nano-beam	Piezoelectric Layer
L	$50nm$	
b	$50nm$	
h	$2nm$	$2nm$
E	$169GPa$	
ν	0.06	0.3
ρ	$2,231Kg/m^3$	$7,500Kg/m^3$
e_{31}	-	-5.2
e_{33}	-	15.1
c_{11}	-	$13.9GPa$
c_{12}	-	$7.78GPa$
c_{33}	-	$11.743GPa$
c_{13}	-	$7.428GPa$
$\varepsilon_3^{\varepsilon}$	-	$5.62\times10^{-9}\ F/m$
$\varepsilon_1^{\varepsilon}$	-	$6.46GPa$
g_o	$10nm$	
ε_o	$8.854187\times10^{-12}\ F/m$	

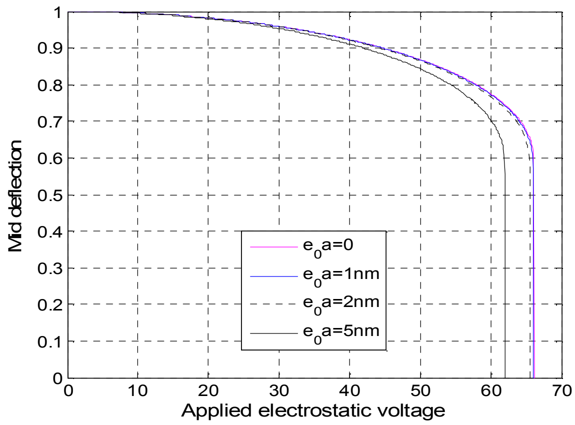


Figure 2. The mid-deflection versus applied electrostatic voltage considering different nonlocal parameters and $V_P = 0\,volt$.

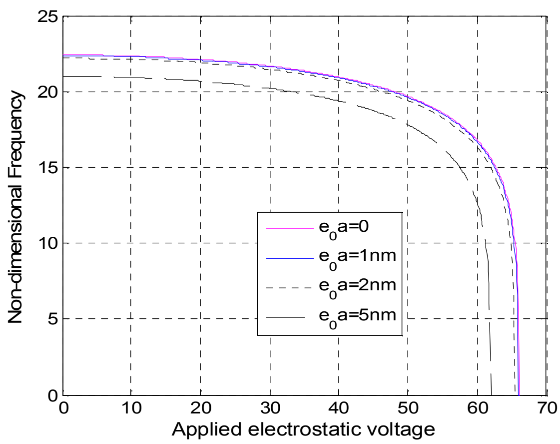


Figure 3. Non-dimensional frequency versus the applied electrostatic voltage considering different nonlocal parameters and $V_P = 0\,volt$.

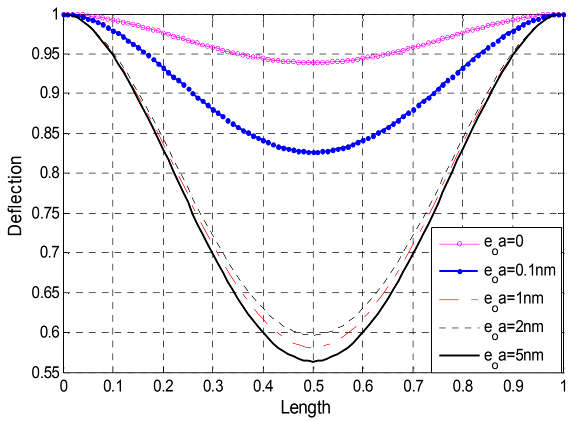


Figure 4. Deflection versus the Length considering different nonlocal parameters and $V_P = 0\,volt$.

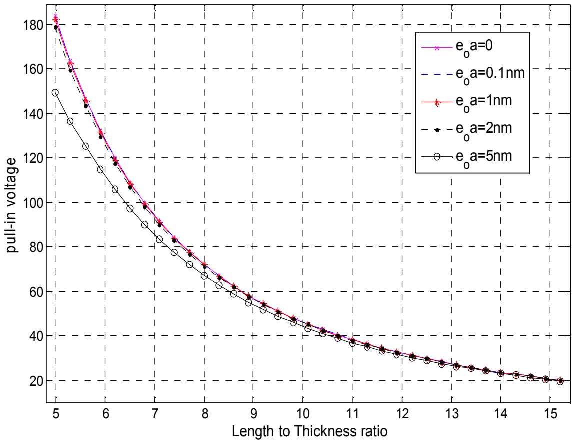


Figure 5. Pull-in voltage versus the Length to thickness ratio considering different nonlocal parameters and $V_P = 0\,volt$.

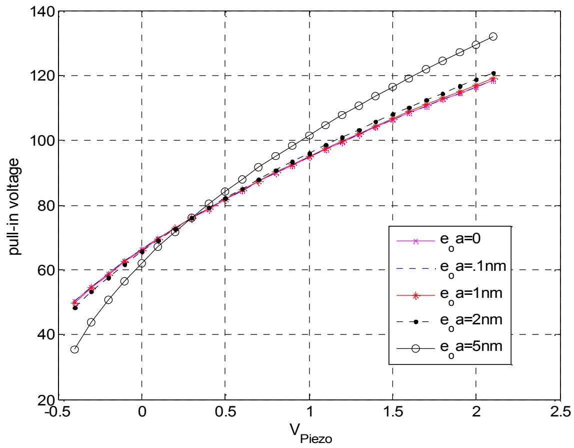


Figure 6. Pull-in voltage versus the Piezoelectric voltage considering different nonlocal parameters.

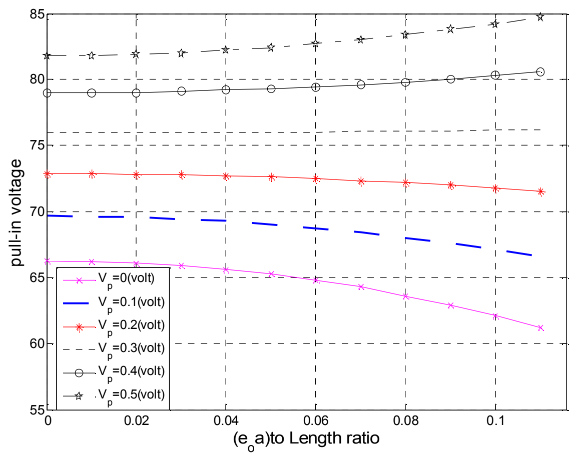


Figure 7. Pull-in voltage versus the Nonlocal parameter to length ratio considering different nonlocal parameters.

In Figure 6 effect of axial force (piezoelectric voltage) on the non-local behavior of the nano-beam is investigated.

By increasing the applied piezoelectric voltages, the pull-in voltage using nonlocal theory has higher increment rate compared to that of the local theory.The important point in Figure 6 is the coincidence of the results of local and nonlocal theories in a special applied piezoelectric voltage, which means, depending on the value of the applied piezoelectric voltage, the pull-in voltages using nonlocal theory may be more or less than the results of the local theory.

Figure 7 shows the effect of nonlocal parameter on the pull-in value for different applied piezoelectric voltages. As shown, increasing the nonlocal parameter for lower applied piezoelectric voltages (lower than ≈ 0.3 volt for the studied case study) decreases the pull-in voltage value but for higher piezoelectric voltages (higher than ≈0.3 volt) the opposite takes place. As one more point, applying particular piezoelectric voltage (in this case study ≈ 0.3 (volt)) the results of the local and nonlocal theories are close together.

5. CONCLUSION

In this paper, mechanical behavior of an electro-statically actuated fixed-fixed nano-beam, sandwiched with two piezoelectric layers and subjected to a tuning voltage was investigated. The coupled equations governing the electromechanical behavior of the nano-beam based on the nonlocal theory of elasticity were obtained and solved numerically. The results showed that increasing the nonlocal parameter for a given applied voltage increases the beam deflection and consequently decreases the pull-in voltage. The effect of the length to thickness ratio of the beam on the difference between results of the classic and nonlocal theories was investigated, and showed that for the lower length to thickness ratios this difference is significant. Furthermore, it was showed that the nonlocal theory has softening effect when the applied piezoelectric voltage is zero, but in the presence of the tuning applied piezoelectric voltage, nonlocal theory has softening and stiffening effect depending on the piezoelectric voltage.

6. REFERENCES

1. Shabani, R., Sharafkhani, N. and Gharebagh, V., "Static and dynamic response of carbon nanotube-based nanotweezers", ***International Journal of Engineering-Transactions A: Basics***, Vol. 24, No. 4, (2011), 377.

2. Basir Jafari, S., Esmaeilzadeh Khadem, S. and Malekfar, R., "Validation of shell theory for modeling the radial breathing mode of a single-walled carbon nanotube", ***International***

Journal of Engineering-Transactions A: Basics, Vol. 26, No. 4, (2013), 447-454.

3. Mohammadimehr, M., Saidi, A. R., Ghorbanpour Arani, A. and Han, Q., "Post buckling equilibrium path of a long thin-walled cylindrical shell (single-walled carbon nanotube) under axial compression using energy method", ***International Journal of Engineering-Transactions A: Basics***, Vol. 24, (2011), 79-86.

4. Shafiei, S. and Salahi, E., "Effect of combination on properties of cu-ag nanocomposites synthesized by heat treatment", ***International Journal of Engineering-Transactions B: Applications***, Vol. 23, No. 3&4, (2010), 209.

5. Eringen, A. C., "Nonlocal polar elastic continua", ***International Journal of Engineering Science***, Vol. 10, No. 1, (1972), 1-16.

6. Eringen, A. C., "On differential equations of nonlocal elasticity and solutions of screw dislocation and surface waves", ***Journal of Applied Physics***, Vol. 54, No. 9, (1983), 4703-4710.

7. Eringen, A. C., "Theory of nonlocal piezoelectricity", ***Journal of Mathematical Physics***, Vol. 25, (1984), 717.

8. Zhou, Z. and Jia, D., "The scattering of harmonic elastic anti-plane shear waves by two collinear symmetric cracks in infinite long strip using the non-local theory", ***Mechanics Research Communications***, Vol. 27, No. 3, (2000), 307-318.

9. Zhou, Z.-G., Du, S.-Y. and Wang, B., "Investigation of anti-plane shear behavior of a griffith crack in a piezoelectric material by using the non-local theory", ***International Journal of Fracture***, Vol. 111, No. 2, (2001), 105-117.

10. Ke, L.-L., Wang, Y.-S. and Wang, Z.-D., "Nonlinear vibration of the piezoelectric nanobeams based on the nonlocal theory", ***Composite Structures***, Vol. 94, No. 6, (2012), 2038-2047.

11. Arani, A. G., Vossough, H., Kolahchi, R. and Barzoki, A. M., "Electro-thermo nonlocal nonlinear vibration in an embedded polymeric piezoelectric micro plate reinforced by dwbnnts using dqm", ***Journal of Mechanical Science and Technology***, Vol. 26, No. 10, (2012), 3047-3057.

12. Rezazadeh, G., Tahmasebi, A. and Zubstov, M., "Application of piezoelectric layers in electrostatic mem actuators: Controlling of pull-in voltage", ***Microsystem Technologies***, Vol. 12, No. 12, (2006), 1163-1170.

13. Azizi, S., Ghazavi, M.-R., Esmaeilzadeh Khadem, S., Yang, J. and Rezazadeh, G., "Stability analysis of a parametrically excited functionally graded piezoelectric, mem system", ***Current Applied Physics***, Vol. 12, No. 2, (2012), 456-466.

14. Azizi, S., Rezazadeh, G., Ghazavi, M.-R. and Khadem, S. E., "Parametric excitation of a piezoelectrically actuated system near hopf bifurcation", ***Applied Mathematical Modelling***, Vol. 36, No. 4, (2012), 1529-1549.

15. Vahdat, A. S., Rezazadeh, G. and Ahmadi, G., "Thermoelastic damping in a micro-beam resonator tunable with piezoelectric layers", ***Acta Mechanica Solida Sinica***, Vol. 25, No. 1, (2012), 73-81.

16. Hosseinzadeh, A. and Ahmadian, M., "Application of piezoelectric and functionally graded materials in designing electrostatically actuated micro switches", ***Journal of Solid Mechanics***, Vol. 2, No. 2, (2010), 179-189.

17. Crawley, E. F. and De Luis, J., "Use of piezoelectric actuators as elements of intelligent structures", ***AIAA Journal***, Vol. 25, No. 10, (1987), 1373-1385.

18. Quek, S. and Wang, Q., "On dispersion relations in piezoelectric coupled-plate structures", ***Smart Materials and Structures***, Vol. 9, No. 6, (2000), 859.

19. Pietrzakowski, M., "Piezoelectric control of composite plate vibration: Effect of electric potential distribution", ***Computers & Structures***, Vol. 86, No. 9, (2008), 948-954.

20. SHAH, M. A. A., REZAZADEH, G. and SHABANI, R., "Effect of electric potential distribution on electromechanical behavior of a piezoelectrically sandwiched micro-beam", ***Journal of Solid Mechanics***, Vol. 4, (2012), 48-58

21. Zhu, M. and Leighton, G., "Dimensional reduction study of piezoelectric ceramics constitutive equations from 3-D to 2-D and 1-D", ***Ultrasonics, Ferroelectrics and Frequency Control, IEEE Transactions on***, Vol. 55, No. 11, (2008), 2377-2383.

22. Jackson, J. D. and Fox, R. F., "Classical electrodynamics", ***American Journal of Physics***, Vol. 67, (1999), 841.

23. "Casimir HBG", ***Proc Kon Ned Akad Wet***, Vol. 51, (1948), 793-800.

24. Guo, J.-G. and Zhao, Y.-P., "Influence of van der waals and casimir forces on electrostatic torsional actuators", ***Microelectromechanical Systems, Journal of***, Vol. 13, No. 6, (2004), 1027-1035.

25. Vel, S. S., Mewer, R. and Batra, R., "Analytical solution for the cylindrical bending vibration of piezoelectric composite plates", ***International Journal of Solids and Structures***, Vol. 41, No. 5, (2004), 1625-1643.

26. Zarei, O. and Rezazadeh, G., "A novel approach to study of mechanical behavior of nem actuators using galerkin method", ***International Journal of Nanosystems***, Vol. 1, (2008), 161-169.

27. Nayfeh, A. and Mook, D., "Nonlinear oscillations ", ***John Willey and Sons, New York***, (1979)

28. Osterberg, P. M. and Senturia, S. D., "M-test: A test chip for mems material property measurement using electrostatically actuated test structures", ***Microelectromechanical Systems, Journal of***, Vol. 6, No. 2, (1997), 107-118.

29. Ramamurty, U., Sridhar, S., Giannakopoulos, A. and Suresh, S., "An experimental study of spherical indentation on piezoelectric materials", ***Acta Materialia***, Vol. 47, No. 8, (1999), 2417-2430.

Mechanical Response of a Piezoelectrically Sandwiched Nano-beam Based on the NonLocal Theory

A. Shah-Mohammadi-Azar , A. Khanchehgardan, G. Rezazadeh, R. Shabani

Department of Mechanical Engineering, University of Urmia, Urmia, Iran

PAPER INFO

Paper history:
Received 03 May 2013
Received in revised form 11 June 2013
Accepted 20 June 2013

Keywords:
NEMS
Nonlocal Elasticity
Electric Potential
Piezoelectric Layer
Piezoelectric Actuation
Electrostatic Actuation

چکیده

در این مقاله رفتار مکانیکی یک نانوتیر دو سر گیردار بررسی شده است. نانو تیراز دو طرف با دو لایه‌ی پیزو الکتریک پوشیده شده است. معادلات الکترومکانیکی کوپل حاکم با توجه به مدل غیرمحلی الاستیسیته بر اساس فرضیات معادله‌ی تیر اولر-بر نولی و همچنین با توجه به مدل غیرمحلی پیزو الکتریسیته بر اساس معادلات الکترواستاتیک ماکسول استخراج شدند. از دو ولتاژ پیزوالکتریک برای تنظیم سخت‌پایی نانوتیر استفاده شده است. معادلات کوپل به دست آمده با استفاده از روش خطی سازی گام به گام و روش گلرکین حل شدند. نتایج به دست آمده با نتایج الاستیسیته کلاسیک مقایسه شده‌اند. همچنین تاثیر ولتاژ پیزوالکتریک (نیروی کشش محوری) بر رفتار غیرمحلی مدل بررسی شده است.

doi: *10.5829/idosi.ije.2013.26.12c.12*