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Convective Heat Transfer from a Heated Rotating Disk at Arbitrary Inclination Angle in Laminar Flow

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ABSTRACT

In this paper, experimental data and numerical results of heat transfer from a heated rotating disk in still air are presented over a large range of inclination angles and a dimensionless correlation is developed for forced, natural and mixed convection. The measured Nusselt number over the rotating disk is compared with the numerical results. The goal of the present research is to develop a semi empirical correlation in the familiar classical form for a rotating disk at any arbitrary inclination angle over a wide range of rotational Reynolds numbers. The results show that the local Nusselt number does not change dramatically with inclination angle.

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NOMENCLATURE

c	Empirical coefficient	T	Temperature, K.
c_p	Specific heat at constant pressure, *J/kgK*	V	Velocity, m/s
d	Diameter, m	V	Volume, m^3
g	Gravitational acceleration, m/s^2.	**Greek Symbols**	
h	Convective heat transfer coefficient, W/m^2K.	β	Thermal expansion coefficient, 1/K
k	Thermal conductivity, W/mK	θ	Tangential co-ordinate
n	Empirical exponent	ν	Kinematic viscosity, m^2/s
Nu	Nusselt number, hd/k_f	ρ	Density, kg/m^3
P	Pressure, Pa	ω	Angular velocity, rad/s
Pr	Prandtl number	**Subscripts**	
Q	Heat transfer rate, W		
$\dot{q}$	Heat flux (Q/A), W/m^2	r	Radial co-ordinate, m
R	Radius, m	m	Mean
Re_ω	Rotational Reynolds number, $R^2\omega/\nu$	f	Fluid
t	Thickness, m	x, y, z	Cartesian co-ordinates

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1. INTRODUCTION

The flow field and convective heat transfer in rotating systems are coupled both experimentally and theoretically. Rotating discs can be found in a broad spectrum of practical applications such as electronic components, rotating heat exchangers, friction pumps, flywheels, turbine blades, brakes and even modern high speed CD-ROMs. Fluid flow and heat transfer characteristics of rotating disk(s) have attracted constant attention of researchers over the past century. The first work in this area was done by von Karman in 1921 [1]. The classical model of von Karman dealt with an infinite rotating disk, and the fluid far away from the disk is assumed to be at rest. He gave a formulation of the problem and then introduced his famous transformations which reduced the governing partial differential equations to ordinary differential equations.

Wagner for the first time, to the knowledge of the authors, developed an expression for the convective heat transfer coefficient from a heated rotating disk [2]. Since the pioneering work of Wagner many authors have studied flow and heat transfer problems due to rotating systems such as Zandbergen and Dijkstra [3], Kreith [4], Owen and Rogers [5] or Dorfmann [6] and so on [7-11]. The distribution of the convective heat transfer coefficient over a disk rotating in still air, in both laminar and turbulent regimes, has been extensively studied in the past; e.g., the papers by Millsaps and Pohlhausen [12], Northrop and Owen [13] and Cardone et al. [14], Sparrow and Gregg [15], Heydari [16] and Kumar et al. [17] are herein recalled. Mainly these investigations can be divided to two groups, namely, free rotating disks with outer forced flows perpendicular to the disks, and enclosed rotating disks. Papers about a rotating disk in a forced flow parallel to the plane are very few. The difference between the above flow classes and this kind of flow studied in the present paper is that the former are axisymmetric and the latter is a non-axisymmetric flow of a special kind. In axisymmetric forced flows parallel to the disk surface, all the derivatives with respect to the angle are neglected, while in the non-axisymmetric flow the derivatives have to be kept with respect to all three spatial co-ordinates.

In laminar flow, the heat transfer coefficient is high at the disc center and then becomes relatively constant for the rest of the disc surface. However, in turbulent flow, the heat transfer coefficient declines from the center, becomes minimum, and then increases with a large slope towards the edge of the disk. For small values of the local Reynolds number based on the local radius, the induced flow is laminar and the boundary layer thickness turns out to be constant over the disk surface. Consequently, the convective heat transfer coefficient is also independent of the local radius and is practically a sole function of the angular speed:

$$h = a\,k\sqrt{\frac{\omega}{\nu}} \tag{1}$$

where, a is a dimensionless constant, h the convective heat transfer coefficient, ω the disk angular speed, k the thermal conductivity and υ the kinematic viscosity of air. The constant a in Equation (1) is a function of the Prandtl number and can be evaluated from the exact solution of the governing equations [12]. Often, Equation (1) is expressed in terms of non dimensional quantities:

$$Nu_r = a\sqrt{\mathrm{Re}_r} \tag{2}$$

where, Nu_r is the local Nusselt number and Re_r is the local Reynolds number, both based on the local radius r. In the majority of prior works, the heat transfer from a rotating disk maintained at a constant temperature was studied for a variety of Prandtl numbers in steady state conditions. Several papers have recently released some new experimental data and empirical correlations for heat transfer from the isothermal rotating disk in still air in laminar flow, These data can be correlated for Pr=0.7 as:

$$Nu_m = 0.33\,\mathrm{Re}_\omega^{0.5} \quad \text{For} \quad 10^3 \le \mathrm{Re}_\omega \le 2\times10^5 \tag{3}$$

the value of C=0.33 agrees with the results of Kreith [4] and it is a little bit smaller then the value C=0.4 suggested by Richardson and Saunders [18] or Dennis et al. [19].

For relatively high values of the local Reynolds number, the flow becomes unstable and transition to turbulent flow eventually occurs. Clearly, the transitional Reynolds number is strongly dependent on the experimental conditions and generally ranges between 200,000 and 320,000. For the transition region, a more sophisticated correlation derived by Cobb and Saunders [20] is also available in the literature. In the turbulent regime the convective heat transfer coefficient is an increasing function of the radius. A suitable correlation for the fully turbulent case calls:

$$Nu_m = 0.015\,\mathrm{Re}_\omega^{0.8} \quad \text{for} \quad \mathrm{Re}_\omega \ge 5\times10^5 \tag{4}$$

Which is in very good agreement with the data for an isothermal disk reported in the literature [4, 17, 18]. Afterwards, the value of C=0.0163 was suggested by Cardone et al. [14]. Other researchers studied the heat transfer near a rotating disk considering different flow and thermal conditions [21-25]. From the standpoint of practical engineering applications, the problems of constant flux can be found in a variety of industrials example. At this time no empirical data exists in the available literature for natural or forced convection from stationary/rotating disks with an imposed heat flux, which is a more realistic condition in many practical applications. This is the objective of the current research. The present paper intends to provide descriptions of flow field and heat transfer characteristics in a rotating disk with this type of boundary condition. Generally, the goal of the present research is to develop an empirical correlation in the familiar classical form:

$$Nu_m = c.\mathrm{Re}_\omega^n \tag{5}$$

For a thin rotating disk, including the influence of inclination angle.

2. EXPEREMENTAL APPARATUS

Sketch of the experimental apparatus are shown in Figures 1a and 1b, respectively. It consists of a thin circular disk with radius equal to R=12.5 cm. The rotational speed of the disk was fixed on ω (rad/s). The rotational Reynolds number is defined as:

$$\mathrm{Re}_\omega = \frac{Vd}{\nu} = \frac{\mathrm{R}\,\omega R}{\nu} = \frac{R^2\omega}{\nu} \tag{6}$$

The temperature difference between the rotating disk and air was varied as an effective parameter. Five situations (inclination angles) were tested for a wide range of rotational Reynolds numbers. The circular disk that was used as the heat transfer medium for collecting the experimental data presented in this paper was commercially available.

The disk was made by pressing the metallic material at high pressure in a round die, to some extent like producing flat coin pieces. The material to make this piece is carbon steel (Mn ≤ 1%, Si ≤ 0.1%) type. Choosing this material makes the electrical flux, and thus heat generation by Joule heating to be uniform in the heat transfer model. An electrical circuit is used for heating the disc. The experimental setup is arranged for indirectly measuring the surface temperature and the convective heat transfer rate. It should be noted that this assumes the temperature to be uniform within and over the surface of the disk. A constant heat flux may be maintained during a test by adjusting the supply voltage and current to a set value for the duration of the experiment. Since the disk is thin, with a uniform potential difference between its flat surfaces, the current flux will be uniform, resulting in a uniform Joule heating within the disk. A digital thermocouple is used to measure the surface temperature of the disk. Because of difficulties associated with mounting a thermocouple on the rotating disk surface for measuring the surface temperature, its temperature is measured in stationary condition at different runs.

The rotational speed was kept constant at each run. After the measurements were completed at a rotational speed, a new rotational speed was selected. Once all speeds were investigated, the inclination angle of the disk was changed and the whole process was repeated. A series of the convective heat transfer coefficients at various inclination angles are obtained. The average convective heat transfer coefficient, h_m can be expressed in terms of the heat transfer rate, Q, the heat transfer area, A, and the temperature difference, $(T_m - T_f)$ between the surface and the fluid, thus:

$$h_m = \frac{Q}{A\left(T_m - T_f\right)} \tag{7}$$

where, as mentioned earlier, the convective heat transfer rate Q can be measured by analyzing the electrical circuit represented. The main objective of the present study is to calculate the mean Nusselt number at steady state conditions, defined as:

$$Nu_m = \frac{h_m d}{k_f} = \frac{\dot{q}\, d}{k_f\left(T_m - T_f\right)} \tag{8}$$

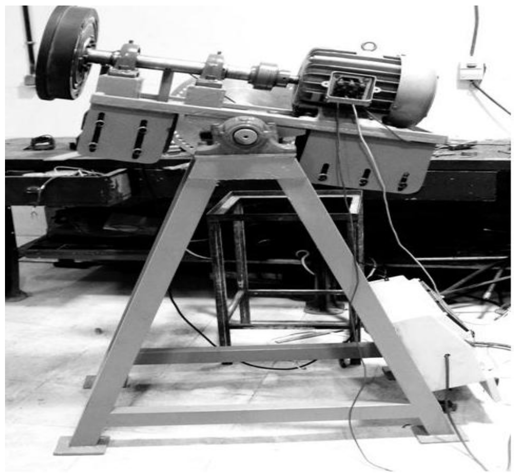


Figure 1a. Experimental apparatus

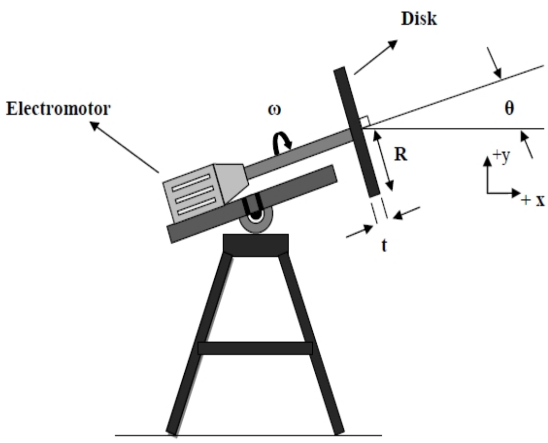


Figure 1b. Schematic diagram of the experimental apparatus.

A dimensional analysis shows that the mean Nusselt number (at steady state) can be defined as:

$$Nu_m = Nu_m(\mathrm{Pr}, \mathrm{Re}_\omega) \tag{9}$$

where, Pr is the Prandtl number and Re_ω the rotational Reynolds number given by Equation (6). In the limiting case of a rotary disk in still air the Nusselt number is typically correlated by an expression given by Equation (5).

3. EXPEREMENTAL UNCERTAINTY

As was mentioned earlier, five different situations were tested, the inclination angle ranging from 0 to 90º. The disk radius was R=125 mm and its thickness to diameter ratio (t/d) were 0.1. Property values for air, which was at atmospheric pressure, were evaluated at the film temperature, T_{film}, where $T_{film} = (T_w + T_f)/2$. The rotational Reynolds numbers, Re_ω, ranged from 10^3 to 7×10^4. The heat transfer rate of the disk was 50W. Experimental uncertainty in the Prandtl number (fluid viscosity, specific heat and thermal conductivity) is assumed to be negligible, since it is primarily a function of air temperature. Experimental uncertainty in Reynolds number is due primarily to uncertainty in the velocity and kinematic viscosity measurements. Experimental uncertainty in the Nusselt numbers is due primarily to uncertainty in the measured convective heat transfer coefficients and surface mean temperatures. Losses due to thermal radiation were also modeled and found to be negligible over the range of all data presented in this paper.

4. PROBLEM FOURMULATION

A thin circular disk with outer radius R, and thickness *t*, rotates at angular velocity ω in the X-Y plane making an angle of θ with horizontal plane and creates a complex flow. The Z-axis was chosen as the axis of rotation; far away from the disk, the velocity field is given by the rather simple potential flow with nearly zero velocity. General considerations show that the flow is three-dimensional, and it must be asymmetric with regard to the X-Z plane, because the action of gravity destroys the mirror symmetry. The disk surface is kept at constant flux. Property values for air, which was at atmospheric pressure, were evaluated at the film temperature. Furthermore, density is assumed to vary with temperature. So, the problem should be considered with fluid density as a function of temperature. The Boussinesq approximation is used to simplify the equations governing the fluid motion in order to facilitate both theoretical analysis and numerical computation that was described in detail by J. Boussinesq [26-29]. In this approximation, the effect of temperature on density affects the momentum balance but not the mass balance. So, density is treated as a constant value in all solved equations, except for the buoyancy term in the momentum equation:

$$\rho g = \rho\beta(T - T_0)g \tag{10}$$

where, ρ is the (constant) density of the fluid, T_0 is the air operating temperature, and β is the thermal expansion coefficient. The thermal expansion coefficient is computed as follows:

$$\beta = -\frac{1}{\rho}\left(\frac{\partial\rho}{\partial T}\right)_P \approx \text{For Ideal Gas} \approx \frac{1}{T_{film}} \tag{11}$$

In the flow domain, using the Boussinesq approximation, the following steady-state equations arise:

$$\frac{1}{r}\frac{\partial}{\partial r}(r v_r) + \frac{1}{r}\frac{\partial}{\partial\theta}(v_\theta) + \frac{\partial}{\partial z}(v_z) = 0 \tag{12}$$

$$\rho\left(v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial\theta} + v_z\frac{\partial v_r}{\partial z}\right) = \rho\beta(T - T_0)g_r + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial\theta^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial\theta} + \frac{\partial^2 v_r}{\partial z^2}\right] \tag{13}$$

$$\rho\left(v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial\theta} + \frac{v_r v_\theta}{r} + v_z\frac{\partial v_\theta}{\partial z}\right) = \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_\theta)\right) + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial\theta^2} + \frac{1}{r^2}\frac{\partial v_r}{\partial\theta} + \frac{\partial^2 v_\theta}{\partial z^2}\right] \tag{14}$$

$$\rho\left(v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial\theta} + v_z\frac{\partial v_z}{\partial z}\right) = \rho\beta(T - T_0)g_z - \frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial\theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right] \tag{15}$$

$$v_r\frac{\partial T}{\partial r} + \frac{v_\theta}{r}\frac{\partial T}{\partial\theta} + v_z\frac{\partial T}{\partial z} = (k/\rho c_P)\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 T}{\partial\theta^2} + \frac{\partial^2 T}{\partial z^2}\right] \tag{16}$$

Equation (12) is the continuity equation for an incompressible fluid, Equations (13) - (15) represent the Navier–Stokes equations for a Newtonian fluid, and Equation (16) represents the energy equation. The flow field and convective heat transfer over disk-type geometry is described efficiently by means of the boundary layer theory [30-33] and is presented in many current heat transfer text books [34-38].

5. NUMERCAL APPROACH

To solve the above system of equations a numerical approach based on the finite-volume-method is chosen.

The discretization of the partial differential equations is performed on a cylindrical mesh with staggered grid arrangement for the velocities. The SIMPLE iterative technique of Patankar [39] for the pressure forces results in coupled sets of equations. The chosen basic numerical method has a formal accuracy that is second-order with respect to space increments; for sake of convergence, a second-order monotonically preserving upwind-difference method is applied [39-41]. The disk with radius R=12.5cm and a thickness to diameter ratio t/d=0.1 is located at the origin in the x, y-plane. At the x-y plane and x-z plane the no-slip condition is prescribed. With regard to the global co-ordinate system the rotation is with angular velocity ω. At the disk surface, the velocity is prescribed by the no-slip condition leading to non-trivial velocity boundary conditions at the obstacle's boundary:

$$z=0: \quad v_r=0, \quad v_\theta = r\omega, \quad v_z=0 \tag{17}$$

The boundary conditions far from the disk were:

$$z=\infty: \quad v_r=0, \quad v_\theta=0 \tag{18}$$

The computational region extends from x=-4.25R to x=4.25R, from y=-4.5R to y=4.5R, and from z=0 to z=1.85R, respectively. These dimensions have to be found sufficient after some preliminary studies. By trial runs, such a domain produced relatively accurate results with a moderate grid mesh. The grid sensitivity study showed that meshes of 244×244×56 and 268×268×56 gave average heat transfer coefficients with a difference between 0.85% to 1.55% for different situations of the disk. Therefore, the mesh of 244×244×56 was adopted for the simulation. A non-uniform grid was used to account for the high gradient of velocity and temperature in the vicinity of surfaces. The mesh size has been refined near the disc, whereas far away from the disk a coarser mesh size has been chosen. Thermal boundary conditions of the disk were consistent with the actual experiment. The properties and flow domain parameters used for calculation are summarized in Table 1.

6. RESULTS AND DISCUSSION

The rotation imparts tangential and outward radial components of velocity to the fluid close to the disk surface. This causes an axial flow of fluids towards the disk. Considerable experimental data and numerical results were obtained in the present research.

As was mentioned earlier, our investigations are limited to a single disk model of diameter, d=25 cm, included vertical, horizontal and three other angles of orientation. The total numerical results for laminar convection heat transfer, ranging in rotational Reynolds number, $1 \leq Re_\omega \leq 2\times10^5$, are presented as double logarithmic plot of the mean Nusselt number, Nu_m ,against the rotational Reynolds number, Re_ω, in Figure 2.

Figure 2 illustrates the cumulative numerical data of all convective heat transfer at various angles of inclination obtained using the proposed numerical simulation. The dots indicate the numerical finite-volume-method results and they are connected by interpolating curves in order to give the reader a clearer picture. The numerical data in Figure 2 clearly demonstrates the relative influence of the inertia and buoyancy forces for convective heat transfer.

TABLE 1 Simulation Parameters

Symbol	Quantity	Unit
ρ	Density of air	$1.19\ kg/m^3$
ν	Kinematic viscosity of air	$15.1\times10^{-6}\ m^2/s$
c_p	Specific heat of air	$1007\ J/kg.K$
k_f	Thermal conductivity of air	$26\ KW/mK$
R	Disk radius	$0.125m$
t/d	Normalized disk thickness	0.1
Re_ω	Rotational Reynolds	1 up to 2×10^5
-	Computational domain in x-direction	-4.25R up to 4.25R
-	Computational domain in y-direction	-4.5R up to 4.5R
-	Computational domain in z-direction	0 up to 1.85

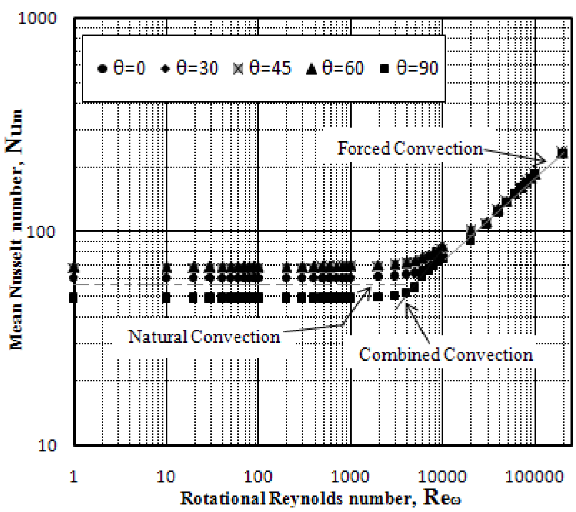


Figure 2. Numerical data of convection heat transfer for rotating disk at various inclination angles.

As the Reynolds number decreases, natural convection and buoyancy force becomes more dominant; asymptotically approaching that of pure natural convection as represented by the dashed lines which were obtained from the natural convection correlation. Mean Nusselt number does not change with rotational Reynolds number in this range. Combined forced and natural convection is complicated because of coupling the inertia and buoyancy forces. It is evident in this figure that buoyancy force dominates and pure natural convection exists when the rotational Reynolds number ranges between 2500-5000 at different inclination angles. In general, heat exchange takes place by pure natural convection in rotational Reynolds numbers less than 5000, forced convection is more dominant in rotational Reynolds numbers more than 10^4 and combined forced and natural convection exists in the range of $5000 \leq Re_\omega \leq 10^4$. For sufficient low values of the rotational Reynolds number, the flow behavior remains laminar. The flow passes through a transition region (for approximately $2\times10^5 \leq Re_\omega \leq 5\times10^5$) and as the rotational Reynolds number increases the flow becomes turbulent. So, in present numerical results, $Re_\omega = 2\times10^5$ is located in transition region.

The experimental data for laminar convection, at rotational Reynolds number of $1000 \leq Re\omega \leq 7\times10^4$, are depicted in dimensionless form in Figure 3, where Nu_m is plotted as a function of the Reynolds number, Re_ω. The data presented in this figure consist of multiple heat transfer model at various angles of inclination. Figure 3 illustrates the cumulative experimental data of all the heat transfer modes. The range of Reynolds numbers given in Figure 3 represented the limits of the existing experimental apparatus and heat transfer model, not necessarily the limit of the existing correlation. The data indicate that heat transfer coefficients are generally higher for disk in inclination angle of 60º than for disk in any other orientations. It is clear from this figure that the influence of inclination angle appears to be continuous, and is greatest at the lowest Reynolds number, and at least at the highest. The influence of inclination angle is diminished for six highest value of the Reynolds number. This figure which illustrates the experimental data, shows very little difference between the vertical, horizontal, or inclined disk orientation (within experimental uncertainty). As can be seen, it does not appear to be an appreciable difference in the heat transfer characteristics for various inclination angles over this range of Reynolds numbers, except for θ=90. Figures 4 and 5 display the experimental data correlation for inclination angles from 0 to 90º from vertical, respectively. The dots indicate the experimental results and the lines represent suitable asymptotic correlations. The numerical data for $1000 < Re_\omega < 70000$ in Figures 4 and 5 are the same type of data as in Figure 3. With the increase of the rotational Reynolds number, the surface average Nusselt numbers are increased. In addition, the surface average Nusselt numbers increased as the inclination angle increased for inclination angles of 0 to 60º.

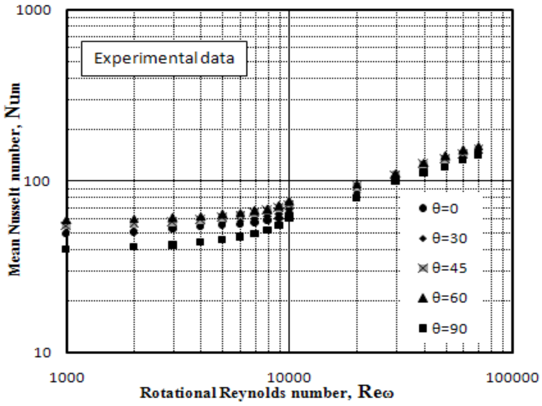


Figure 3. Experimental data of convection heat transfer for rotating disk at various inclination angles.

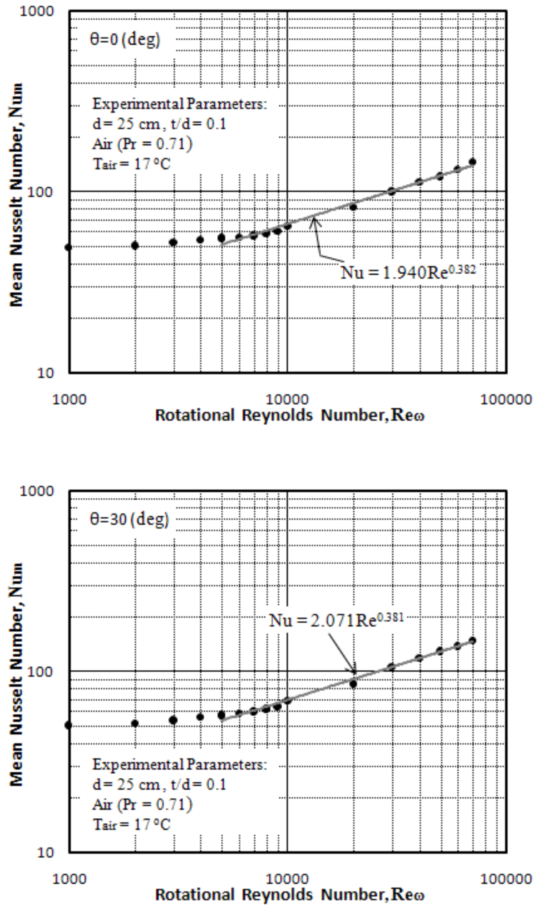
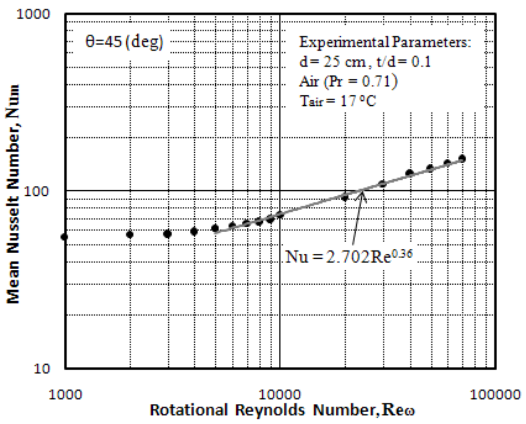
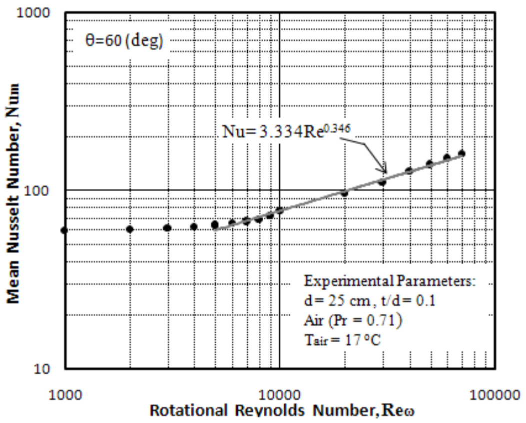


Figure 4. Comparison of experimental data of convection heat transfer with correlation for rotating disk at inclination angles of 0º and 30º.

As it was mentioned earlier, in Figures 4 and 5 pure natural convection exists in rotational Reynolds numbers between 2500-5000 at different inclination angles. The correlations in Figures 4 and 5 are valid for 5000≤ Re_ω ≤70000 (combined and force convection heat transfer range), where the coefficient *C* and exponent *n* are presented in Equation 5. So, the Nusselt-Reynolds correlations expressed by Equation 5 yield very good estimation between the mean Nusselt number and rotational Reynolds number.

However, our investigation was limited to a single disk model of diameter, d = 250 mm, and a thickness-to-diameter ratio, t/d = 0.1. Although the experimental data covers several orders of magnitude in the Reynolds number, it would be interesting to see if there are inclination angle effects at lower Reynolds numbers. The above correlation has the advantage of being valid over the full range of inclination angles between the vertical and horizontal limits. Thus, in this study only the comparison between horizontal and vertical positions can be distinguished, and it can be clarified the influence of inclination angle on heat transfer rate in a rotating disk.





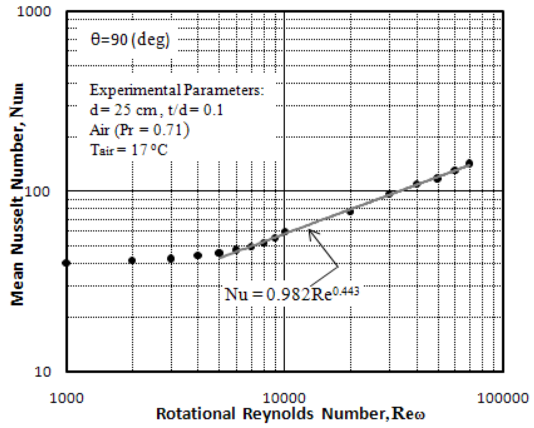


Figure 5. Comparison of experimental data of convection heat transfer with correlation for rotating disk at various angles of inclination from 45º to 90º.

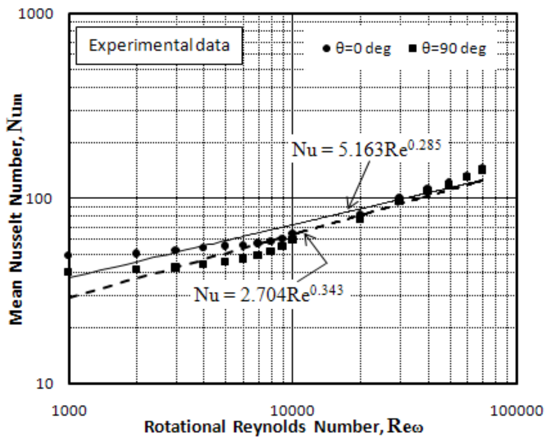


Figure 6. Comparison of experimental convection heat transfer for vertical and horizontal rotating disk.

Referring to Figure 6, all data were for vertical (θ=0) and horizontal (θ=90) positions. In this figure, convection heat transfer in both vertical and horizontal positions for 1000≤ Re_ω ≤70000 are compared and empirical correlations are presented in this range of rotational Reynolds numbers. The exponent *n*, in vertical and horizontal positions are 0.285 and 0.343, respectively. Having verified the present code performance via solving different test cases, the code was employed to investigate convective heat transfer from a heated rotating iso-flux circular disk in still air. Comparisons between the results of the present simulations and the experimental results, carried out in this study only for verification purposes, are presented in Figures 7 and 8. The numerical results and experimental data are compared for several inclination angles in Figures 7 and 8. The numerical and experimental results are in good agreement, but at lower rotational Reynolds numbers the agreement is less.

Therefore, excellent agreement exists between the experimental data and the numerical results for 1000< Re_ω < 70000. Figures 7 and 8 represent the same type of data as Figures 2 and 3. The range of Reynolds numbers given in Figures 7 and 8 represent the limits of the existing experimental apparatus and heat transfer model. The data indicate that heat transfer coefficients are generally higher for disk in inclination angle of 60º than for disk in other orientations. With the increase of the rotational Reynolds number, the surface average Nusselt number is increased. Moreover, the surface average Nusselt number is increased as the inclination angle is increased for inclination angles of 0 to 60º. It is clear from this figure that the influence of inclination angle is greatest at the lowest Reynolds number, and at least at the highest. The influence of inclination angle is diminished for highest value of the Reynolds number. This figure which illustrates the compatibility of numerical and experimental results, shows very little difference between them (within experimental uncertainty).

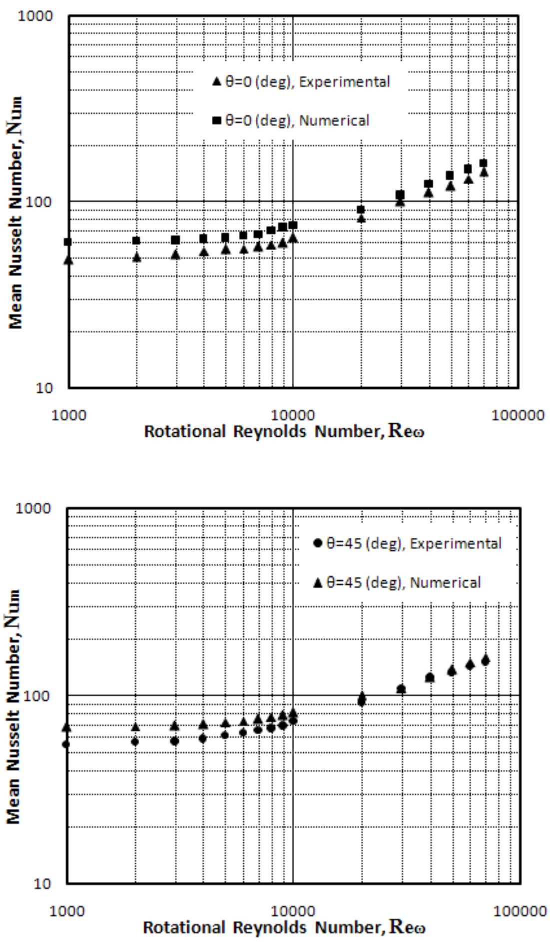


Figure 7. Comparison of convection heat transfer for experimental data and Numerical results at inclination angles of 0º and 45º.

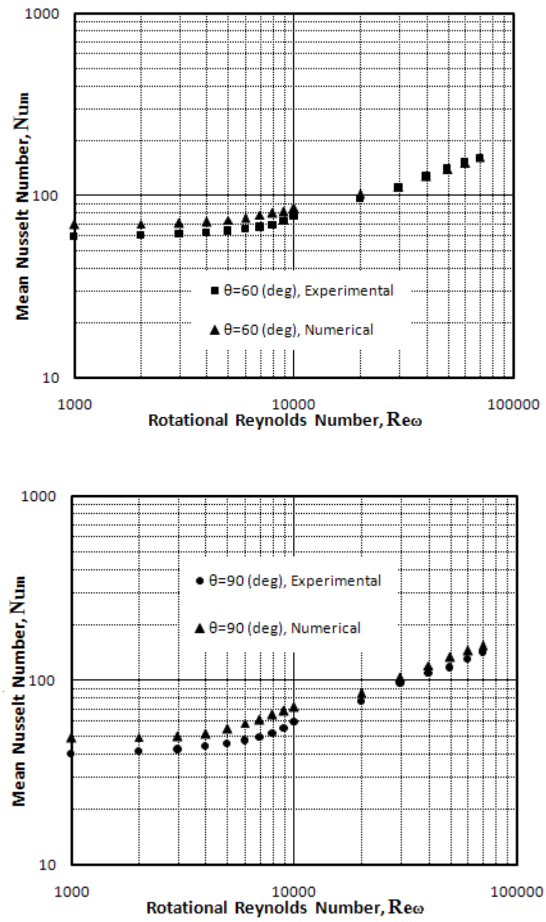


Figure 8. Comparison of convection heat transfer for experimental data and Numerical results at inclination angles of 60º and 90º.

For Re_ω > 10000, the experimental heat transfer data are virtually indistinguishable from the numerical results. Generally, the average difference between numerical and experimental data in the stationary regime was 4.8 percent in Nu_m. The numerical code was also validated in one case (for θ=0) with the analytical solution of Cochran [30], the average deviation of 0.65 percent was observed. Therefore, there is a rational and logical difference between experimental and numerical results in the heat transfer characteristics for the various inclination angles over this range of Reynolds numbers. Thus, acceptable variance exists between experimental and numerical data in the present research.

6. CONCLUSION

Experimental heat transfer data and numerical results were presented. A dimensionless correlation was also proposed for convection heat transfer from heated rotationally iso-flux circular disk over a wide range of rotational Reynolds numbers at various inclination angles. This investigation was limited to a single disk model of diameter, d = 250 mm, and a thickness-to-diameter ratio, t/d = 0.1. The numerical results showed that natural convection and buoyancy force becomes more dominant with the increase of the rotational Reynolds number and heat exchange takes place by pure natural convection at rotational Reynolds numbers less than 5000. Comparison between the simulation and the experimental results was carried out in this study to verify the numerical results. There was very little difference between the two sets of results for the various inclination angles over this range of rotational Reynolds numbers and acceptable variance exists between them. The experimental heat transfer data appear to be lower than the numerical results in this range. For Re_ω > 10000, the experimental heat transfer data are virtually indistinguishable from the numerical results. The majority of experimental data presented in this research agreed well with the proposed empirical correlation over a wide range of Reynolds numbers and angles of orientation. In addition, interest is the similarity in the coefficients for the experimental and numerical-empirical correlations. This interesting result leads encouragement for the future research. Additional data must be obtained at arbitrary inclination angles. This conclusion was obtained in one case (for θ=0) and agreed with the work of Stefan aus der Wiesche [42].

Since only air was tested, the current correlation is recommended for Prandtl numbers near unity, which includes most common gases. The correlation may be valid for Prandtl numbers outside this range, however, this is not known at this time since no experimental data is available. Furthermore, the maximum aspect ratio recommended should not be much more than the majority of the heat transfer models tested, thus (t/d)<0.1. The influence of larger aspect ratios should also be the subject of ongoing research. According to the experimental results in the current research, disk orientation cannot greatly affect convection heat transfer. This effect is completely diminished for Re_ω >10000.

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Convective Heat Transfer from a Heated Rotating Disk at Arbitrary Inclination Angle in Laminar Flow

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چکیده

در این تحقیق به ارائه داده‌های تجربی و نتایج عددی برای یک دیسک گرم که درون هوا می‌چرخد در محدوده وسیعی از زوایا پرداخته می‌شود. در ادامه روابطی با کمک انطباق بر داده‌های تجربی در رژیم‌های جابجایی طبیعی، ترکیبی و اجباری بدست می‌آید. داده‌های عددی با داده‌های تجربی مقایسه می‌شود و در نتیجه یک رابطه نیمه تجربی که هدف این تحقیق است در محدوده وسیعی از زوایا و عدد رینولد به فرم کلاسیک بدست می آید. نتایج نشان می‌دهد عدد ناسلت محلی با زاویه دیسک بطور شدید تغییر نمی‌کند.

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