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MHD Flow of Blood through Radially Non-symmetric Stenosed Artery: a Hershcel-Bulkley Model

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ABSTRACT

The purpose of this study is to develop a mathematical model for studying the magnetic field effect on blood flow through an axially non-symmetric but radially symmetric atherosclerotic artery. Herschel-Bulkley equation has been taken to represent the non-Newtonian character of blood. The response of magnetic field, stenosis height, shape parameter on velocity, volumetric flow rate in stenotic section and wall shear stress at the surface of stenosis are revealed analytically and graphically.

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1. INTRODUCTION

Now a days, magnetic therapy is widely used for curing various diseases. The Blood which is considered as a magnetohydrodynamics (MHD) fluid will help in controlling blood pressure and has potential therapeutic use in the disease of heart and blood vessel. Using an appropriate magnetic field, it can become effective to conditions such as poor circulation, travel sickness, pain, headaches, muscle sprains, strains and joint pain.

The idea of electromagnetic fields in medical research was firstly given by Kolin [1] and later, Korchevskii et al. [2] discussed the possibility of regulating the movement of blood in human system by applying magnetic field. Vardhanyan [3] studied the effect of magnetic field on blood flow. Halder [4] analyzed the effect of magnetic field on blood flow through an indented tube in presence of erythrocytes. A mathematical model for biomagnetic fluid dynamics, suitable for the description of the Newtonian blood flow under the action of an applied magnetic field has been proposed by Tzirtzilakis [5]. Barcroft and Edholm [6] studied the effect of temperature on blood flow and deep temperature in the human forearm. They gave the variation in blood flow as a result of changes in temperature of the surrounding water.

Abbas et al. [7] investigated two dimensional magneto-hydrodynamic (MHD) flow of upper convected Maxwell fluid in a porous channel. Arterial wall shear stress is considered to be an important factor in the localization of atherosclerotic. Jain et al. [8] developed a mathematical model for studying oscillatory flow of blood in a stenosed artery under the influence of transverse magnetic field through porous medium. They examined effects of various parameters particularly magnetic number and porosity constant on the blood flow through stenosis. Rathod and Tanveer [9] studied the plusatile flow of blood through a porous medium under the influence of periodic body acceleration by considering blood as a couple stress, incompressible, electrical conducting fluid in presence of magnetic field. Mustapha et al. [10] investigated flow of an electrically conducting fluid characterizing blood through the arteries having irregular shaped multi-stenoses in the environment of a uniform transverse magnetic-field. Sankar and Lee [11] developed a computational model to analyze the effects of magnetic

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field in a pulsatile flow of blood through narrow arteries with mild stenosis, treating blood as Casson fluid model. They investigated that the velocity and flow rate decrease with the increase of the Hartmann number. They also observed that the skin friction and longitudinal impedance increase with the increase of the amplitude parameter of the artery radius. Varshney et al. [12] proposed a mathematical model for the blood flow through stenosed artery in the presence of magnetic field. They also studied the effect of externally applied magnetic field on velocity, flow rate, flow resistance and wall shear stress. The effect of externally imposed body acceleration and magnetic field on peristaltic flow of blood through an arterial segment having stenosis has been investigated by Shit and Roy [13]. Their studies pertain to a situation in which blood obeying micropolar fluid model, where the effect of heat transfer phenomena has been taken into account. Bhargava et al. [14] numerically studied the pulsatile flow and mass transfer of an electrically conducting Newtonian bio fluid via a channel with porous medium.

Kumari and Prasad [15] studied the fully developed free convection flow of a third grade fluid in a vertical channel under the effect of magnetic field. They used perturbation technique to solve governing non-linear equations for the velocity field and temperature field. Sarma and Sut [16] developed a numerical model to study the effect of magnetic field on pulsatile flow of blood in a porous channel. They observed that when the Hartmann number increases, the fluid velocity as well as magnitude of mass flux decreases. Das and Saha [17] studied the effect of an externally applied uniform transverse magnetic field on pulsatile flow of blood containing particles through a rough thin-walled elastic tube. They derived analytical expressions for the phase velocity ratio and the reduction of amplitude and represented their natures graphically with the frequency parameter in the presence of magnetic field. Wahab and Salem [18] studied the flow of an incompressible, viscous, electrically conducting fluid in the presence of transverse magnetic field. They noticed that the effect of the magnetic field is to decrease the velocity profile and flow rate. Therefore, the velocity profile becomes more parabolic.

Our work is an extensive study of Bali and Awasthi [19] which is on MHD flow of blood through multiple stenosed artery assuming blood as Casson Fluid. Singh [20] observed that flow resistance decreases as shape parameter and parameter β increases and it increases as stenosis height and length increase. He also studied that wall shear stress increases up to mid axial distance and then it decreases for increasing values of β . The main object of the present work is to study the effect an externally applied uniform magnetic field on the axially non-symmetric but radially symmetric atherosclerotic artery with core region. Blood is modeled as a Herschel-Bulkley fluid by properly accounting for yield stress of blood in small blood vessels.

2. MATHEMATICAL FORMULATION

Consider the motion of blood following Hershcel-Bulkley equation through an axially symmetric but radially non -symmetric stenosed artery under the influence of an external applied uniform transverse magnetic field. In such case the radius of artery $R(z)$ can be written as:

$$R^*(z) = R_0\left[1 - A\left\{(l_0^*)^{(s-1)}(z^* - d^*) - (z^* - d^*)^s\right\}\right] \quad ; d^* \le z^* \le d^* + l_0^*$$

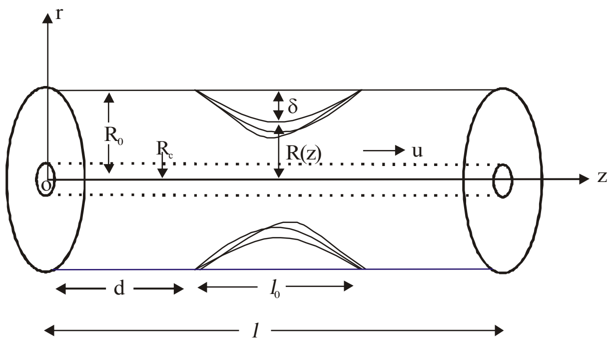
$$= R_0 ; \quad \text{Otherwise} \tag{1}$$

where, $A = \dfrac{\delta}{R_0 (l_0^*)^s} \dfrac{s^{s/(s-1)}}{(s-1)}$

The Navier-Stoke equation is

$$-\frac{\partial p^*}{\partial z^*} + \frac{1}{r^*}\frac{\partial}{\partial r^*}(r^*\tau^*) + \mu_0 M \frac{\partial H^*}{\partial z^*} = 0 \tag{2}$$

where, r^* and z^* be the radial and axial co-ordinates, μ_0 magnetic permeability, M magnetization H^* is magnetic field intensity, p^* is pressure and τ^* be shear stress.



The constitutive equation is given by

$$(\tau^* - \tau_0^*)^n = K\left(-\frac{\partial u^*}{\partial r^*}\right); \tau^* \ge \tau_0^* \tag{3}$$

$$\frac{\partial u^*}{\partial r^*} = 0; \tau^* \le \tau_0^* \tag{4}$$

where, τ_0^* be the yield stress and K be the viscosity coefficient of blood.

The boundary conditions pertaining to the problem

$$u^* = 0 \text{ at } r^* = R^*(z) \tag{5a}$$

$$\tau^* \text{ is finite at } r^* = 0 \tag{5b}$$

In the core region $u^* = u_c^*$ at $r^* = R_c^*$ (5c)

where, u_c^* is the core velocity.

3. SOLUTION OF THE PROBLEM

Introducing the following non-dimensional scheme

$$r = \frac{r^*}{R_0},\ z = \frac{z^*}{l},\ R = \frac{R^*}{R_0},\ P = \frac{P^*}{\rho u_0^2},\ u = \frac{u^*}{u_0},\ \tau = \frac{\tau^*}{\rho u_0^2}$$
$$d = \frac{d^*}{l},\ l_0 = \frac{l_0^*}{l},\ H = \frac{H^*}{H_0} \tag{6}$$

where, H_0 is the external transverse uniform constant magnetic field. The geometry of the stenosis in non-dimensional form is given by

$$R(z) = 1 - A\left[(l_0)^{(s-1)}(z-d) - (z-d)^s\right];\ d \le z \le d + l_0 \tag{7}$$

=1; Otherwise

where, $A = \frac{\delta}{R_0 l_0^s}\frac{s^{s/(s-1)}}{(s-1)}$

Equations (2) and (4) reduce to:

$$-\frac{\partial p}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}(r\tau) + f_1\frac{\partial H}{\partial z} = 0 \tag{8}$$

$$(\tau - \tau_0)^n = f_2\left(-\frac{\partial u}{\partial r}\right);\ \tau \ge \tau_0 \tag{9}$$

$$\frac{\partial u}{\partial r} = 0;\ \tau \le \tau_0 \tag{10}$$

where, $f_2 = \frac{K}{\rho^n u_0^{2n-1} R_0}$, $f_1 = \frac{\mu_0 M H_0}{\rho u_0^2}$

The bondary conditions (5a-5c) will now become

$$u = 0 \text{ at } r = R(z) \tag{11a}$$

$$\tau \text{ is finite at } r = 0 \tag{11b}$$

In the core region $u = u_c$ at $r = R_c$ (11c)

Using analytic method in Equations (8-10) and using boundary conditions (11a-11c), the expression for velocity u and u_c are:

$$u = -\frac{1}{2^n f_2\left(\frac{\partial p}{\partial z} - f_1\frac{\partial H}{\partial z}\right)}\left[\left\{\left(\frac{\partial p}{\partial z} - f_1\frac{\partial H}{\partial z}\right)r - 2\tau_0\right\}^{n+1} - \left\{\left(\frac{\partial p}{\partial z} - f_1\frac{\partial H}{\partial z}\right)R - 2\tau_0\right\}^{n+1}\right] \tag{12}$$

If $u = u_c$ at $r = R_c$

$$u_c = -\frac{1}{2^n f_2\left(\frac{\partial p}{\partial z} - f_1\frac{\partial H}{\partial z}\right)}\left[\left\{\left(\frac{\partial p}{\partial z} - f_1\frac{\partial H}{\partial z}\right)R_c - 2\tau_0\right\}^{n+1} - \left\{\left(\frac{\partial p}{\partial z} - f_1\frac{\partial H}{\partial z}\right)R - 2\tau_0\right\}^{n+1}\right] \tag{13}$$

The volumetric flow rate Q rate is given by

$$Q = 2\pi\int_0^{R_c} r u_c\, dr + 2\pi\int_{R_c}^{R} r u\, dr = Q_c + Q_1 \tag{14}$$

where, Q_c and Q_1 are the flow rate in core and annular region of the stenotic artery. Using u and u_c from Equations (12) and (13) in Equation (14), then flow rate Q is:

$$Q = \gamma\left[\theta(\alpha^{n+2}\eta - \beta^{n+2}\kappa) + \frac{1}{2}(\beta^{n+1}R^2 - \alpha^{n+1}R_c^2)\right] \tag{15}$$

where,

$$\alpha = \left[\left(\frac{\partial P}{\partial z} - f_1\frac{\partial H}{\partial z}\right)R_c - 2\tau_0\right],\ \beta = \left[\left(\frac{\partial P}{\partial z} - f_1\frac{\partial H}{\partial z}\right)R - 2\tau_0\right]$$

$$\gamma = \frac{\pi}{2^n f_2(n+1)\left(\frac{\partial P}{\partial z} - f_1\frac{\partial H}{\partial z}\right)}$$

$$\theta = \frac{1}{\left(\frac{\partial P}{\partial z} - f_1\frac{\partial H}{\partial z}\right)^2(n+2)(n+3)}$$

$$\eta = \left(\frac{\partial P}{\partial z} - f_1\frac{\partial H}{\partial z}\right)(n+2)R_c - 2\tau_0$$

$$\kappa = \left(\frac{\partial P}{\partial z} - f_1\frac{\partial H}{\partial z}\right)(n+2)R - 2\tau_0$$

The wall shear stress

$$\tau_R = -K\left(\frac{\partial u}{\partial r}\right)_{r=R} \quad ...(16)$$

where, $K = \mu$

Now differentiating Equation (12) with respect to r and substituting in Equation (16), then:

$$\tau_R = \frac{\mu}{2^n f_2}\left[\left(\frac{\partial P}{\partial z} - f_1\frac{\partial H}{\partial z}\right)R - 2\tau_0\right]^n \tag{17}$$

4. RESULTS AND DISCUSSION

It is difficult to handle the problem with the extended ideas considered herein, but computation with MATLAB 7.0 makes it easier to describe the numerical results graphically for the present investigation. Figures 1, 2, 3 depict that velocity curve shifts towards the origin with increases in radial distance. Further that velocity of blood reduces as magnetic field intensity increases in Figure 1. The variation of velocity with parameter determining the shape of stenosis is also shown in Figure 2. The result presented in this figure indicates that velocity increase with the increase in shape parameter. The Figure 3 illustrates that the velocity diminishes with the increase in stenosis height.

Variation of core velocity (u_c) with the stenosis height for different values of induced magnetic field gradient ($H = dH/dz$) and shape parameter (s) are shown in Figures 4 and 5, respectively. The results show that core velocity decreases as the stenosis height and induced magnetic field increase. Core velocity increases with the increase in shape parameter.

Variations of volumetric flow rate with axial distances for various stenosis height and magnetic field gradient are presented in Figures 6 and 7, respectively. The study reveals that with increasing stenosis height, rate of flow diminishes for increasing axial distance z. It is also observed that flow rate becomes higher in the absence of magnetic field and it gradually diminishes with increasing magnetic field.

The results of variation of wall shear stress (τ_R) with axial distance z for different values of magnetic field intensity $(H = dH/dz)$ and yield stress (τ_R) have shown in Figures 8 and 9, respectively. It is noted that the wall shear stress increases as the axial distance z increases form 0 to 0.5 and then it decreases as z increases from 0.5 to 1. This is due to large velocity gradient and therefore the severity of the stenosis significantly affects the wall shear stress characteristics.

The maximum wall shear stress occurs at the middle of the stenosis. It is also obvious from these figures wall shear stress decreases for increasing values of magnetic field and yield stress.

5. CONCLUDING REMARKS

The present study deals with the response of magnetic field intensity on the velocity of blood, core velocity, volumetric flow rate and wall shear stress. Blood is represented by Hershcel-Bulkley (non-Newtonian) fluid model. In brief, we have observed that presence of magnetic field reduces the above flow characteristics. The MHD principle may be used to deaccelerate the flow of blood in a human arterial system. Therefore, it is helpful in treatment of certain cardiovascular diseases. Magnetic field applied is also affecting the flow of blood which is helpful for the problem like blood pressure, hypertension.

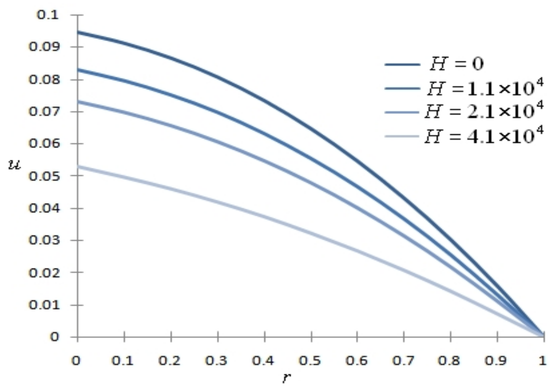


Figure 1. Variation in velocity of blood with radial distance for different magnetic field intensity

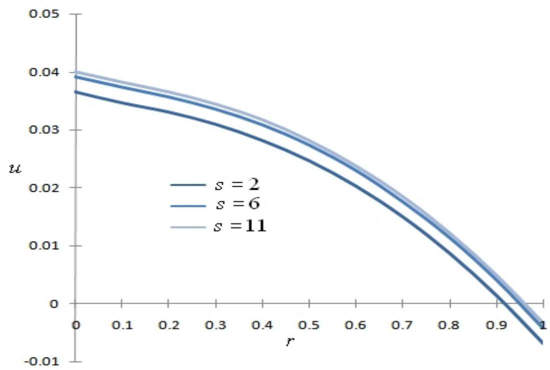


Figure 2. Variation in velocity of blood with radial distance for different values of stenosis shape parameter

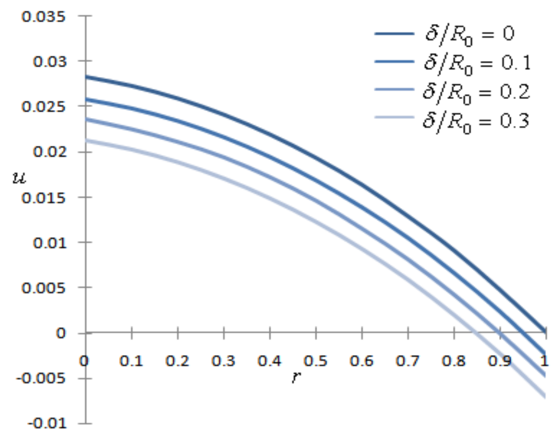


Figure 3. Variation in velocity of blood with radial distance for different values of stenosis height

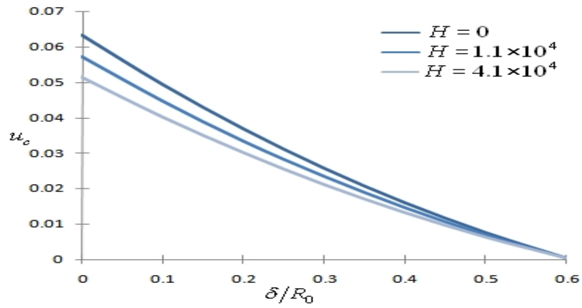


Figure 4. Variation in core velocity with stenosis height for different values of magnetic field intensity

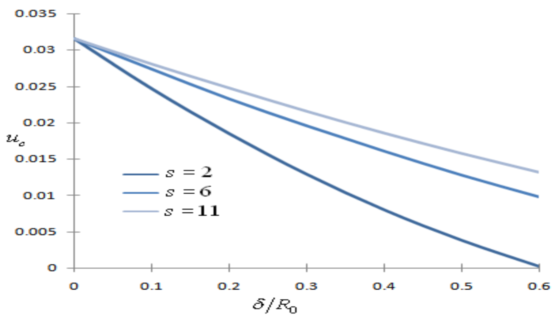


Figure 5. Variation in core velocity with stenosis height for different values of stenosis shape parameter

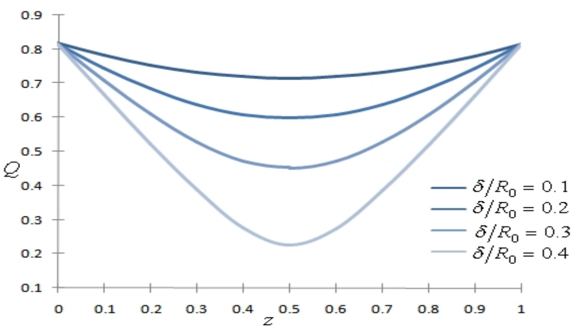


Figure 6. Variation in volumetric flow rate with axial distance for different values of stenosis height

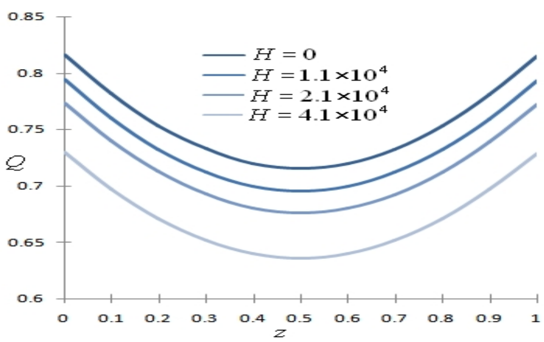


Figure 7. Variation in volumetric flow rate with axial distance for different values of magnetic field intensity

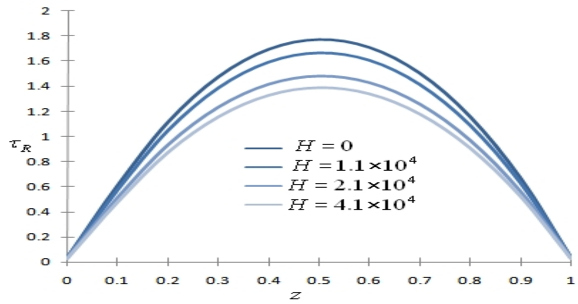


Figure 8. Variation in wall shear stress with axial distance for different values of magnetic field intensity

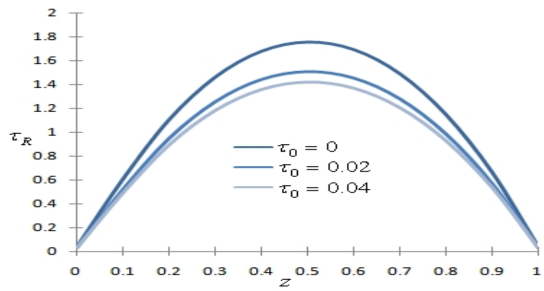


Figure 9. Variation in wall shear stress with axial distance for different values of yield stress

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چکیده

هدف از این مطالعه، توسعه یک مدل ریاضی برای مطالعه اثر میدان مغناطیسی روی جریان خون از طریق سرخرگ تصلب شریان غیر متقارن محوری اما متقارن شعاعی است. معادله Herschel-Bulkley برای نشان دادن خصوصیت غیر نیوتنی خون استفاده شده است. پاسخ میدان مغناطیسی، ارتفاع انقباض، پارامتر شکل روی سرعت، سرعت حجمی جریان در بخش انقباض و تنش کششی دیواره در سطح انقباض به صورت تحلیلی و ترسیمی نشان داده شده است.

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