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An Analytical Approach to the Effect of Viscous Dissipation on Shear-driven Flow between two Parallel Plates with Constant Heat Flux Boundary Conditions

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ABSTRACT

An investigation has been made to analyze the effects of viscous dissipation on the heat transfer characteristics for both hydro-dynamically and thermally fully developed laminar shear-driven flow between two infinitely long parallel plates, where the upper plate is moving in an axial direction at a constant speed. On the basis of some routine assumptions made in the literature, a closed-form analytical expression of Nusselt numbers for the flow of Newtonian fluid with constant properties has been developed for three different cases of constant heat flux boundary conditions. The significant effect of the viscous dissipation as compared to other terms in the energy equation is manifested by the Brinkman number. In order to have a generalized idea about the viscous-heating effect on the heat transfer analysis, different definitions of the Brinkman number have been used in the present study. Here, focus is on the viscous dissipative effect due to the shear produced by the moving upper plate apart from the viscous heating due to internal fluid friction for the flow of a Newtonian fluid. The prominent role of the viscous dissipation on the heat transfer characteristics has been discussed in detail for the problem under consideration subjected to different thermal boundary conditions.

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NOMENCLATURE

$a_1 - a_3$	Constant
Br	Brinkman number
Br_m	Modified Brinkman number
c_p	Specific heat at constant pressure (J/g K)
h_1	Limiting heat transfer coefficient at upper plate (W/m^2-K)
H	Channel height (m)
k	Thermal conductivity (W/mK)
Nu_H	Nusselt number at the upper plate
q_1	Upper wall heat flux (W/m^2)
q_2	Lower wall heat flux (W/m^2)
T	Temperature (K)
T_1	Upper wall temperature (K)
T_2	Lower wall temperature (K)
u	Velocity (m/s)
U	Dimensionless velocity
U_P	Velocity of the moving plate (m/s)
x	Axial coordinate direction (m)
y	Vertical coordinate direction (m)
Y	Dimensionless vertical coordinate
Greek Symbols	
θ	Dimensionless temperature
θ_m	Dimensionless bulk mean temperature
μ	Dynamic viscosity (kg/m-s)
ρ	Density (kg/m^3)
Subscripts	
f	Uniform fluid
m	Mean
w	Wall

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1. INTRODUCTION

In the realm of macro-flows, there are many practical applications where heat and fluid transport involves the movement of boundaries, e.g., extrusion, hot rolling, drawing, and continuous casting and so on. The thermo-fluidic transport through those systems, especially transfer of heat either from moving boundaries to the surrounding fluid or the other way around, is of immense importance. However, the moving boundary deforms the fluid velocity profile, and shears the fluid layer near the boundary, and thus, results in local changes in velocity gradient. Hence, the viscous dissipation effects cannot be neglected in heat transfer analysis of a system associated with moving boundaries. In order to obtain the actual heat transfer rate in the application of moving boundaries for proper design of the system, it is important to take into account the effects of viscous dissipations by using accurate velocity distribution.

One of the earliest theoretical works has dealt with the forced convective heat transfer in the entry length through an asymmetrically heated parallel plate channel [1]. Following a numerical analysis, the influence of viscous dissipation for the flow of a Newtonian fluid in a parallel plate channel has been investigated in detail [2]. In another study, the combined forced and free convective heat transfer, including the effect of viscous dissipation, in the fully developed region of a parallel plate vertical channel, has been analyzed [3]. An investigation through numerical analysis has been made to scrutinize the heat transfer of a viscous fluid squeezed and extruded between two parallel plates [4]. The study has considered constant wall temperatures, and revealed the interactive effects of squeezing and extrusion parameters on the velocity, the temperature profile and the heat transfer characteristics. Lahjomri et al. [5] have used a functional analysis method to investigate the influence of viscous dissipation on the heat transfer of a thermally-developing laminar Hartman flow through a parallel plate channel with the aid of a magnetic field. In a study of thermal development of forced convection in a parallel plate channel filled with porous medium, Nield et al. [6] have presented the effects of viscous dissipation with the thermal boundary condition of uniform wall temperature, including axial conduction effects. The effect of viscous heating on the stability of Taylor-Couette flow has been investigated experimentally by White and Muller [7]. The study has considered Newtonian and viscoelastic fluid for experimental exploration of the critical Reynolds number from the onset of the transition to a new state.

Many researchers have explored the effects of viscous dissipation in a forced convective heat transfer phenomenon in parallel plate channels and annular ducts, which significantly alter the thermo-fluidic transport and hence, cannot be neglected even for the flow of an ordinary fluid through macro channel or pipes. The analysis of laminar forced convection in a pipe for Newtonian fluid of constant properties has been made by Aydin [8, 9] considering the effect of viscous dissipation. In Part-1, both hydro-dynamically and thermally fully developed convections have been studied; Part-2 of the study considers the hydro-dynamically developed but thermally developing case. In both cases, two different types of thermal boundary conditions, e.g., constant heat flux (CHF) and constant wall temperature (CWT), have been considered. The variations of the dimensionless radial temperature and the Nusselt number have been observed for different values of the Brinkman number. The influence of viscous dissipation on heat transfer has been found to be strong for higher values of Brinkman number (Br >1), while the influence has been negligible for lower values of Brinkman number. In the thermally developing case, comparing the temperature distribution with that of the same obtained by neglecting the viscous dissipation, it has been observed that the temperature distribution increases in the axial direction, which is attributed to the effect of viscous dissipations. In their study, an important role of the Brinkman number (for the CWT case) and the modified Brinkman number (for the CHF case) has also been investigated on the development of the Nusselt number. The analytical work done by Aydin and Avci [10] has dealt with a convective heat transfer problem for a plane Poiseuille flow with an emphasis on the viscous dissipation effect. The energy equation has been solved for thermally developed and developing cases separately with the boundary condition of CWT and CHF, respectively. In both cases, the flow has been considered to be hydro-dynamically developed. It has been found from the study that with increasing intensity of viscous dissipation (i.e., increase in the Brinkman number), the heat transfer decreases up to a critical value, and that is attributed to the internal heat generation due to the viscous dissipation effect. In another work, Aydin and Avci [11] have studied the laminar forced convective heat transfer problem in a Couette- Poiseuille flow with an emphasis on the viscous dissipation effect. The effect of viscous dissipation on the forced convective heat transfer in fully developed flows of Newtonian fluid through annular ducts has been analyzed in an analytical framework [12]. The study has considered two different cases of thermal boundary conditions and has shown the influence of viscous dissipation on the temperature distribution and the Nusselt number.

In another study, Francisa and Tso [13] have extended the work of Aydin and Avci [10] and have investigated the viscous dissipation effects on the hydrothermal behavior of a Newtonian fluid flowing between two fixed parallel plates with constant-heat-flux thermal boundary conditions at both the plates. The study has revealed that the viscous heating plays an important role in the forced convective heat transfer.

Several approaches have been presented in the literature to obtain various analytical expressions of the Nusselt number as a function of the Brinkman number. The steady state laminar heat transfer to a plane Poiseuille-Couette flow of a Newtonian fluid with simultaneous pressure gradient and axial movement of one of the plates has been investigated by several researchers [14-17]. In all these cases, the energy equation containing viscous dissipation term has been solved numerically to obtain the effects of viscous dissipation on the heat transfer characteristics for the two alternate cases of thermal boundary conditions, e.g., temperature specified at both the plates, or a specified temperature at the stationary plate with the moving plate insulated. All works reported above have discussed about the effects of viscous dissipation on the hydrothermal characteristics in macroscale flows. However, in all cases, it is important to observe the point of singularities on the variation of the Nusselt number with the Brinkman number or the modified Brinkman number. The appearance of those points of singularities has been highlighted considering energy balance between wall surface heat and the heat generation due to viscous dissipation during the thermo-fluidic transport.

Following a numerical analysis, Lin [18] has studied the laminar heat transfer to a non-Newtonian Couette flow with pressure gradient, using the power-law model. Davaa et al. [19] have performed a numerical study to investigate the influence of viscous dissipation on fully developed laminar heat transfer in a non-Newtonian fluid flowing between two parallel plates while one of the plates moving axially. Quite recently, an investigation has been made using an analytical method to probe the effects of viscous dissipation on forced convective heat transfer in a power-law fluid within fixed parallel plates [20]. Considering the constant heat flux boundary condition in the analysis, the study has shown that the heat transfer characteristics depend on the power-law index, and the Brinkman number. The study has also revealed that pseudo-plastic and dilatant fluids exhibit different heat transfer characteristics in a viscous-dissipative environment. The effect of viscous dissipation on a fully developed forced convection Couette flow through parallel plate channel, partially filled with porous media, has also been analyzed [21]. The study has shown that the dimensionless temperature , and consequently, the Nusselt number decreases with increasing Brinkman number. Lately, through a numerical analysis, Ramiar and Ranjbar [22] have investigated the effects of viscous dissipation on the forced convective heat transfer of Al_2O_3 – water nanofluid in a microchannel. The study has explored that the viscous dissipation decreases the Nusselt number. Another very recent study has revealed the effect of viscous dissipation on the temperature profile and the heat transfer characteristics of a Couette-Poiesuille flow of pseudo-plastic fluids [23]. The study has considered both hydro-dynamically and thermally fully-developed flow, while asymmetric heat flux thermal boundary conditions have been used in the analysis. Through a semi-analytical formulation, the study has shown that the modified Brinkman number and the power law index strongly influence the temperature distribution and the Nusselt number.

Most of the above-mentioned studies have concentrated on either Poiseuille flow or a combined Couette-Poiseuille flow. No work, so far, has been reported on the laminar forced convection for a simple shear-driven flow, suggesting a quantitative relation between the different performance index parameters of heat transfer, including viscous dissipation, in a comprehensive way. In another work published quite recently, an effort has been made for analyzing the effects of viscous dissipation on the limiting Nusselt number for a hydro-dynamically fully developed laminar shear-driven flow through an asymmetrically heated annulus of two infinitely long concentric cylinders [24], and parallel plates [25].

The aim of the present study is to explore analytically the heat transfer characteristics for a fully developed shear-driven flow of a Newtonian fluid between two parallel plates. For this, a detailed and rigorous study is carried out to investigate the effect of viscous dissipation on the heat transfer characteristics for a shear-driven flow between two parallel plates subjected to the constant heat-flux boundary condition. In the analysis part, two definitions of the Brinkman number [10], i.e., temperature based and heat flux based, have been used along with the intermediate steps, to whet interest amongst the researchers.

2. PROBLEM FORMULATION AND ANALYSIS

2. 1. Physical Considerations Here, a channel between two parallel plates of infinite length, of height H and width b, with $b>>H$, is considered as shown in Figure 1. Fluid is flowing in the axial (x) direction, while the flow is influenced by the movement of the upper plate. The flow is fully developed - both hydro-dynamically and thermally. The no-slip boundary conditions are assumed to be valid at both the plates for both hydro-dynamic and thermal considerations. In addition to the consideration that the flow is fully developed, few more assumptions considered for the study are given below:

- Newtonian fluid;
- Incompressible fluid flow;
- There is no heat generation and thermo-physical properties are constant;

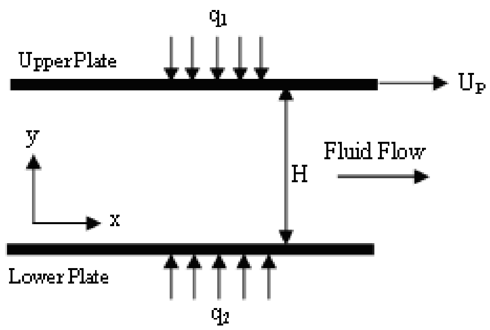


Figure 1. Schematic diagram describing the problem

- Axial conduction is neglected in the fluid and through the wall;

2. 2. Analysis of the Problem

The continuity, momentum and energy equations for incompressible fluid flow are found to be relevant to this study. They are as follows:

Continuity equation:

$$\frac{\partial u}{\partial x} = 0 \tag{1}$$

Momentum equation:

$$\mu\left(\frac{d^2u}{dy^2}\right) = 0 \tag{2}$$

Energy equation:

$$\rho C_p u\frac{\partial T}{\partial x} = k\frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial u}{\partial y}\right)^2 \tag{3}$$

where the last term on the right hand side of the above equation denotes the viscous dissipation.

2. 3. Both Plates at Different Constant Heat Fluxes

Both plates are kept at different constant heat fluxes. Also, the upper plate is at constant heat flux q_1 and the lower plate is at constant heat flux q_2. However, in order to express Equations (2) and (3) in a non-dimensional framework, it is essential to define the non-dimensional parameters suitably. From the physical considerations discussed above, following non-dimensional parameters are chosen:

$$U = \frac{u}{U_P},\ Y = \frac{y}{H},\ \text{and}\ \theta = \frac{(T - T_1)}{\frac{q_1 H}{k}};$$

where T_1 is the temperature of the upper plate.

The non-dimensional fully developed velocity profile is expressed as:

$$U = Y \tag{4}$$

However, with the aid of the above non-dimensionless parameters and using Equation (4), Equation (3) may be normalized to yield the following:

$$\frac{q_1}{H}\frac{d^2\theta}{dY^2} + \frac{\mu U_P^2}{H^2} = \frac{\rho C_p U_P\ y}{H}\frac{\partial T}{\partial x} \tag{5}$$

where $\frac{\partial T}{\partial x} = \frac{dT_1}{dx}$ and $Br_m = \frac{\mu U_P^2}{q_1 H}$ is the modified Brinkman Number based on the upper plate heat flux q_1. By defining $\left(\frac{\rho C_p U_P H}{q_1}\right)\frac{dT_1}{dx} = a_1$, Equation (5) can finally be rewritten as:

$$\frac{d^2\theta}{dY^2} = a_1 Y - Br_m \tag{6}$$

However, in order to get the temperature profile, following thermal boundary conditions, imposed on the plates, are utilized. In a non-dimensional form, the above set of boundary conditions may be expressed as given below:

$$Y = 1,\ \begin{cases}\theta = 0\\ \frac{d\theta}{dY} = 1\end{cases} \tag{7a}$$

$$Y = 0,\ \frac{d\theta}{dY} = -\frac{q_2}{q_1} \tag{7b}$$

Solving Equation (6) with the above set of boundary conditions, the general expression of the temperature profile is obtained as:

$$\theta = \left(1 + \frac{q_2}{q_1}\right)\frac{Y^3}{3} + Br_m\left(\frac{Y^3}{3} - \frac{Y^2}{2} + \frac{1}{6}\right) + \frac{q_2}{q_1}\left(\frac{2}{3} - Y\right) - \frac{1}{3} \tag{8}$$

In order to obtain a deeper insight into the heat transfer characteristics, the bulk mean fluid temperature T_m is defined as:

$$T_m = \frac{\int_{y=0}^{H} uT\,dy}{\int_{y=0}^{H} u\,dy} \tag{9}$$

Heat transfer at the lower plate is expressed as :

$$q_1 = h_1(T_1 - T_m) = k\left.\frac{\partial T}{\partial y}\right|_{y=0} \tag{10}$$

where h_1 is the convective heat transfer coefficient. Hence, the Nusselt number comes out to be:

$$Nu_H = \frac{hH}{k} = \frac{Hq_1}{(T_1 - T_m)k} = -\frac{1}{\theta_m} \tag{11}$$

The non-dimensional mean temperature is given by:

$$\theta_m = \frac{T_m - T_1}{q_1 H / k} = 2\left(\frac{q_2}{15 q_1} + \frac{Br_m}{40} - \frac{1}{10}\right) \quad (12)$$

Finally, the expression of Nusselt number using the Equations (11) and (12) is obtained as:

$$Nu_H = -\frac{60}{\left(8\frac{q_2}{q_1} + 3Br_m - 12\right)} \quad (13)$$

Based on the expression of Nusselt number obtained at the unequal constant heat flux condition as mentioned above, the expression of the Nusselt number can now be derived for some limiting cases to understand the heat transfer characteristics in a viscous-dissipative environment. Some of the cases are discussed in the next sub-sections.

2. 3. 1. Upper Plate at Constant Heat Flux q_1 and Lower Plate Insulated The Nusselt number in this condition from Equation (13), for $q_2 = 0$ is given below:

$$Nu_H = \frac{20}{(4 - Br_m)} \quad (14)$$

The above equation suggests a new way of expressing the Nusselt number as compared to what is available in the literature till date.

2. 3. 2. Both Plates at Equal Constant Heat Flux q_1

In this case, one can express the Nusselt number from Equation (13), for $q_1 = q_2$, as given below:

$$Nu_H = \frac{60}{(4 - 3Br_m)} \quad (15)$$

The above equation also expresses the Nusselt number in a different manner as compared to what is available at present in the literature.

2. 4. Solution Using Temperature Difference

Here, a different kind of analytical method is adopted, and the Brinkman number is defined such as to obtain the closed-form solution of the temperature difference, and, subsequently, the expression of the Nusselt number.

2. 4. 1. Upper Plate at Constant Heat Flux and Lower Plate Insulated In this section, a case is considered where the upper plate is at constant heat flux q_1 and the lower plate is insulated, which resembles Figure 1 with $q_2 = 0$. Moreover, it is assumed that the temperatures of the upper and lower plates are T_1 and T_2, respectively, when both temperatures vary along x direction. However, for this case, following non-dimensional parameters are defined to obtain the thermal energy equation in a non-dimensional framework.

$$Y = \frac{y}{H}\text{; and } \theta = \frac{(T - T_1)}{(T_1 - T_2)} \quad (16)$$

With the aid of the above non-dimensional quantities, the energy equation obtained as:

$$k\frac{\Delta T}{H^2}\frac{d^2\theta}{dY^2} + \mu\frac{U_P^2}{H^2} = \rho C_P U_P Y \frac{dT_1}{dx} \quad (17)$$

The Equation (17) can be simplified as:

$$\frac{d^2\theta}{dY^2} = a_2 Y - Br \quad (18)$$

where $a_2 = U_P \frac{H^2}{\alpha \Delta T}\frac{dT_1}{dx}$, $\Delta T = (T_1 - T_2)$ and the Brinkman Number

$$Br = \frac{\mu U_P^2}{k \Delta T} \quad (19)$$

However, Equation (18) is subjected to the boundary conditions as below:

$$Y = 0, \begin{cases} \frac{d\theta}{dY} = 0 \\ \theta = -1 \end{cases} \quad (20a)$$

$$Y = 1, \ \theta = 0 \quad (20b)$$

Solving Equation (18) with the above set of boundary conditions, the temperature profile is obtained as:

$$\theta = Y^3 + \frac{Br}{2}\left(Y^3 - Y^2\right) - 1 \quad (21)$$

However, by using Equation (20) the expression of the mean temperature in a dimensionless form is obtained as:

$$\theta_m = \frac{T_m - T_1}{T_1 - T_2} = -\left(\frac{Br}{20} + \frac{3}{5}\right) \quad (22)$$

Now, from the heat flux given at the upper plate, the expression of Nusselt number comes out to be:

$$Nu_H = \frac{h_1 H}{k} = \frac{(T_1 - T_2)}{(T_1 - T_m)}\left[\frac{\partial \theta}{\partial Y}\right]_{Y=1} = \frac{\left(3 + \frac{Br}{2}\right)}{\left(\frac{Br}{20} + \frac{3}{5}\right)} \quad (23)$$

However, it is interesting to note from the above expression of the Nusselt number that when $Br = 0$, $Nu = 5$. This is identical to the result obtained under different-heat-flux condition when $Br_m = 0$.

2. 4. 2. Both Plates at Equal Constant Heat Fluxes

In this section, a case is considered where both plates are maintained at the same constant heat flux q_1 (Figure 1 with $q_2 = q_1$). Considering symmetry of the problem, the temperature of both plates is assumed to be T_w, varying along x-direction. However, for this case, following non-dimensional parameters are defined to make the thermal energy equation dimensionless.

$$Y = \frac{y}{H}\text{; and } \theta = \frac{(T - T_w)}{(T_f - T_w)} \tag{24}$$

where T_f is the uniform fluid temperature at the centerline.

With the aid of the above non-dimensional quantities, the energy equation is obtained as:

$$k\frac{\Delta T}{H^2}\frac{d^2\theta}{dY^2} + \mu\frac{U_P^2}{H^2} = \rho C_P U_P Y \frac{dT_w}{dx} \tag{25}$$

The Equation (25) can be rewritten as:

$$\frac{d^2\theta}{dY^2} = a_3 Y - Br \tag{26}$$

where $a_3 = U_P \frac{H^2}{\alpha \Delta T}\frac{dT_w}{dx}$, $\Delta T = (T_f - T_w)$ and the Brinkman Number

$$Br = \frac{\mu U_P^2}{k \Delta T} \tag{27}$$

However, Equation (26) is subjected to the boundary conditions as below:

$$Y = \frac{1}{2}, \begin{cases} \frac{d\theta}{dY} = 0 \\ \theta = 1 \end{cases} \tag{28a}$$

$$Y = 0,\ \theta = 0 \tag{28b}$$

The solution of Equation (26) subjected to the above set of boundary conditions is:

$$\theta = -2Y^3 + \frac{3}{2}Y + \frac{Br}{2}\left(Y^3 - Y^2 + \frac{1}{4}\right) \tag{29}$$

However, the expression of the mean temperature in the dimensionless form is found to be:

$$\theta_m = \frac{T_m - T_w}{T_f - T_w} = \left(\frac{Br}{30} + \frac{1}{5}\right) \tag{30}$$

Now, for the heat flux given at the upper plate, the expression of the Nusselt number reduces to:

$$Nu_H = \frac{h_1 H}{k} = \frac{(T_f - T_w)}{(T_w - T_m)}\left[\frac{\partial \theta}{\partial Y}\right]_{Y=1} = \frac{\left(\frac{9}{2} - \frac{5}{8}Br\right)}{\left(\frac{Br}{30} + \frac{1}{5}\right)} \tag{31}$$

3. RESULTS AND DISCUSSIONS

In order to bring out the effect of viscous dissipation on the Nusselt number and the temperature profile, three different particular cases are presented to investigate the heat transfer characteristics. Using the analytical technique described above, some expressions of the Nusselt number and the temperature profile are obtained. In this section, several plots are presented and discussed in brief.

3. 1. Plates at Different Constant Heat Fluxes q_1 and q_2

The Brinkman number is an important parameter governing the heat transfer and the fluid flow between two parallel plates. Effects of viscous dissipation in a fluid flow and heat transfer phenomenon is explained by the Brinkman number. The present study aims in finding out the influence of the viscous dissipation effects on the temperature profile, and the resulting Nusselt numbers. Figure 2 depicts the dimensionless temperature profile within the flow field for different Br_m, pertaining to the case where plates are kept at different constant heat flux conditions obtained from Equation (8). One may observe that with increasing value of Br_m, the temperature increases as expected. Positive values of Br_m are compatible with the wall heating case, which indicates heat transfer to the fluid across the wall. Therefore, in the cases with positive Br_m, the fluid temperature increases as evident from the above figure. However, the temperature profile close to the upper plate shows an increasing trend. The increasing trend of temperature profile nearer to the upper plate is attributed to the effect of shear in the fluid layer produced by the movement of the upper plate.

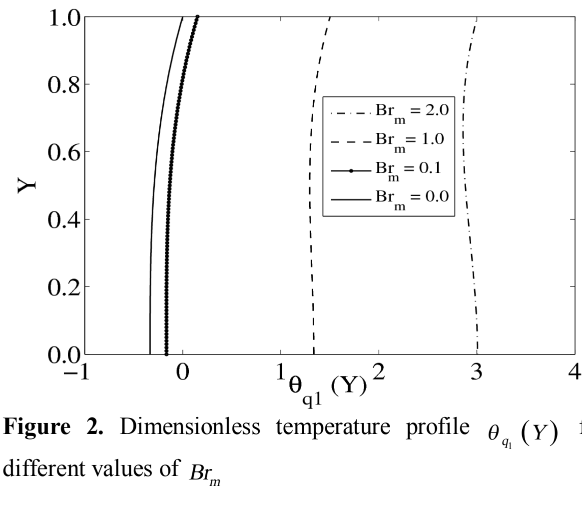


Figure 2. Dimensionless temperature profile $\theta_{q_1}(Y)$ for different values of Br_m

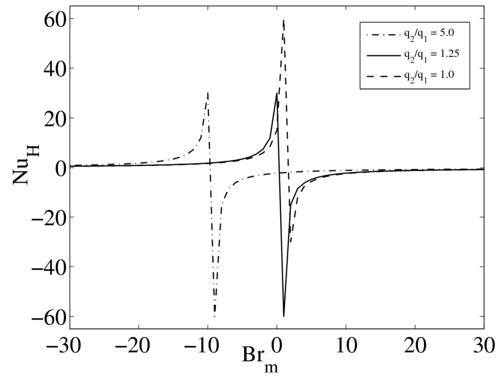


Figure 3(a). The influence of Br_m on the Nu_H for different q_2/q_1

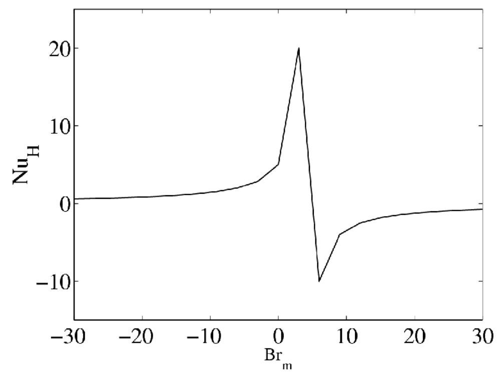


Figure 3(b). The influence of Br_m on the Nu_H for $q_2/q_1 = 0$

The main physical quantity of interest is the Nusselt number which represents the heat transfer rate at the wall of the plate. The variation of the Nusselt number with the Brinkman number needs to be investigated. To demonstrate the effect of viscous dissipation on the Nusselt number, Equation (13) is considered. However, the variation of the Nusselt number with Br_m is shown in Figures 3a and b for heat flux ratio $q_2/q_1 = 1$, 1.25, 5 and for $q_2/q_1 = 0$, respectively. The choice of different heat flux ratios represents different cases. The ratio $q_2/q_1 = 1$ corresponds to the case, where both plates are at equal constant heat flux. Similarly, 0 corresponds to the case of an insulated lower plate. The ratio 1.25 indicates the special case occurring due to the point of singularity at the origin.

One may notice form the above figures that the variation of the Nusselt number with Br_m is not continuous for all the cases considered in the study; rather a clear existence of the point of singularity is observed in each case at a different point at a different Br_m, as suggested by Equation (13). The different locations of the point of singularity are due to the different ratios of heat flux considered, and, at this point, the shear heating balances the heat supplied by the wall. However, from this point of singularity as Br_m increases in the positive direction ($Br_m > 0$), the Nusselt number decreases because of the decrease in the driving potential of the heat transfer, and it finally attains different constant values asymptotically, (when $Br_m \to \infty$), for all the cases of heat flux taken into account. The negative value of Br_m represents the wall-cooling problem and with the increasing value of Br_m in the negative direction, the Nusselt number decreases and an asymptote appears at different constant values of Nu_H for different cases as $Br_m \to -\infty$.

3. 2. Lower Plate Insulated and Upper Plate at Constant Heat Flux In this section, the graphical plots of the variation of the dimensionless temperature profile and the Nusselt number using the Brinkman number defined in Equation (19) are presented. The temperature variation is plotted in Figures 4a - b where as Figure 5 shows the variation of the Nusselt number with Br.

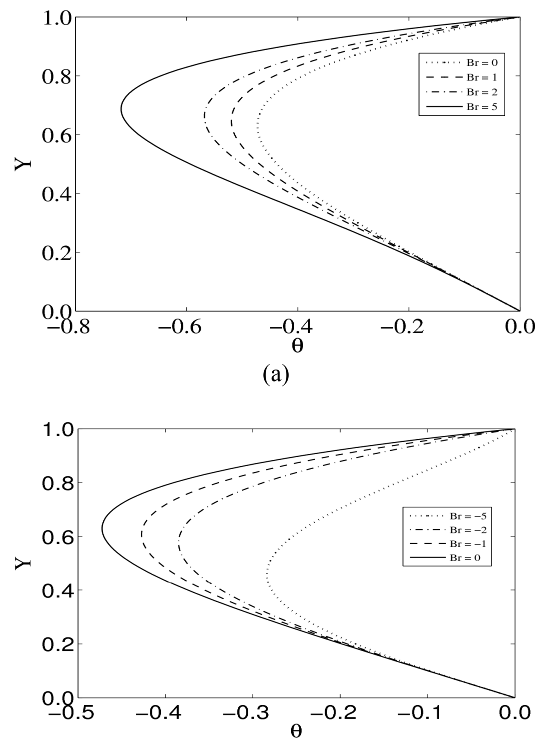


Figure 4. Dimensionless temperature profile $\theta(Y)$ versus Y for different values of Br for the case of insulated lower plate: (a) hot wall (b) cold wall

Figure 4a corresponds to the wall-heating case and, as expected, the bulk temperature of the fluid increases with increasing values of Br. This indicates that as dissipation increases, the fluid temperature increases due to the internal fluid friction. On the contrary, one may observe from Figure 4b that for the wall-cooling case, with increasing Br, the bulk temperature of the fluid decreases compared to the case with a negligible Br. Actually, the wall-cooling case is applied to reduce the fluid temperature, and it is important to note from Figure 4b that even at higher value of Br, the temperature of the fluid decreases, which can be attributed to the movement of the upper plate. However, this shows similarity with the viscous heating effect between parallel plates [15]. Interestingly, one can make an important observation from Figures 4a - b that the viscous dissipation effects become prominent in a zone, close to the upper plate, due to the high shear rate over there.

The variation of Nusselt number as depicted in Figure 5 shows similarity with the results of the published literature [13]. Increasing Br makes the bulk temperature of the fluid to increase and hence, the driving potential of the heat transfer is reduced, which is reflected on the variation of the Nusselt number as Br increases in the positive direction. However, the Nusselt number decreases asymptotically as $Br \to \infty$. As explained, the negative value of Br represents the wall-cooling problem, and with the increasing value of Br in the negative direction, the Nusselt number decreases and an asymptote appears as $Br \to -\infty$. It is important to observe the existence of the point of singularity at $Br = -12$, which is quite clear from Equation (23).

3. 3. Both Plates at Equal Constant Heat Flux

Here, the variation of the dimensionless temperature profile and the Nusselt number using the Brinkman number, defined in Equation (27), is discussed through presentation of their graphical plots obtained from Equations (29) and (31). The temperature variation is plotted in Figures 6a - b; whereas Figure 7 shows the variation of the Nusselt number with Br.

Viscous dissipation always generates a distribution of heat source stimulating the internal energy in the fluid, and hence the temperature profile gets distorted, which is envisaged from the above figures. Figure 6a depicts the dimensionless temperature profile within the flow field for the wall-heating case. As explained earlier that for wall-heating case the fluid temperature increases, where as the reverse is true for the wall-cooling case. Interestingly, one can see from above figure that in case of equal constant heat flux, the dimensionless temperature profile exhibits usual trend of increasing temperature with positive values of Br, up to a certain distance from the lower plate at $Y = 0.3$; then it is followed by a decreasing trend even at positive values of Br up to the upper plate. The increasing temperature with positive Br fairly matches the temperature profile of a viscous fluid squeezed and extruded between two parallel plates [4].

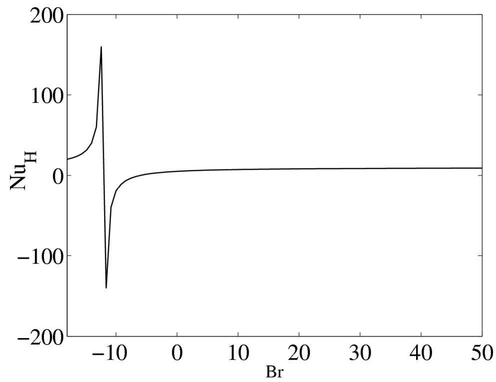
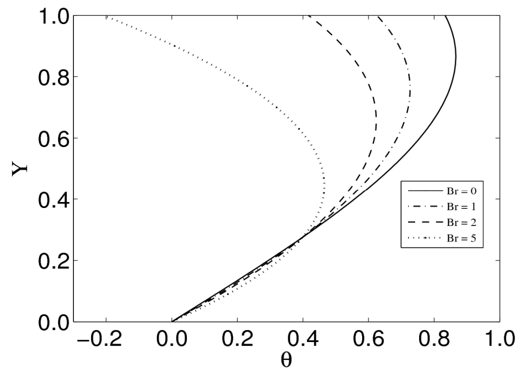
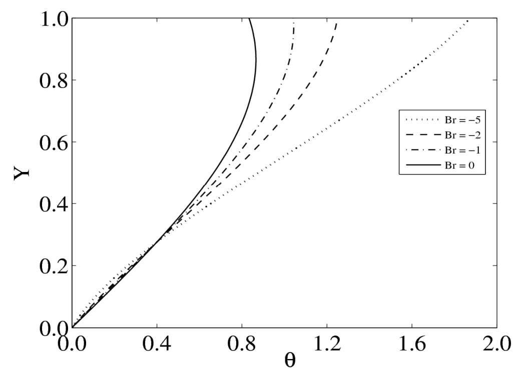


Figure 5. The influence of Br on the Nu_H for the case of insulated lower plate



(a)



(b)

Figure 6. Dimensionless temperature profile $\theta(Y)$ versus Y for different values of Br for the case of equal constant heat fluxes: (a) hot wall (b) cold wall

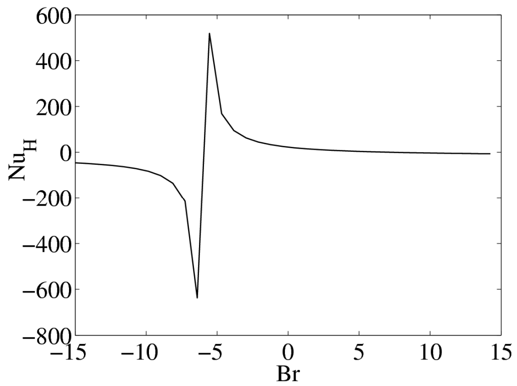


Figure 7. The influence of *Br* on the Nu_H for the case of equal constant heat fluxes

A reverse explanation holds true for negative values of *Br*, which one can also observe from Figure 6b. This contradictory behaviour of the temperature profile with *Br* for any particular case of wall heating as seen from the above figures is owing to the movement of the upper plate, and the thermal boundary condition considered in this case.

Figure 7 exhibits the variation of the Nusselt number with Br. However, compared to cases with an insulated lower plate, the variation of the Nusselt number shows a distinct feature as Br changes in case of the equal constant heat flux condition. It is important to observe the existence of the point of singularity on the variation at $Br = -6$, as expected from Equation (23). However, from the point of singularity the Nusselt number reaches a constant value in either direction asymptotically.

4. CONCLUSIONS

In this work, influence of the viscous dissipation on the heat transfer characteristics in a Newtonian fluid flowing between two parallel plates is investigated. Here, an analytical approach is presented in an exhaustive way to suggest explicit expressions of the Nusselt number, utilizing two definitions of the Brinkman number for three different cases of constant-heat-flux boundary conditions. To obtain the temperature profile, and the resulting Nusselt number, variable separation method has been used twice in the analysis. Also, different cases are demonstrated and expressions of the temperature profile and the Nusselt number are presented in different sub-sections. The influential role of viscous dissipation is found to be of great importance in the heat transfer analysis; hence, an emphasis on the viscous dissipation is given to include the effect of the shear stress induced by axial movement of the upper plate in addition to the effect of the viscous dissipation due to the internal fluid heating.

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An Analytical Approach to the Effect of Viscous Dissipation on Shear-driven Flow between two Parallel Plates with Constant Heat Flux Boundary Conditions

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چکیده

این تحقیقات برای تجزیه و تحلیل تاثیر تغییرات گران‌روی بر روی ویژگی های انتقال حرارت در جریان آرام کاملاً توسعه یافته‌ی هیدرودینامکی و حرارتی بر روی جریان بین دو صفحه‌ی بی نهایت موازی طولانی که در آن صفحه‌ی بالا در جهت محوری در یک سرعت ثابت در حال حرکت است، انجام شده است. بر اساس برخی از مفروضات معمول در مقالات، بیان اعداد نوسلت به فرم بسته تحلیلی برای جریان سیال نیوتنی با خواص ثابت برای سه مورد مختلف از شرایط مرزی با شار ثابت گرمایی مورد استفاده قرار گرفته است. اهمیت تغییرات گران‌روی در مقایسه با شرایط دیگر در معادله‌ی انرژی، توسط عدد برینکمن آشکار می شود. به منظور داشتن یک ایده کلی در مورد اثر حرارتی گران‌روی بر روی تجزیه و تحلیل انتقال حرارت، تعاریف مختلفی از اعداد در مطالعه حاضر مورد استفاده قرار گرفته است. در این جا، تمرکز بر روی اثر اتلاف گران‌روی به دلیل تنش برشی تولید شده توسط صفحه‌ی بالایی در حال حرکت می‌باشد و از حرارت ناشی از گران‌روی به علت اصطکاک داخلی سیال در جریان یک سیال نیوتنی چشم‌پوشی شده است. نقش برجسته‌ی اتلاف گران‌روی بر روی ویژگی‌های انتقال حرارت به صورت جزئی بیان شده است. برای حل این مشکل، با شرایط مرزی حرارتی مختلف مورد بحث قرار گرفته است.

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