



## Simulation of Cold Rolling Process Using Smoothed Particle Hydrodynamics (SPH)

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### ABSTRACT

A mesh-free 2-D numerical approach was examined for cold rolling process to gain a more simpler and faster way to simulate such complicated case. Regarding the wide range of gained reports about the simplicity and computationally efficient characters of the smoothed particle hydrodynamics (SPH) method, the computational simulation stands on SPH technique in current numerical study. In this paper, the computational efforts not only confirm the advantages of using the SPH, but also reveal some improvements in simulation efficiencies. The rolling test was performed for an aluminum strip: Al 6061. In this way, the rolls assumed to behave as rigid bodies and, the aluminum strip assumed to behave as an elastic-plastic continuum. In order to achieve the required assurance of the employed technique, the computed stress distribution patterns were compared with those reported from a finite element study. The comparison reveals good agreements of the two computed results. Moreover, as the final test case, the effect of some parameters that have been reported to play the main role in case; roll diameter, percentage of thickness reduction of the strip, and the rolling speed has been studied. Regarding all parts of the current work, it may be concluded that the SPH can be sufficient tool to gain a rapid and simple simulation for such complicated cases.

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## 1. INTRODUCTION

A wide range of metal forming processes such as extrusion, rolling, drawing and forging are used to produce metal products. The rolling process plays an important role in manufacturing process for making different parts with a variety of dimensions. In General, in about 30% of metal products in the world, the cold rolling process is used. The main advantage of rolling technique with respect to other thickness-reduction processes, such as strip drawing, is that it exerts a tri-axial compression stress to ductility. This may cause a larger reduction per unit mass. In this procedure, the internal raw material, especially steels and aluminum, transform into desirable shape with the aid of at least two rolls. Based on work hardening on metal, rolling is categorized into hot and cold rolling. The work hardening happens only in cold rolling.

In the most of the numerical studies of metal forming, the Finite-element approaches are the main available tool. In literature, some studies were focused nonlinear finite element for cold rolling to eliminate

void defects in materials [1]. Moreover, genetic algorithms, based on numerical estimations, were employed to predict the optimal design points [2]. Further, neural network models based on finite element results on cold rolling were performed to predict the velocity field and the neutral plane's locus [3]. Other studies such as finite-difference (FDM) were also examined for the mechanical studies of the cold strip rolling process [4]. These methods, despite of many advantages, have some considerable restrictions, such as simplification of kinetic assumptions, materials' behaviors, mesh distortion due to large plastic deformation and time-consuming character. Regarding to these considerable restrictions, using mesh-free methods seems to be useful. Due to mesh-free nature, in such large deformations, it seems that the Smoothed Particle Hydrodynamic (SPH) may become a capable tool. SPH is a Lagrangian particle-based method that material is represented by a set of particles follows the motion and advection material quantities such as mass and momentum. This method has been originated in 1977 for astrophysics [5, 6] and gradually is being improved and extended to model a wide range of problems, including shock wave. A Modified Compressible Smoothed Particle Hydrodynamics

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(MCSPH) is used to model shock wave and elastic-plastic deformations of solids [7].

A new algorithm of smoothed particle hydrodynamics (SPH) was proposed to simulate high velocity impact and elastic-plastic deformation [8]. A modified SPH method is also improved for fluid-structure interaction (FSI) problems combined with solid-rigid contacts [9].

SPH is a robust mesh-free method that its high capability is proved in metal forming, extrusion and forging using LS-DYNA software [10] and cutting [11].

Some software's such as DEFORM take the advantage of finite element method [12] to simulate rolling process. In comparison, the SPH is less examined than the FEM. It seems that SPH is a young mesh-free method even it doesn't have the restrictions of mesh-based method.

The purpose of this paper is to develop and examine the efficiency of the SPH in of the complicated metal forming process. It is noticeable that the simulation of rolling by SPH has not been reported in the technical literature.

The current work focuses on a 2-D simulation of the cold rolling by SPH. The radial return plasticity model proposed by Wilkins [13] was also used to gain a more reliable mechanical simulation. The unloading condition was set as one of the most concerned conditions in rolling process. Simultaneously, the variability some factors such as diameter, angular velocity and thickness has been examined to achieve a more realistic study. To set such conditions, some special treatments in SPH must be performed.

## 2. SPH FORMULATION

Smoothed Particle Hydrodynamic (SPH) is a mesh free method which is based on interpolation theory in which the interpolation function is called Kernel.

For more details, referred to Monaghan et al. [14] is recommended. Referring to Monaghan et al. [15], the preferred form of continuity equation is:

$$\frac{d\rho_a}{dt} = \rho_a \sum_b \frac{m_b}{\rho_b} V_{ab} \nabla_a w_{ab} \quad (1)$$

where,  $\rho$ ,  $V$  and,  $m$  are density, velocity and mass, respectively.  $b$  is the neighbor index of the particle  $a$ .

The advantage of this form of the continuity equation  $m_b / \rho_b$  becomes constant, and consequently,  $\nabla \cdot V = 0$  is satisfied.

The SPH momentum equation used for elastic and fracture problems is:

$$\frac{dv_a^i}{dt} = \sum_b m_b \left( \frac{\sigma_a^{ij}}{\rho_a^2} + \frac{\sigma_b^{ij}}{\rho_b^2} + \Pi_{ab} + R_{ab}^{ij} \Gamma^n \right) \frac{\partial w_{ab}}{\partial x_a^j} + g^i \quad (2)$$

where,  $v_a$ ,  $p_a$  are the velocity and density of particle  $a$  and  $m_b$  is the mass of particle  $b$ . The term  $\Pi_{ab}$  produces a shear and bulk viscosity (for more details see [15, 16]).

$W_{ab} = w(r_{ab}, h)$  is the interpolation kernel with smoothing length  $h$  and  $r_{ab} = |r_a - r_b|$  is the defined distance between particle  $a$  and  $b$ . In this paper, cubic spline kernel is used which have following form ( $q = \frac{r}{h}$ ):

$$\begin{cases} w(r, h) = \frac{10}{7\pi h^2} \left( 1 - \frac{3}{2} q^2 + \frac{3}{4} q^3 \right) & \text{for } 1 > q \\ w(r, h) = \frac{5}{14\pi h^2} (2 - q)^3 & \text{for } 1 < q < 2 \\ w(r, h) = 0 & \text{for } q > 2 \end{cases} \quad (3)$$

$\sigma_{ij}$  is the stress tensor which can be calculated as:

$$\sigma^{ij} = -p \delta^{ij} + S^{ij} \quad (4)$$

where,  $P$  is the pressure and  $S$  is deviatoric stress. The evaluation equation for the deviatoric stress  $S$  is:

$$\frac{ds^{ij}}{dt} = 2\mu \left( \varepsilon^{ij} - \frac{1}{3} \delta^{ij} \varepsilon^{kk} \right) + s^{ik} R^{jk} + R^{ik} s^{kj} \quad (5)$$

where,  $\mu$  is shear modulus and the pressure is determined as follows:

$$P = c_0^2 (\rho - \rho_0) \quad (6)$$

Here,  $c_0$  is the speed of sound and  $\rho_0$  is the reference density.

The plasticity model used is radial return which was proposed by wilkins [13]. Initial response of deviatoric stress is assumed to be elastic. In the applied model:

$$s^{ij} = \alpha s_{tr}^{ij} \quad (7)$$

where,  $s_{tr}^{ij}$  and  $s^{ij}$  are trial and final deviatoric stresses at the end of time step, respectively.  $\alpha$  is proportionality constant coefficient defined as:

$$\alpha = 1 - \frac{3\mu_s d\bar{\varepsilon}_p}{\bar{\sigma}^{tr}} \quad (8)$$

$\bar{\sigma}^{tr}$  is the effective stress in von-misses yield criterion. In the equivalent plastic strain:

$$d\bar{\varepsilon}_p = \frac{\bar{\sigma}^{tr} - \sigma_n^y}{3G + H} \quad (9)$$

where,  $\sigma_n^y$  is the final yield stress and  $H$  is the hardening modulus. The plastic strain is sum of  $d\varepsilon_p$  of each time step as:

$$\bar{\varepsilon}_p = \bar{\varepsilon}_p + d\bar{\varepsilon}_p \quad (10)$$

### 3. SPH MODELING OF THE ROLLING PROCESS

A plain strain rolling simulation is performed on Al 6061. Figure 1 shows a general view of process and parameters such as bite angle ( $\alpha$ ), neutral plan, deformed and undeformed thickness of strip.

The initial particles positions of the rolls and the strip is shown in Figure 2. An alternating digital tree (ADT) algorithm was used to search nearest neighbor particles within the distance of  $3h$  ( $h$  is smoothing length and  $h = 1.5 r_{ab}$ ). A time step of  $1e-6$  Sec was used to ensure numerical stability [7]. Table 1, shows the material properties used in this study. The geometric properties of test case are to values introduced in Table 2.

TABLE 1. Properties of Al 6061

Roll diameter (mm)	60
Angular velocity (rad/s)	33.42
Initial thickness (mm)	3.17
Reduction (%)	27.44
Coefficient of friction	0.13

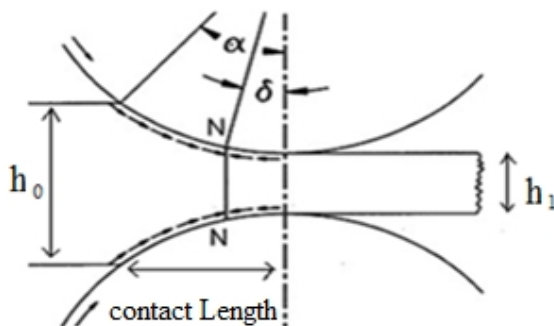


Figure 1. The Schematic cold rolling process configuration.

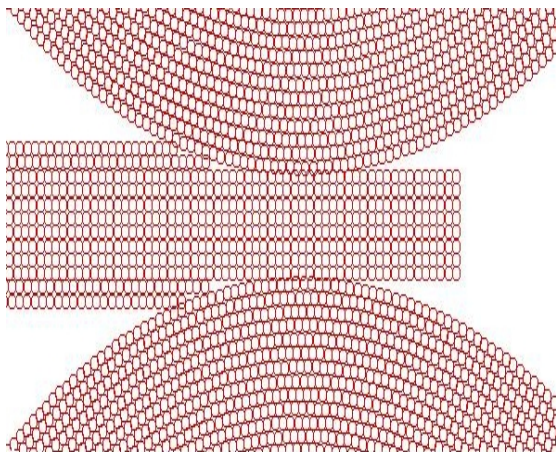


Figure 2. Initial particle position of the rolls and the strip.

TABLE 2. The geometric properties of test case.

Bulk modulus (GPa)	70.0
Shear Modulus (GPa)	27.0
Initial Yield Stress (MPa)	55.2
Hardening Modulus (MPa)	1.67
Density ( $kg/m^3$ )	2700.0

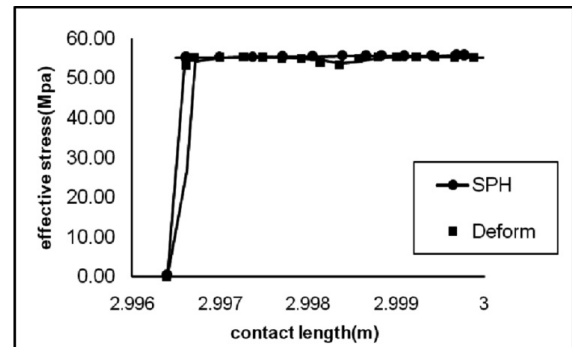


Figure 3. Stress distribution comparison of SPH and DEFORM.

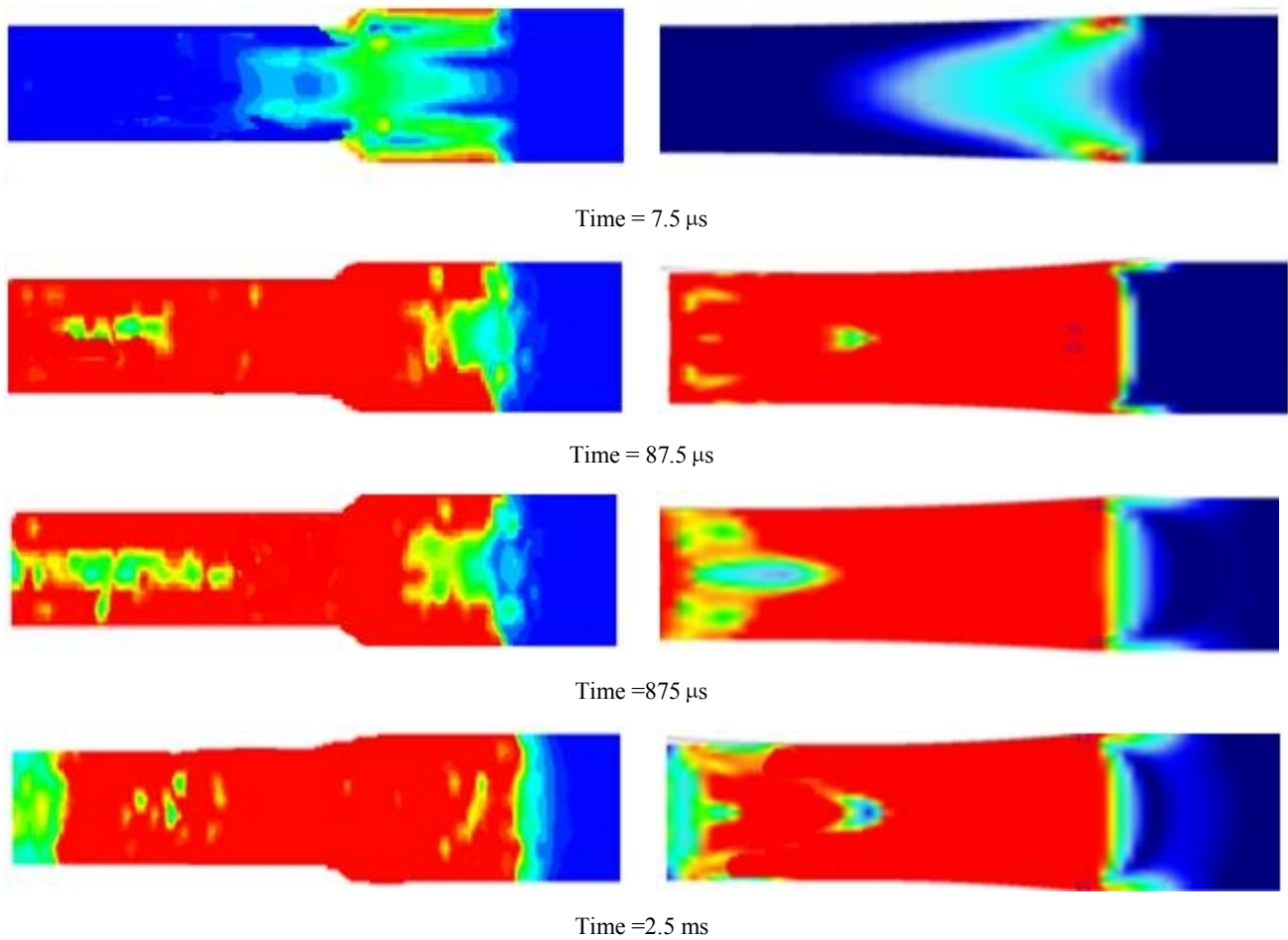
### 4. THE NUMERICAL VALIDATIONS

The numerical simulation is validated by a 2D rolling model simulated in commercial finite element software, DEFORM-2D. It can be observed in Figure 3 that the stress distribution of SPH and DEFORM are fairly in agreement. The stress contours obtained from the two techniques, in different time- steps are compared with each other in Figure 4. This figure indicates that stress starts from the first contact point of roll and strips (at bite angle) and gradually spreads through the surface close to rolls and strip's center.

### 5. THE FINAL NUMERICAL TESTS

Unloading or deduction of load exerted on work piece is achieved by removing work piece from rolls gap in actual process. In mesh based methods, unloading is complicated because at the end of each increment, each element of work piece must be checked for passing rolls' gap. In contrast, simulation of rolling using SPH is not conditional and such control is not required. This is because of interface information transfer nature of the SPH, which stands on the simple interchange of particles by transferring of pressure to neighbor particles even between two materials.

Regarding the test protocol mentioned before, the numerical test concerns on variable effective parameters, and each one were described alternatively in this section.



**Figure 4.** Stress distribution using SPH (left) and DEFORM model (right) at 17.5  $\mu$ s, 87.5  $\mu$ s, 875  $\mu$ s and 2.5 m $\mu$ s. The colors is set as: At left side, red is 53.0 Mpa, green 26.5 Mpa, light blue 15.9 Mpa and dark blue corresponding 0.0, for the right side the same colors are set for 55.2 Mpa, 27.6 Mpa, 16.6 Mpa and 0.0 for dark blue, respectively.

**5. 1. Diameter** Augmentation of roll diameter exerts more pressure on sheet and let plastic deformation of metal strip starts earlier. Reduction of diameter (diameter = 40mm) causes the pressure on the strip to decrease (see Figure 5). Because of this deduction, plastic deformation starts with more delay and the gradient of elastic part of stress reduces (see Figure 6).

**5. 2. Thickness Reduction** More reduction in thickness may causes more complicated and coarse grid system and consequently may cause a significant CPU time in the mesh-based methods. Alternatively, in the SPH this difficulty reduces to minor CPU time increases. Figure 7 shows a comparison of pressure distribution of rolling in two cases of thickness reduction. More reduction of thickness (37%) means more deformation of the work piece; therefore, more pressure is needed for deformation. Figure 7 also depicts larger deformations cause larger gradient.

More reduction of thickness in mesh based methods causes more distortion in elements and this seems to be a significant deficiency of these methods. So, re-meshing or use of lateral methods would be crucial. SPH is a particle method and ability to overcome this problem is the superiority of this method.

**5. 3. Rotation Speed Reduction** Reduction of angular velocity (rotation speed =16.71 rad/s) doesn't have any effect on the distribution of pressure for non sensitive materials. However, the speed reduction causes more gentle deformations as well as reduction in the gradient of elastic part of the stress. As Figure 9 shows, the reduction of angular velocity doesn't affect pressure distribution on the strip. The stress distribution in reduction of the angular velocity case is shown in Figure 10. It is evident that the slowing down causes a delay in plastic deformation.

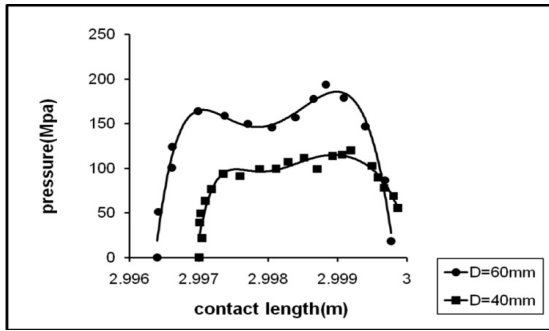


Figure 5. Comparison of pressure distribution of rolling in reduction of the diameter.

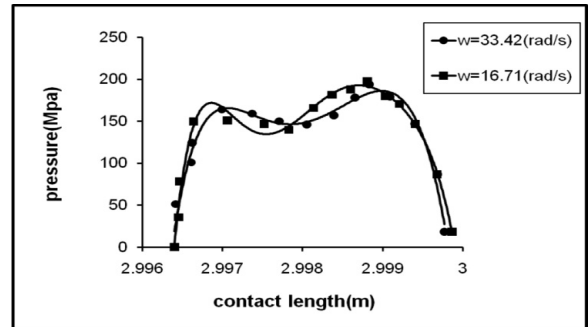


Figure 9. Comparison of pressure distribution of rolling in reduction of angular velocity.

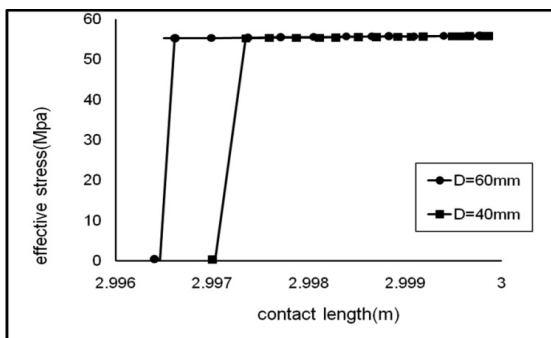


Figure 6. Comparison of stress distribution of rolling in reduction of diameter.

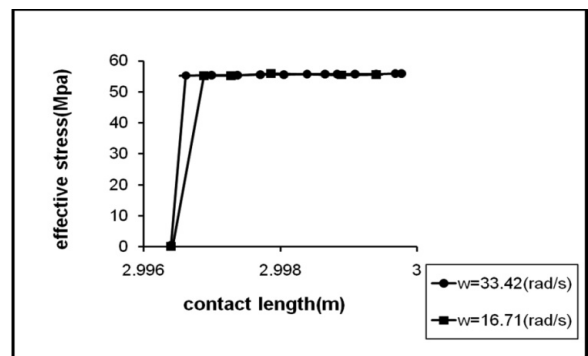


Figure 10. Comparison of stress distribution of rolling in reduction of angular velocity

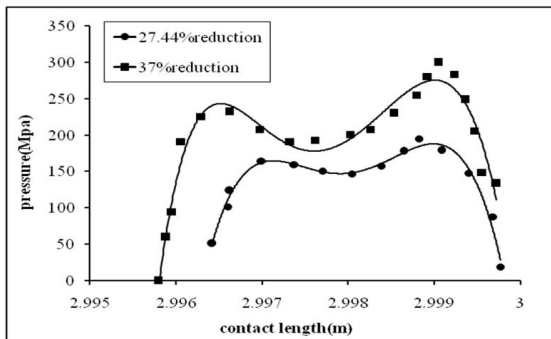


Figure 7. Comparison of pressure distribution of rolling in reduction of the thickness.

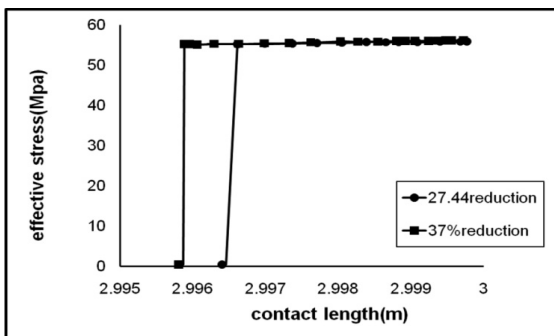


Figure 8. Comparison of stress distribution of rolling in reduction of thickness

## 6. CONCLUSIONS

In this paper, stress and pressure distribution on the plain strain rolling was investigated. The results of simulation of stress distribution using SPH and DEFORM were in relatively agreement with each other. The objectives, such as reduction of diameter, thickness and angular velocity showed that the reduction of the angular velocity could not affect the pressure, but the reduction of diameter and the thickness had decreasing and increasing effects on the pressure, respectively. In addition, the reduction of the angular velocity and the diameter caused initiation of plastic deformation after a little while. In contrast, thickness-reduction augments the gradient of elastic deformation in stress.

The agreement fortified the idea of achieving much more simple future numerical approaches for such complicate and time consuming cases. In other hand, the variable smoothing distance used in current study can be in outer edges of the strip may be found out as a novel trick for applying the SPH in narrow strips to overcome the force of denser particles. Meanwhile, the rigid assumption of the rolls forced the authors to use an energy balancer for the SPH technique to overcome the artificial energy accumulation in rolls.

The SPH approach in comparison with mesh based method was less time-consuming because of its flexible mesh free nature. In mesh based methods, unloading is complicated due to mesh distortion and in contrast, simulation of rolling using SPH is not conditional and such control are not required. This described superiority proves capability of using the SPH approach in deformation case studies.

Finally, by the advances mentioned above, and by preparing a more flexible and fast solver the physical variable conditions were studied to show the capability of setting a wide range test protocol.

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بر اساس تواناییهای گزارش شده و ساده سازی های روش هیدرودینامیک ذرات هموار به عنوان یک روش عددی بدون شبکه در فرآیند های تغییر شکل، یک تقریب دوبعدی در فرآیند نورد سرد در نظر گرفته شده است. با استفاده و آزمودن روش هیدرودینامیک ذرات هموار (SPH) در مورد فرآیند نورد سرد نه تنها باعث پیشرفت های موردی در خود روش SPH گردید بلکه حقایق فیزیکی در خور توجه ای نیز در مورد فیزیک مورد بحث به دست داد. یک نوار آلومینیومی ۶۰۶۱ برای مطالعه نورد در نظر گرفته شد. در این ارتباط دیسک نورد صلب در نظر گرفته شد و فرض بر این بنا شد که نوار قابلیت انعطاف الاستیک - پلاستیک داشته باشد. برای نیل اطمینان از روش به کار برده شده نتایج محاسباتی تنش با نتایج حاصل از روش اجزای محدود مقایسه شده است و نتایج حاکی از مطابقت قابل قبولی است. برای نتایج نهایی اثر بعضی از پارامترهای موثر مانند قطر دیسک، درصد کاهش ضخامت نوار آلومینیومی و سرعت چرخش مورد بررسی قرار گرفت. در این مطالعات نه تنها روش مورد استفاده توسعه نسبی یافته است، بلکه تواناییهای برای یک تقریب نسبتا سهل و دقیق در پدیده های تغییر شکل پیچیده نیز حاصل گردید.

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