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Soret Dufour Driven Thermosolutal Instability of Darcy-Maxwell Fluid

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ABSTRACT

In this paper, linear stability of double diffusive convection of Darcy-Maxwell fluid with Soret and Dufour effects is investigated. The effects of the Soret and Dufour numbers, Lewis number, relaxation time and solutal Darcy Rayleigh number on the stationary and oscillatory convection are presented graphically. The Dufour number enhances the stability of Darcy-Maxwell fluid for stationary convection while it has a stabilizing character for overstability. The Soret number is to destabilize the system in both cases of stationary and oscillatory modes. In the limiting case some previous results have been recovered.

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NOMENCLATURE

C	solute concentration	L_{e_D}	Dufour number $\left(=\frac{D_{12}\Delta C}{D_{11}\Delta T}\right)$
d	height of the fluid layer	L_{e_S}	Soret number $\left(=\frac{D_{21}\Delta T}{D_{22}\Delta C}\right)$
D_a	Darcy number $\left(=\frac{k}{d^2}\right)$	m	horizontal wave number in y direction
D_{11}	thermal diffusivity	M	ratio of heat capacities
D_{12}	Dufour coefficient	N	Buoyancy ratio $\left(=\frac{\alpha'\Delta C}{\alpha\Delta T}\right)$
D_{21}	Soret coefficient	p	pressure
D_{22}	solutal diffusivity	**q**	velocity vector (u, v, w)
g	gravitational acceleration	R_{a_D}	Darcy Rayleigh number $\left(=\frac{g\alpha d\Delta T k}{\nu D_{11}}\right)$
k	permeability	R_{s_D}	Solutal Darcy Rayleigh number $\left(=\frac{gk\alpha'\Delta C d}{\nu D_{22}}\right)$
k_1	$(0,0,1)$	t	time
l	horizontal wave number in x direction	T	temperature

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L_e	Lewis number $\left(=\frac{D_{11}}{D_{22}}\right)$	W_0	dimensionless amplitude of velocity perturbation
Greek Symbols		β	horizontal wave number $\left(=\sqrt{l^2+m^2}\right)$
ΔT	temperature difference between the walls	μ	effective viscosity
ΔC	concentration difference between the walls	$\bar{\lambda}$	stress relaxation characteristic time
ϕ	$=\varepsilon L_e$	ρ	density
θ_0	dimensionless amplitude of temperature perturbation	ε	porosity
Γ_0	dimensionless amplitude of concentration perturbation	α	coefficient of thermal expansion
σ	growth rate $(=\sigma_r+i\sigma_i)$	α'	coefficient of solutal expansion
ν	kinematic viscosity $\left(=\frac{\mu}{\rho_0}\right)$	λ	dimensionless relaxation number $\left(=\bar{\lambda}\frac{D_{11}}{d^2}\right)$
Subscripts			
b	basic state	o	reference value
i	imaginary	r	real
Superscripts			
′	perturbed quantity	st	stationary
*	dimensionless quantity	over	overstability (oscillatory convection)

1. INTRODUCTION

A large number of studies related to monodiffusive (Thermal) convection and double diffusive (Thermosolutal) convection have been carried out in a continuous as well as in porous medium (Chandrasekhar [1], Drazin and Reid [2], Ingham and Pop [3,4] and Nield and Bejan [5] have set some of the milestones).

Historically, studies of thermal convection in a fluid saturated porous layer uniformly heated from below, a problem analogous to Rayleigh-Benard problem, were first put forward by Horton and Rogers [6] and later, independently by Lapwood [7]. The problem was generalized to double diffusive convection by Veronis [8] who demonstrated that subcritical instabilities may set in at a Rayleigh number smaller than that given by monodiffusive instability theory. The effect of double diffusive convection arises from the fact that heat diffuses more rapidly than a dissolved substance creating temperature and concentration difference under gravity. For example, in stellar interiors, the helium acts like salt in raising the density and in diffusing more slowly than the heat. This competition between the thermal and solutal buoyancy forces plays an important role in many physical phenomena. (See, for example, Mamou [9] and other references therein). In the case of flow of fluid through porous media, both fluid and solid regions are responsible for the transfer of heat, while the solute transfers by diffusion and convection only through the fluid region. Thus, the properties of the porous medium can affect the transient flow behaviour even when the thermal and solutal diffusivities are equal.

The problems of double diffusive convection in porous media occur in a broad spectrum of disciplines such as in the insulation of buildings and equipments, energy storage and recovery, geothermal energy extraction, dispersion of pollutants in the environment (like underground disposal of nuclear and non-nuclear waste), material and food processing, circulation in planetary atmosphere and growth of metal crystals.

Double diffusive convection for a porous medium was first analyzed by Nield [10]. The critical Rayleigh numbers for the onset of stationary and overstable convection were obtained for different thermal and solutal boundary conditions. Rudraiah et al. [11] considered a porous layer with isothermal and isosolutal boundaries. They investigated the finite amplitude flow for opposing buoyancy forces and the threshold for subcritical convection was obtained as a function of the ratios of diffusivities. Furthermore, finite amplitude convection in a porous layer near the threshold was considered by Brand and Steinberg [12]. Recently, thermosolutal convection in different types of viscoelastic fluids through porous media has been considered by a number of researchers.

In double diffusive convection when an external temperature gradient is imposed, a chemical potential gradient is produced. Likewise, an analogous effect regarding temperature is produced corresponding to external concentration gradient. In heat-salt pair, there are many physical phenomena where it has been observed that the flux of salt is a function of temperature gradient as well as salinity gradient. This effect of temperature gradient on salt flux is termed as thermophoresis or Soret effect [13, 14]. Reciprocally, diffusion thermo or Dufour effect [15, 16] is observed which corresponds to specific differentiation developing in temperature submitted to the concentration gradient.

Thermosolutal convection with and without Soret-Dufour effects so far are considered to be identical by converting the equations and boundary conditions of Soret-Dufour thermosolutal convection problems into those for thermosolutal convection without these effects with the help of a linear transformation [17]. But the fact is that Soret effect itself is of great importance in achieving difficult purifications in isomeric substance of various types such as, in mixture between gases with very light molecular weight, such as H_2 or He and of medium molecular weight such as N_2 or air. Soret effect is also important to predict the composition profile of oil fields. Similarly several cases of Dufour effect of considerable magnitude have been prescribed by Eckert and Drake [18]. The Dufour driven thermosolutal convection was first examined by Veronis [8] while Tewfik et al. [19] were the first reserchers on Soret-Dufour driven thermosolutal convection, followed by Sparrow et al. [20].

In the last few decades, extensive theoretical and experimental investigations have been performed on double diffusive convection caused by coupled molecular diffusion. Rudraiah and Malashetty [21] have investigated the influence of coupled molecular diffusion on double diffusive convection in a porous medium. Weaver and Viskanta [22] have analyzed the Soret-Dufour effect on the natural convection in heat and mass transfer in a cavity due to combined horizontal temperature and concentration gradients. They have pointed out that when the differences of temperature and concentration are large, or when the difference of molecular mass of the two elements in a binary mixture is great, the coupled interaction is significant. They have established the important result that the total mass flux through the cavity due to Soret effect can be as much as 10 – 15%, and energy transfer due to Dufour effect can be of appreciable magnitude compared to heat conduction. Recently, the effect of the cross coupled diffusion in a system with horizontal temperature and concentration gradients has been examined by Malashetty and Gaikwad [23]. Magherbi et al. [24] have investigated the Dufour effect on entropy generation in double diffusive convection. Among the recent contributions in the area of thermosolutal diffusion with Soret and Dufour effects, some of the milestones are due to Postelnicu [25] who studied the simultaneous heat and mass transfer by natural convection from a vertical flat plate embedded in an electrically conducting fluid saturated in a porous medium, Kim et al. [26] who analyzed Soret-Dufour effect on convective instabilities in nanofluids for a Darcy Boussinesq model, Gaikwad et al. [27] who studeid Soret-Dufour effect on the double diffusive convection in a two component couple stress fluid layer using both linear and non-linear stability analysis and Narayana and Murthy [28, 29] who investigated combined Soret - Dufour effect on free convection heat and mass transfer from a horizontal plate in a Darcy porous medium as well as in a doubly stratified Darcy porous medium.

During recent years, various problems regarding non-Newtonian fluids saturating porous media have received much attention [see review by Wang and Tan [30]]. Maxwell [31] was first who developed a model for a visco-elastic type fluid. Using this model, Tan and Masuoka [32] studied the monodiffusive stability of a fluid saturated in a porous medium. An analogous model, called modified Darcy-Maxwell model of the same fluid was introduced by Khuzayorov et al. [33]. Wang and Tan [30] used this modified Darcy-Maxwell model to investigate the instability of a double diffusive mixture.

In view of the above studies, and keeping in mind that thermophoresis (Soret effect) and diffusion thermo (Dufour effect), however small they may be, are present in thermosolutal convections and are equally important, the aim of the present study is to theoretically examine the linear stability of double diffusive convection in Darcy-Maxwell fluid [33] confined between two parallel plates in the presence of Soret and Dufour effects, when both stationary and oscillatory convections occur.

Therefore, the aim of the present paper is to extend the study of Wang and Tan [30] for Soret - Dufour phenomena, i.e. to investigate the effect of thermal diffusion (Soret) and diffusion thermo (Dufour) on the linear stability of modified Darcy-Maxwell model [33].

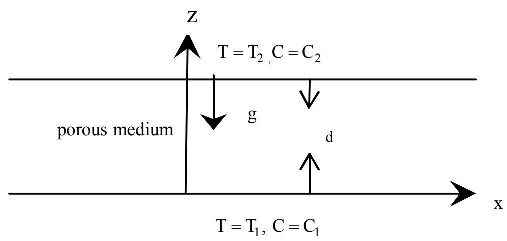

Figure 1. Physical Configuration

2. FLOW STRUCTURE AND THE MATHEMATICAL FORMULATION

A layer of Maxwell fluid is setup in a porous medium of thickness d by maintaining the horizontal boundaries at different temperatures and concentrations $(T_1 > T_2, C_1 > C_2)$ as shown in Figure 1.

The several assumptions used in the present paper are:

(a) The boundaries are horizontal, parallel and infinite.
(b) The porous medium is isotropic and homogeneous.
(c) The saturating fluid is incompressible, non-Newtonian, Darcy-Maxwellian and assumed to be everywhere in total thermodynamic equilibrium together with the porous medium.
(d) The layer is heated and soluted from below.
(e) The Boussinesq approximation is valid which states that the variations of density in the equations of motion can safely be ignored everywhere except in its association with the external force [1].

In view of these assumptions, the governing equations describing the conservation of mass and momentum [30] can be written in the vector form as:

$$\nabla \cdot \mathbf{q} = 0 \tag{1}$$

$$\frac{\mu \mathbf{q}}{k} = \left(1 + \bar{\lambda}\frac{\partial}{\partial t}\right)\left(-\nabla p - \rho \mathbf{g}\right) \tag{2}$$

where, $\mathbf{q} = (u, v, w)$ is the Darcian velocity, k is the permeability of the porous medium, μ is the effective viscosity of fluid in the porous medium, $\bar{\lambda}$ is the stress relaxation characteristic time constant, p pressure, ρ is the density, and $\mathbf{g}$ is the acceleration due to gravity.

Following Groot [34] and McDougall [35], the phenomenological equations involving Soret-Dufour effects on the fluxes of heat and mass to the thermal and solute gradients present in the assumed fluid may be written as:

$$\left(\frac{1}{M}\frac{\partial}{\partial t} + \mathbf{q}.\nabla\right)T = D_{11}\nabla^2 T + D_{12}\nabla^2 C \tag{3}$$

$$\left(\in \frac{\partial}{\partial t} + \mathbf{q}\cdot\nabla\right)C = D_{22}\nabla^2 C + D_{21}\nabla^2 T \tag{4}$$

where, D_{11} is thermal diffusivity, D_{22} solutal diffusivity, D_{12} and D_{21} quantify the contributions to the heat flux due to concentration gradient (Dufour coefficient) and mass flux due to temperature gradient (Soret coefficient) respectively, ε the normalized porosity, M the ratio of heat capacities [36] and T and C are temperature and concentration respectively.

The boundary conditions of the problem are: $w = Dw = 0$ at $z = 0$ and $z = d$.

The equation of state is:

$$\rho = \rho_0\left[1 - \alpha(T - T_0) + S\alpha'(C - C_0)\right] \tag{5}$$

where, ρ and ρ_0 are the densities at the current and reference state respectively, α and α' are the coefficients of thermal and solutal expansions and $S = +1$ (or -1) depending on whether the density of diffusing component is greater (or less) than that of the solvent.

2. 1. Basic State

The basic state of the system is steady and is given by:

$$q_b = (0,0,0),\ \ p = p_b(z),\ \ \rho = \rho_b(z), T = T_b(z) \text{ and } C = C_b(z) \tag{6}$$

Substituting Equations (6), Equations (1) – (5) yield:

$$\frac{dp_b}{dz} + \rho_b g = 0 \tag{7}$$

$$T_b = -\frac{D_{12}}{D_{11}}C_b + \left(T_1 - \frac{\Delta T}{d}z\right) + \left(C_1 - \frac{\Delta C}{d}z\right)\frac{D_{12}}{D_{11}} \tag{8}$$

$$C_b = -\frac{D_{21}}{D_{22}}T_b + \left(C_1 - \frac{\Delta C}{d}z\right) + \left(T_1 - \frac{\Delta T}{d}z\right)\frac{D_{21}}{D_{22}} \tag{9}$$

$$\rho_b = \rho_0\left[1 - \alpha(T_b - T_0) + S\alpha'(C_b - C_0)\right] \tag{10}$$

Here, $\Delta T = T_1 - T_2 > 0$ and $\Delta C = C_1 - C_2 > 0$.

2. 2. Perturbed State

On the basic state we superimpose perturbations in the form

$$q = q_b + q',\ \ p = p_b(z) + p', \rho = \rho_b(z) + \rho',\ \ T = T_b(z) + T' \text{ and } C = C_b(z) + C' \tag{11}$$

where, primes indicate perturbations which are functions of space as well as time.

Introducing Equation (11) into Equations (7) – (10), and ignoring the second and higher order terms of the perturbations, we have:

$$\nabla \cdot \mathbf{q}' = 0 \tag{12}$$

$$\frac{\mu \mathbf{q}'}{k} = -\left(1 + \bar{\lambda}\frac{\partial}{\partial t}\right)\left[\nabla p' - \mathbf{k_1} g\rho_0(\alpha T' - \alpha' C')\right] \tag{13}$$

$$\frac{1}{M}\frac{\partial T'}{\partial t} - \frac{\Delta T}{d}w' = D_{11}\nabla^2 T' + D_{12}\nabla^2 C' \tag{14}$$

$$\in \frac{\partial C'}{\partial t} - \frac{\Delta C}{d}w' = D_{22}\nabla^2 C' + D_{21}\nabla^2 T' \tag{15}$$

where, $\mathbf{k_1} = (0,0,1)$ and $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$.

Using the following non-dimensional parameters:

$$\left.\begin{aligned} &\mathbf{x^*} \equiv \mathbf{x}\left(x^*, y^*, z^*\right) = \left(\frac{x}{d}, \frac{y}{d}, \frac{z}{d}\right), \\ &T^* = \frac{T'}{\Delta T},\ C^* = \frac{C'}{\Delta C},\ \mathbf{q}^* = \frac{d}{D_{11}} q', \\ &t^* = \frac{D_{11} M}{d^2} t,\ p^* = \frac{k}{\mu D_{11}} p' \end{aligned}\right\} \tag{16}$$

assuming M = 1 and ignoring terms with the asterisks, the resulting equations in non-dimensional form are obtained as:

$$\nabla \cdot \mathbf{q} = 0 \tag{17}$$

$$\mathbf{q} = \left(1 + \lambda \frac{\partial}{\partial t}\right)\left[-\nabla p + \mathbf{k_1} R_{a_D} (T - SN)\right] \tag{18}$$

$$\frac{\partial T}{\partial t} - w = \nabla^2 T + L_{e_D} L_e \nabla^2 C \tag{19}$$

$$\in \frac{\partial C}{\partial t} - w = \nabla^2 C + L_{e_S} L_e^{-1} \nabla^2 T \tag{20}$$

where, $L_e = \frac{D_{11}}{D_{22}}$: (Lewis number)

$L_{e_D} = \frac{D_{12}\Delta C}{D_{11}\Delta T}$: (Dufour number)

$L_{e_s} = \frac{D_{21}\Delta T}{D_{22}\Delta C}$:(Soret number)

$N = \frac{\alpha'\Delta C}{\alpha \Delta T}$:(Buoyancy Ratio)

$R_{a_D} = \frac{g\alpha d \Delta T k}{\nu D_{11}}$:(Darcy Rayleigh number)

$\lambda = \bar{\lambda}\frac{D_{11}}{d^2}$:(non-dimensional Relaxation number)

3. LINEAR STABILITY THEORY

Following normal mode analysis, we assume the perturbations to be periodic waves of the form:

$$\begin{bmatrix} \mathbf{q}(x, y, z, t) \\ T(x, y, z, t) \\ C(x, y, z, t) \end{bmatrix} = \begin{bmatrix} \mathbf{V}(z) \\ \theta(z) \\ \Gamma(z) \end{bmatrix} \exp(\sigma t + ilx + imy) \tag{21}$$

where, σ is the growth rate and, in general, a complex quantity $(\sigma = \sigma_r + i\sigma_i)$, and l and m are dimensionless horizontal wave numbers in x and y directions, respectively. Stability or instability of the system is governed by $\mathrm{Re}(\sigma)$ such that the system is stable if $\mathrm{Re}(\sigma) < 0$ for all modes, and unstable if $\mathrm{Re}(\sigma) > 0$, even for a single mode. The marginal state is prescribed by $\mathrm{Re}(\sigma) = 0$.

Substituting Equation (21) in linearized Equations (17) - (20), and simplifying the resulting equations for vertical component W of **V,** we get:

$$\left(D^2 - \beta^2\right) W = -R_{a_D} \beta^2 (1 + \lambda\sigma)(\theta - SL_e N\Gamma) \tag{22}$$

$$\sigma\theta - W = \left(D^2 - \beta^2\right)\theta + L_e L_{e_D}\left(D^2 - \beta^2\right)\Gamma \tag{23}$$

$$\in \sigma L_e \Gamma - W = \left(D^2 - \beta^2\right)\Gamma + L_e^{-1} L_{e_s}\left(D^2 - \beta^2\right)\theta \tag{24}$$

where, $\beta = \sqrt{l^2 + m^2}$ is the horizontal wave number and $D \equiv \frac{d}{dz}$.

It is to be noted that in the absence of Soret-Dufour effects ($L_{e_D} = 0$ and $L_{e_s} = 0$) and for S = 1, the eigen value problem given by Equations (22)-(24) converts into one previously given by Wang and Tan [30]. Furthermore, when solute alone is taken to be absent, i.e. for $R_{s_D} = 0$, it converts into one discussed by Nield [10].

In agreement with boundary conditions, we choose the solutions of the form:

$$\left.\begin{aligned} W(z) &= W_0 \cos(n\pi z), \\ \theta(z) &= \theta_0 \cos(n\pi z) \\ \text{and} \quad \Gamma(z) &= \Gamma_0 \cos(n\pi z). \end{aligned}\right\} \tag{25}$$

where n = 1,2,3,4------. Substituting Equations (25) into Equations (22)-(24), we obtain:

$$QW_0 - R_{a_D}\beta^2(\lambda\sigma + 1)\theta_0 + R_{a_D}\beta^2 \times SNL_e(\lambda\sigma + 1)\Gamma_0 = 0 \tag{26}$$

$$W_0 - (\sigma + Q)\theta_0 + L_e L_{e_D} Q\Gamma_0 = 0 \tag{27}$$

$$W_0 - L_e^{-1} L_{e_s} Q\theta_0 - (\in \sigma L_e + Q)\Gamma_0 = 0 \tag{28}$$

where, $Q = \pi^2 + \beta^2$ for the critical model n = 1. For non - trivial solution of W_0, θ_0, and Γ_0, we require:

$$\begin{vmatrix} Q & -R_{a_D}\beta^2(\sigma\lambda + 1) & R_{a_D}\beta^2 SNL_e(\lambda\sigma + 1) \\ 1 & -(\sigma + Q) & -L_e L_{e_D} Q \\ 1 & -L_e^{-1} L_{e_s} Q & \in \sigma L_e + Q \end{vmatrix} = 0 \tag{29}$$

Solving Equation (29), we get:

$$A_2\sigma^2 + A_1\sigma + A_0 = 0 \tag{30}$$

where,

$$A_2 = \phi Q + R_{s_D}\lambda\beta^2 S - R_{a_D}\lambda\phi\beta^2 \tag{31}$$

$$\begin{aligned} A_1 = {} & (1 + \phi)Q^2 + R_{s_D} S(1 + \lambda Q)\beta^2 - R_{a_D} \times \\ & \left(\phi + \lambda Q - \lambda L_e L_{e_D} Q + \lambda SNL_{e_s} Q\right)\beta^2 \end{aligned} \tag{32}$$

$$A_0 = Q^3 - Q^3 L_{e_D} L_{e_s} + R_{s_D}\beta^2 QS - R_{a_D}\beta^2 Q \times \left(1 - L_e L_{e_D} + SNL_{e_s}\right) \tag{33}$$

with $\phi = \in L_e$ and $R_{s_D} = \frac{gk\alpha'\Delta C d}{\nu D_{22}}$ (Solutal Darcy Rayleigh number).

4. ANALYSIS AT THE MARGINAL STATE

To analyze the marginal state, substituting $\sigma = i\sigma_i$, where σ_i is real, in Equation (30) and separating the real and imaginary parts of the resulting equation, we get:

$$\left(\phi Q + R_{s_D}\lambda S\beta^2 - R_{a_D}\lambda\phi\beta^2\right)\sigma_i^2 - \left[Q^3 - Q^3 L_{e_D} L_{e_S} + R_{s_D} S\beta^2 Q - R_{a_D}\beta^2 Q \times \left(1 - L_e L_{e_D} + SNL_{e_s}\right)\right] = 0 \quad (34)$$

$$\left[(1+\phi)Q^2 + R_{s_D} S(1+\lambda Q)\beta^2 - R_{a_D}\left(\phi + \lambda Q - \lambda L_e L_{e_D} Q + \lambda SN\, L_{e_s} Q\right)\beta^2\right]\sigma_i = 0 \quad (35)$$

It is clear from Equation (34) that for S = 1 at the margin of stability, stationary Darcy Rayleigh number $R_{s_D}^{st}$, is obtained in the form:

$$R_{a_D}^{st} = \frac{Q^2 - Q^2 L_{e_D} L_{e_s} + R_{s_D}\beta^2}{\beta^2\left(1 - L_e L_{e_D} + NL_{e_s}\right)} \quad \text{or}$$

$$R_{a_D}^{st} = \frac{\left[\left(\pi^4 + 2\pi^2\beta^2 + \beta^4\right)\left(1 - L_{e_D} L_{e_s}\right) + R_{s_D}\beta^2\right]}{\left(1 - L_e L_{e_D} + NL_{e_s}\right)\beta^2} \quad (36)$$

The minimum value of $R_{a_D}^{st}$ can be obtained by setting $\partial R_{a_D}^{st} / \partial\beta = 0$, which results in a critical wave number $\beta_c = \pi$, and hence the critical Darcy Rayleigh number for the stationary convection, denoted by $R_{a_{D(critical)}}^{st}$ reduces to:

$$R_{a_{D(critical)}}^{st} = \frac{1}{\left(1 - L_e L_{e_D} + NL_{e_s}\right)} \times \left[4\pi^2\left(1 - L_e L_{e_s}\right) + R_{s_D}\right] \quad (37)$$

It is to be noted that both the critical wave number and the critical Darcy Rayleigh number are independent of the relaxation time λ.

In the absence of Dufour effect $\left(L_{e_D} = 0\right)$, Equation (37) reduces to:

$$R_{a_{D(critical)}}^{st} = \frac{1}{\left(1 + NL_{e_s}\right)}\left[4\pi^2 \times \left(1 - L_e L_{e_s}\right) + R_{s_D}\right] \quad (38)$$

while in the absence of Soret effect $\left(L_{e_s} = 0\right)$, it reduces to:

$$R_{a_{D(critical)}}^{st} = \frac{4\pi^2 + R_{s_D}}{\left(1 - L_e L_{e_D}\right)} \quad (39)$$

whereas in the absence of both Soret and Dufour effects $\left(L_{e_D} = 0, L_{e_S} = 0\right)$, the critical Darcy Rayleigh number is given by:

$$R_{a_{D(critical)}}^{st} = 4\pi^2 + R_{s_D} \quad (40)$$

a result also given by Wang and Tan [30].

It is clear from Equations (38), (39) and (40) that whereas the Soret effect reduces critical Darcy Rayleigh number, the Dufour number has a reverse effect. This analytical analysis confirms the findings of Gaikwad et al. [27] who showed that the Dufour effect has a stabilizing character, while the Soret effect has a destabilizing character.

5. OSCILLATORY CONVECTION

Oscillatory convection, at the marginal state $(\sigma_r = 0)$, is characterized by $\sigma_i \neq 0$. Therefore, for oscillatory convection, Darcy Rayleigh Number is obtained in the form:

$$R_{a_D}^{over} = \frac{Q^2(1+\phi) + R_{s_D}(1+\lambda Q)\beta^2}{\beta^2\left(\phi + \lambda Q - \lambda L_e L_{e_D} Q + \lambda NL_{e_s} Q\right)} \quad (41)$$

Consequently, frequency for overstability is obtained as Equation (42) (see Equation (42)). From Equation (43) it is clear that oscillatory motions are excluded if I > 0, and not excluded if I < 0. (see Equation (43)).

$$\sigma_i^2 = \frac{\left[\left\{(\lambda Q - 1) + (\phi - \lambda Q + 1)\left(L_e L_{e_D} - L_{e_S}\right) - L_{e_D} L_{e_S}(\lambda Q + \phi)\right\}Q^2 + R_{S_D}\left(\phi - 1 + L_e L_{e_D} - NL_{e_S}\right)\beta^2\right]}{\left[\phi^2(1 - \lambda Q) - \phi\lambda Q\left(L_e L_{e_D} - NL_{e_S}\right) + R_{S_D}\left(1 - \phi - L_e L_{e_D} + NL_{e_S}\right)\lambda^2\beta^2\right]} \quad (42)$$

$$\begin{aligned} I = {} & \left(1 - \phi - L_e L_{e_D} + NL_{e_S}\right)^2 \lambda^2 R_{S_D}^2 \beta^4 + \left(1 - \phi - L_e L_{e_D} + NL_{e_S}\right)\left[\left\{(1 - \lambda Q) - (\phi + 1 - \lambda Q)\left(L_e L_{e_D} - NL_{e_S}\right) + \right.\right. \\ & \left.\left. L_{e_D} \times L_{e_S}(\lambda Q + \phi)\right\}\lambda^2 Q^2 + \phi^2(1 - \lambda Q) - \phi\lambda Q\left(L_e L_{e_D} - NL_{e_S}\right)\right] R_{S_D}\beta^2 + \left\{(\lambda Q - 1)(\phi - \lambda Q + 1)\left(L_e L_{e_D} - NL_{e_S}\right)\right. \\ & \left. - L_{e_D} L_{e_S}(\lambda Q + \phi)\right\}\left\{\phi^2(\lambda Q - 1) + \phi\lambda Q \times \left(L_e L_{e_D} - NL_{e_S}\right)\right\}Q^2 \end{aligned} \quad (43)$$

6. NUMERICAL RESULTS AND DISCUSSION

For S = 1 in the absence of Soret and Dufour effects ($L_{e_s}=0$ and $L_{e_D}=0$) the critical Darcy Rayleigh number for stationary convection and that for oscillatory convection reduce to

$$R^{st}_{a_{D(critical)}} = 4\pi^2 + R_{s_D} \tag{44}$$

and

$$R^{over}_{a_D} = R_{s_D}\frac{1+\lambda Q}{\phi+\lambda Q} + \frac{(1+\phi)Q^2}{(\phi+\lambda Q)\beta^2} \tag{45}$$

respectively.

Minimum σ_i^2 with respect to β, from Equation (42) reduces to:

$$\sigma_i^2 = \frac{(\lambda Q-1)Q^2 + R_{s_D}(\phi-1)\beta^2}{\phi^2(1-\lambda Q) + R_{s_D}(1-\phi)\lambda^2\beta^2} \tag{46}$$

and oscillatory convection is excluded if

$$\lambda^2\beta^4(1-\phi)^2 R^2_{s_D} + \beta^2(1-\phi)(1-\lambda Q)\times\left(\lambda^2Q^2+\phi^2\right)R_{s_D} + \phi^2Q^2(1-\lambda Q)^2 > 0 \tag{47}$$

and not excluded if

$$\lambda^2\beta^4(1-\phi)^2 R^2_{s_D} + \beta^2(1-\phi)(1-\lambda Q)\times\left(\lambda^2Q^2+\phi^2\right)R_{s_D} + \phi^2Q^2(1-\lambda Q)^2 < 0 \tag{48}$$

The results from Equations (44) to (48) are exactly the same as obtained by Wang and Tan [30] for the thermosolutal convection of Darcy Maxwell fluid in a porous medium. It is important to note that it is only due to Soret-Dufour effects, represented by L_{e_s} and L_{e_D} respectively that, contrary to Wang and Tan [30], the Darcy Rayleigh number R_{a_D} at the marginal state depends upon the Lewis number L_e. This effect of Lewis number and that of other parameters on the behaviour of neutral stability curves have been depicted in Figures (2) – (5). The important observations regarding neutral stability curves in $R_{a_D}-\beta$ plane for different values of β are:

- For fixed $L_{e_s}(=0.25)$, $L_{e_D}(=2)$, $R_{s_D}(=25)$ and $N(=10)$, critical Darcy Rayleigh number R_{a_D} becomes larger with increasing Lewis number (Figure 2),
- For fixed $L_{e_s}(=0.25), R_{s_D}(=25)$, $L_e(=0.75)$ and $N(=10)$, critical Darcy Rayleigh number R_{a_D} decreases with increasing Dufour number (Figure 3),
- For fixed, $L_{e_D}(=2)$, $R_{s_D}(=25)$, $L_e(=0.75)$ and $N(=10)$, critical Darcy Rayleigh number R_{a_D} decreases with increasing Soret number (Figure 4),
- For fixed $L_e(=0.75)$, $L_{e_D}(=2)$, $L_{e_S}(=0.25)$, and N (=10), critical Darcy Rayleigh number R_{a_D} increases with increasing Solutal Darcy Rayleigh number R_{s_D} (see Figure 5).

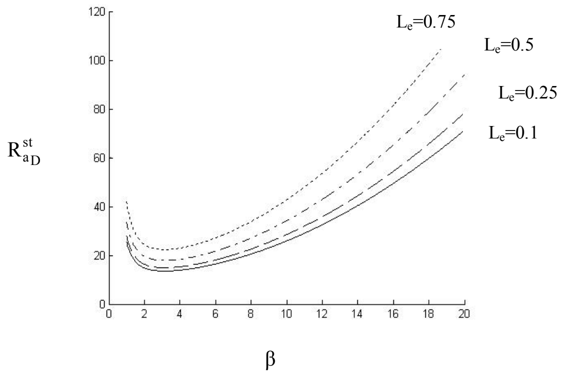

Figure 2. Variations of critical Rayleigh number with wave number for different values of Lewis number

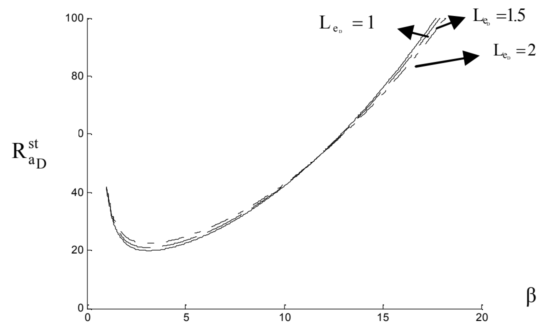

Figure 3. Variations of critical Rayleigh number with wave number for different values of Dufour number

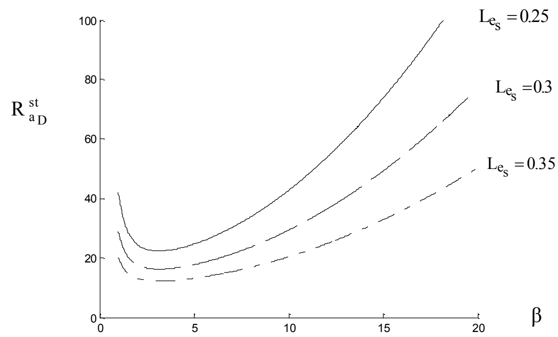

Figure 4. Variations of critical Rayleigh number with wave number for different values of Soret number

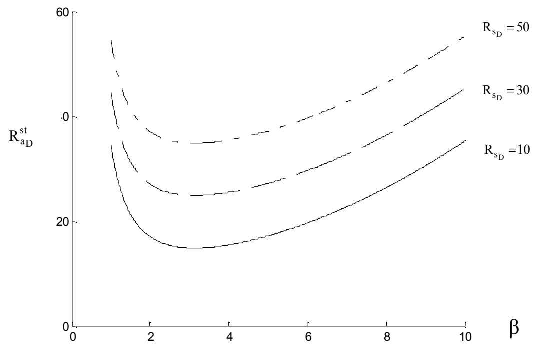

Figure 5. Variations of critical Rayleigh number with wave number for different values of Solutal Darcy Rayleigh number.

The above observations indicate that in case of stationary convection effects of Lewis number and solutal Darcy Rayleigh number are stabilizing, whereas, the Soret number and Dufour number act as catalysts to instability. However, Dufour phenomena, considered individually (in the absence of Soret effect), is shown analytically to postpone instability. Similarly, the variations of oscillatory critical Darcy Rayleigh number $R^{over}_{a_D}$ with wave number β are depicted in Figures (6) – (11) under different situations and it has been observed that:

- For fixed $L_{e_D}(=2)$, $L_{e_S}(=0.25), N(=10)$, $R_{s_D}(=25)$ and $\lambda=(0.05)$, an increase in the value of l_e increases $R^{over}_{a_D}$ at corresponding values of ϕ (see Figure 6).
- For fixed $L_{e_D}(=2)$, $L_{e_S}(=0.25), N(=10)$, $L_e(=0.5)$, $R_{s_D}(=25)$ and $\lambda=(0.05)$, an increase in the value of ϕ continuously increases $R^{over}_{a_D}$ (see Figure 7).
- For fixed $L_{e_S}(=0.25), N(=10), L_e(=0.5)$, $\phi(=0.25)$, $R_{s_D}(=25)$ and $\lambda=(0.05)$, an increase in the value of L_{e_D} (dufour number) increases $R^{over}_{a_D}$ (see Figure 8).
- For fixed $N(=10), L_e(=0.5), \phi(=0.25)$, $R_{s_D}(=25)$ and $\lambda=(0.05)$, $R^{over}_{a_D}$ decreases with the increasing Soret number (see Figure 9). The role of Soret number is observed to remain the same for stationary convection as well as for overstability. However, for a given Soret number, while for stationary convection the curves in (R_{a_D},β) plane decrease in small wave number range and then increase beyond this (see Figure 4), such curves decrease continuously for overstability (see Figure 9).
- For fixed $L_{e_S}(=0.25), N(=10), L_e(=0.5)$, $L_{e_D}(=2)$, $\phi(=0.25)$ and $\lambda=(0.05)$, $R^{over}_{a_D}$ increases with the increasing solutal Darcy Rayleigh number (see Figure 10).
- For fixed $L_{e_S}(=0.25), N(=10), L_e(=0.5)$, $L_{e_D}(=2)$, $\phi(=0.25)$ and $\lambda=(0.05)$, $R_{s_D}(=25)$, $R^{over}_{a_D}$ decreases with the increasing relaxation time λ (see Figure 11). $\lambda\to 0$ lead to the Newtonian overstable Darcy Rayleigh number. Variations in $R^{over}_{a_D}$ with β tends to reduce as λ increases after decline in short wave number range, thus reducing the curvature of Newtonian curve. The point of critical Darcy Rayleigh number ceases to exist for large λ.

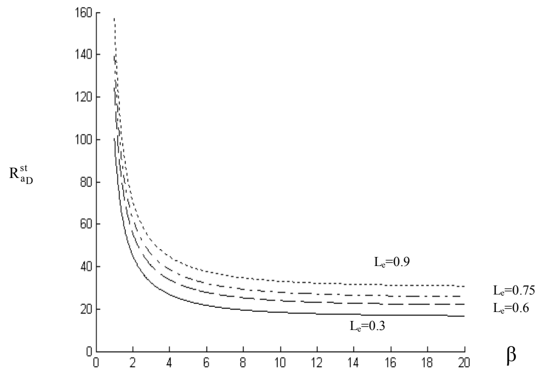

Figure 6. Effect of Lewis number on Darcy Rayleigh number for overstability

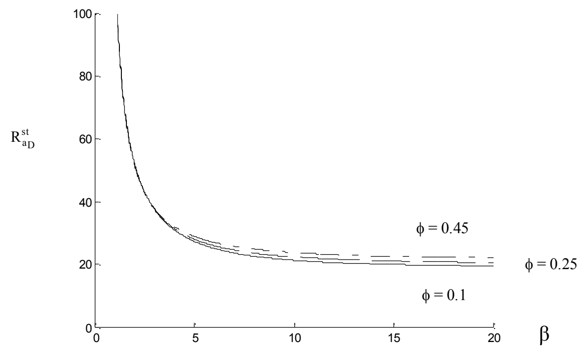

Figure 7. Effect of $\phi(=\varepsilon L_e)$ on the Darcy Rayleigh number for overstability

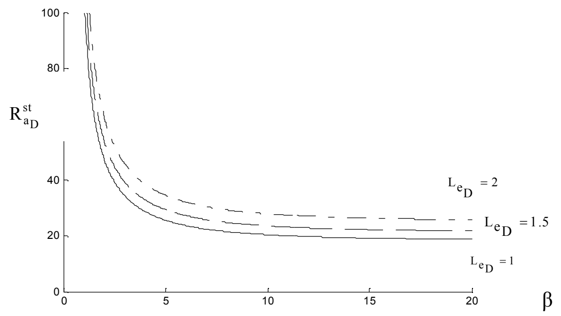

Figure 8. Effect of Dufour number on the Darcy Rayleigh number for overstability

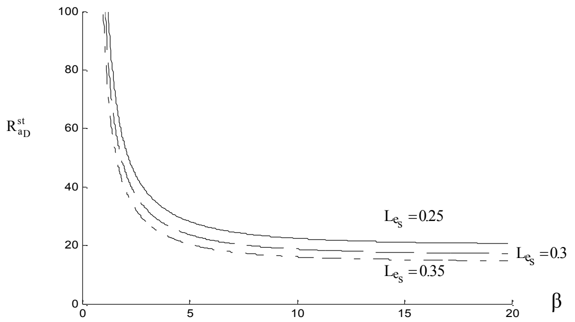

Figure 9. Effect of Soret number on the Darcy Rayleigh number for overstability

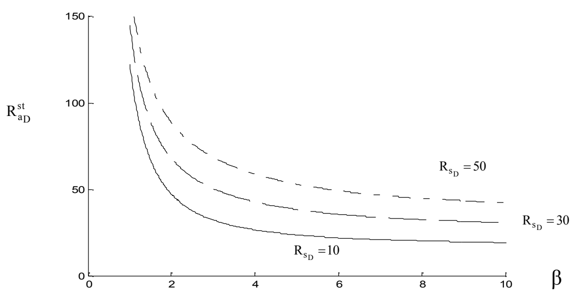

Figure 10. Effect of solutal Darcy Rayleigh number on the Darcy Rayleigh number for overstability.

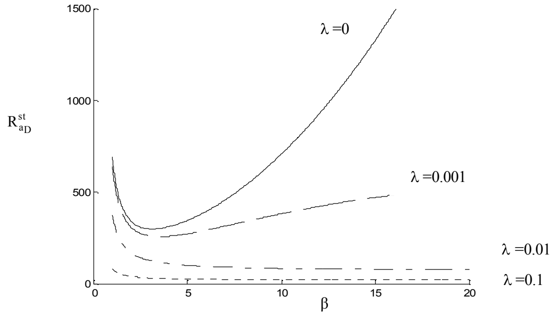

Figure 11. Effect of the relaxation time λ on the Darcy Rayleigh number for overstability

7. CONCLUSION

The onset of double diffusive convection in a Darcy Maxwell model with Soret-Dufour effects has been studied using linear stability theory. The following conclusions are drawn:

1. For stationary convection, the critical wave number β_c is not changed due to the presence of Soret and Dufour phenomena, however, the critical Darcy Rayleigh number depends upon both the Soret and Dufour numbers. It is observed that when considered individually, the Soret number reduces the critical Darcy Rayleigh number, while the Dufour number has a reverse effect. Whereas when both Soret and Dufour phenomena are present, Soret number as well as Dufour number act as catalysts to instability for stationary convection, while Dufour number has a stable effect for overstability.
2. The Lewis number enhances stability in both cases of stationary convection and overstability.
3. The relaxation time shows the same effect as in the absence of Soret and Dufour effects.
4. The existence of critical Darcy Rayleigh number in stationary convection is not seen in overstability. For oscillatory convection, the numerical computations indicate an asymptotic behaviour of R_{a_D} for large values of wave number, β, for all cases. As such, critical wave number can not be ascertained. It also ensures that R_{a_D} continuously decreases as β increases, implying thereby that small wave length perturbations are more unstable.

8. REFERENCES

1. Chandrasekhar, S. "Hydrodynamic and Hydromagnetic Stability", Dover, New York, (1981).
2. Drazin, P.C. and Reid, W.H., "Hydromagnetic Stability", Cambridge University Press, Cambridge, (1981).
3. Ingham, D. and Pop I. (Eds), "Transport Phenomenon in Porous Media-I", Pergamon, Oxford, (1998).
4. Ingham, D. and Pop, I., (Eds), "Transport Phenomenon in Porous Media-II", Pergamon, Oxford, (2002).
5. Nield, D.A. and Bejan, A. (Ed), "Convection in Porous Media", Second Edition, Springer, New York, (1999).
6. Horton, C.W. and Rogers, F.T. Jr., "Convection currents in a porous medium", ***Journal of Applied Physics***, Vol. 16, No. 6, (1945), 367-370.
7. Lapwood, E.R., "Convection of a fluid in a porous medium", ***Proceedings of the Cambridge Philosophical Society***, Vol. 44, No. 1, (1948), 508-521.
8. Veronis, G., "On finite amplitude instability in a thermohaline convection", ***Journal of Marine Research***, Vol. 23, No. 4, (1965), 1-17.

9. Mamou, M., "Stability analysis of the perturbed rest state and of the finite amplitude steady double-diffusive convection in a shallow porous enclosure", ***International Journal of Heat Mass Transfer,*** Vol. 46, No. 12, (2003), 2263-2277.

10. Nield, D.A., "Onset of thermohaline convection in porous medium", ***Water Resources Research***, Vol. 4, No. 3, (1968), 553-560.

11. Rudraiah, N., Srimani, P.K. and Friedrich, R., "Finite amplitude convection in a two component fluid saturated porous layer", ***International Journal of Heat Mass Transfer***, Vol. 25, No. 5, (1982), 715-722.

12. Brand, H. and Steinberg, V., "Nonlinear effects in the convective instability of a binary mixture in a porous medium near threshold", ***Physics Letters A***, Vol. 93, No. 7, (1983), 333-336.

13. Soret, C.H., Archives of Physical and Natural Sciences Geneve-2, (1879), 48-61.

14. Soret, C.H., Archives of Physical and Natural Sciences Geneve-4, (1880), 209-213.

15. Dufour, L., Archives of Physical and Natural Sciences Geneve-45, (1872), 9.

16. Dufour, L., "On the diffusion of gases through porous walls and the accompanying changes in temperature", ***Annals of Physics***, Vol. 224, (1873), 490.

17. Knobloch, E., "Convection in binary fluids", ***The Physics of Fluids,*** Vol. 23, No. 9, (1980), 1918-1920.

18. Eckert, E.R.G. and Drake, R.M., "Analysis of Heat and Mass Transfer", McGraw Hill, New York (1972).

19. Tewfik, O.E. and Yang, J.W., "The thermodynamics coupling between heat and mass transfer in free convection", ***International Journal of Heat Mass Transfer***, Vol. 6, No. 10, (1963), 915-923.

20. Sparrow, E.M., Minkowycz, W.J. and Eckert, E.R.G., "Transpiration induced buoyancy and thermal diffusion - diffusion thermo in helium air free convection boundary layer", ***ASME Journal of Heat Transfer***, Vol. 86, No.4, (1964), 508-520.

21. Rudraiah, N. and Malashetty, M.S., "The influence of coupled molecular diffusion on double diffusive convection in a porous medium", ***ASME Journal of Heat Transfer***, Vol. 108, No. 4, (1986), 872-876.

22. Weaver, J.A. and Viskanta, R., "Natural convection due to horizontal temperature and convection gradients-1. Variable thermo physical property effects", ***International Journal of Heat Mass Transfer,*** Vol. 34, No. 12, (1991), 3107-3120.

23. Malashetty, M.S. and Gaikwad, S.N., "Effect of cross diffusion on double diffusive convection in the presence of horizontal gradients", ***International Journal of Engineering Sciences***, Vol. 40, No. 7, (2002), 773-783.

24. Magherbi, M. Hidouri, N. and Abbassi, H., "Influence of Dufour effect on entropy generation in double diffusive convection", ***InternationalJournal of Energy***, Vol. 4, No. 3, (2007), 227-252.

25. Postelnicu, A., "Influence of a magnetic field on heat and mass transfer by natural convection from vertical surface in porous media considering Soret and Dufour effects", ***International Journal of Heat Mass Transfer***, Vol. 47, No. 6-7, (2004), 1467-1472.

26. Kim, J., Kang, Y.T. and Choi, C.K., "Analysis of convective instability and heat transfer characteristic of nanofluids", ***The Physics of Fluids,*** Vol. 16, No. 7, (2004), 2395-2401.

27. Gaikwad, S.N., Malashetty, M.S. and Prasad, K.R., "An analytical study of linear and non-linear double diffusive convection with Soret and Dufor effects in couple stress fluid", ***International journal of non-linear mechanics***, Vol. 42, No.7, (2007), 903-913.

28. Narayana, P.A.L. and Murthy, P.V.S.N., "Soret and Dufour effects on free convection heat and mass transfer in a doubly stratified Darcy porous medium", ***Journal of Porous Media,*** Vol. 10, No. 6, (2007), 613-624.

29. Narayana, P.A.L. and Murthy, P.V.S.N., "Soret and Dufour effects on free convection heat and mass transfer from a horizontal flat plate in a Darcy porous medium", ***ASME Journal of Heat Transfer,*** Vol. 130, No. 10, (2008), 1045041-1045045.

30. Wang, S.W. and Tan, W.C., "Stability analysis of double diffusive convection of maxwell fluid in a porous medium heated from below", ***Physics Letters A,*** Vol. 372, No. 17, (2008), 3046-3050.

31. Maxwell, J.C., "On the dynamical theory of gases", ***Philosophical transactions of the Royal Society of London,*** Vol.157, (1867), 49-88.

32. Tan, W.C. and Masuoka, T., "Stoke's first problem for a second grade fluid in a porous half-space with heated boundary", ***International journal of non-linear mechanics,*** Vol. 40, No. 4, (2005), 515–522.

33. Khuzhayorov, B., Auriault, J.L. and Royer, P., "Derivation macroscopic filtration law for transient linear fluid flow in porous media", ***International Journal of Engineering Sciences***, Vol. 38, No. 5, (2000), 487-504.

34. Groot, S.R.De. and Mazur, P., "Non-equilibrium Thermodynamics", North Holland Publishing Company, Amsterdam, (1962).

35. McDougall, T.J., "Double diffusive convection caused by coupled molecular diffusion", ***Journal of Fluid Mechanics,*** Vol. 126, (1983), 379-397.

36. Lombardo, S., Mulone, G. and Straughan, B., "Non-linear stability in the Bénard problem for a double-diffusive mixture in a porous medium", ***Mathematical methods in the applied sciences,*** Vol. 24, No. 16, (2001), 1229-1246.

Soret Dufour Driven Thermosolutal Instability of Darcy-Maxwell Fluid

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چکیده

پایداری خطی ضریب نفوذ دوتایی سیال Darcy-Maxwell با تاثیرات Dufour و Soret بررسی شده است. تاثیرات اعداد Dufour و Soret و عدد لوئس ، زمان ارامش و عدد Darcy Rayleigh solutal در حالات ثابت و نوسانی به صورت نمودار نشان داده شده است. عدد Dufour ، پایداری Darcy-Maxwell سیال را برای یک همرفت ایستا افزایش می دهد در حالیکه ان یک خصوصیت پایدار کننده برای حالت فراپایدار است. عدد Soret برای بی ثبات کردن سیستم در موارد نوسانی وپایدار می باشد. در بعضی از موارد محدود، بعضی از نتایج قبلی دوباره به دست امده است.

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