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A Facility Location Problem with Tchebychev Distance in the Presence of a Probabilistic Line Barrier

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ABSTRACT

This paper considers the Tchebychev distance for a facility location problem with a probabilistic line barrier in the plane. In particular, we develop a Mixed-Integer Nonlinear Programming (MINLP) model for this problem that minimizes the total Tchebychev distance between a new facility and the existing facilities. A numerical example is solved to show the validity of the developed model. Because of difficulty in solving this problem while increasing the number of existing facilities, we propose and design an efficient meta-heuristic algorithm, namely differential evolution (DE), for the given problem. Finally, the associated results are compared with the exact solution and lower bound for the different-sized problems.

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1. INTRODUCTION

In this paper, we present a new mathematical model for a Weber location problem with the Tchebychev distance in the presence of a probabilistic line barrier, which is known as $1 / \mathbb{R}^2 / \mathcal{B} = \text{a probabilsitic line}/d_\infty^B/\Sigma$ in Hamacher and Nickel [1]. We introduce a statement of the problem as follows. In facility location problems, the class of l_p distance metrics, called l_p-norm, is the most common features of discussion among researchers. l_p distance metrics for the n-dimensional space location problem is shown by:

$$l_p = (\textstyle\sum_{i=1}^{n}(|X_i - X_t|^p))^{1/p} \tag{1}$$

where X_i are the coordinates of the existing facilities and X_t is the n dimensional vector location of the new facility. It is clear that for $n = 2$ and $p = \infty$, we encounter with the planar Tchebychev distance metric. However, for $p = 1$ and $p = 2$, the rectilinear and Euclidean distance metrics are appeared, respectively.

$$l_{p=1} = |x_1 - y_1| + |x_2 - y_2| \tag{2}$$

$$l_{p=2} = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2} \tag{3}$$

$$l_{p=\infty} = \max\{|x_1 - y_1|, |x_2 - y_2|\} \tag{4}$$

In this paper, we focus on the Tchebychev distance metric where in spite of widely applications in the fields of material handling systems operations, CNC tool path planning, and manufacturing in real location problems, it is rarely mentioned by researchers in this area. Following are some of the related reviews.

Special feature of a continuous or Weber location problem is that for example, many researchers consider some restrictions in R_n, in which the locating or travelling may be prohibited. Most of the common restricted planar location problems, in which establishing and/or travelling through some area is not permitted, fall into one of the following categories; firstly, *forbidden regions* (e.g., national parks or other protected areas, in which the establishing of a facility is prohibited but traveling through the regions is permitted), secondly, *congested regions* (e.g., big lakes or forest, in which establishing of a facility is prohibited but travelling through the region is possible with a penalty), thirdly, *barrier regions* (e.g., military areas, mountain ranges, big rivers and the lake, where both establishing and travelling are forbidden). Hamacher and Nickel [2] surveyed location problems with forbidden regions extensively. Based on the classification of location problems in Hamacher and

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Nickel [1], the above problem statement is presented as follows: $1 \,/\, \mathbb{R}^2 \,/\, \mathcal{B} = 1$ probabilsitic line$/d_\infty^B/\Sigma$, where the first position shows the number of new facility, the second position indicates the solution space, the third position shows the special features of location problems (e.g., forbidden regions R or barriers B in the planar case or a ● if no special features are to be considered), the fourth position represents the information about the relation between of new and existing facilities, and the fifth position contains the objective function.

Consider a plane with m existing facilities and a line-shaped barrier, which randomly occurs on a pre-determined horizontal route. It is desired to find a new facility location in the plane such that the total Tchebychev barrier distance from the new facility to the existing facilities is minimized. In general, the p-norm distance Weber location problem with a barrier can be written as:

$$l_p = \min \sum_{i=1}^{m} w_i . d_p^B \,(X, X_i) \tag{5}$$

where w_i is the positive weight between the i-th existing facilities and the new facility. The coordinates of the new and existing facilities in the plane are presented as $X = (x, y)$ and $X_\text{i} = (x_\text{i}, y_\text{i})$, respectively. To identify the concept of d_p^B, called the p-norm barrier distance, consider two arbitrary points $X, Y \in \mathcal{F}$, where $1 \leq p \leq \infty$. d_p^B (X, Y) is defined as $inf\{\, l(\mathbf{P}_{X\text{-}Y})$: $\mathbf{P}_{X\text{-}Y}$ is feasible X,Y path$\}$, where $l(\mathbf{P}_{X\text{-}Y})$ is the length of the feasible X-Y path. Let $d_p\,(X,Y)$ be the p-norm distance between $X, Y \in \mathcal{F}$. Two arbitrary points (i.e.,X and $Y \in \mathcal{F}$) are called *p-visible* if the p-norm barrier distance between X and Y is equal to the p-norm distance (i.e., $d_p^B(X, Y) = d_p\,(X, Y)$, that is the presence of a barrier has no effect on visibility of two points X and Y.

On the other hand, if the p-norm barrier distance between the two points X and Y is greater than the p-norm distance, $d_p^B(X, X_i) > d_p\,(X, X_i)$, then the p-norm distance between $X, Y \in \mathcal{F}$ is called *p-shadow* (i.e., barrier affects on the p-norm distance between two points $X, Y \in \mathcal{F}$). Considering this definition, for one feasible point $X \in \mathcal{F}$, the set of *visible* points is defined as:

$$visible_p\,(X) = \{\, Y \in \mathcal{F}: d_p^B(X, Y) = d_p\,(X, Y)\}. \tag{6}$$

In other words, points from the feasible region, Y, which are *p-visible* with a given feasible point X. For a feasible point $\in \mathcal{F}$, the set of *shadow* points is defined as:

$$Shadow_p(X) = \{Y \in \mathcal{F}: d_\text{p}^B(X, Y) > d_\text{p}\,(X, Y)\} \tag{7}$$

It means that if distance between X and Y is *p-shadow*, then it becomes barrier distance, (see Klamroth, [3]). In this paper, the Tchebychev distance metric is emphasized (i.e., $p = \infty$). For a feasible point $X \in \mathcal{F}$ the set of *shadow* points is defined as:

$$Shadow_\infty(X) = \{Y \in \mathcal{F}: d_\infty^B(X, Y) > d_\infty\,(X, Y)\} \tag{8}$$

and the set of *visible* points are defined as:

$$visible_\infty\,(X) = \{\, Y \in \mathcal{F}: d_\infty^B(X, Y) = d_\infty\,(X, Y)\}. \tag{9}$$

Therefore, the Tchebychev distance single facility Weber location problem in the presence of the probabilistic line barrier can be stated by:

$$\min \sum_{i=1}^{m} w_i . d_\infty^B \,(X, X_i) \tag{10}$$

The organization of the paper is presented as follows. In Section 2, we provide a literature review of the papers related to the location problems with barriers. In Section 3, we present the Mixed-Integer Nonlinear Programming (MINLP) model and the expected barrier distance function is given. In Section 4, we illustrate a numerical example. Section 5 proposes a meta-heuristic algorithm based on Differential Evolution (DE). Section 6 introduces a lower bound for the presented model. Section 7 compares the computational results of these algorithms for different size samples of this problem. Finally, Section 8 contains the remarking conclusions and further research directions.

2. LITERATURE REVIEW

Because of the mathematical nature of the Tchebychev distance, many researchers in the field of operations research applied the transformation for this metric to other metrics for the ease of solving the problem. Francis et al. [4] introduced a technique for the minimax location problem with the help of the diamond-covering problem using the transformation the Tchebychev space to rectilinear space. They found that the rectilinear distance between any two points of the diamond was the same as the Tchebychev distance between any two points of the square. Hwang and Lim [5] described a single facility location problem in an automated storage and retrieval system (AS/RS). This problem is converted to the Tchebychev minimax location problem. Gaboune et al. [6] discussed on the distances between points distributed uniformly and pairs of co-planar rectangles as well as pairs of rectangular parallelepipeds. They derived the expected distances using the rectilinear, the Euclidean, and the Tchebychev metric. Gass and Witzgall [7] introduced the Tchebychev minimax problem and an approximation of this problem using linear programming techniques to find a circle that is the closest to a given set of customer points. Nickel and Puerto [8] and Farahani and Hekmatfar [9] introduced the transformation rules of each lp-norm to the others in details. They stated that transformation is useful while analyzing and solving a problem with a given metric distance is difficult. Parthasarathy et al. [10] expanded a single facility location problem with the Tchebychev distance metric in the three-dimensional space and developed a new algorithm for solving the elusive three-dimensional case. Then, they found near-optimal solutions in practical computational times.

Although facility location problems with a barrier are so applicable in the real world (e.g., industrial plant design and urban planning), relatively little attention has been received in the location literature, especially with Tchebychev distance. Katz and Cooper [11] studied the Euclidean Weber problem and one circular barrier for the first time. To solve the problem, they proposed a heuristic method based on the sequential unconstrained minimization technique (SUMT). Bischoff and Klamroth [12] took the advantage of the Weiszfeld technique and genetic algorithm (GA) to solve the same problem. Aneja and Parlar [13] studied the Euclidean Weber problem, in which two cases of forbidden and barrier regions are considered. The authors considered a solution procedure that generates some candidate locations using simulated annealing. Then, they constructed a visibility graph to evaluate possible solutions in the barrier case. McGarvey and Cavalier [14] modified the Big Square Small Square (BSSS) method to approximate the global optima of the Euclidean Weber problem with the convex polyhedral barriers. This method, which works based on the Branch-and-Bound (B&B) technique, was originally proposed by Hansen et al. [15] in order to solve continuous location problems. Batta et al. [16] generalized the results of Larson and Sadiq [17] by considering both arbitrary shaped barriers and convex forbidden regions. A similar discretization for a general class of distance functions was derived by Hamacher and Klamroth [18].

Dearing and Segars [19] found an equivalent result for single facility location problems with the rectilinear distance in the presence of barriers. Dearing and Segars [20] improved the computational efficiency of these methods significantly.

They explained that the consideration of a reduced dominating set is sufficient to solve the problem. Subsequently, Dearing et al. [21] studied the same problem using the block norm distances in place of the rectilinear distances. Savas et al. [22] first considered the location of a single finite-size facility in the presence of the arbitrary shaped barriers with the Manhattan (i.e., rectilinear) distance. Klamroth [23] presented the Weber problem in the presence of line barriers with a finite number of passages. She proved that the time complexity of the problem exponentially grows by increasing the number of passages. Klamroth and Wiecek [24] proposed an algorithm for multi-criteria location problems with line barriers considering various distance functions.

For the first time, Canbolat and Wesolowsky [25] introduced the rectilinear distance Weber problem with a probabilistic line barrier and proposed an algorithm to solve the problem.

In this paper, we consider the Tchebychev distance Weber problem with a probabilistic line barrier. Canbolat and Wesolowsky [25] did not present any mathematical model and they just solved small-sized problem. Amiri-Aref [26] and Amiri-Aref [27] considered the same problem in the case of center problem and multi-period respectively. However, we generate and solve an MINLP model with the LINGO software. In addition, we propose a meta-heuristic algorithm based on Differential Evolution (DE) for finding a good solution for the large-sized problems that cannot be solved optimally by any exact solution in reasonable time. To the best of our knowledge, there is no published paper in this field. We utilize the proposed DE because of its efficiency. DE is a well-known meta-heuristic algorithm than can be applied for non-differential nonlinear models. It is first introduced by Storn and Price [28] to solve global optimization over continues space in order to fulfill the following four requirements expected from a practical minimization technique:

- ❖ Ability to handle non-differentiable, nonlinear, and multimodal cost functions.
- ❖ Parallelizability to cope with computation intensive cost functions.
- ❖ Ease of use (i.e., few control variables to steer the minimization). These variables should also be robust and easy to choose.
- ❖ Good convergence properties (i.e., consistent convergence to the global minimum in consecutive independent trials).

Lampinen and Zelinka [29] utilized DE to manufacturing optimization problems with mixed-integer discrete-continuous variables that design mechanical elements (e.g., gear train, pressure vessels and springs). Babu and Sastry [30] used DE for the estimation of effective heat transfer parameters in tricklebed reactors using radial temperature profile measurements fermentation process. Ponsich and Coello Coello [31] proposed the DE method to solve a Mixed-Integer Non-Linear Programming (MINLP) for batch plant design problems and compared its results with an exact optimization method (i.e., B&B) and with a Genetic Algorithm (GA). The results repeatability was found for the DE method is much better than for the GA. Lampinen and Zelinka [29], Babu, and Sastry [30] also used DE for a number of test problems and showed that DE is very robust in obtaining the global minimum in comparison to other direct search methods.

3. PROBLEM FORMULATION

In the following, some indices and parameters to produce a mathematical programming model for the desired problem are introduced.

3. 1. Indices and Parameters

i Index of the existing facilities

m Number of the existing facilities

L Length of the barrier

w_i Weight of the i-th facility

x_i x-coordinate of the i-th facility

y_i y-coordinate of the i-th facility

B Location of the barrier

L_1 Lower limitation of the uniform distribution

L_2 Upper limitation of the uniform distribution

$$S_i = \begin{cases} 1 & \text{if facility } i \text{ locates in the above halfplane } b. \\ 0 & \text{Otherwise} \end{cases}$$

3. 2. Decision Variables

X x-coordinate of the new facility

Y y-coordinate of the new facility

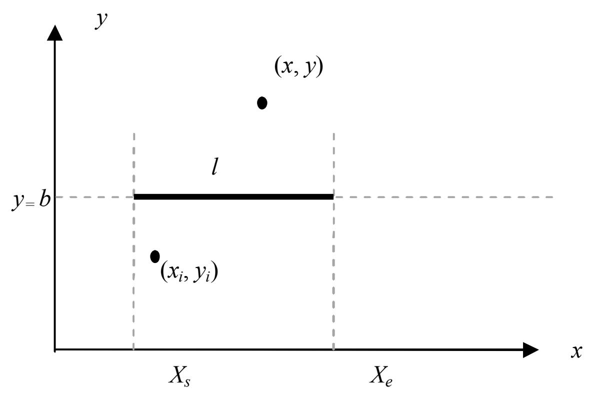

Figure 1. Probabilistic line barrier.

In this section, a general model for the Weber problem is presented that minimizes the total traveled Tchebychev distance in the presence of a probabilistic line barrier. Since there is a probabilistic barrier, calculating the expected barrier distance should be considered. So, the objective Function (5) can be reformulated by:

$$f(X) = \min_{X\in\mathcal{F}} \sum_{i=1}^{m} w_i . E[d_\infty^B(X, X_i)] \quad (11)$$

Following is the computation of $E[d_\infty^B(X, X_i)]$. We also consider a horizontal line barrier with length l that has a fixed y-coordinate at b and a probabilistic x-coordinate. Let X_s, which is a continuous random variable with known parameters, be the starting point of the line barrier. Then, the ending point of the line barrier, called X_e, can be calculated by $X_e = X_s + l$ (see Figure 1). Canbolat and Wesolowsky [25] considered all possible cases of locating the existing and the new facility related to a line barrier position on the plane. They showed when the conditions $0 \le x\text{-}X_s \le l$ and $0 \le x_i\text{-}X_s \le l$ are met, then the distance between new and existing facility can be called the barrier distance. Since the barrier randomly occurs on a horizontal line barrier, they proved that the barrier conditions can be represented in the following form.

$$\max\{x - l, x_i - l\} \le X_s \le \min\{x, x_i\}, \quad \forall\, i. \quad (12)$$

It means that the barrier can affect on the x-coordinate of the distance between two arbitrary points. In other word, the barrier cannot affect the y-coordinate of the distance. So, the y-coordinate of the distance becomes the regular Tchebychev distance.

$$\begin{aligned} &\min\ \sum_{i=1}^{m} w_i . \max\ \{E[d_\infty^B(x, x_i)], E[d_\infty^B(y, y_i)]\} = \\ &\min\ \sum_{i=1}^{m} w_i . \max\ \{E[d_\infty^B(x, x_i)], |y - y_i|\} \end{aligned} \quad (13)$$

Furthermore, computing the expected barrier distance is completely discussed in Canbolat and Wesolowsky [25]. They illustrated that the expected barrier distance of x from x_i can be generally written by:

$$E[d_1^B(x, x_i)] = \begin{cases} \frac{(l-|x-x_i|)^2}{2r} + |x - x_i|; & |x - x_i| < l \\ |x - x_i|; & |x - x_i| \ge l \end{cases}; \ \forall\, i \quad (14)$$

Some auxiliary variables, which check the barrier conditions, are introduced as follows:

1) ***Condition I*** checks the visibility of the new facility and existing facility i.

$$a_i = \begin{cases} 1 & |x - x_i| < l \\ 0 & |x - x_i| \ge l \end{cases} \quad \forall\, i \quad (15)$$

2) ***Condition II*** checks the correlation of the new facility and existing facility i, in which they are located in the same half-plane or in different half-planes.

$$t_i = \begin{cases} 1 & |p - s_i| = 1 \\ 0 & \text{otherwise} \end{cases} \quad ; \quad \forall\, i \quad (16)$$

where

$$s_i = \begin{cases} 1 & y_i > b \\ 0 & y_i \leq b \end{cases} \quad ; \quad \forall i \qquad (17)$$

$$p = \begin{cases} 1 & y > b \\ 0 & y \leq b \end{cases} \qquad (18)$$

3) ***Condition III*** determines the length of the distance between new facility and existing facility i according to the Tchebychev distance function. It means that if for a given i we have $\max\{|y - y_i|, |x - x_i|\} = |y - y_i|$, then regular Tchebychev distance is considered and the line barrier has not affect on the distance; else, the barrier may influence on the distance, related to the two conditions stated before.

$$z_i = \begin{cases} 1 & \max\{|y - y_i|, |x - x_i|\} = |y - y_i| \\ 0 & \max\{|y - y_i|, |x - x_i|\} = |x - x_i| \end{cases} \qquad (19)$$

In general, the objective Function (6) can be reformulated in the form of the MINLP model.

$$\min \quad \sum_{i=1}^{n} w_i \times \{ (\frac{(l-|x-x_i|)^2}{2(l_2-l_1)}) \times t_i \times a_i + |x - x_i|) \times (1 - z_i) + z_i \times |y - y_i|\} \qquad (20)$$

s.t.

$$|x - x_i| \times (2 \times z_i - 1) \leq (2 \times z_i - 1) \times |y - y_i| \;;\; \forall i \qquad (21)$$

$$|x - x_i| \times (2 \times a_i - 1) \leq (2 \times a_i - 1) \times l \quad ; \quad \forall i \qquad (22)$$

$$y \times (2 \times p - 1) \geq (2 \times p - 1) \times b \qquad (23)$$

$$y_i \times (2 \times s_i - 1) \geq (2 \times s_i - 1) \times b \quad ; \quad \forall i \qquad (24)$$

$$|s_i - p| = t_i \quad ; \quad \forall i \qquad (25)$$

$$z_i, a_i, p, t_i \in \{0,1\} \quad ; \quad \forall i \qquad (26)$$

$$x, y \geq 0 \qquad (27)$$

Objective Function (15) minimizes the total expected weighted Tchebychev barrier distance between new and existing facility, in which the first term of this function is calculated when the three barrier conditions stated before are satisfied simultaneously. When the barrier conditions are not met together, the second term of this objective function will be computed. Constraint (16) specifies the distance from the new facility to each existing facility, which is the reformulation of *Condition III*. Constraint (17) determines the visibility condition between the new facility and each existing facility that verify the *Condition I*. Constraints (18) and (19) investigate the position of the new and each existing facility whether in the upper or lower half plane, respectively. They are the restatement form of Constraints (12) and (13). Constraint (20) checks the position of the new and each existing facility to each other that checks *Condition* II. The binary and nonnegative variables are expressed in Constraints (21) and (22).

4. A NUMERICAL EXAMPLE

For better understanding, we consider an example that we find the location of the new facility between eight existing facilities and a probabilistic line barrier with the fixed length 4. The x-coordinate of a barrier has the uniform distribution between (3, 8) and y-coordinate fixed at 6. Table 1 provides the data for this example. Then, we solve this example with LINGO 9 and illustrate the example in Figure 2. Furthermore, we compare the results obtained from the problem without a barrier. Table 2 shows the optimal location of a new facility and their objective function in the presence of a barrier and without barrier.

TABLE 1. Data of the existing facilities.

i	x_i	y_i	w_i
1	11.16	4.48	1
2	7.59	9.62	1
3	9	11.61	1
4	9.81	5.5	1
5	6.06	4.8	1
6	10.03	8	1
7	12	7.5	1
8	6	9.46	1

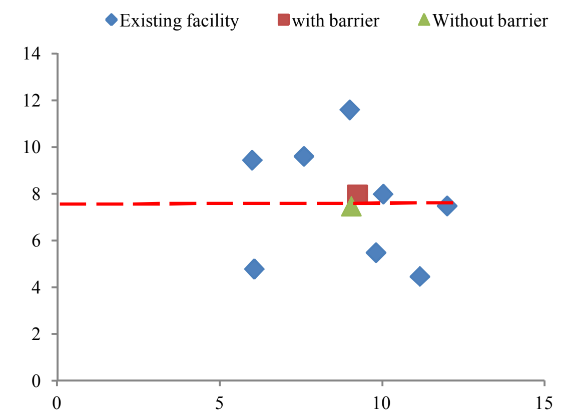

Figure 2. Optimal location of a new facility and location of the existing facilities.

TABLE 2. Optimal location of a new facility and its objective function.

	x^*	y^*	f^*
With barrier	9.235	7.975	21.22
Without barrier	9.048	7.488	15.38

TABLE 3. Parameters of the proposed DE.

Name	Symbol	Explanations
Mutation rate	F	Difference weighting factor
Population size	Np	Number of vectors in each generation
Stopping condition	G_{Max}	Usually select a certain number for the maximum iteration of a generation

1	2	.....	Np
X_1	X_2	.....	X_{np}
Y_1	Y_2	.....	Y_{np}

Figure 3. Representation of the initial population.

5. DIFFERENTIAL EVOLUTION

The differential evolution (DE) is a well-known meta-heuristic algorithm, which is applicable for the non-differential nonlinear models. It is first introduced by Storn and Price [28] to solve the global optimization over the continues space. Lampinen and Zelinka [29], Babu, and Sastry [30] have also used DE for a number of test problems and showed that DE is very robust in obtaining the global minimum in comparison to other direct search methods.

The DE algorithm is a parallel direct search method, which utilizes a number of *d*-dimensional parameter vectors as the initial population (*Np*) chosen randomly and is fixed during the minimization process. DE generates new vectors for the next generation by three operations, namely (1) mutation that adds the weighted difference between two population vectors to a third vector to yield the so-called *noisy vector*, (2) crossover where the noisy vector's parameters are then combined with the parameters of *target vectors*, in which the result vector is called the *trial vector*, and (3) selection. If the objective function of the trial vector is lower than the target vector, it replaces the target vector in the following generation. In each generation, each of *Np* vectors should to be a target vector once. There are different strategies of DE that differences are in the target vector selection and difference vector creation. We use the DE/rand/1 strategy in this paper. We explain Parameters of the proposed DE in Table 3.

The modification of the DE algorithm for the given problem is briefly introduced below.

5. 1. Representation Let *Np* be the population size and *d* be the number of dimensions. So, a new facility location should be shown with a vector with two dimensions. We create a set of *Np* vector randomly as initial population (*pop*) as shown in Figure 3.

5. 2. Mutation For each target vector, we select three vector randomly (r_1, r_2 and r_3). Calculate the noisy vector $u_{i,j,G}$ as follows.

$$u_{i,j,G+1} = pop_{r_3,j,G} + F \times \left(pop_{r_1,j,G} - pop_{r_2,j,G}\right) \tag{28}$$

5. 3. Crossover Because of increasing the diversity of the perturbed parameter vectors, crossover is proposed. With probability *Cr*, we select the parameters value from the noisy vector; otherwise, from target vector. The result vector is trial vector $x_{i,j,G}$ (i=1,…,np; j=1,…,d; G=1,…,G_{Max}).

5. 4. Selection We compare the cost function value of the trial and target vectors. The one with a lower cost survive for the next generation.

5. 5. Repairing Strategy The proposed DE algorithm has also a repair strategy when the illegal vector is created. After initialization and each operation (i.e., mutation and crossover) in the proposed DE algorithm, if the value of parameter vector for each population is out of the range of the existing facilities, we replace it by a random legal vector in this range. We find the best combination of the DE parameters by experiment that we fix the population size and a number of the existing facilities, and then tune between the difference weighting factor (*F*) and the crossover rate (*Cr*). We run the proposed DE for some combination of DE parameters and compare the objective function. Finally, we achieve the best combination of *Cr* = 0.7 and *F* = 0.9. The pseudo code of the proposed DE is presented in Appendix 1.

6. LOWER BOUND

Since location problems with barriers are generally difficult global optimization problems, problem relaxations allowing for the determination of good bounds are essential for their solution. Klamroth [3] considered a simple way for finding a lower band for the location problem with a barrier that is the problem without the corresponding constraints of a barrier. Due to this definition, we solve the problem of type $1/\mathbb{R}^2/\bullet/d_\infty/\Sigma$ instead of the problem of type $1/\mathbb{R}^2/\mathcal{B}-1\,probabilistic\ line\ /d_\infty^B/\Sigma$. The mathematical model of this problem is presented in Appendix 2.

7. COMPUTATIONAL RESULT

In this section, we compare the computational results obtained from the DE algorithm and the lower bound as well as SLP (one of the strategy directions for solving non-linear problems in the LINGO software) for the variety size of test problems, which are shown in Table 4. In all test problems, we assume that the length of a line barrier is equal to 50 and its y-coordinate is 55. It is supposed that the start point of the line barrier follows the uniform distribution function U (30, 80). We use MATLAB.7.5.0 software for running algorithms. The system utilized is a personal computer with Intel Pentium (R) Dual CPU E2180 @ 2GHZ and 2 GB RAM. Following are the explanation of Table 4.The result shows that SLP algorithm found the optimal solution except test problems 7 to 10. The results found by DE show the same value as SLP until test problem 6. We can state that our DE algorithm is robust because the variance of five iterations of each test problem is zero. We also show the DE convergence rate in Figure 4. It can be observe that the DE converges to optimal value after almost ten generations. We then compare DE and SLP running time duration in Figure 5. SLP cannot achieve global optimum in a reasonable time for a larger size of the given problems (for example, in test problem 6, LINGO is able to find the optimal value after 2.36 hours). We also run our proposed DE algorithm for a larger size of this problem for five hours without stopping the condition criteria. The computational results are the same as shown in Table 5. For test problems 7-10 that the exact solution cannot find the global optimum, we compare our DE results with a lower bound (i.e., column LB in Table 5). Furthermore, we can state our proposed DE algorithm is an efficient algorithm to solve both small and large-sized problems.

TABLE 4. explanation of columns

Column		**Explanation**
No.		Number of problem
M		Number of facility
LB		Lower Bound
MINLP:	**z^*(SLP)**	The objective function value obtained from the SLP solver of the LINGO
	Time	Computing time of LINGO
DE:	**Best**	Best objective values for five times iteration of the DE algorithm
	Mean	Mean objective values for five times iteration of the DE algorithm
	STDV	Standard deviation are computed for each sample problem and are depicted
	CN	Closeness ratio are computed for each sample problem
	Time	Computation time of DE algorithm for each sample problem

TABLE 5. Computational result

No	***m***	***LB***	**MINLP:**		**DE:**				
			z^*(SLP)	**Time (s)**	**Best**	**Mean**	**STDV**	**CN**	**Time (s)**
1	10	224.72	226.52	17	226.521	226.521	0	0	.32
2	20	440.26	449.34	112	449.338	449.338	0	0	.59
3	40	718.165	956.17	550	956.173	956.173	0	0	1.28
4	60	1236.7	1453.2	1944	1453.15	1453.15	0	0	2.7
5	80	1424.4	1829.8	5127	1829.8	1829.8	0	0	3.31
6	100	1749.2	2331.7	8498	2331.7	2331.7	0	0	4.16
7	150	2584.9	2909.5*	10800	3531	3531	0	0	5.39
8	200	3648.4	1801.1*	10800	4929.5	4929.5	0	0	6.39
9	250	4549.5	554.4*	10800	5842.2	5842.2	0	0	9.67
10	500	9178.9	1115.2*	10800	11931	11931	0	0	12.48

* Returned by the optimization software as an objective bound after 3-hour computational time.

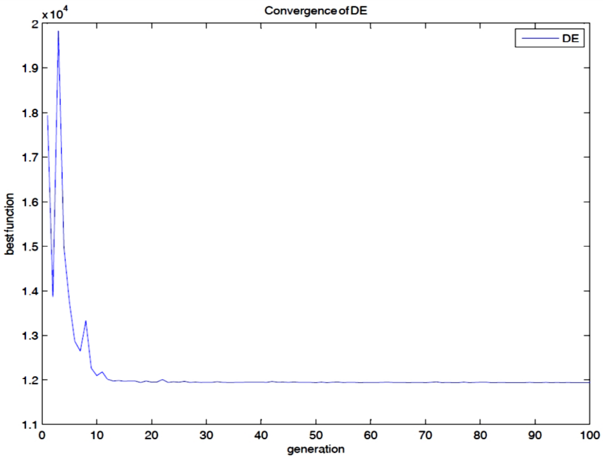

Figure 4. DE convergence

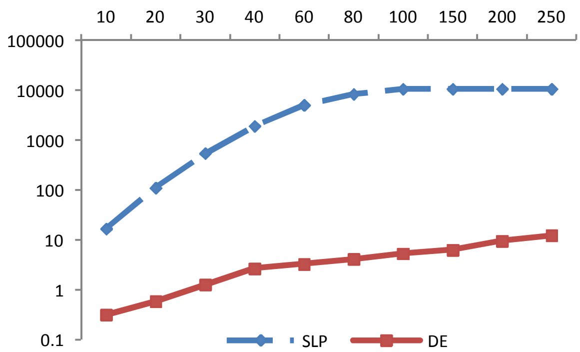

Figure 5. DE and SLP executive time for different number of existing facility

8. CONCLUSION

The purpose of this paper was to find the optimal location of a new facility in the presence of a probabilistic line barrier under the Tchebychev distance norm. After investigating the visible and shadow region as well as their relevance to a barrier condition, we developed a mathematical model for this problem. We use the SLP algorithm to solve this model. The computational results showed that this exact algorithm could not achieve any solution but an objective bound. Therefore, we proposed the differential evolution (DE) algorithm for the presented model and compared its result with the exact solution and lower bound. As future research, considering another type of a barrier or combination more than one barrier can be the extension for this problem. Another extension can be the use of another meta-heuristic algorithm for this problem. In addition, it can be supposed that the location of the existing facilities or the barrier length is not crisp.

9. REFERNCES

1. Hamacher, H.W., Nickel, S., "Multicriteria planar location problems", ***European Journal of Operational Research***, Vol. 94, (1996), pp. 66–86.
2. Hamacher, H.W., Nickel, S., "Restricted planar location problems and applications", ***Naval Research Logistics***, Vol. 42, (1995), pp. 967–992.
3. Klamroth, K., "Single-Facility Location Problems with Barriers", Springer Series in Operations Research, 2002.
4. Francis, R. L., McGinnis, L.F., White, J.A., "Facility layout and location: An analytical approach", Prentice Hall, New Jersey, 1992, pp. 184–238.
5. Hwang, H., Lim, J.M., "Deriving an optimal dwell point of the storage/retrieval machine in an automated storage/retrieval system", ***International Journal of Production Research***, Vol. 31, (1993), pp. 2591–2602.
6. Gaboune, B., Laporte, G., Soumis, F., "Expected distances between two uniformly distributedrandom points in rectangles and rectangular parallelepipeds", ***Journal of the Operational Research Society***, Vol. 44(5), (1993), pp. 513–519.
7. Gass, S. L., Witzgall, C., "On an approximate minimax circle closest to a set of points", ***Computers and Operations Research***, Vol. 31, 2004, pp. 637–643.
8. Nickel, S., Puerto, J., "Location theory: a unified approach", Springer-Verlag, Berlin Heidelberg, 2005.
9. Zanjirani Farahani, R., Hekmatfar, M., (Eds.), "Facility location: Concepts, models, algorithms and case studies", Berlin,Germany: Springer-Verlag, Vol. 549, 2009.
10. Parthasarathy, M., Hale, T., Blackhurst, J., Frank, M., "The three-dimensional Fermat-Weber problem with Tchebychev distances", ***Advanced Modeling and Optimization***, Vol. 8(1), (2006), pp. 65–71.
11. Katz, I., Cooper, L., "Formulation and the case of Euclidean distance with one forbidden circle", ***European Journal of Operational Research***, Vol. 6, (1981), 166–173.
12. Bischoff, M., Klamroth, K., "An efficient solution method for Weber problems with barriers based on genetic algorithms", ***European Journal of Operational Research***, Vol. 177(1), (2007), pp. 22–41.
13. Aneja, Y.P., Parlar, M., "Algorithms for Weber facility location in the presence of forbidden regions and or barriers to travel", ***Transportation Science***, Vol. 28(1), (1994), pp. 70–76.
14. McGarvey, R.G., Cavalier, T.M., "A global optimal approach to facility location in the presence of forbidden regions", ***Computers and Industrial Engineering***, Vol. 45(1), (2003), pp. 1–15.
15. Hansen, P., Peeters, D., Thisse, J., "On the location of an obnoxious facility", ***Sistemi Urbani***, Vol. 3, (1981), pp. 299–317.
16. Batta, R., Ghose, A., Palekar, U., "Locating facilities on the manhattan metric with arbitrarily shaped barriers and convex forbidden regions", ***Transportation Science***, Vol. 28(1), (1989), pp. 70–76.
17. Larson, R.C., Sadiq, G., "Facility location with the manhattan metric in the presence of barriers to travel", ***Operations Research***, Vol. 31, (1983), pp. 652–669.
18. Hamacher, H.W., Klamroth, K., "Planar Weber location problems with barriers and block norms", ***Annals of Operations Research***, Vol. 96, (2000), pp. 191–208.

19. Dearing, P.M., Segars, J.R., "Solving rectilinear planar location problems with barriers by a polynomial partitioning", ***Annals of Operations Research***, Vol. 111(1–4), (2002a), pp. 111–133.

20. Dearing, P.M., Segars, R., "An equivalence result for single facility planar location problems with rectilinear distance and barriers", ***Annals of Operations Research***, Vol. 111(1–4), (2002b), pp. 89–110.

21. Dearing, P.M., Klamroth, K., Segars, R., "Planar location problems with block distance and barriers", ***Annals of Operations Research***, Vol. 136(1), (2005), pp. 117–143.

22. Savas, S., Batta, R., Nagi, R., "Finite-size facility placement in the presence of barriers to rectilinear travel", ***Operations Research***, Vol. 50(6), (2002), pp. 1018–1031.

23. Klamroth, K., "Planar Weber location problems with line barriers", ***Optimization***, Vol.49 (5–6), (2001), pp. 517–527.

24. Klamroth, K., Wiecek, M.M., "A bi-objective median location problem with a line barrier", ***Operations Research***, Vol. 50(4), (2002), pp. 670–679.

25. Canbolat, M., Wesolowsky, G., "The rectilinear distance Weber problem in the presence of a probabilistic line barrier", ***European Journal of Operational Research***, Vol. 202, (2010), pp. 114–121.

26. M.Amiri-Aref, N.Javadian, R. Tavakkoli-Moghaddam, M.B. Aryanezhad, A multi-period 1-center location problem in the presence of a probabilistic line barrier, ***Amirkabir International Journal of Modeling, Identification, Simulation and Control***, Vol. 44(1), (2012), pp. 47-55.

27. M.Amiri-Aref, N.Javadian, R. Tavakkoli-Moghaddam, M.B. Aryanezhad, The center location problem with equal weights in the presence of the probabilistic line barrier, ***International Journal of Industrial Engineering and Computation***, Vol. 2(4), (2011), pp. 793-800.

28. Storn, R., Price, K., "Differential evolution – A simple and efficient heuristic for global optimization over continuous spaces", ***Journal of Global Optimization***, Vol. 11, (1996), pp. 341–359.

29. Lampinen, J., Zelinka, I., "Mechanical engineering design optimization by differential evolution", in: Corne, D., Dorigo, M., Glover, F. (Eds.) *New Ideas in Optimization*, McGraw-Hill, London, (1999), pp. 127–146.

30. Babu, B.V., Sastry, K.K.N., "Estimation of heat transfer parameters in a trickle-bed reactor using differential evolution and orthogonal collocation", ***Computers & Chemical Engineering***., Vol. 23(3), (1999), pp. 327–339.

31. Ponsich, A., Coello Coello, C.A. "Differential Evolution performances for the solution of mixed-integer constrained process engineering problems", ***Applied Soft Computing,*** vol. 11(1), (2011), pp. 399–409.

APPENDIX 1

Following is the pseudo code of the proposed DE.

Begin
G=0
Create a random initial population $pop_{i,G}$; $\forall i$ *(i=1,…,np)*
Evaluate f($pop_{i,G}$*)* ; $\forall i$ *(i=1,…,np)*
For G=1 to G_{Max} *do*
For i=1 to np do
Slecct randomly $r_1 \neq r_2 \neq r_3$
j_{rand}*=randint(1,d)*
For j=1 to d do
$u_{i,j,G+1} = pop_{r_3,j,G} + F \times (pop_{r_1,j,G} - pop_{r_2,j,G})$
If($rand_j[0,1) \leq Cr$ *or j=*j_{rand} *) then*
$x_{i,j,G+1} = u_{i,j,G+1}$
Else
$x_{i,j,G+1} = pop_{i,j,G}$
End If
End For
If ($f(x_{i,G+1}) \leq f(pop_{i,G})$ *) then*
$pop_{i,G+1} = x_{i,G+1}$
Else
$pop_{i,G+1} = pop_{i,G}$
End If
End for
G=G+1
End for
End

APPENDIX 2

Following is the mathematical model for the lower band of our problem.

$$\min \ \sum_{i=1}^{n} w_i \times \{ \ |x - x_i| \times (1 - z_i) + z_i \times |y - y_i| \} \tag{20}$$

$$|x - x_i| \times (2 \times z_i - 1) \leq (2 \times z_i - 1) \times |y - y_i| \ ; \ \forall i \tag{30}$$

$$z_i \in \{0,1\} \qquad ; \ \forall i \tag{31}$$

$$x, y \geq 0 \tag{32}$$

A Facility Location Problem with Tchebychev Distance in the Presence of a Probabilistic Line Barrier

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چکیده

این مقاله، یک مسأله مکانیابی تک تسهیل را در حضور یک مانع خطی در صفحه با فاصله از نوع چبیشف در نظر می‌گیرد. به‌طور خاص، یک مدل برنامه‌ریزی عدد صحیح غیر خطی مختلط برای این مسأله توسعه داده می‌شود، به‌نحوی که مجموع فاصله وزن‌دار بین تسهیل جدید و تسهیلات موجود را کمینه می‌شود. برای صحه گذاری مدل ریاضی، یک مثال عددی حل می‌شود. به‌علت سختی ایجاد شده در حل این مسأله با افزایش تعداد تسهیلات موجود، یک الگوریتم فراابتکاری کارا به‌نام تکامل تفاضل برای حل این مسأله در اندازه بزرگ پیشنهاد و طراحی می‌شود. در خاتمه، مقایسه نتایج مربوطه با روشهای دقیق و حد پایین برای اندازه‌های مختلف از این مسأله ارائه می‌شود.

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