EFFECT OF WAVY WALL ON CONVECTION HEAT TRANSFER OF WATER-AL_2O_3 NANOFLUID IN A LID-DRIVEN CAVITY USING LATTICE BOLTZMANN METHOD

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Abstract In the present study, the effects of wavy wall's properties on mixed convection heat transfer of Water-Al_2O_3 Nanofluid in a lid-driven cavity are investigated using the Lattice Boltzmann Method (LBM). The Boundary Fitting Method with second order accuracy at both velocity and temperature fields is used to simulate the curved boundaries in the LBM. The problem is investigated for different Richardson numbers ($0.1 \le Ri \le 10$), volume fractions of nanoparticles ($0.01 \le \varphi \le 0.05$), curve amplitudes ($0.05 \le A \le 0.25$) and phase shifts of corrugated wall ($0 \le \Lambda \le 270$) when the Reynolds number is equal to 25. The results represent the effective role of corrugated wavy wall on the rate of nanofluid heat transfer. It is observed that increasing the wavy wall's amplitude leads to a decrease in the average Nusselt number in high Richardson number. It is found that increasing the volume fraction of nanoparticles enhances the rate of heat transfer. Results also show that adding nanoparticles to the base fluid has significant effects on both fluid flow and temperature field of the mixed convection, especially for low Richardson number.

Keywords Lattice Boltzmann method; Boundary fitting method; Wavy surface; Nanofluid; Mixed convection; Richardson number .

چکیده در مطالعه حاضر، اثر دیواره ی موجی شکل بر انتقال حرارت جابجایی ترکیبی نانوسیال آب–اکسید آلومینیوم در یک محفظه با درب متحرک با استفاده از روش شبکه بولتزمن مورد بررسی قرار می گیرد. برای مدلسازی مرزهای منحنی شکل در روش شبکه بولتزمن از روش انطباق مرزها که دقتی مرتبه دو را برای میدان جریان و دما فراهم می آورد استفاده شده است. مسئله برای مقادیر متفاوت عدد ریچاردسون ($0.1 \le Ri \le 10$)، کسر حجمی نانوذرات ($0.01 \le \varphi \le 0.05$)، دامنه موج دیواره ($0.05 \le A \le 0.25$) و اختلاف فاز دیواره ($0 \le \Lambda \le 270$) در حالیکه عدد رینولدز برابر ۲۵ می باشد حل شده است. نتایج بدست آمده نشان دهنده نقش موثر دیواره موجی شکل در نرخ انتقال حرارت نانوسیال می باشند. مشاهده می شود که افزایش دامنه دیواره موجی باعث کاهش مقدار متوسط عدد ناسلت در عدد ریچاردسون های بالا می شود. فهمیده می شود که افزایش کسر حجمی نانوذرات باعث بهبود نرخ انتقال حرارت می شود. نتایج همچنین نشان می دهند که اضافه کردن نانوذرات به سیال پایه تاثیرات قابل توجهی بر میدان جریان و دمای جابجایی ترکیبی می گذارد که این تاثیرات در عددهای ریچاردسون کم بیشتر می باشد.

1. INTRODUCTION

The enhancement of fluids heat transfer is a very interesting topic for different kinds of industrial and engineering problems. A well-known way for the enhancement of the rate of convection heat transfer in various applications is adding very fine conductive particles in nano scale to base fluids that don't have sufficient conductivity. Because of the high thermal conductivity of metal particles, they can improve conductivity of the suspensions systematically. Nowadays, nanoparticle added fluids are known as nanofluid that first time Choi [1] named this kind of fluid suspensions. He presented the enhancement of convection heat transfer by adding nanoparticles to the fluids.

In recent years, many studies have been conducted to investigate the heat transfer of nanofluids numerically and experimentally [1-10]. Khanafer et al. [2] presented a heat transfer enhancement by adding nanoparticles to fluid in a two dimensional enclosures for different Grashof numbers. Iulian et al. [3] conducted an experimental study and represented heat transfer enhancement possibility of coolants with suspended Al_2O_3 nanoparticles dispersed in water inside a radial flow cooling device. Abu-Nada et al. [7] did a numerical solution for natural convection heat transfer enhancement in horizontal concentric annuli using nanofluids. They found that the addition of various types and volume fractions of nanoparticles, had adverse effects on natural convection heat transfer characteristics.

Although the heat transfer enhancement is one of the main reasons of using nanoparticles in thermal applications, on other hand there are some studies that reported heat transfer decreasing by adding nanoparticles [8-10]. Nnanna et al. [8] reported that adding alumina nanoparticle decreases natural convection heat transfer. Santra et al. [9] investigated the effect of Cu nanoparticles on behavior of Water as a cooling medium at laminar natural convection in a square cavity. In their results, it was observable that the heat transfer decreased with increase of the volume fraction of nanoparticles for a constant Rayligh number. Putra et al. [10] analyzed natural convection in a horizontal cylinder in the presence of nanoparticles experimentally. They showed that the presence of nanoparticles in water led to the decrease of natural convection systematically.

The progress of using the Lattice Boltzmann Method (LBM) as a numerical technique to simulate the heat transfer and fluid flow has been obvious in the last decade [11-16]. The LBM has well-known advantages such as easy implementation, possibility of parallel coding and simulating of complex geometries[12] and fluid dynamic problems such as melting [13], turbulent fluids [14], fuel cell[15], porous media[16], nanofluids[17,18] and etc. Xuan and Yao [17] discussed the irregular motion of the nanoparticles and dynamic behavior of nanofluids and described a LBM for modeling the nanofluids. Nemati et al [18] investigated the mixed convection in a lid-driven cavity in the presence of different Water based nanofluids using LBM.

The LBM utilizes two distribution functions, for the flow and temperature fields. It uses modeling of movement of fluid particles to define macroscopic parameters of fluid flow. The basic form of LBM applies uniform Cartesian cells to discrete problem domain. Each cell of the grid contains a constant number of distribution functions, which represent the number of fluid particles movement in these separated directions. The distribution functions are obtained by solving the Lattice Boltzmann equation (LBE), which is a special form of the kinetic Boltzmann equation.

Because of using Cartesian uniform lattices in basic form of LBM, modeling of curved boundary in this method needs some stairs. Boundary Fitting Method is a treatment to simulate curved boundaries in LBM which was presented by Fillipova and Hanel [19] and improved by Mei et al. [20]. The difficulty of curved boundary conditions is to find a formulation for particles distribution function at the neighboring of solid curvy walls. An extrapolation of the velocity field in the Boundary Fitting Method is reported by Guo [21] which is an expansion of the treatment proposed by Chen et al. [22].

Convection heat transfer from wavy surfaces is observed in many engineering and scientific applications such as electronic devices, solar collectors and wavy condensers in refrigerators. In recent years, many studies about convection heat transfer and fluid flow in wavy enclosures and channels have been done [23-26]. Al-Amiri et al. [23] have conducted a study on mixed convection heat transfer in a lid-driven cavity with a sinusoidal bottom wall using a finite element approach based on the Galerkin method. They studied the effect of the Richardson number, amplitude of the surface, and number of undulations on the fluid flow and heat transfer. Al-Zoubi and Brenner [24] investigated the fluid flow over a sinusoidal surface using LBM. They studied the effect of Reynolds number and geometrical dimensions on the velocity distributions and flow factors.

Despite many numerical studies carried out on wavy wall channels or cavities using CFD methods [27], the lack of using LBM to simulate the convection heat transfer in wavy corrugated cavity filled with nanofluids is sensible. Therefore, the main aim of the present study is to investigate the effect of wavy wall on mixed convection of Water-Al_2O_3 nanofluid in a lid-driven cavity using Boundary Fitting Method in LBM. The curve amplitude and phase shift of wavy walls are studied in different conditions to exhibit the role of the wavy wall shape on both the heat transfer rate and fluid flow of the base fluid and the nanofluids. Also, the effect of the volume fraction of the nanoparticles on the local Nusselt number distribution, average Nusselt number, streamlines and temperature contours is investigated for various Richardson numbers, when Reynolds number is fixed at 25.

2. LATTICE BOLTZMANN METHOD

The basic form of the LBE with an external force by introducing BGK approximations can be written as follows for the both flow and temperature fields:

$$f_\alpha(x+e_\alpha\Delta t, t+\Delta t) = f_\alpha(x,t)+\frac{\Delta t}{\tau_m}[f_\alpha^{eq}(x,t)-f_\alpha(x,t)]+\Delta t e_\alpha F_\alpha$$
$$g_\alpha(x+e_\alpha\Delta t, t+\Delta t) = g_\alpha(x,t)+\frac{\Delta t}{\tau_t}[g_\alpha^{eq}(x,t)-g_\alpha(x,t)] \quad (1)$$

where $f_\alpha(x,t)$, e_α and F_α are a distribution function on the mesoscopic level, the discrete lattice velocity and the external force term in α direction, respectively. f_α^{eq} and g_α^{eq} are equilibrium distribution functions that are calculated as follows:

$$f_\alpha^{eq}=w_\alpha\rho[1+\frac{e_\alpha u}{c_s^2}+\frac{1}{2}\frac{(e_\alpha u)^2}{c_s^4}-\frac{1}{2}\frac{u^2}{c_s^2}]$$
$$g_\alpha^{eq}=w_\alpha T[1+\frac{e_\alpha u}{c_s^2}] \quad (2)$$

where ρ and w_α are, respectively, the lattice fluid density and weighting factor which has these values $w_0=4/9$ for $|c_0|=0$, $w_{1-4}=1/9$ for $|c_{1-4}|=1$ and $w_{5-9}=1/36$ for $|c_{5-9}|=\sqrt{2}$ in the D2Q9 model. To model buoyancy force, the force term in Eq.1 needs to be assumed as below in the needed direction:

$$F_\alpha=3w_\alpha g_\alpha\beta(T-T_{ref}) \quad (3)$$

The LBM Eqs. (1) and (2) are solved in two important steps that are called collision and streaming steps. The collision step is as follows:

$$\tilde{f}_\alpha(x,t+\Delta t)=-\frac{1}{\tau_m}[f_\alpha(x,t)-f_\alpha^{eq}(x,t)]-f_\alpha(x,t)$$
$$\tilde{g}_\alpha(x,t+\Delta t)=-\frac{1}{\tau_t}[g_\alpha(x,t)-g_\alpha^{eq}(x,t)]-g_\alpha(x,t) \quad (4)$$

The streaming step can be written as follows:

$$f_\alpha(x+e_\alpha\Delta t, t+\Delta t)=\tilde{f}_\alpha(x,t+\Delta t)$$
$$g_\alpha(x+e_\alpha\Delta t, t+\Delta t)=\tilde{g}_\alpha(x,t+\Delta t) \quad (5)$$

where $\tilde{f}_\alpha$ and $\tilde{g}_\alpha$ denotes the post-collision distribution function. Macroscopic variables can be calculated in terms of these variables, with the following formula for flow density, momentum and temperature, respectively.

$$\rho=\sum_\alpha f_\alpha\ ,\ \rho u_i=\sum_\alpha f_\alpha c_{i\alpha}\ ,\ T=\sum_\alpha g_\alpha \quad (6)$$

At present study, nanofluid is assumed as a single phase fluid. The added nanoparticles to the base fluid changes the thermal conductivity, viscosity and other basic characteristics of the base fluid, thus innovated nanofluid has special characteristics due to the combination of the pure fluid and added nanoparticles.

Thermal diffusivity, density, heat capacitance, thermal expansion and viscosity of the nanofluid can be defined as of nanofluid as follows [28]:

$$\alpha_{nf}=\frac{k_{nf}}{(\rho c_p)_{nf}},\qquad \rho_{nf}=(1-\varphi)\rho_f+\varphi\rho_s$$
$$(\rho c_p)_{nf}=(1-\varphi)(\rho c_p)_f+\varphi(\rho c_p)_s$$
$$\beta_{nf}=(1-\varphi)\beta_f+\varphi\beta_s,\qquad \mu_{nf}=\frac{\mu_f}{(1-\varphi)^{2.5}} \quad (7)$$

where the applied model for nanofluid viscosity is the Brinkman model the nanofluid containing a dilute suspension of small rigid spherical particles. The effective thermal conductivity presented by Patel et al. [30] is used as follows:

$$\frac{k_{nf}}{k_f} = 1 + \frac{k_s A_s}{k_f A_f} + c k_s Pe \frac{A_s}{k_f A_f} \qquad (8)$$

where,

$$\frac{A_s}{A_f} = \frac{d_s}{d_f}\frac{\varphi}{1-\varphi}, \quad Pe = \frac{u_s d_s}{\alpha_f}, \quad u_s = \frac{2k_b T}{\pi \mu_f d_s^2} \qquad (9)$$

where, u_s is the Brownian motion term of nanoparticle velocity and k_b is the Boltzmann constant.

3. BOUNDARY FITTING METHOD

For simulating a curved boundary, the used method is Boundary Fitting Method which is the same as those presented by Mei et al. [20] for the velocity field and those reported by Guo et al. [21] for the temperature field, that these treatments supply second order accuracy for both the velocity and temperature fields in curved boundaries.

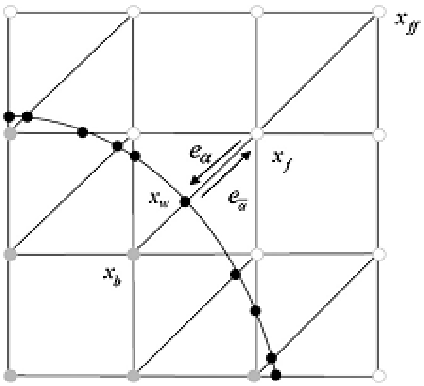


Figure 1. Schematic representation of curved boundary.

Figure 1 reveals an arbitrary part of a curved wall boundary. The black points indicate points on the boundary x_w, the white points represent the boundary nodes in the fluid region x_f and the grey points show those in the solid region x_b. The fraction of an intersected link in the fluid region Δ is defined as follows:

$$\Delta = \frac{|x_f - x_w|}{|x_f - x_b|} \qquad (10)$$

From the computational point of view, the numerical procedure for definition of Δ at all lattice directions which is needed in each boundary node is one of the important steps in modeling of curve boundaries using Boundary Fitting Method in LBM. To calculate the distribution function in the solid region ($\tilde{f}_{\bar{\alpha}}(x_b,t)$) based on the boundary nodes in the fluid region, the procedure used is the same as what reported by Mei et al. [20]. The Chapman-Enskog expansion for the post-collision distribution function is:

$$\tilde{f}_{\bar{\alpha}}(x_b, t+\Delta t) = (1-\lambda)\tilde{f}_{\alpha}(x_f, t+\Delta t) + \lambda f_{\alpha}^{o}(x_b, t+\Delta t) - 2\frac{3}{c^2} w_{\alpha} \rho(x_f, t+\Delta t) e_{\alpha} u_w \qquad (11)$$

where,

$$f_{\alpha}^{o}(x_b, t+\Delta t) = f_{\alpha}^{eq}(x_f, t+\Delta t) + \frac{3}{c^2} w_{\alpha} \rho(x_f, t+\Delta t) e_{\alpha}(u_{bf} - u_f) \qquad (12)$$

where u_{bf} and λ are defined the aspect, the value of Δ is as follows:

$$\begin{cases} u_{bf} = u_{ff}, \lambda = \dfrac{2\Delta - 1}{\tau_m - 2} if 0 < \Delta \le 1/2 \\ u_{bf} = (1 - \dfrac{3}{2\Delta})u_f + \dfrac{3}{2\Delta}u_w, \lambda = \dfrac{2\Delta - 1}{\tau_m + 1/2} if 1/2 < \Delta \le 1 \end{cases} \qquad (13)$$

where u_w is the velocity of the solid wall, u_{bf} is the assumed velocity for interpolations and $e_{\bar{\alpha}} \equiv -e_{\alpha}$. For the temperature field in the curved boundary in the present study, the applied method is based on a combination of the methods presented by Guo et al. [21]. The distribution function for temperature is divided into two parts, equilibrium and non equilibrium:

$$g_{\bar{\alpha}}(x_b, t) = g_{\bar{\alpha}}^{eq}(x_b, t) + g_{\bar{\alpha}}^{neq}(x_b, t) \qquad (14)$$

So, the collision step is obtained as follow:

$$\tilde{g}_{\bar{\alpha}}(x_b, t+\Delta t) = g_{\bar{\alpha}}^{eq}(x_b, t) + (1 - \frac{1}{\tau_t}) g_{\bar{\alpha}}^{neq}(x_b, t) \qquad (15)$$

Obviously, to calculate $\tilde{g}_{\alpha}(x_b, t+\Delta t)$, both $g_{\bar{\alpha}}^{eq}(x_b,t)$ and $g_{\bar{\alpha}}^{neq}(x_b,t)$ are required. Equilibrium and non equilibrium parts of Eq.11 are define as:

$$g_{\alpha}^{eq}(x_b,t) = w_{\alpha}T_b^*(1+\frac{3}{c^2}e_{\alpha}u_b) \quad (16)$$

Then, T_b^* is determined by linear extrapolation using either of the realations:

$$\begin{cases} T_b^* = T_{b1} if \Delta \geq 0.75 \\ T_b^* = T_{b1} + (1-\Delta)T_{b2} if \Delta \leq 0.75 \end{cases} \quad (17)$$

where T_{b1} and T_{b2} are defined as follows with respect to Δ :

$$T_{b1} = [T_w + (\Delta - 1)T_f]/\Delta$$
$$T_{b2} = [2T_w + (\Delta - 1)T_{ff}]/(1+\Delta) \quad (18)$$

where T_f and T_{ff} denote the fluid temperature in node x_f and x_{ff} , respectively. The next task is to determine the $g_{\alpha}^{neq}(x_b,t)$. Second-order approximation is also used and $g_{\alpha}^{neq}(x_b,t)$ is evaluated as:

$$g_{\alpha}^{neq}(x_b,t) = \Delta g_{\alpha}^{neq}(x_f,t) + (1-\Delta)g_{\alpha}^{neq}(x_{ff},t) \quad (19)$$

4. RESULTS AND DISCUSSION

The vertical walls of the cavity are assumed to be insulated while the wavy bottom wall is maintained at a uniform temperature (T_h) higher than the top lid temperature (T_c). The lid wall has a constant velocity motion in a left to right direction (see Figure 2). The Prandtl number of base fluid (Water) at reference temperature ($22^{\circ}C$) is 6.57.

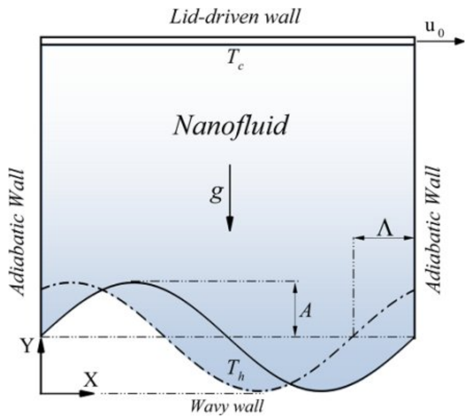


Figure 2. Schematic geometry of the problem.

It is assumed that the nanoparticles are spherical with uniform diameter and they are at thermal equilibrium state with base fluid. The particles in nanofluid have the same velocity as the base fluid thus there is no sleep velocity between added particles and molecules of the base fluid. Thermo-physical properties of particles and Water at reference temperature are tabulated in Table 1.

TABLE 1. Thermo-physical properties of the base fluid and nanoparticles.

Property	Water	Al_2O_3
$c_p(J/KgK)$	4179	765
$\rho(Kg/m^3)$	997.1	3970
$k(W/mK)$	0.613	25
$\beta \times 10^5 (K^{-1})$	21	0.85
$d_p(nm)$	0.384	47

The wavy wall is defined as follows:

$$Y = A(1+Sin(2\pi X + \frac{\Lambda}{180}\pi)) \quad (20)$$

where Λ shows the phase shift of wall function and A is the non-dimensional curve amplitude. The Nusselt number is defined along the wavy wall:

$$Nu = -L(\frac{k_{nf}}{k_f})\frac{\partial T}{\partial n} , \quad (21)$$

where n is the coordinate direction normal to the wavy wall and L is the length of wavy wall which is assumed as characteristic length. By integrating the local Nusselt number along the wavy surface, the average Nusselt number is calculated as follows:

$$Nu_{ave} = \frac{1}{L}\int_0^L Nuds \quad (22)$$

where s shows the integral calculated along the wavy line.

4.1. Grid Check and Results Convergence A uniform regular Cartesian mesh with 160×160 grid points is used. This number of nodes is selected with a mesh independency treatment that is indicated in Table 2 for a special issue.

TABLE 2. Average Nusselt number along the wavy wall.

$Ri=10$, $A=0.15$, φ=0.01 and $\Lambda=0$				
Grid	100×100	125×125	160×160	180×180
Nu_{ave}	6.758	6.976	7.100	7.006

To get a steady state result in the numerical solution, the results are compared at different time steps. Figure 3 shows the local Nusselt number along the wavy wall for various numbers of time steps. As indicated, when the number of time steps is more than 5×10^4 the results converge to a steady state and increasing of the time step after this condition has no clear effect on the values of needed parameters.

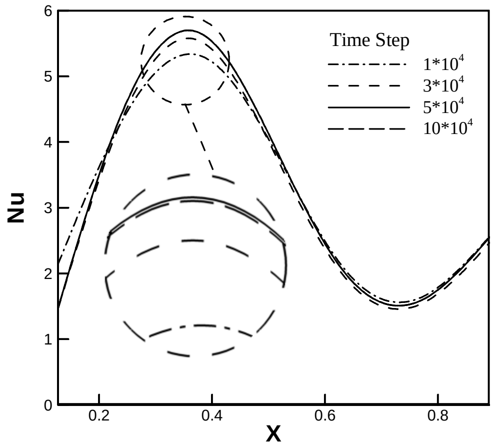


Figure 3. Nusselt number distribution at different time steps ($Ri=1$, φ=0.05 , $A=0.15$ and $\Lambda=0$).

4.2. Code Validation The numerical code is validated by the results presented by Al-Amiri et al. [23] for mixed convection in a lid-driven cavity with wavy sinusoidal bottom surface.

Table 3 shows the comparison of the values of average Nusselt number of this study and those presented by Al-Amiri et al. [23] which were in good agreement.

TABLE 3. Average Nusselt number along the wavy.

$Gr=10^4$, $Pr=1$, $A=0.05$, $\varphi=0$ and $\Lambda=90$			
	Present study	Al-Amiri et al. [23]	Deviation
Ri=0.1	7.242	7.412	2.30%
Ri=1	3.200	3.192	0.25%
Ri=10	2.829	2.694	5.01%

In addition, the streamline and temperature contours at $Gr=10^4$, $Ri=10$, $Pr=1$ and $A=0.05$ for $\Lambda=90$ are compared with results reported by Al-Amiri et al. [23] (See Figure 4).

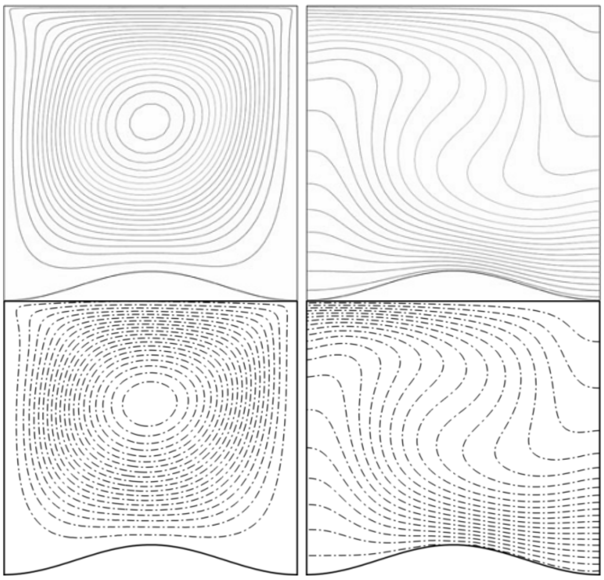


Figure 4. Streamline and temperature contours ($Gr=10^4$, $Ri=10$, $Pr=1$, φ=0 , $A=0.05$ and $\Lambda=270$).
Top)Al-Amiri et al. [23] down)Present study.

4.3. Flow and Thermal Fields The presence of dense particles with high thermal conductivity in suspensions has different effects on temperature and flow fields. Improvement of thermal conductivity of the base fluid and increasing of the flow momentum are some of the advantages of the presence of nanoparticles in the base fluid. These increases of flow momentum and thermal conductivity lead to heat transfer enhancement of the base fluid. Although, the overall shape of flow field is the same for both plain and nano-fluids, it should be mentioned that the value of stream-function increases by increasing the volume fraction of added nanoparticles (Figures 5 and 6). The value of stream-function at the core of the main vortex ($\psi_{\min}$) increases by increase of the volume fraction of the nanoparticles which can enhance the heat transfer rate.

Figure 5 shows streamlines and temperature contours for different volume fractions at $Ri=0.1$, $Ri=1$ and $Ri=10$ when $A=0.15$ and $\Lambda=0$. It is visible that the addition of nanoparticles to the base fluid have more effects on flow field in

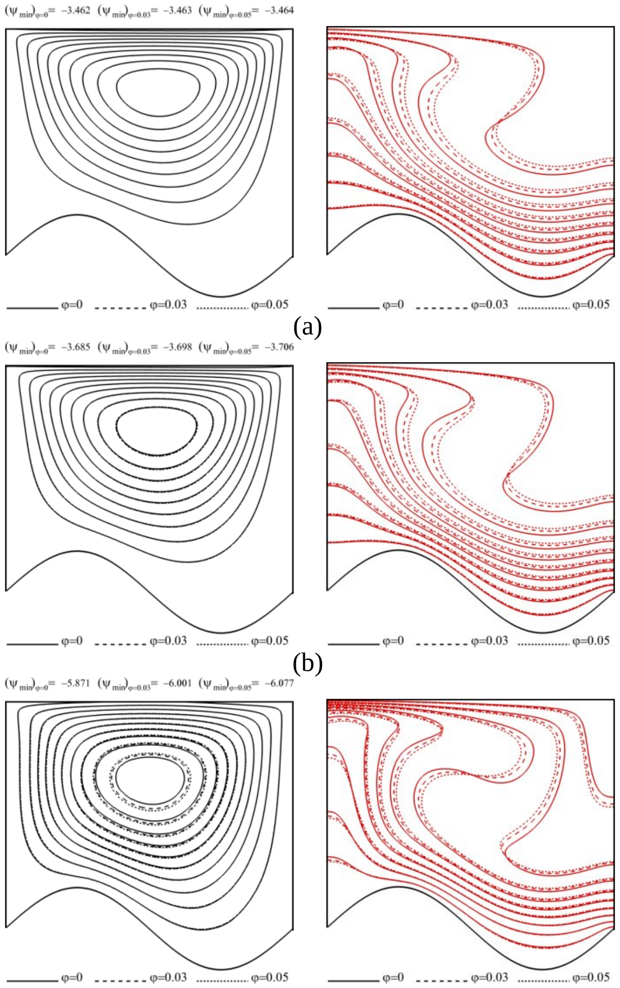


Figure 5. Streamlines and temperature contours at $A = 0.15$ and $\Lambda = 0$.
a) $Ri = 0.1$ b) $Ri = 1$ c) $Ri = 10$.

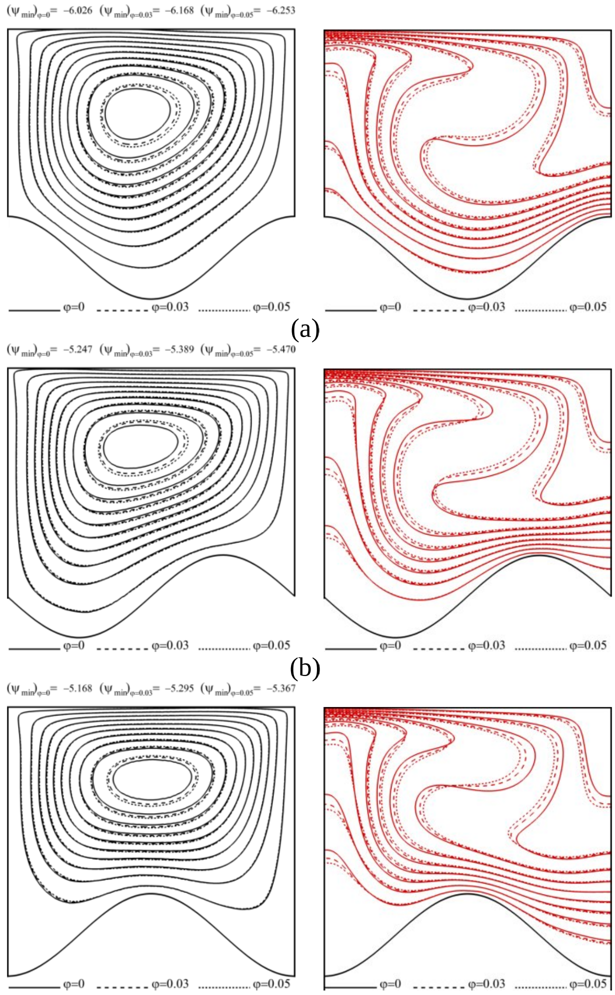


Figure 6. Streamlines and temperature contours at $Ri = 10$ and $A = 0.15$.
a) $\Lambda = 0$ b) $\Lambda = 90$ c) $\Lambda = 180$

high Richardson numbers (Figure 5) which can be due to the buoyancy force which is increased by the addition of nanoparticles.

4.4. Heat Transfer Characteristics To reveal the details of heat transfer enhancement by the addition of nanoparticles to the base fluid at different Richardson numbers, the values of average Nusselt number for different volume fractions and Richardson numbers are plotted in Figure 7.

It is observed that the presence of nanoparticles in the base fluid at low Richardson numbers makes better heat transfer enhancement in comparison to the high Richardson numbers.

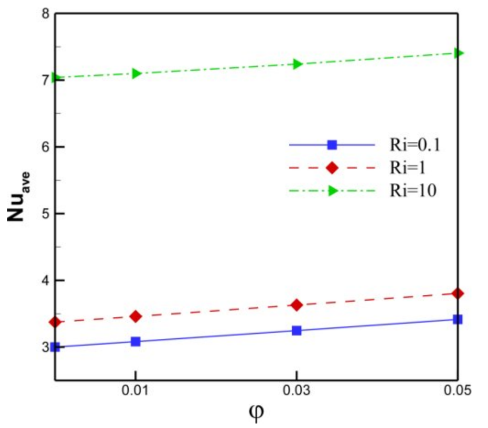


Figure 7. Average Nusselt number number at different Ri and φ ($A = 0.15$ and $\Lambda = 0$).

The increase of the conduction heat transfer by the addition of conductive nanoparticles to the base fluid is one of the main reasons of heat transfer enhancement of the base fluid which is more effective for low Richardson numbers that forced convection is dominant. Adding the 0.05 volume fraction of Al_2O_3 nanoparticles at $Ri = 0.1$ enhances the average Nusselt number about 14%, 12% and 5% at $Ri = 0.1$, $Ri = 1$ and $Ri = 10$, respectively, when $A = 0.15$ and $\Lambda = 0$.

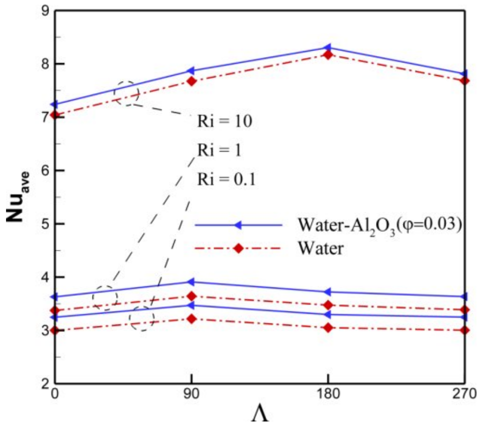


Figure 8. Average Nusselt number at different Ri and Λ ($\varphi = 0.03$ and $A = 0.15$).

To investigate the role of wavy surface shapes in mixed convection of nanofluids, phase shift (Λ) of the wavy wall is tested for different values from 0 to 270. The base shape is a sinusoidal form that is assumed as $\Lambda = 0$ (see Eq.20).

The effect of wall phase shift on average Nusselt number for different fluids Richardson numbers is exhibited in Figure 8. In all cases, it is visible that wavy wall with $\Lambda = 90$ has maximum value of Nu_{ave} at low Richardson numbers ($Ri \leq 1$). On other hand, the best value of Nu_{ave} in comparison to other phase shifts is achieved for a wavy wall with $\Lambda = 180$ at high Richardson number($Ri = 10$).

In high Richardson numbers, the momentum value of fluid flow decreases by an increase of the curve amplitude (Figure 9). Heat transfer decreases by increasing the amplitude of the wavy wall at high Richardson number (Figure 10).

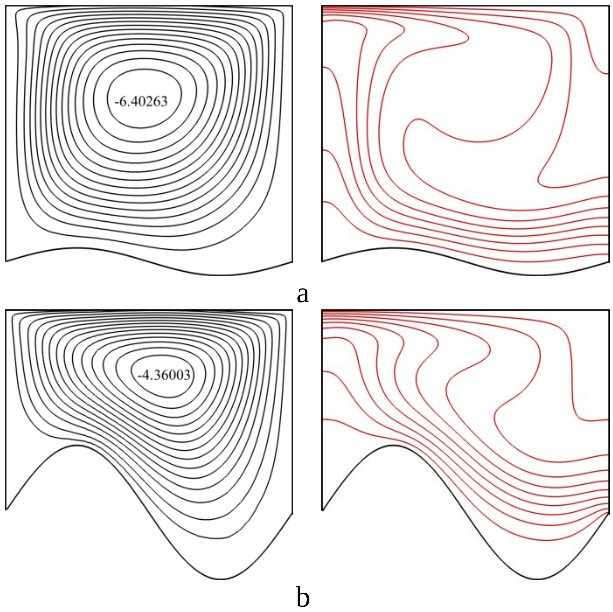


Figure 9. Streamlines and temperature contours at $Ri = 10$, $\varphi = 0.01$ and $\Lambda = 0$.
a) $A = 0.05$ b) $A = 0.25$.

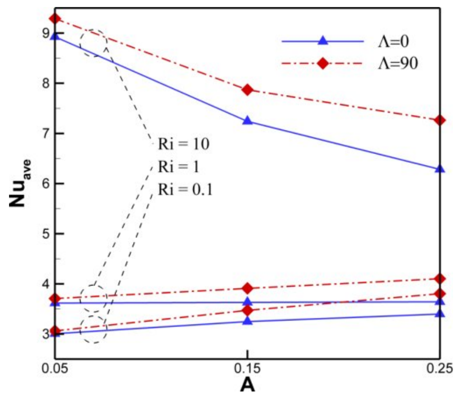
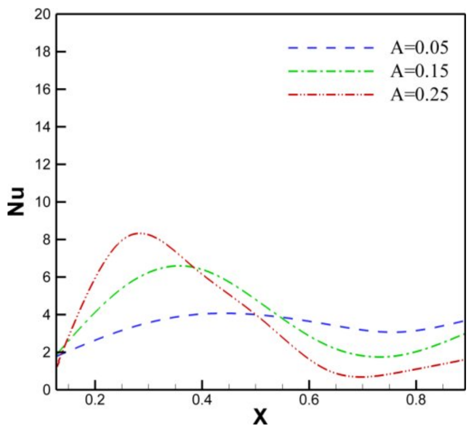


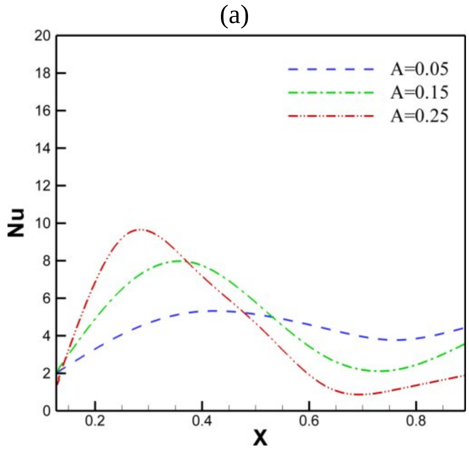
Figure 10. Average Nusselt number at different A, Ri and Λ for $\varphi = 0.03$.

To delineate the details of heat transfer reduction by increasing the curve amplitude in high Richardson numbers, the Nusselt number distribution along the wavy wall is presented in Figure 11 for different Richardson numbers.

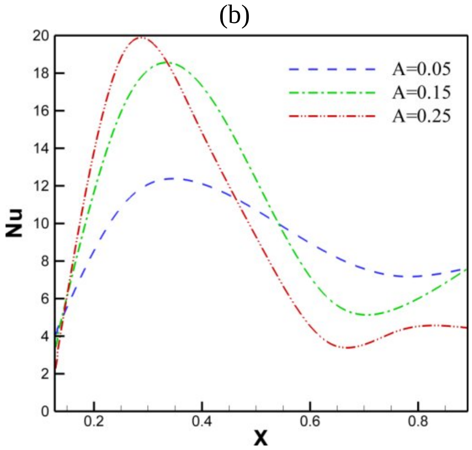
As can be seen in Figure 9, the isotherm lines are more compact at near of the wavy wall peak and lose their compactness at the dip of wavy wall. This trend of isotherm lines causes that the Nusselt number distribution be similar to wavy wall pattern (Figure 10). The increases of average Nusselt number by adding 0.05 volume fraction of the nanoparticles are about 2%, 5%, and 6% when A is equal to 0.05, 0.15, 0.25, respectively at $Ri = 10$ and $\Lambda = 0$. It is found that the best effect of adding different nanoparticles in enhancement of heat transfer rate of the base fluid occurs in $A = 0.25$.



(a)



(b)



(c)

Figure 11. Nusselt number distribution at different A ($\varphi = 0.03$ and $\Lambda = 0$).
a) $Ri = 0.1$ b) $Ri = 1$ c) $Ri = 10$.

5. CONCLUSION

The Boundary Fitting Method in the LBM has been used to simulate fluid flow and heat transfer of different Water-based Al_2O_3 nanofluid in a Lid-driven cavity with a corrugated wall. This study emphasizes on authority of LBM to model fluid flow and convection heat transfer problems in complex geometries. The effect of volume fraction of nanoparticles on mixed convection is investigated for different Richardson numbers, curve amplitudes and phase shifts of wavy wall in the range of 0.1-10, 0.05-0.25 and 0-270, respectively. Some of the most important results that have been achieved in this study are as follows:

1. Increase of nanoparticles volume fraction enhances the average Nusselt number of the suspension for different Richardson numbers.
2. Richardson number plays an important role in mixed convection of nanofluids from wavy surfaces.
3. Heat transfer enhancement by adding nanoparticles is more visible for low Richardson number in comparison to high Richardson number.
4. The increase of curve amplitude decreases the average Nusselt number when natural convection is dominant ($Ri = 10$), while this increase causes the increase of Nu_{ave} at low Richardson numbers ($Ri \leq 1$).
5. The maximum rate of heat transfer of nanofluid is observed for $\Lambda = 90$ at low Richardson numbers and $\Lambda = 180$ at high Richardson numbers.

Nomenclature

A	Amplitude of wavy wall
c	Discrete lattice velocity
c_s	Speed of sound in Lattice scale
c_p	Specific heat at constant pressure $(kj.kg^{-1}.k^{-1})$
F	External force in direction of lattice velocity
f , g	Distribution function
g_y	Acceleration due to gravity $(m.s^{-2})$
Gr	Grashof number $(g.\beta.(T_h - T_c).L^3.\nu^{-2})$

k	Thermal conductivity $(W.m^{-1}.k^{-1})$
L	Characteristic length (m)
Nu	Nusselt number
Pr	Prandtl number $(\nu.\alpha^{-1})$
Re	Reynolds number $(u_0.L.\nu^{-1})$
Ri	Richardson number $(Gr.Re^{-2})$
T	Dimensionless temperature $\left(T=(\theta-\theta_c)/(\theta_h-\theta_c)\right)$
u_0	Velocity of lid $(m.s^{-1})$
X,Y	Dimensionless coordinate

Greek symbols

α	Discrete lattice direction
β	Thermal expansion coefficient $(1.k^{-1})$
Δt	Lattice time step
θ	Non-dimensionless temperature (k)
Λ	Phase shift of wavy wall
ρ	Density $(kg.m^{-3})$
τ	Lattice relaxation time
φ	Solid volume fraction
ψ	Dimensionless stream-function
w	Weighting factor

Subscripts

ave	Average
c	Cold
eq	Equilibrium
f	Fluid
h	Hot
nf	Nanofluid
s	Solid particle

6. REFERENCES

1. Choi, U.S., "Enhancing thermal conductivity of fluids with nanoparticles developments and application of non-Newtonian flows", ***ASME Journal of Heat Transfer***, Vol. 66, (1995), 99-105.
2. Khanafer, K., Vafai, K. and Lightstone, M., "Buoyancy-driven heat transfer enhancement in a two-dimensional enclosure utilizing nanofluids", ***International Journal of Heat and Mass Transfer***, Vol. 46, (2003), 3639-3653.
3. Gherasim, I., Roy, G., Nguyen, C.T. and Vo-Ngoc, D., "Experimental investigation of nanofluids in confined laminar radial flows", ***International Journal of Thermal Science***, Vol. 48, (2009), 1486-1493.
4. Heyhat, M.M. and Kowsary, F., "Numerical Simulation of Forced Convection of Nanofluids by a Two-Component Non-homogeneous Model", ***International Journal of Engineering Transactions A: Basics***, Vol. 23, (2010), 89-99.
5. Ghaffari, O., Behzadmehr, A. and Ajam, H., "Turbulent mixed convection of a nanofluid in a horizontal curved tube using a two-phase approach", ***International Communications in Heat and Mass Transfer***, Vol.37, (2010), 1551-1558.
6. Sheikhzadeh, G.A, Arefmanesh, A., Kheirkhah, M.H. and Abdollahi, R., "Numerical Study of Natural Convection in an Inclined Cavity with Partially Active Side Walls Filled With Cu-Water Nanofluid", ***International Journal of Engineering Transactions B: Application***, Vol. 24, (2011), 279-292.
7. Abu-Nada, E., Masoud, Z. and Hijazi, A., "Natural convection heat transfer enhancement in horizontal concentric annuli using nanofluids", ***International Communications in Heat and Mass Transfer***, Vol. 35, (2008), 657-665.
8. Nnanna, A.G.A. and Routhu, M., "Transport phenomena in buoyancy-driven nanofluids - Part II", ***ASME Heat Transfer Conference***, San Francisco, California, USA , (July. 17-22, 2005), 2005.
9. Santra, A.K., Sen, S. and Chakraborty, N., "Study of heat transfer augmentation in a differentially heated square cavity using copper-water nanofluid", ***International Journal of Thermal Science***, Vol. 47, (2008), 1113-1122.
10. Putra, N., Roetzel, W. and Das, S.K., "Natural convection of nanofluids", ***Heat Mass Transfer***, Vol. 39, (2003), 775-784.
11. Mohamad, A., "Applied Lattice Boltzmann Method for Transport Phenomena, Momentum, Heat and Mass Transfer", ***Sure***, Calgary, (2007).
12. Fattahi, E., Farhadi, M. and Sedighi, K., "Lattice Boltzmann simulation of natural convection heat transfer in eccentric annulus", ***International Journal of Thermal Science***, Vol. 49, (2010) 2353-2362.
13. Gao, D. and Chen, Z., "Lattice Boltzmann simulation of natural convection dominated melting in a rectangular cavity filled with porous media", ***International Journal of Thermal Science***, Vol.50, (2011), 493-501.
14. Liu, H., Zhou, J.G. and Burrows, R., "Numerical modeling of turbulent compound channel flow using the lattice Boltzmann method", ***International Journal for Numerical Methods in Fluids***, Vol. 59, (2009), 753-765.
15. Aghajani Delavar, M., Farhadi, M. and Sedighi, K., "Numerical simulation of direct methanol fuel cells using lattice Boltzmann method", ***International journal of hydrogen energy***, Vol. 35, (2010), 9306-9317.
16. Jahanshahi, E., Gandjalikhan Nassab, S.A. and Jafari, S., "Numerical Simulation of a Three-Layered Radiant Porous Heat Exchanger Including Lattice Boltzmann Simulation of Fluid Flow", ***International Journal of Engineering Transactions A: Basics***, Vol. 24, (2011), 301-319.
17. Xuan, Y. and Yao, Z., "Lattice Boltzmann model for nanofluids", ***Heat Mass Transfer***, Vol. 41, (2005), 199-205.
18. Nemati, H., Farhadi, M., Sedighi, K. and Fattahi, E., "Lattice Boltzmann simulation of nanofluid in lid-driven cavity", ***International Communications in Heat and Mass Transfer*** , Vol. 37, (2010) 1528-1534.
19. Filippova, O. and Hänel, D., "Boundary fitting and local grid refinement for lattice-BGK models", ***International Journal of Modern Physics C***, Vol. 9, (1998), 1271-1279.
20. Mei, R., Luo, L.S. and Shyy, W., "An Accurate Curved

Boundary Treatment in the Lattice Boltzmann Method", ***Journal of Computational Physics***, Vol. 155, (1999), 307-330.
21. Guo, Z.L., Zheng, C. and Shi, B.C., "An extrapolation method for boundary conditions in lattice Boltzmann method", ***Physics of Fluids***, Vol. 14, (2002), 2007-2010.
22. Chen, S., Martinez, D. and Mei, R., "On boundary conditions in lattice Boltzmann methods", ***Physics of Fluids***, Vol. 8, (1996), 2517–2536.
23. Al-Amiri, A., Khanafer, K., Bull, J. and Pop, I., "Effect of sinusoidal wavy bottom surface on mixed convection heat transfer in a lid-driven cavity", ***International Journal of Heat and Mass Transfer***, Vol. 50, (2007), 1771-1780.
24. Al-Zoubi, A. and Brenner, G., "Simulating fluid flow over sinusoidal surfaces using the lattice Boltzmann method", ***Computers and Mathematics with Applications*** , Vol. 55, (2008), 1365-1376.
25. Heidary, H. and Kermani, M.J., "Effect of nanoparticles on forced convection in sinusoidal-wall channel", ***International Communications in Heat and Mass Transfer***, Vol. 37, (2010), 1520-1527.
26. Saboonchi, A. and Mohammadi Z.M., "Thermal Contact Resistance of Wavy Surfaces", ***International Journal of Engineering Transactions A: Basics***, Vol. 17, (2004), 293-300.
27. Arefmanesh, A., Najafi, M. and Nikfar, M., "MLPG application of nanofluid flow mixed convection heat transfer in a wavy wall cavity", ***Computer Modeling in Engineering and Sciences***, Vol. 69, 2010, 91-117.
28. Wang, X.Q. and Mujumdar, A.S., "Heat transfer characteristics of nanofluids: a review", ***International Journal of Thermal Science***, Vol. 46, (2007), 1-19.
29. Patel, H.E., Pradeep, T., Sundararajan, T., Dasgupta, A., Dasgupta, N. and Das, S.K., "A micro convection model for thermal conductivity of nanofluid", ***Pramana-Journal of Physics***, Vol. 65, (2005), 863-869