

# EFFECT OF LOCATION OF EXPLANATORY VARIABLE ON MONITORING POLYNOMIAL QUALITY PROFILES

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**Abstract** The quality is typically modeled as the univariate or multivariate distribution of quality characteristic/s. In recent applications of statistical process control, quality profiles in which the relationship between a response and explanatory variable/s is captured and monitored are increasingly used to model the quality. Several techniques have been developed to enhance the speed of detecting changes in parameters of polynomial profiles. In this paper, we consider the effect of allocating the explanatory variable X in optimizing the performance of one of the well-known methods referred to as EWMA4 method. An optimization model is built and solved using the genetic algorithm to find the optimal location of X- values that minimizes the average of the run length distribution (ARL). The effect of location optimization is studied using a simulation study, and the results are compared with non-optimized strategies in terms of average run length criterion.

**Keywords** Average run length; Polynomial profiles; Explanatory variable; EWMA4 method.

**چکیده** کیفیت کالا و خدمات عموماً با استفاده از توزیع‌های یک یا چند متغیره مشخصه‌های کیفی مدل‌سازی میشود. با این وجود، در کاربردهای نوین کنترل فرایند آماری، به‌طور فزاینده‌ای از پروفایل‌های کیفی برای مدل‌سازی و پایش رابطه بین متغیرهای مستقل و وابسته استفاده می‌شود. در رابطه با پروفایل‌های چند متغیره که از پرکاربردترین پروفایل‌های کیفی می‌باشند، روش‌های متعددی برای بهبود سرعت کشف تغییرات ارائه شده است. در این تحقیق، تأثیر موقعیت مکانی متغیرهای مستقل در بهبود عملکرد نمودار EWMA4، که از متدوال‌ترین نمودارهای کنترل پروفایل‌های چند متغیره می‌باشند، مورد بررسی قرار گرفته است. در این یک رابطه مدل بهینه‌سازی برای تعیین مکان بهینه متغیرهای مستقل که منجر به کمینه شدن طول دنباله می‌شود طراحی و با استفاده از الگوریتم ژنتیک حل شده است. همچنین با استفاده از شبیه‌سازی آماری عملکرد روش پیشنهادی با سایر روش‌های موجود مقایسه گردیده است.

## 1. INTRODUCTION

Over the last decade, several charting techniques have been developed and examined in terms of their effectiveness in detecting changes in process parameters when the quality function might be explained by a simple linear model. The applications of profile monitoring in statistical process control (SPC) studies have been extended to cover both phases I and II. In phase I, practitioners are usually interested in investigating the statistical stability of process parameters and

estimating their nominal values while phase II is devoted to detecting any changes in the parameters which are assumed to be known. Areas for future research on profile monitoring topic can be found in Woodall et al. [1].

Several studies have relied on practical applications in examining their suggested methods for detecting anticipated changes in process or product quality level. Mestek et al. [2] studied the stability of calibration curves in the photometric determination of  $Fe^{3+}$  with sulfosalicylic acid. Stover and Brill [3] considered the linear

calibration of the multilevel ion chromatography and showed how to determine the instrument response stability and the most proper calibration frequency. Kang and Albin [4] presented two other different examples. One of these is an illustrative example where the relationship between the amount of dissolved artificial sweetener aspartame and the temperature levels is better explained by a nonlinear model. In the other one, they used a simple linear model to describe the semiconductor manufacturing process. An example of polynomial profiles is considered by Montgomery [5]. In this work, he used a second-order polynomial profile to describe the relation between the automobile engine speed and the torque produced by an engine. Another application of polynomial profile is presented in Amiri et al. [6].

The majority of the previous researches gave a considerable attention to phase II studies while literatures considering phase I are not of that abundance. Mestek et al. [2] suggested a  $T^2$  control chart to study the stability of linear calibration curves in the photometric determination of  $\text{Fe}^{3+}$  with sulfosalicylic acid. Two phase I strategies for determining instrument response stability were presented and examined by Stover and Brill [3]. These two monitoring strategies were applied to multilevel ion chromatography. Mahmoud and Woodall [7] suggested a phase I method using global F-test to observe the model parameters and a Shewhart control chart to monitor changes in process variability. Another phase I approach based on the segmented technique for testing the constancy of the regression parameters in stage I for linear profile data has been investigated by Mahmoud et al. [8]. This method is referred to as a change point approach.

Kang and Albin [4] suggested two phase II methods for monitoring simple linear profiles. The first method is a multivariate Hotelling  $T^2$  chart based on successive vectors of the least squares estimators of the intercept and slope. The second one is an exponentially weighted moving average control chart (EWMA) in conjunction with R control chart; they referred to this method as EWMA/R. Kim et al. [9] suggested the use of three separate EWMA control charts (EWMA3); in order to remove the correlation between regression parameters, the average of the explanatory variable is set to zero. Phase II comparative study between

the National Institute of Standard and Technology (NIST) method proposed by Croarkin and Varner [10] and the KMW method- the letters K, M and W refer to the initials of the authors, suggested by Kim et al. [9] was performed by Gupta et al. [11]. In this work, they replaced the three EWMA control charts of KMW method by three separate Shewhart control charts. Niaki et al. [12] proposed a control chart based on the generalized linear test (GLT) to monitor coefficients of the linear profiles and an R-chart to monitor the error variance, the combination of which is called GLT/R chart. Zou et al. [13] considered the case when the number of available samples is not sufficiently large for proper estimation of simple linear function parameters; this approach is referred to as a self-starting approach. An approach using a single chart integrating likelihood ratio statistic with the EWMA procedure for monitoring linear profiles was developed by Zhang et al. [14]. For further performance improvement, they added the variable sampling interval (VSI) feature to their suggested technique. The speed of detecting changes in regression parameters of simple linear profiles was investigated using a cumulative sum statistic; the results of this study were presented by Saghaei et al. [15]. Zhu and Lin [16] proposed new approach for observing changes in the slope of the simple linear functions.

Another recent contribution by Noorossana et al. [17] explored the performance of three control chart schemes when several correlated characteristics might be modeled as a set of linear functions of one independent variable. They referred to this situation as multivariate simple linear profiles structure. Li and Wang [18] suggested the use of an exponentially weighted moving average control chart with variable sampling intervals for monitoring linear profiles (VSI-EWMA3). They examined the performance of this strategy using a real set of data from an optical imaging system.

Sometimes, more sophisticated models such as polynomial, nonlinear, or multiple linear profiles rather than the simple linear profiles better describe the quality. Three different phase I methods for monitoring polynomial profiles were examined by Kazemzadeh et al. [19]; they also provided an approach based on likelihood ratio test to identify the location of the shift.

Kazemzadeh et al. [20] considered the second-order polynomial profiles and introduced a new technique based on the idea of transforming the polynomial model to the orthogonal form. Then, the three regression parameters will be independent, and one can use three individual EWMA control charts in conjunction with another EWMA chart for monitoring the residuals. Ding et al. [21] presented a phase I technique for monitoring nonlinear profiles. This strategy consists of two main components: a data-reduction component and a data-separation technique that can detect both single and multiple shifts. Williams et al. [22] suggested and investigated the use of the  $T^2$  chart to monitor the coefficients of a parametric nonlinear regression model. Chicken et al. [23] developed and tested a semi-parametric wavelet method for monitoring changes in nonlinear profiles. Zhang and Albin [24] developed a method that deals with the quality profiles as vectors and applies a  $\chi^2$  control chart to determine outliers. Zou et al. [25] proposed a novel multivariate exponentially weighted moving average control chart (MEWMA) for monitoring general linear profiles. Mahmoud [26] introduced a phase I approach for monitoring multiple linear profiles. Jensen et al. [27] considered the correlation structure between linear profiles and investigated the effectiveness of a new technique accounting this issue.

Some other researchers have shown interest in studying the impact of violating some of the basic assumptions in modeling the corresponding process. For example, Noorossana et al. [28] considered the effect of not fulfilling the normality assumption. Noorossana et al. [29] studied the case when quality profiles are not independent of each other. Soleimani et al. [30] presented an analytical study to examine how the speed of catching changes in the regression parameters of simple linear profiles is influenced by autocorrelation. The effect of correlation on nonlinear profiles using nonlinear mixed models can be found in Jensen and Birch [31].

This work aims to investigate and examine the effect of locating the explanatory variable in enhancing the statistical performance of EWMA4 method, developed by Kazemzadeh et al. [20], for monitoring changes in  $k$ -order polynomial quality functions. This paper is organized as follows:

Section 2 discusses the orthogonal polynomial method (EWMA4). Section 3 describes the procedure used to find the orthogonal polynomials when the values of the explanatory variable are unequally spaced; an illustrative example is presented. In Section 4, we describe the relation between the coefficients of the original and the orthogonal polynomial forms. Our optimization model and a comparative study between a regular plan and a set of optimized plans are considered in Section 5. Conclusions and recommendation are reported in Section 6.

## 2. ORTHOGONAL POLYNOMIALS METHOD

This is a phase II method proposed by Kazemzadeh et al. [20]. It is based on the use of orthogonal polynomial regression to monitor the polynomial quality functions. Since this method uses  $k + 2$  control charts where  $k$  is the order of the polynomial function, it is not recommended to be used with large  $k$ 's.

Under the state of the statistical control, the relationship between the outgoing quality (Y) and the process input (X) is described by Equation (1).

$$y_{ij} = A_0 + A_1x_i + A_2x_i^2 + \dots + A_kx_i^k + \varepsilon_{ij} \quad (1)$$

$$i = 1,2,3,\dots,n \quad j = 1,2,3,\dots$$

The nominal values of process parameters are represented by  $A_0, A_1, A_2, \dots, A_k$  and  $\varepsilon_{ij} \sim N(\mathbf{0}, \sigma^2)$ . The following is the transformed model:

$$y_{ij} = B_0P_0(x_i) + B_1P_1(x_i) + B_2P_2(x_i) + \dots + B_kP_k(x_i) + \varepsilon_{ij}$$

$$i = 1,2,3,\dots,n \quad j = 1,2,3,\dots \quad (2)$$

The value  $B_l$  represents the coefficient of the  $l^{\text{th}}$  order orthogonal polynomial  $P_l(x_i)$ . This technique uses Equation 3 to calculate the least square estimators of  $B_0, B_1, \dots, B_k$ .

Kazemzadeh et al. [20] reported that these estimators are normally distributed such that  $E(\hat{B}_{lj}) = B_l$  and  $Var(\hat{B}_{lj}) = \sigma^2 / \sum_{i=1}^n P_l^2(x_i)$ ; where  $j$  is the sequential number of the profile and  $l$  is the order orthogonal polynomial ( $l = 1,2,3, \dots, k$ ).

$$\hat{B}_{lj} = \frac{\sum_{i=1}^n P_l(x_i) y_{ij}}{\sum_{i=1}^n P_l^2(x_i)} \quad (3)$$

$$j = 1, 2, 3, \dots \quad l = 0, 1, 2, \dots, k$$

For monitoring the changes in the coefficients of the regression model, three individual EWMA control charts are proposed. The EWMA statistic and the control limits can be calculated as follows:

$$EWMA_l(j) = \theta \hat{B}_{lj} + (1 - \theta) EWMA_l(j - 1) \quad (4)$$

$$UCL_l = B_l + K_l \sqrt{\frac{\theta \sigma^2}{(2 - \theta) \sum_{i=1}^n P_l^2(x_i)}} \quad (5)$$

$$LCL_l = B_l - K_l \sqrt{\frac{\theta \sigma^2}{(2 - \theta) \sum_{i=1}^n P_l^2(x_i)}} \quad (6)$$

$$j = 1, 2, 3, \dots \quad l = 0, 1, 2, \dots, k$$

where,  $EWMA_l(\mathbf{0})$  is set equal to  $B_l$ , and the smoothing constant ( $\mathbf{0} < \theta \leq \mathbf{1}$ ) and the multiplier ( $k_l > \mathbf{0}$ ) are selected to give a specific expected value of the in-control average run length (ARL). The fourth chart of EWMA4 is a one-side EWMA control chart used to monitor the variability in the process. The  $EWMA_E$  statistic and the control limit are as follows:

$$EWMA_E(j) = \text{Max}\{\theta(MSE_j - 1) + (1 - \theta)EWMA_E(j - 1), \mathbf{0}\} \quad (7)$$

$$UCL_E = K_E \sqrt{\frac{\theta \text{Var}(MSE_j)}{(2 - \theta)}} \quad j = 1, 2, 3, \dots \quad (8)$$

### 3. CONSTRUCTING ORTHOGONAL POLYNOMIALS

The orthogonal polynomials for a set of equally spaced independent variables are usually provided in references that cover the regression topic, e.g. Montgomery [5].

When it comes to the unequally spaced  $X$ -values, one should use one of several techniques developed for this purpose. For example, Kendall [32] studied the analysis of equally spaced  $X$ -values as a special case of the general usage of

orthogonal polynomials. Another method for finding these polynomials can be found in Robson [33]. In this work, he introduced a simple recursive method to find appropriate orthogonal polynomial. Grandage [34] presented a recursive procedure that requires the solution of a set of linear equations. The motivation behind the use of Robson's technique in this paper is the recommendations reported in an analytical and comparative research in using the polynomial orthogonal curves in Dental Healthcare Industry developed by Toby et al. [35].

In this section, we will briefly state the outlines of Robson's method followed by an illustrative example.

**3.1. Using Robson's Approach** Assume that Equations 1 and 2 are the best models describing the relationship between the quality response ( $Y$ ) and the unequally spaced explanatory variable ( $X$ ).

Using the Robson's technique, the function  $P_l(x_i)$  can be calculated from the relation; see [33]:

$$P_l(x_i) = \frac{1}{c_l} \left[ x_i^l - \sum_{v=0}^{l-1} P_v(x_i) \sum_{i=1}^n x_i^l P_v(x_i) \right] \quad (9)$$

where,  $c_l$  represents the normalizing factor (constant), and it can be calculated as follows; see [33]:

$$c_l = \left\{ \sum_{i=1}^n \left[ x_i^l - \sum_{v=0}^{l-1} P_v(x_i) \sum_{i=1}^n x_i^l P_v(x_i) \right]^2 \right\}^{\frac{1}{2}} \quad (10)$$

**3.2. An Illustrative Example** This example shows the procedures for constructing the orthogonal polynomials when the relationship between  $Y$  and  $X$  is described by a second order polynomial function. Let us assume that the original form is:

$$Y_{ij} = 3 + 2x_i + x_i^2 + \varepsilon_{ij} \quad (11)$$

where, the fixed values of the independent variable ( $X$ ) are 1, 2, 4 and 7. Then,

$$\text{For } l = 0, c_0 = \sqrt{n} = \sqrt{4} \text{ and } P_0(x_i) = 1/\sqrt{n} = 1/\sqrt{4}$$

hence,

$$c_0 P_0(x_i) = 1 \quad (12)$$

For  $l = 1$ ,  $c_1 = \sqrt{\sum_{i=1}^4 (x_i - 3.5)^2}$

and

$$P_1(x_i) = \frac{x_i - 3.5}{\sqrt{\sum_{i=1}^4 (x_i - 3.5)^2}} \quad (13)$$

then,

$$c_1 P_1(x_i) = x_i - 3.5 \quad (14)$$

For  $l = 2$ ,

$$c_2 P_2(x_i) = x_i^2 - \frac{1}{4} \sum_{i=1}^4 x_i^2 - (x_i - 3.5) \frac{\sum_{i=1}^4 x_i^2 (x_i - 3.5)}{\sum_{i=1}^4 (x_i - 3.5)^2}$$

Table 1 shows the calculations for finding the values of the orthogonal polynomials using Robson's technique when  $X$ -values are unequally spaced.

**TABLE 1.** Orthogonal polynomials computation for  $X=\{1,2,4,7\}$

$x_i$	1	2	3
	$c_0 P_0(x_i)$	$c_1 P_1(x_i)$	$c_2 P_2(x_i)$
1	1	-10/4	1-70/4+5(342)/84=1296/336
2	1	-6/4	4-70/4+3(342)/84= -432/336
4	1	2/4	16-70/4-1(342)/84= -1872/336
7	1	14/4	49-70/4-7(342)/84=1008/336

The common factors within each column of  $c_i P_i(x_i)$  should be removed to get integer values. For simplicity, the integer values of column 2 can be found by multiplying each element in this column ( $c_1 P_1(x_i)$ ) by the sample size ( $n$ ) and refer to it as  $c_1' P_1'(x_i)$ . Column 3 can be converted to integer values through two or more steps as follows:

**Step 1:** Start by multiplying all elements in column 3 ( $c_2 P_2(x_i)$ ) by the sum of squares of the integer values of column 2 ( $\sum_{i=1}^n (c_1 P_1(x_i))^2$ ).

**Step 2:** Round off the results of step 1 to the closest integer values.

**Step 3:** Divide all resulted elements by the absolute value of the minimum value in this column. If all values are integers then stop; other else go to step 4.

**Step 4:** Start searching for common factors within the new  $c_2 P_2(x_i)$  values.

**Step 5:** Divide  $c_2 P_2(x_i)$  values by the common factor and rename this column as  $c_2' P_2'(x_i)$ .

The results of these calculations are shown in Table 2. Another example for unequally spaced  $X$ -values case can be found in Robson [33].

**TABLE 2.** Final values of orthogonal polynomials

$i$	$X_i$	$c_0' P_0'(x_i)$	$c_1' P_1'(x_i)$	$c_2' P_2'(x_i)$
1	1	1	-5	9
2	2	1	-3	-3
3	4	1	1	-13
4	7	1	7	7

The orthogonal polynomials of order 3, the  $c_3 P_3(x_i)$  values can be determined from  $c_0 P_0(x_i)$ ,  $c_1 P_1(x_i)$ , and  $c_2 P_2(x_i)$  as follows:

$$c_3 P_3(x_i) = x_i^3 - P_0(x_i) \sum_{i=1}^n x_i^3 P_0(x_i) - P_1(x_i) \sum_{i=1}^n x_i^3 P_1(x_i) - P_2(x_i) \sum_{i=1}^n x_i^3 P_2(x_i) \quad (16)$$

#### 4. ANALYZING ORTHOGONAL POLYNOMIAL MODELS

Some of the linear regression works such as Montgomery [5] considered the orthogonal transformation and provided the orthogonal polynomials for a set of equally spaced  $X$ -values. In this section, we present these orthogonal polynomials and describe their relation with the regression coefficients of the original model. The orthogonal polynomials shown in Equation 2 can be written as follows; (see Montgomery [5] and Kazemzadeh et al. [20]):

$$P_0(x) = 1 \quad (17)$$

$$P_1(x) = \frac{\varphi_1}{d} (x - \bar{x}) \quad (18)$$

$$P_2(x) = \varphi_2 \left[ \left( \frac{x - \bar{x}}{S} \right)^2 - \left( \frac{n^2 - 1}{12} \right) \right] \quad (19)$$

where,  $\varphi_j$  is a constant, and we choose its value to have integer polynomials, and  $S$  represents the distance between the  $X$ -values. We estimate the regression parameters in the transformed model as follows; see Montgomery [5] and Kazemzadeh et al. [20]:

$$B_0 = A_0 + \bar{x}A_1 + \left[ \bar{x}^2 + \left( \frac{n^2 - 1}{12} \right) S^2 \right] A_2 \quad (20)$$

$$B_1 = \frac{S}{\varphi_1} (A_1 + 2A_2\bar{x}) \quad (21)$$

$$B_2 = \frac{A_2 S^2}{\varphi_2} \quad (22)$$

Now, we analyze these relationships in order to see how the parameters of the orthogonal model are affected by the shift in the original regression coefficients; refer to Kazemzadeh et al. [20]. Table 3 summarizes these relations.

**TABLE 3.** Changes in orthogonal polynomial models

Original coefficients	Changes in the orthogonal form		
	$B_0$	$B_1$	$B_2$
$A_0$	$\lambda\sigma$	-	-
$A_1$	$\bar{x}\beta\sigma$	$\frac{S\bar{x}\beta\sigma}{\varphi_1}$	-
$A_2$	$\delta\sigma \left( \bar{x}^2 + \left( \frac{n^2 - 1}{12} \right) S^2 \right)$	$2\bar{x}\delta\sigma$	$\frac{S^2\delta\sigma}{\varphi_2}$

Table 3 shows that the amount of change in regression coefficients in orthogonal model is affected by some of design parameters such as  $\bar{x}$ ,  $n$  and  $S$ . In this paper, we only consider the effect of location represented by  $\bar{x}$  and  $S$ . Our approach adopts the idea of allocating the  $X$ -values in positions such that the shift in orthogonal parameters is maximized.

## 5. OPTIMIZING THE LOCATION OF X'S USING GA APPROACH

The genetic approach (GA) is widely used in stochastic optimization technique. Several authors have utilized this technique in SPC applications to find the optimal design parameters or examine the performance of several charting techniques (e.g.  $\bar{X}$ , Hotelling  $T^2$ , and EWMA control charts).

**5.1. Setting the Parameters of the Genetic Approach** In this paper, the MATLAB software is used for coding and running the genetic approach. Since the quality of the genetic approach output is affected by the values of its parameters, two Taguchi experiments were conducted to determine

the optimal setting of the GA: 1- population size (PS), 2- crossover probability (CP), and 3- mutation rate (MR).

These inputs along with the amount of shift in the first and the second parameters ( $A_1, A_2$ ) were analyzed in L9 ( $3^4$ ) orthogonal arrays. The settings of these experiments are shown in Tables 4 and 5.

**TABLE 4.** L9 design ( $3^4$ ) when  $A_1$  shifts to  $A_1 + \beta\sigma$

Run	PS	CP	MR	$\beta$
1	50	0.1	0.05	0.01
2	50	0.3	0.10	0.05
3	50	0.5	0.25	1.00
4	75	0.1	0.10	1.00
5	75	0.3	0.25	0.01
6	75	0.5	0.05	0.05
7	100	0.1	0.25	0.05
8	100	0.3	0.05	1.00
9	100	0.5	0.10	0.01

**TABLE 5.** L9 ( $3^4$ ) design when  $A_2$  shifts to  $A_2 + \delta\sigma$

Run	PS	CP	MR	$\delta$
1	50	0.1	0.05	0.001
2	50	0.3	0.10	0.005
3	50	0.5	0.25	0.100
4	75	0.1	0.10	0.100
5	75	0.3	0.25	0.001
6	75	0.5	0.05	0.005
7	100	0.1	0.25	0.005
8	100	0.3	0.05	0.100
9	100	0.5	0.10	0.001

After the experiments, the optimal levels of the three parameters of the genetic approach were determined and presented in Table 6, which were the same for the shift in the second and the third parameters of the polynomial model.

**TABLE 6.** Optimal setting of GA approach

Shift	PS	CP	MR
$\beta$	100	0.5	0.1
$\delta$	100	0.5	0.1

In this article, the GA approach is employed to specify the optimal location of a set of  $n$  explanatory variables such that the off-target average run length (ARL) is minimized. The optimization model is as follows:

$$\text{Min } ARL_{d \neq 0}(d, n) \quad (23)$$

subject to:

$$x_1 \geq ll \quad (24)$$

$$x_n \leq ul \quad (25)$$

$$k + 1 \leq g \leq n \quad x \in Z^{+n} \quad (26)$$

where,  $d$  is the amount of shift in the orthogonal least square estimators ( $B_0, B_1, \dots, B_k$ ),  $l$  is the order of the orthogonal polynomial ( $l = 0, 1, \dots, k$ ),  $g$  represents the number of locations of the independent variable, and  $ll$  and  $ul$  are the lower and upper limits of the practical range of  $X$ . Since  $\sum_{i=1}^n p_{i0} = n$ , the shift in  $B_0$  is not sensitive to the location of  $X$ -values. This paper only considers the shift in  $B_1, B_2 \dots B_k$ .

### 5.2 Running the optimization model and estimating ARL

To study the effect of location on the statistical performance of EWMA4 method while not losing generality, the model in Equation 17 is considered.

The four control charts of EWMA4 technique were designed to have the over all in-control ARL

of 200. The smoothing factor ( $\theta$ ) of EWMA charts is set equal to 0.2. The multipliers of EWMA charts of least square estimators ( $K_L$ ) and EWMA chart of residuals ( $K_E$ ) are set equal to 3.105 and 3.595, respectively.

In order to better display the effect of location, a simulation study with 50,000 runs was conducted to estimate out-of-control ARL values of optimization model output under different shifts in  $A_1$  and  $A_2$ . Table 7 shows the off-target ARL values of two types of sampling strategies. The first type is designed with regular location of  $X$ -values and the second is sampling plans optimized in terms of the location of  $X$ . Under the shift in the first parameter ( $A_1$ ) from  $A_1$  to  $A_1 + bs$  and at any level of shifts considered, there is a set of optimized plans performing much better than the regular one. As it is shown in the reduction column, optimizing the location of  $X$ -values will lead to a high significant reduction in sampling efforts and costs.

**TABLE 7.** ARL comparisons between regular and optimized plans under shift in second parameter from  $A_1$  to  $A_1 + bs$

Shift	ARL (sample)			Reduction (%)	Independent Variable									
	Regular	Optimized			$X_1$	$X_2$	$X_3$	$X_4$	$X_5$	$X_6$	$X_7$	$X_8$	$X_9$	$X_{10}$
0.01	144.9	118.4	18.27	2.00	3.00	7.00	7.00	7.00	9.00	10.0	10.0	10.0	10.0	
		124.9	13.73	2.00	3.00	6.00	7.00	7.00	7.00	9.00	10.0	10.0	10.0	
		126.9	12.36	1.00	2.00	3.00	7.00	7.00	9.00	10.0	10.0	10.0	10.0	
0.02	69.89	37.26	46.69	4.00	8.00	8.00	9.00	9.00	9.00	10.0	10.0	10.0	10.0	
		39.32	43.74	3.00	5.00	6.00	9.00	9.00	10.0	10.0	10.0	10.0	10.0	
		42.80	38.76	3.00	4.00	5.00	9.00	9.00	9.00	9.00	10.0	10.0	10.0	
0.03	36.58	22.21	39.28	1.00	4.00	5.00	6.00	8.00	9.00	9.00	10.0	10.0	10.0	
		26.02	28.87	1.00	3.00	4.00	5.00	6.00	8.00	9.00	9.00	10.0	10.0	
		30.01	17.95	1.00	2.00	3.00	3.00	5.00	8.00	9.00	9.00	10.0	10.0	
0.04	20.75	9.200	55.65	3.00	8.00	8.00	10.0	10.0	10.0	10.0	10.0	10.0	10.0	
		15.41	25.76	2.00	2.00	2.00	6.00	6.00	7.00	9.00	10.0	10.0	10.0	
		18.58	10.49	1.00	2.00	3.00	5.00	5.00	7.00	8.00	9.00	9.00	10.0	
0.05	13.47	6.200	53.96	7.00	8.00	8.00	9.00	9.00	10.0	10.0	10.0	10.0	10.0	
		8.150	39.50	1.00	6.00	7.00	7.00	8.00	8.00	9.00	10.0	10.0	10.0	
		10.87	19.34	1.00	1.00	2.00	5.00	7.00	8.00	8.00	10.0	10.0	10.0	
0.06	9.720	6.420	33.96	1.00	4.00	6.00	6.00	8.00	9.00	9.00	10.0	10.0	10.0	
		6.470	33.46	1.00	4.00	6.00	8.00	9.00	9.00	9.00	10.0	10.0	10.0	
		6.100	37.21	1.00	3.00	4.00	5.00	5.00	9.00	9.00	10.0	10.0	10.0	
0.07	7.570	4.160	45.02	7.00	8.00	8.00	8.00	9.00	9.00	9.00	9.00	10.0	10.0	
		4.270	43.65	5.00	8.00	8.00	8.00	9.00	9.00	9.00	9.00	10.0	10.0	
		5.550	26.69	1.00	3.00	4.00	6.00	8.00	8.00	9.00	10.0	10.0	10.0	
0.08	6.290	3.540	43.64	1.00	8.00	9.00	9.00	9.00	10.0	10.0	10.0	10.0	10.0	
		3.480	44.60	3.00	8.00	9.00	9.00	9.00	9.00	10.0	10.0	10.0	10.0	
		3.750	40.37	2.00	6.00	8.00	8.00	9.00	9.00	10.0	10.0	10.0	10.0	
0.09	5.290	2.790	47.24	7.00	8.00	9.00	10.0	10.0	10.0	10.0	10.0	10.0	10.0	
		2.860	46.06	7.00	7.00	9.00	9.00	10.0	10.0	10.0	10.0	10.0	10.0	
		3.710	29.99	1.00	2.00	6.00	7.00	7.00	10.0	10.0	10.0	10.0	10.0	
0.10	4.540	2.600	42.74	5.00	8.00	9.00	9.00	9.00	10.0	10.0	10.0	10.0	10.0	
		3.350	26.20	3.00	4.00	5.00	7.00	8.00	8.00	8.00	9.00	10.0	10.0	
		3.990	12.13	1.00	1.00	2.00	2.00	6.00	7.00	8.00	9.00	10.0	10.0	

The question someone may ask is - will the optimized plan perform better than the regular one if the locations are optimized at a shift value ( $\beta'$  or  $\delta'$ ) not reflecting the actual process shift

( $\beta$  or  $\delta$ )?

To answer this question, two analytical studies are conducted. The results are illustrated in Tables 8 and 10.

**TABLE 8.** ARL comparison between regular and two optimized plans at  $\beta'=0.01$

Strategy Type	X-Values	b									
		0.010	0.015	0.020	0.025	0.030	0.040	0.050	0.15	0.25	
Regular	1 2 3 4 5 6 7 8 9 10	144.89	109.0	69.89	51.50	36.50	21.1	13.5	2.70	1.70	
Optimized-1	2 3 7 7 7 9 10 10 10 10	118.4	76.11	46.85	30.88	20.49	12.14	8.29	2.06	1.14	
Optimized-2	2 3 6 7 7 7 9 10 10 10	124.9	80.03	50.29	33.11	23.51	13.50	9.04	2.17	1.21	

**TABLE 9.** ARL comparisons between regular and optimized plans under shift in third parameter from  $A_2$  to  $A_2 + \delta S$

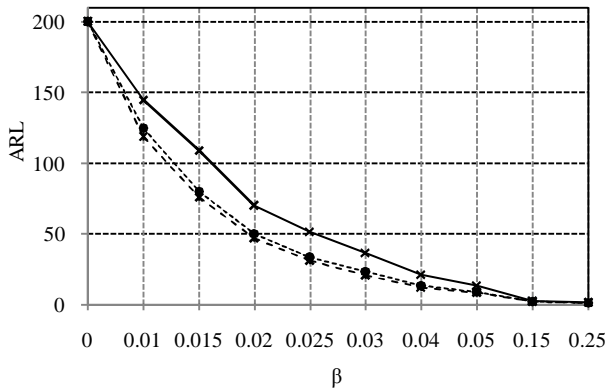
Shift	ARL (sample)			Reduction (%)	Independent Variable									
	Regular	Optimized			X <sub>1</sub>	X <sub>2</sub>	X <sub>3</sub>	X <sub>4</sub>	X <sub>5</sub>	X <sub>6</sub>	X <sub>7</sub>	X <sub>8</sub>	X <sub>9</sub>	X <sub>10</sub>
0.001	160.3	126.1	21.35	2.00	3.00	7.00	7.00	7.00	9.00	10.0	10.0	10.0	10.0	
		131.4	18.03	1.00	2.00	3.00	7.00	7.00	9.00	10.0	10.0	10.0	10.0	
		137.0	14.52	2.00	3.00	6.00	7.00	7.00	7.00	9.00	10.0	10.0	10.0	
0.002	96.42	43.67	54.71	4.00	8.00	8.00	9.00	9.00	9.00	10.0	10.0	10.0	10.0	
		46.19	52.10	3.00	5.00	6.00	9.00	9.00	10.0	10.0	10.0	10.0	10.0	
		51.50	46.59	3.00	4.00	5.00	9.00	9.00	9.00	10.0	10.0	10.0	10.0	
0.003	54.69	29.22	46.56	1.00	4.00	5.00	6.00	8.00	9.00	9.00	10.0	10.0	10.0	
		38.35	29.88	1.00	2.00	3.00	3.00	5.00	8.00	9.00	9.00	10.0	10.0	
		25.30	53.74	1.00	4.00	8.00	8.00	9.00	9.00	9.00	9.00	10.0	10.0	
0.004	34.03	27.09	20.41	1.00	2.00	3.00	5.00	5.00	7.00	8.00	9.00	9.00	10.0	
		10.04	70.49	3.00	8.00	8.00	10.0	10.0	10.0	10.0	10.0	10.0	10.0	
		20.57	39.54	2.00	2.00	2.00	6.00	6.00	7.00	9.00	10.0	10.0	10.0	
0.005	21.77	6.990	67.89	7.00	8.00	8.00	9.00	9.00	10.0	10.0	10.0	10.0	10.0	
		10.50	51.77	1.00	6.00	7.00	7.00	8.00	8.00	9.00	10.0	10.0	10.0	
		13.72	36.97	1.00	1.00	2.00	5.00	7.00	8.00	8.00	10.0	10.0	10.0	
0.006	15.52	8.070	48.03	1.00	4.00	6.00	6.00	8.00	9.00	9.00	10.0	10.0	10.0	
		7.370	52.54	1.00	4.00	6.00	8.00	9.00	9.00	9.00	10.0	10.0	10.0	
		9.320	39.96	1.00	3.00	4.00	5.00	5.00	9.00	9.00	10.0	10.0	10.0	
0.008	9.420	3.800	59.71	1.00	8.00	9.00	9.00	9.00	10.0	10.0	10.0	10.0	10.0	
		3.880	58.85	3.00	8.00	9.00	9.00	9.00	9.00	10.0	10.0	10.0	10.0	
		4.300	54.38	2.00	6.00	8.00	8.00	9.00	9.00	10.0	10.0	10.0	10.0	
0.010	6.710	2.840	57.66	5.00	8.00	9.00	9.00	9.00	10.0	10.0	10.0	10.0	10.0	
		2.830	57.88	3.00	4.00	5.00	7.00	8.00	8.00	9.00	10.0	10.0	10.0	
		5.380	19.89	1.00	1.00	2.00	2.00	6.00	7.00	8.00	9.00	10.0	10.0	
0.100	1.000	1.000	0.000	1.00	1.00	1.00	2.00	4.00	6.00	7.00	7.00	9.00	10.0	
		1.000	0.000	2.00	2.00	3.00	4.00	6.00	7.00	8.00	8.00	10.0	10.0	
		1.000	0.000	2.00	3.00	3.00	4.00	6.00	7.00	7.00	8.00	10.0	10.0	

**TABLE 10.** ARL comparisons between regular and two optimized plans at  $\delta'=0.0002$

Strategy Type	X -Values	d				
		0.001	0.003	0.005	0.01	0.05
Regular	1 2 3 4 5 6 7 8 9 10	160.00	54.70	21.90	6.65	1.00
Optimized-1	2 3 7 7 7 9 10 10 10 10	129.86	26.49	10.25	3.73	1.00
Optimized-2	1 2 3 7 7 9 10 10 10 10	133.91	28.94	11.00	3.99	1.00



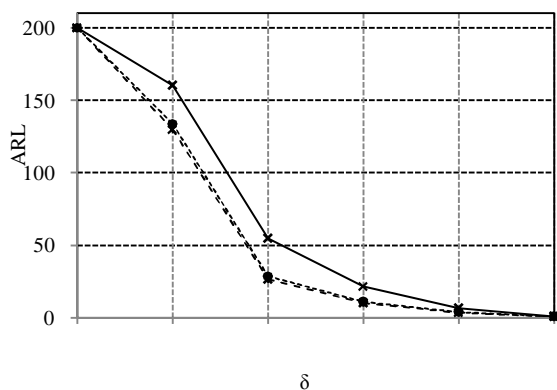
Table 8 and Figure 1 show the results of comparing one regular with two plans optimized at  $\beta'=0.01$ . The results show that if the location of  $X$  of a certain sample of size  $n$  is optimized at a shift in  $A_1$  equals  $\beta'$ , this sample can statistically work better than the regular one at all levels of shift higher than  $\beta'$ .



**Figure 1.** ARLs comparison to study the effect of optimizing the  $X$ -values at shift value equal to  $\beta'$

We also considered optimizing the location of the independent variable when the shift occurs in the third parameter  $A_2$ . Table 9 illustrates the simulation results when  $A_2$  shifts from  $A_2$  to  $A_2 + \delta\sigma$ . Compared to regular plans, optimized plans perform perfectly at all shift levels in  $A_2$ .

Again, to examine the performance of the proposed approach when the locations are optimized at a shift level equal to  $(\delta')$ , two optimized strategies ( $\delta' = 0.0002$ ) are compared versus a regular plan with  $n = 10$ . The results are shown in Table 10 and graphically presented in Figure 2.



**Figure 2.** Comparison of ARLs to study the effect of optimizing the  $X$ -values at shift value equal to  $\delta'$

As one may have noted, Table 10 and Figure 2 reveal that the optimized plans have a distinctive performance compared with the regular plan at all levels ( $\geq \delta'$ ).

## 6. CONCLUSION

In this paper, we provided a systematic approach for the purpose of enhancing the statistical performance of one of well-known phase II polynomial profiling techniques referred to as EWMA4. The suggested approach considers the effect of allocating the explanatory variable  $X$  in minimizing the average of the run length distribution (ARL). Finding the optimal location of the independent variable is formulated as an optimization problem solved by using the genetic algorithm (GA). The effect of location optimization is examined using a simulation study, and the results are compared with non-optimized strategies in terms of average run length criterion. The results reveal the potentials of optimizing the location of  $X$ -values in improving the performance of EWMA4 method.

Such methodology can be utilized to reduce the cost and effort of sampling and minimize the time that a process stays off-target.

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