# THE OPTIMAL ENERGY CARRIERS' SUBSTITUTES IN THERMAL POWER PLANTS: A FUZZY LINEAR PROGRAMMING MODEL

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**Abstract** In this paper, a dynamic optimization approach for optimal choice of energy carriers in thermal power plants is proposed that analyzes the substitution of energy carriers in short-term planning of a power plant. The model is based on linear programming method with objective of minimizing costs under constraints of resource availability, energy balances, environmental regulations and electricity production requirements. The restriction of resource availability in cold months due to depletion of gas pressure is considered. This research ( as case studies) demonstrates the application of the model for determination of efficient substitutes and optimization of their consumption in two thermal power plants in Iran. In these case studies, the reasonable solutions for dynamic planning of substitution of energy carriers in two power plants have been obtained. Effect of uncertainties of fuel price on the model was examined. Thus, a fuzzy linear programming model with fuzzy objective coefficients was formulated and two fuzzy methods were used: Zimmerman max-min and TH methods. The model solved for one of the power plants and its results were compared to obtained results of the crisp model. Finally results of both fuzzy methods were compared to each other.

**Keywords** Energy conservation; Alternative fuel; Fuzzy linear programming; Optimization; Thermal power plant.

**چکیده** در این مقاله یک رویکرد بهینه سازی پویا برای تعیین بهینه حامل های انرژی در نیروگاههای حرارتی ارائه شده که مسئله جایگزینی حامل های انرژی در نیروگاه را در برنامه ریزی کوتاه مدت مورد بررسی قرار می دهد. مدل ارائه شده یک مدل برنامه -ریزی خطی با هدف حداقل سازی هزینه ها تحت محدودیتهای دسترسی به منابع، موازنه انرژی و زیست محیطی و منطبق با نیازهای تولید برق است. همچنین در این مدل محدودیت سوخت در فصل سرد سال (به علت افت فشار گاز) در نظر گرفته شده است . به عنوان مطالعه موردی سعی شده است که کاربرد مدل برای تعیین جا نشین های کارا ( مناسب) و میزان بهینه مصرف آنها در دو نیروگاه حرارتی درایران مطرح گردد . نتایج حاصل از مطالعه موردی نشان می دهد که جوابهای منطقی از مدل پویای جایگزینی حامل های انرژی برای هر دو نیروگاه بدست آمده است . به علاوه دراین مقاله اثر عدم قطعیت قیمت سوخت بر روی مدل نیز بررسی شده است. بنابراین یک مدل برنامه ریزی خطی فازی باضرایب تابع هدف فازی فرموله شده واز دو روش فازی استفاده شده است: روش حداکثر–حداقل زیمرمن ,TH. سپس مدل برای یک نیروگاه به عنوان مطالعه موردی حل شده و نتایج آن با نتایج مدل قطعی وهمچنین نتایج دو روش فازی با هم مقایسه می شوند.

## 1. INTRODUCTION

In today's world, nonrenewable energy resources such as oil, natural gas and coal are still the most noticeable sources of energy in different economic sectors; particularly in the electric power sector. Economic growth, increasing population and improvement of life standards have led to increases in energy consumption patterns. In oil and gas producing countries, domestic demands highly decrease the capacity of these countries' energy exports. Therefore, energy resource management is of great importance and has a large effect on economic development of a society. Of course, efficient energy carriers for different end-users with the objective of optimal consumption, minimum costs and minimum environmental hazards need to be determined.

Jebaraj and Iniyan [1] have reviewed different types of energy models. Energy substitution is a well- established topic in the empirical literature. Marchetti [2] has developed a synthetic model for primary energy substitution. In this model, societal efficiency, literacy and mineral resources were used as variables. Kydes [3], Iniyan et al. [4] and Kamimura et al. [5] have investigated fuel substitution implementing different mathematical models.

During the last two decades, there have also been several academic investigations about optional resource allocation [6-18]. In most cases, optimization methods and decision analysis have been used as the main methodology. Energy resource allocation alternatives have been evaluated and compared from different points of view in their works.

The energy resource consumers in a region typically include several sectors, such as transportation, residential, commercial, agricultural, industrial, and electrical sectors. Among them, the electrical sector seems to have more opportunities for fuel substitution. The Power plants as electricity producers consume various fossil fuels to generate electricity. Fuel-substitution may involve a temporary fuel switching to minimize short-term or seasonal environmental or fuel price impacts, and permanent shifts to more efficient fuels or to lower polluting forms of the same fuel (e.g., from high to low sulfur coal).

Söderholm [19] has analyzed short-run fuel substitution in west European power generation, and the impact of system load factors on fossil fuel choice, within a translog cost function. Dahl and Ko [20] have estimated inter-fuel substitution using two flexible functional forms (the translog and the logit) on monthly data of US power plants. Tauchman [21] has analyzed two different aspects of fuel choice: the investment decisions which determine the fuel mix in the long run, and fuel consumption based on existing generation capacities, i.e. short-run inter-fuel substitution. The empirical analysis is based on panel data of major German utilities with the consideration of environmental regulations. Chaaban et al.[22] have presented an econometric model that can be used for evaluating and comparing the alternatives, so as to determine the most economically feasible option taking into account various cost parameters. The optimal fuel choice for power plants depends on fuel costs, plant location, resource availability, environmental pollution caused by burning of fuels, and even mid-term and long-term policies governing the energy sector. In Iran, energy sector faces challenging problems in the availability of nonrenewable energy resources (NRE), especially in cold months when gas pressure is highly depleted due to increases in household consumption. To deal with these limitations, the state energy authority sets restrictions on the maximum resources available to the countries' power plants for the last four months of the year.

Moreover, uncertainties affect energy planning. The dramatic fluctuations and the unpredictability of NRE (mainly crude oil) prices in recent years, make it necessary to account for these uncertainties. To a large extent, these price fluctuations are due to the impacts of a number of political and international factors. These uncertainties can alter planning decisions.

In the past, energy planners mostly applied stochastic programming to confront uncertainties. This methodology is effective when (a) the variables are in nature, stochastic and independent of other variables such as time and (b) sufficient data from different situations in different periods is available. A stochastic version of the dynamic linear programming model has been presented by Messner et al. [23]. This approach explicitly incorporates the uncertainties in the model, endogenizing interactions between the decision structure and uncertainties involved.

Another approach used in recent years to take uncertainties into account is the fuzzy approach, especially fuzzy optimization. In the last three decades, various contributions have been published on fuzzy optimization and fuzzy linear programming (FLP). Despite this extensive literature, the application of these methods to energy planning is relatively recent [24]. One of the early-published contributions in this field is Canz [25]. The basic purpose of this research was to evaluate how the methodology of FLP can support the decision-making process in energy system planning under uncertainty. The research reported in the paper provided an overview of the methods and tools used for supporting decision making in energy system planning in Germany. Other notable contributions on this subject include Mavrotas et al. [26], Chedid et al. [27], Agrawal and Singh [28], Sadeghi and Mirshojaeian [24] and Güngör and Arikan [29].

Mavrotas et al. [26] used a linear programming (LP) model, including both continuous and integer variables which demonstrated energy flows and discrete energy technologies in a large hotel unit near Athens. The model included fuzzy parameters in order to adequately handle the uncertainties regarding energy costs.

Chedid et al.'s [27] study was concerned with applications of multi-objective linear programming and fuzzy theory to the analysis of energy allocation problems. Agrawal and Singh [28] have presented a fuzzy multi-objective model for an energy resource allocation problem with linear fuzzy objectives, including economic, environmental, technical, and other concerns. The cooking energy sources for the household sector of Uttar Pradesh (India) are considered as an illustration of the approach.

Sadeghi and Mirshojaeian [24] have demonstrated an application of FLP for the optimization of the energy supply system in Iran.

Finally, Güngör and Arikan [29] have used three fuzzy preference models to evaluate a set of alternatives (e.g. Natural gas, coal and nuclear) for further development in long-term planning for power plants.

The choice of focus in this paper is also motivated by reviewing the paper by Huang et al. [30]. This work proposed an integrated dynamic optimization approach for nonrenewable energy resource management under uncertainty and proposes a hybrid inexact chance-constrained mixed-integer linear programming (ICCMILP) method. The application of this method was demonstrated in four industrial sectors for long-term planning.

However, our investigation differs from the aforementioned study in the sense that we study the optimization of fuel consumption in power plants rather than in the industrial sector in which boiler systems used fossil fuels. Moreover, the load factor and electricity generation demand need to be taken into account for fuel substitution in power plants which we did in our work. Also, our work focuses on short-term planning in thermal power plants rather than long-term planning.

The other contribution of our paper is to study the effect of uncertainties of the fuel price on the optimization model, so a fuzzy linear programming model with the fuzzy objective coefficients based on two fuzzy approaches (max-min operator and the TH method) is proposed and its results are compared with crisp model.

The rest of the paper is organized as follows: The model formulation is described in section 2. The methodological issues of the proposed fuzzy mathematical programming approach are discussed in section 3. Case studies of two thermal power plants in Iran are described and discussed in section 4. Finally, section 5 summarizes the main conclusions.

## 2. MODEL FORMULATION

The problem under consideration is related to k types of limited resources that are to be allocated to a thermal power plant over a planning horizon, with the objective of optimizing the system performance.

**2.1. Assumptions** According to the facts about non-renewable energy resource allocation in the power plant and without any loss of generality, we can impose some assumptions for simplification:

i. Each unit of the power plant can consume all different kinds of fuels at the same time.

ii. It is possible to switch available fuels in the beginning of each period.

iii. In cold months, the state energy authority limits fuel consumption in power plants, especially natural gas, due to an increase in household fuel consumption and decrease in gas pressure.

iv. Even if the power plant does not experience any problems with gas pressure depletion, it must still comply with the state-imposed limitations.

v. We only consider the effects of the two major harmful pollutants: $NO_x$ and $SO_2$.[i]

**2.2. Decision Variables** The decision variables include the amount of energy type j allocated to the power plant in period k, denoted by $x_{jk}$.

Fossil-fueled thermal power plants usually consume natural gas, fuel oil and gas oil. In those thermal power plants which use mainly natural gas and fuel oil. The gas oil is not used for electricity generation because it is expensive and hard to obtain. It is only used for startup. Therefore, in our investigation of steam power plants, the fuel alternatives reduce to two options: Natural gas and fuel oil. The length of each period is taken to be one month.

**2.3. Objective Function** The objective is to minimize the overall system cost in a power plant during one year planning horizon. Major costs related to fuels comprise fuel prices, capital costs (gas and oil pipeline costs), operating costs, and emission costs. In this work, we consider the emission costs as the social environmental costs incurred due to the disruptive effects of pollution.

*Min F* = total cost of energy consumption + operating costs + social costii of $SO_2$ emission + social cost of $NO_x$ emission + capital costs

$$\sum_{j=1}^{2}\sum_{k=1}^{12}(EP_{jk}\,x_{jk} + EP_{jk}\,x_{jk} + SO_{jk}\,x_{jk}\,STC_k + NO_{jk}\,x_{jk}\,PTC_k) + CP_1 + CP_2 \quad (1)$$

where $EP_{jk}$=market price of fuel j during period k (US$/l for fuel oil and US$/m$^3$ for natural gas); $OP_{jk}$=operating cost of fuel j in period k (US$); $SO_{jk}$=emission factor of $SO_2$ from fuel j during period k (g/l for fuel oil and g/m$^3$ for natural gas), $STC_k$= cost of $SO_2$ emission during period k (US$/gr); $NO_{jk}$=emission factor of $NO_x$ from fuel j during period k (g/l for fuel oil and g/m$^3$ for natural gas); $PTC_k$= cost of $NO_x$ emission during period k (US$/g); j=type of fuel, j=1: Natural Gas, j=2: fuel oil; k=time period(month). $CP_1$=Capital cost of natural gas (US$); $CP_2$=Capital cost of fuel oil (US$).

It is necessary to emphasize that $CP_1$ and $CP_2$ are fixed costs during the year. Moreover, the $STC_k$ and $PTC_k$ are fixed parameters which are obtained from EPA website[iii]. EPA is the US Environmental Protection Agency which its important mission is to protect and improve air quality in order to avoid or mitigate the consequences of air pollution's harmful effects.

**2.4. Constraints** The constraints of the model include resource availability limitations, energy balances, and environmental constraints.

**2.4.1. Resource Availability Constraints** Resource availability constraints can be external and internal.

***External constraint***

Upon the onset of the cold season and the increase in urban gas consumption, the state determines limits on the consumption of natural gas and fuel oil which are imposed in the final four months of a year. Thus, we have the following constraint:

$$\sum_{k=9}^{12} x_{jk} \le EO_j \quad , j = 1,2 \quad (2)$$

where $EO_j$ is the maximum allowed consumption of fuel j for the entire four month period at the end of the year.

***Internal Constraint***

In our model we denote the maximum consumption of fuel j by each unit for each day by $O_{jk}$ and assume it remains constant during the entire period k. This is calculated based on the peak load requirements predicted for period k. Total maximum fuel consumption in different months of a year basically depends on daily maximum consumption and monthly load factor. So we have:

$$EA_{jk} = O_{jk} \times \eta_k \times N \times n_k \quad (3)$$

Where: $n_k$= the number of days of period k (29, 30 and 31)[iv] ; $O_{jk}$= maximum daily consumption of fuel j during period k for a unit (m$^3$/h for natural gas, l/h for fuel oil), N= number of units; $\eta_k$ = load factor of period k; $EA_{jk}$= maximum consumption of fuel j during period k for the entire plant.

The optimal amount of fuel j in period k should be a percentage of maximum fuel consumption ($EA_{jk}$) in that period:

$$x_{jk} \le EA_{jk} \qquad \forall j, k \quad (4)$$

**2.4.2. Energy Balances** The amount of thermal energy from fuels should be equal to or more than the thermal energy obtained from power generation. The thermal energy obtained from 1kwh electricity is 860 kcal. Thus, the energy balance for each period (month) is expressed by the following equation:

$$\eta \times \sum_{j=1}^{2} HV_j \times x_{jk} \ge 860 \times PE_k \ , k = 1,2,\ldots,12 \quad (5)$$

where η = efficiency of the process; HVj = heating value of fuel j ( kcal/l for fuel oil and kcal/$m^3$ for natural gas) ; PEk = power generation in period k (kwh).

### 2.4.3. Environmental Constraints

***Environmental Concerns***

Today, fossil fuels are still the predominant source of energy in most economic worldwide sectors, particularly in the electric power sector. Ironically, the majority of harmful pollutants are released by fossil-fuelled power plants. Among these pollutants, the most important are sulfur dioxide ($SO_2$) and nitrogen oxides ($NO_x$). Social and environmental concerns necessitate the need to implement strategies and plans to mitigate the impacts of energy conversion process in general, and power plant emissions in particular [22]. Switching to fuels with a low carbon to hydrogen ratio, such as natural gas, is regarded as one effective option for reducing greenhouse emissions and decreasing the concentration of $SO_2$ in the atmosphere.

***Model of Environmental Constraints***

We may enforce environmental constraints to control and prevent emissions from fossil fuel burning. Thus, the total emissions from the fuels should not exceed the maximum allowable emission level for pollutants, as follows:

$$\frac{\sum_{j=1}^{2} SO_{jk} x_{jk}}{10^{-9} \times \sum_{j=1}^{2} 4186 \times HV_j \times x_{jk}} \leq SEC_k \qquad (6)$$

$$k = 1,2,\ldots,12$$

$$\frac{\sum_{j=1}^{2} NO_{jk} x_{jk}}{10^{-9} \times \sum_{j=1}^{2} 4186 \times HV_j \times x_{jk}} \leq NEC_k \qquad (7)$$

$$k = 1,2,\ldots,12$$

where $SEC_k$ and $PEC_k$ are maximum allowable emission levels for $SO_2$ and $NO_x$ and are derived from EPA website. It provides standards air pollutants. Since $SEC_k$ and $PEC_k$ are in terms of g/Gj, the amounts of emissions in Equations (6) and (7) are divided by the sum of thermal energy from the two fuels.

## 3. METHODOLOGICAL APPROACH AND FUZZY CONSIDERATIONS

**3.1. LP Models** Mathematical programming and mainly LP models are the most widely applied tools in energy planning and resource allocation problems which aim at minimizing the discounted cost (capital and operational) or maximizing benefit while satisfying energy demand over the planning horizon. The constraints of the problem usually represent the demand of various energy consuming sectors, resource availability, energy balances, and environmental regulations. Sometimes other constraints can be imposed (e.g., use of local resources due to regulatory obligations).

**3.2. Fuzzy Approach** Fuzzy set theory has been applied in a vast variety of areas, since it was presented by Zadeh [31]. This is because fuzzy sets can effectively handle the imprecision or vagueness in encountered by humans and systems in decision situations. The fuzziness of a property lies in the lack of well defined boundaries for the set of objects to which this property applies. One of the most common and practical fields of fuzzy set theory is fuzzy mathematical programming, especially fuzzy linear programming.

In the next section, various kinds of FLP will be introduced and defuzzification methods of FLP models with fuzzy objective function coefficients will be described. Fuzzy mathematical programming can be divided into three groups based on the type of uncertainty involved [32]:

1. *Fuzzy mathematical programming with vagueness* was developed in the 70s involves decision problems under fuzzy goals and constraints; known as "flexible programming". Many papers on the development of this method were reviewed by Zimmermann [33, 34].
2. *Fuzzy mathematical programming with ambiguity* handles fuzzy and imprecise coefficients of objective functions and constraints but not fuzzy goals and constraints. This type of fuzzy mathematical programming is also known as "possibilistic programming".
3. *Fuzzy mathematical programming with vagueness and ambiguity* involves both fuzzy coefficients and vague preferences. This fuzzy programming, first introduced by Negoita et al. [35] is called "Robust programming".

In this paper, we use the second type (fuzzy mathematical programming with ambiguity), since our model involves fuzzy coefficients of the objective function. We do not, however, consider fuzzy coefficients in our constraints.

**3.3. Fuzzy Linear Programming with Fuzzy Objective Function Coefficients** Consider an FLP model with fuzzy objective function coefficients [36]:

$$Max\ Z = \tilde{C}X \tag{11}$$

$$\text{s.t.}\quad AX \leq b \tag{12}$$

$$X \geq \mathbf{0} \tag{13}$$

Fuzzy parameters can be represented by fuzzy numbers (triangular, trapezoidal, Gaussian and sigmoidal). For simplicity and without loss of generality, we assume that $\tilde{C}_i$ are triangular fuzzy numbers. We denote these fuzzy coefficients by $C = (C^p, C^m, C^o)$ where $C^m$ is the mean value of $\tilde{C}$, $C^P$ is the left value (pessimistic) and $C^o$ is the right value (optimistic). Therefore, the fuzzy linear programming model reduces to the following:

$$max\ Z = \sum_{j=1}^{n} (C_j^p, C_j^m, C_j^o) x_j \tag{14}$$

$$\text{s.t.}\quad \sum_{j=1}^{n} a_{ij}\, x_j \leq b_i,\ i = 1, \dots, m \tag{15}$$

$$x_j \geq 0, \qquad \forall j \tag{16}$$

There are different methods for solving this problem. We will describe two well-known methods. The first method combines three objective function parameters into a single crisp parameter, and as a result, transforms the problem into a crisp linear programming model. One combination method uses the following factor, called the "strong probability factor":

$$C_j = \frac{C_j^p + 4C_j^m + C_j^o}{6} \qquad \forall j \tag{17}$$

The second method considers the fact that our purpose in an optimization problem is maximizing $C^mX$, minimizing $(C^m\text{-}C^P)X$ and maximizing$(C^o\text{-}C^m)X$, instead of simultaneous maximization of the triangular fuzzy coefficients, (i.e., $(C^P,C^m,C^o)X$). Therefore, our problem becomes a crisp multi-objective linear programming problem modeled as follows:

$$Min\ Z_1 = (C^m\text{-}C^p)X \tag{18}$$

$$Max\ Z_2 = C^mX \tag{19}$$

$$Max\ Z_3 = (C^o\text{-}C^m)X \tag{20}$$

$$s.t.\quad AX \leq b \tag{21}$$

$$X \geq 0 \tag{22}$$

One way to solve this multi-objective programming problem is to determine membership functions for three objective functions and maximize their alpha-cuts. For this purpose, we first calculate $Z^l$ and $Z_i^u$ which are optimal lower and upper limits for the objective functions, respectively:

$$Z^{l} = min\ (C^m\text{-}C^p)X \tag{23}$$

$$s.t.\quad AX \leq b \tag{24}$$

$$X \geq \mathbf{0} \tag{25}$$

$$Z_2^{l} = min\ C^mX \tag{26}$$

$$s.t.\quad AX \leq b \tag{27}$$

$$X \geq \mathbf{0} \tag{28}$$

$$Z_3^{l} = min\ (C^O\text{-}C^m)X \tag{29}$$

$$s.t.\quad AX \leq b \tag{30}$$

$$X \geq \mathbf{0} \tag{31}$$

$$Z_1^{u} = max\ (C^m\text{-}C^p)X \tag{32}$$

$$s.t.\quad AX \leq b \tag{33}$$

$$X \geq \mathbf{0} \tag{34}$$

$$Z_2^{u} = max\ C^mX \tag{35}$$

$$s.t.\quad AX \leq b \tag{36}$$

$$X \geq \mathbf{0} \tag{37}$$

$$Z_3^{u} = max\ (C^O\text{-}C^m)X \tag{38}$$

$$s.t.\quad AX \leq b \tag{39}$$

$$X \geq \mathbf{0} \tag{40}$$

Now, the membership function for $Z_1$ is:

$$\mu_{Z_1}(Z_1)=\begin{cases}1 & Z_1\le Z_1^{\ell}\\ \dfrac{Z_1^u-Z_1}{Z_1^u-Z_1^{\ell}} & Z_1^{\ell}\le Z_1\le Z_1^u\\ 0 & Z_1\ge Z_1^u\end{cases} \tag{41}$$

And for $Z_2$ we have:

$$\mu_{Z2}(Z_2)=\begin{cases}1 & Z_2> Z_2^{u}\\ \dfrac{Z_2-Z_2^{\ell}}{Z_2^u-Z_2^{\ell}} & Z_2^{\ell}\le Z_2\le Z_2^u\\ 0 & Z_2< Z_2^{\ell}\end{cases} \tag{42}$$

Similarly, the membership function for $Z_3$ is defined akin to $Z_2$. Finally, the problem reduces to a crisp single-objective linear programming model as follows:

$$max \quad \alpha \tag{43}$$

$$s.t. \quad \alpha(Z_1^u - Z_1^l) \le Z_1^u - (C^m - C^p)X \tag{44}$$

$$\alpha(Z_2^u - Z_2^l) \le C^m X - Z_2^l \tag{45}$$

$$\alpha(Z_3^u - Z_3^l) \le (C^p - C^m)X - Z_3^l \tag{46}$$

$$AX\le b \quad X\ge 0 \tag{47}$$

where α is the minimum of the membership functions [37].

In the next section, the model is applied in two cases for two power plants individually. Then, for one of the power plants, the effects of uncertainties on fuel oil price are examined. In this case, only the fuel oil price coefficient in the objective function becomes fuzzy as in the following formulation:

$$F=\sum_{j=1}^{2}\sum_{k=1}^{12}\left(\widetilde{EP}_{jk}x_{jk}+CP_{jk}y_{jk}+OP_{jk}x_{jk}STC_k+NO_{jk}x_{jk}PTC_k\right)+CB+OB \tag{48}$$

For solving the fuzzy model, the second method described above is chosen due to its axiomatic strength and also because membership functions and α are obtained. The fuzzy model is solved and the results are compared with the crisp model results in the next section.

It is noteworthy that the researchers recently have shown that the max-min operator often does not produce the efficient and unique solutions [38-41]. For that reason, quite a few methods were developed by researchers to fix this deficit. First, Lai and Hwang [39] presented the improved max–min approach named LH method, then Selim and Ozkarahan [42] proposed a modified version called MW method, and Li et al. [40] developed a two-phase fuzzy approach identified as LZL method.

Torabi and Hassini [38] proposed a novel interactive fuzzy approach (called TH method) to solve the multi-objective linear model (MOLP) and finding a preferred compromise solution. In this work, we also apply TH method to the problem to test our results and to compare it with that of max-min operator.

The TH method is formulated as shown below:

$$max\ \lambda(v)=\gamma\lambda_0+(1-\lambda)\textstyle\sum_h\theta_h\,\mu_h(v) \tag{49}$$

$$s.t.\quad \lambda_0\le\mu_h(v),\quad h=1,\dots,4 \tag{50}$$

$$v\in F(v), \lambda_0\ and\ \gamma\in[0,1] \tag{51}$$

where $\mu_h(v)$ and $\lambda_0=\min\{\mu_h(v)\}$ indicate the satisfaction degree of $h^{th}$ objective function and the minimum satisfaction degree of objectives, respectively. This method gives the decision maker more reliability to have efficient solution by changing the value of some controllable parameters such as $\gamma$.

## 4. CASE STUDIES

We examine two thermal power plants in Iran. These plants are the main providers of electricity for Tehran megacity with a population in excess of 7 million. These power plants consume fuels for the process of heating and steam generation.

The first one, Montazer Ghaem, is located in the city of Karaj nearby Tehran. It was established in 1974 and has four steam generation units, each with a capacity of 156.25 (Mwh).

The second one, Besat, is one of oldest power plants in Tehran. This power plant was established in the south of Tehran in 1964. The location was chosen based on its close proximity to a heavy fuel oil refinery, major gas pipelines and underground water. It also has three steam units with a capacity of 82.5 (Mwh) per unit. Our data was collected directly from these two plants and also from the Tavanir organization, which is the state authority for electric power.

Twelve months of 2007 are considered as the planning horizon. Because of fluctuating fuel prices, limited non-renewable energy resource availability and environmental concerns, the plants could utilize fuel switching in order to improve their economic and environmental efficiency. The proposed problem is how to effectively plan the energy consumption pattern for the power plants under a number of economic, resource availability, and environmental constraints while minimizing the overall system costs.

**4.1. Results and discussion** The formulated LP model (described in section 2) and the fuzzy model (described in section 3) were individually solved for each plant using the LINDO software package.

Table1 shows the optimal solutions obtained from the model for power plant 1 (Montazer Ghaem). According to this table, in periods 1-7, the power plant should consume natural gas only and in periods 8-11, the power plant should mainly use fuel oil (77, 88, 44 and 91%, for periods 8, 9, 10, and 11 respectively). It should consume fuel oil only in period 12.

The plant should consume natural gas due to its high availability, low costs (price, capital and operating costs) as well as low $SO_2$ and $NO_x$ emissions. The causes for selecting fuel oil in periods 8-12 lies mainly in the state-imposed limits on gas supply due to depletion of gas pressure and increase in domestic consumption. It should be noted environmental constraints can be satisfied only up to period 8. After that, because of the nature of fuel oil, these constraints would not be satisfied as shown in Table 2. Therefore, it is somewhat paradoxical that the state concerns for nationwide gas pressure results in environmental damage. The total cost is calculated to be $2.979828×10^{12}$.

Table 3 indicates optimal solutions for the second power plant (Besat). In periods 1-6, the model prescribes that this power plant should only use natural gas due to its advantages outlined above. In periods 7 and 8 the plant should continue to use 87 and 80% natural gas (13 and 20% of fuel oil), respectively.

**TABLE 1.** Solutions of model (Power plant1:Montazer Ghaem)

| Period | Fuel Option | $X_{ij}$ | Solution |
|---|---|---|---|
| 1 | gas | $X_{11}$ | 77870016 |
| | Fuel oil | $X_{21}$ | 0 |
| 2 | gas | $X_{12}$ | 79994880 |
| | Fuel oil | $X_{22}$ | 0 |
| 3 | gas | $X_{13}$ | 89994240 |
| | Fuel oil | $X_{23}$ | 0 |
| 4 | gas | $X_{14}$ | 95868864 |
| | Fuel oil | $X_{24}$ | 0 |
| 5 | gas | $X_{15}$ | 104993280 |
| | Fuel oil | $X_{25}$ | 0 |
| 6 | gas | $X_{16}$ | 89119296 |
| | Fuel oil | $X_{26}$ | 0 |
| 7 | gas | $X_{17}$ | 92292480 |
| | Fuel oil | $X_{27}$ | 0 |
| 8 | gas | $X_{18}$ | 20384902 |
| | Fuel oil | $X_{28}$ | 65765784 |
| 9 | gas | $X_{19}$ | 9558819 |
| | Fuel oil | $X_{29}$ | 66243684 |
| 10 | gas | $X_{110}$ | 5699985.5 |
| | Fuel oil | $X_{210}$ | 81058848 |
| 11 | gas | $X_{111}$ | 7941195 |
| | Fuel oil | $X_{211}$ | 77542864 |
| 12 | gas | $X_{112}$ | 0 |
| | Fuel oil | $X_{212}$ | 38279688 |

**TABLE 2.** $NO_x$ and $SO_2$ emission for power plant1 for period 8 to 12 (g/Gj)

| Period / Emission | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|
| $SO_2$ | 902 | 1018.5 | 1080.8 | 1052.9 | 1147.6 |
| $NO_x$ | 236.2 | 244 | 248 | 264.7 | 253.2 |

In the final four months of the year, the power plant should solely use fuel oil, because it is not allowed to use any natural gas. Again, environmental constraints would not be satisfied in the last four months due to the usage of fuel oil which results in high emissions of $SO_2$ and $NO_x$. During four months (9-12), the amount of emissions of $SO_2$ and $NO_x$ were 1147.607 and 253.29 g/Gj, respectively.

**TABLE 3.** Solution of the model (power plant 2: Besat)

| Period | Fuel Option | $X_{ij}$ | Solution |
|---|---|---|---|
| 1 | gas | $X_{11}$ | 43415192 |
| | Fuel oil | $X_{21}$ | 0 |
| 2 | gas | $X_{12}$ | 40679036 |
| | Fuel oil | $X_{22}$ | 0 |
| 3 | gas | $X_{13}$ | 46461872 |
| | Fuel oil | $X_{23}$ | 0 |
| 4 | gas | $X_{14}$ | 53709452 |
| | Fuel oil | $X_{24}$ | 0 |
| 5 | gas | $X_{15}$ | 53826632 |
| | Fuel oil | $X_{25}$ | 0 |
| 6 | gas | $X_{16}$ | 54195752 |
| | Fuel oil | $X_{26}$ | 0 |
| 7 | gas | $X_{17}$ | 40250592 |
| | Fuel oil | $X_{27}$ | 5999598 |
| 8 | gas | $X_{18}$ | 31320722 |
| | Fuel oil | $X_{28}$ | 8012069 |
| 9 | gas | $X_{19}$ | 0 |
| | Fuel oil | $X_{29}$ | 39349800 |
| 10 | gas | $X_{110}$ | 0 |
| | Fuel oil | $X_{210}$ | 44055900 |
| 11 | gas | $X_{111}$ | 0 |
| | Fuel oil | $X_{211}$ | 48591900 |
| 12 | gas | $X_{112}$ | 0 |
| | Fuel oil | $X_{212}$ | 38624608 |

**TABLE 4.** Fuzzy model Solutions (power plant 2:Besat)

| Period | Fuel Option | $X_{ij}$ | Solution |
|---|---|---|---|
| 1 | gas | $X_{11}$ | 0 |
| | Fuel oil | $X_{21}$ | 43415192 |
| 2 | gas | $X_{12}$ | 0 |
| | Fuel oil | $X_{22}$ | 40679036 |
| 3 | gas | $X_{13}$ | 0 |
| | Fuel oil | $X_{23}$ | 46461872 |
| 4 | gas | $X_{14}$ | 53709452 |
| | Fuel oil | $X_{24}$ | 0 |
| 5 | gas | $X_{15}$ | 53826632 |
| | Fuel oil | $X_{25}$ | 0 |
| 6 | gas | $X_{16}$ | 0 |
| | Fuel oil | $X_{26}$ | 54195752 |
| 7 | gas | $X_{17}$ | 40250592 |
| | Fuel oil | $X_{27}$ | 5999598 |
| 8 | gas | $X_{18}$ | 31320722 |
| | Fuel oil | $X_{28}$ | 8012069 |
| 9 | gas | $X_{19}$ | 0 |
| | Fuel oil | $X_{29}$ | 39349800 |
| 10 | gas | $X_{110}$ | 0 |
| | Fuel oil | $X_{210}$ | 44055900 |
| 11 | gas | $X_{111}$ | 0 |
| | Fuel oil | $X_{211}$ | 48591900 |
| 12 | gas | $X_{112}$ | 0 |
| | Fuel oil | $X_{212}$ | 38624608 |

According to Table 4, the results of the fuzzy and crisp models are equal in most periods and differ only in periods 1, 2, 3, and 6. The fuzzy model proposes to use fuel oil instead of natural gas in periods 1, 2, 3, and 6. For other periods, the optimal consumption pattern is equal in both fuzzy and crisp models.

The minimum total cost in the fuzzy model is about 15 million dollars more than the crisp model. The reason for this is that actual fuel oil prices were spread more to the right (right spread is equal to 25% of the fuel price) than to the left (left spread is equal to 12% of the fuel price), so the total fuel consumption cost and consequently the total cost is greater in the fuzzy model than in the crisp model. The total cost at minimum and maximum fuel oil price is 16,376,734 and 181,639,570 dollars.

For TH method, the decision maker provided the relative importance of objectives for $Z_1$, $Z_2$ and $Z_3$ linguistically as: $\theta_2 > \theta_1 = \theta_3$ and based on this relationships we set the objectives weight vector as: $\theta = (0.25, 0.5, 0.25)$. By doing some initial experiments, the stopping criteria for solving the problem as well as controllable parameters were set as $\gamma = 0.4$. It is noted that the reason for selecting $\gamma = 0.4$ is that the $Z_2$ is the most important objective and also $Z_1$ and $Z_3$ are actually relative measures from $Z_2$. Thus the somewhat unbalanced compromise solution with highest satisfaction degree for $Z_2$ is of particular interest. The result of the model is shown in Table 5.

By solving the model, $\lambda_0 = 1$, $\lambda(v) = 2.7$ have been obtained. According to Table 5, the results of the fuzzy TH and crisp models are equal in most periods and differ in periods 2, 6, 7, and 8. The

**TABLE 5.** TH fuzzy model solution (power plant2: Besat)

| Period | Fuel Option | $X_{ij}$ | Solution |
|---|---|---|---|
| 1 | gas | $X_{11}$ | 43415192 |
| | Fuel oil | $X_{21}$ | 0 |
| 2 | gas | $X_{12}$ | 0 |
| | Fuel oil | $X_{22}$ | 40679036 |
| 3 | gas | $X_{13}$ | 46461872 |
| | Fuel oil | $X_{23}$ | 0 |
| 4 | gas | $X_{14}$ | 53709452 |
| | Fuel oil | $X_{24}$ | 0 |
| 5 | gas | $X_{15}$ | 53826632 |
| | Fuel oil | $X_{25}$ | 0 |
| 6 | gas | $X_{16}$ | 0 |
| | Fuel oil | $X_{26}$ | 54195752 |
| 7 | gas | $X_{17}$ | 0 |
| | Fuel oil | $X_{27}$ | 46250192 |
| 8 | gas | $X_{18}$ | 0 |
| | Fuel oil | $X_{28}$ | 39332792 |
| 9 | gas | $X_{19}$ | 0 |
| | Fuel oil | $X_{29}$ | 39349800 |
| 10 | gas | $X_{110}$ | 0 |
| | Fuel oil | $X_{210}$ | 44055900 |
| 11 | gas | $X_{111}$ | 0 |
| | Fuel oil | $X_{211}$ | 48591900 |
| 12 | gas | $X_{112}$ | 0 |
| | Fuel oil | $X_{212}$ | 38624608 |

fuzzy model proposes to use fuel oil instead of natural gas in periods 2, 6, 7, and 8. For other periods, the optimal consumption pattern is equal in both fuzzy and crisp models. Moreover, the results of two fuzzy models are different for four periods. But, a more reliable solution is obtained by the TH method with high satisfaction degree for some objectives with higher relative importance without any attention paid to the satisfaction degree of other objectives (i.e., yielding unbalanced compromise solutions). We come to the conclusion that TH method is more efficient method than the Zimmermann max–min method. This method can generate both unbalanced and balanced efficient solutions based on the decision maker's preferences together with offering appropriate flexibility to provide different solutions to help the decision maker in selecting the final preferred compromise solution [38].

## 5. CONCLUDING REMARKS

In this paper we proposed a mathematical model to determine efficient energy carriers for fuel substitution in power plants. We subsequently incorporated the effects of the uncertainty of fuel oil price in our model with a modification of the objective function using a fuzzy LP method.

The presented LP model provides solutions for decision variables which represent the optimal choice of fuel and the optimal amount of each fuel to be consumed by the plant. This methodology was applied in two real case studies concerning two thermal power plants in Iran. The results reveal that natural gas is highly recommended except in cold months, especially in the crisp models. In the fuzzy model, the results are similar to the crisp one except for four months. Moreover, in this work we applied the TH method to the problem to test our results and to compare it with the max-min operator.

Economic efficiency and environmental benefits can be obtained through the effective planning and allocation of energy resources. In further research, this model can develop further both by more comprehensive constraints and additional objective functions. The uncertainty in parameters like load factors, power generation, and other factors could also be considered. Also, optimization of energy carriers for several power plants at the same time can be attempted. This study presents a new model and its application to fossil fuel allocation in power plants but the proposed methodology can also be applied to the allocation of other resources.

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[i] We ignore $CO_2$ because of Iran's minor share in global $CO_2$ emissions. However, this parameter can be easily incorporated into the model if necessary.

[ii] Social cost is the cost to society as a whole from an event, action, or policy change. It includes negative externalities and does not count costs that it transfers to others. Environmental pollution will result in social cost to the society.

[iii] www.epa.gov

[iv] In the Iranian calendar, the first 6 months are 31 days long, the next five are 30 days long, and the last month of the year is 29 days long.