APPLICATION OF GAME THEORY IN DYNAMIC COMPETITIVE PRICING WITH ONE PRICE LEADER AND N DEPENDENT FOLLOWERS

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Abstract In this research, data from five major Iranian firms active in breakfast cheese business were used to determine the pricing status of Ultra Filtration (UF) cheese. Because the leader firm (firm A) claimed that in all work periods, its product was sold under the ceiling price, we decided to calculate the optimal price of each firm in four different states (according to the firm A's claim) based on the game theory principles to verify this claim. In addition, the price in each period was predicted with a time series approach and compared with the calculated optimal price of game theory approach. Consequently, after price calculation, the leader firm's claim was accepted.

Keywords Static Game; Dynamic Game; Complete Information; Competitive Pricing; Stackelberg Game; Collusion

چکیده در این تحقیق از داده‌های پنج شرکت عمده ایرانی تولید کننده پنیرهای صبحانه در ایران برای بررسی وضعیت قیمت گذاری پنیر UF استفاده شد. از آنجا که شرکت رهبر (شرکت A) ادعا می‌کرد که همواره در تمامی دوره‌های کاری، محصولش زیر قیمت سقف به فروش رسیده است، تصمیم گرفتیم تا قیمت بهینه هر شرکت را در چهار حالت مختلف (بر طبق ادعای شرکت A) براساس اصول نظریه بازی‌ها محاسبه کنیم تا این ادعا را ارزیابی کنیم. همچنین، قیمت در هر دوره به وسیله دیدگاه سری‌های زمانی پیش‌بینی و با قیمت بهینه محاسبه شده از دیدگاه نظریه بازی‌ها مقایسه شد. در نهایت پس از محاسبه قیمت، ادعای شرکت رهبر پذیرفته شد.

1. INTRODUCTION

In a study in Japan, 70% of respondents stated that they would prefer rational prices to high quality products [1]. Price is one of the marketing mixture factors that are affected rapidly by the management decisions. Mc Kinsey firm counselors believe that the quickest and most efficient way for a firm to achieve the maximum benefit, is by setting appropriate prices for its products or services. These counselors also reported that for the 2462 firms that they studied, each 1% improvement in price results in an average of 11.1% increased in benefits [2].

Price leadership has long been recognized as an important and frequently occurring phenomenon [3]. U.S. steel production firms maintained parallel price changes before the arrival of foreign competitors [3]. General Motors acted as a price leader for many years, and its prices were followed by Chrysler and American Motors [4].

The effects of the actions and marketing behavior of one brand can be distributed among competitors' market shares in a complex manner. In 1988, Carpenter, Cooper, Hanssens, and Midgley presented methods for modeling brand competition and brand strategies in markets where competitive effects can be distributed differentially and asymmetrically. In that paper, empirical studies were discussed, model parameters were estimated and competitive strategy implications of the proposed model were explained. Price and advertisement competition between brands of an Australian home appliance firm is used to illustrate the application of these procedures [4]. For example, the Wall Street Journal reported that Chrysler's pricing strategy seems to follow its competitor, Ford [5].

Examples of documented cases of gasoline market price leaders forcing companies to cooperate with price cuts are Shell Oil in California, and Standard Oil in Ohio [3]. Other markets with price leadership patterns include air travel agencies, turbo generators, personal computers, and some packaged consumer goods such as cigarettes and breakfast cereal [6, 7].

Analyzing and anticipating competitor moves is central to modern competitive strategy. In contexts involving intense inter-firm interaction, the value of a particular strategy depends largely on how competitors will react to it. Despite many developments, anecdotal evidence indicates that the effective use of techniques to gauge decisions based on competitive considerations has been scant in practice. Moura et al intended to fill this void. Using data from the auto insurance industry in Brazil, they contrast strategies that do and do not anticipate competitor reactions. Basically, they show that it pays to anticipate those reactions. An optimal strategy will explore both demand elasticity and competitors' patterns of reaction. They show that such "strategic" policy is expected to outperform a "myopic" approach that ignores competitor reactions. They also develop a methodology to compute demand elasticity and reaction functions and numerically compute optimal reaction strategies [8].

Kwon et al. [9] described the dynamics of demand as a continuous time differential equation based on an evolutionary game theory perspective. They input real sales data into the equation to obtain estimates of parameters that govern the evolution of demand, and then refined the parameters on a discrete time scale. The resulting model takes the form of a differential variation inequality. They presented an algorithm based on a gap function for the differential variation inequality and reported its numerical performance for an example revenue optimization problem [9].

We employ a model presented by Roy, Hanssens and Raju in their paper "Competitive Pricing by a Price Leader". We modify the preceding paper's model to more accurately model market conditions. In the preceding paper, it is assumed that the market consists of two firms, labeled firm 1 and firm 2. This research focuses on the pricing problem of the leader (firm 1), although, the analysis can also be used for optimal pricing rule for the follower (firm 2). It is also assumed that pricing decisions are made in discrete time periods. For example, forecasts of future demand are obtained every month and prices are set based on these forecasts. Each period is presented by subscript *t*, where *t*=1, 2, 3… T. The original model from the preceding paper with the objective function:

$$\min \sum_{t=1}^{T} \rho^{t-1} [(q_t^1 - q^{11*})^2 + (q_t^2 - q^{12*})^2] \quad (1)$$

Subject to:

$$q_t^1 = a_{11} q_{t-1}^1 + a_{12} q_{t-1}^2 - b_{11} p_t^1 + b_{12} p_t^2 + u_t^1 \quad (2)$$

$$q_t^2 = a_{21} q_{t-1}^1 + a_{22} q_{t-1}^2 - b_{22} p_t^2 + b_{21} p_t^1 + u_t^2 \quad (3)$$

where q_t^i is the sale of firm *i* in period *t*, and p_t^i is the price of firm *i* in period *t*. It is assumed that sales in period *t* depend on the sales in period (*t-1*), prices in period *t*, and other exogenous factors that are not known completely at the beginning of period *t*. In these equations, b_{11} reflects the effect of the firm's own price in period *t* on demand, and its symbol is consistent with the basic intuition that an increase in own price, p_t^1, reduces demand in period *t*. b_{12} captures the effect of the competitor's price. An increase in the competitor's price, p_t^2, increases demand for firm 1. Equation 2 also assumes that sales in period (*t-1*) affect sales in period *t*, implying that there is a carryover effect from the past. This carryover effect may be due to a problem in the distribution system, or due to the fact that consumer preferences do not change instantaneously. Finally, demand in period *t* is dependent on exogenous factors such as changes in a customer's preferences or changes related to a design or advertisement that is determined by u_t^1 [7].

It is also assumed that the leader's objective is to keep unit sale as close as possible to a preset target in each period. The follower is assumed to have a similar objective. It focuses on firm 1 here, because the process is identical for firm 2. Firm 1's objective is to set price p_t^1 at the beginning of each period so that the actual sales are as close as possible to a preset target q^{11*}. Firm 1 also expects a certain level of q^{12*} for firm 2's sales and plans based on this expectation. In other words, q^{11*} is the target that firm 1 wants to meet, and q^{12*} is the part of industry demand that firm 1 expects firm 2 to meet; therefore, the objective is very similar to achieving a pre-set market share of expected industry sales [7].

The target sales vectors of firm 1 and firm 2 are $q^{1*} = (q^{11*}, q^{12*})$ and $q^{2*} = (q^{21*}, q^{22*})$, respectively. If expectations and targets are in line, then $q^{11*} = q^{21*}$ and $q^{12*} = q^{22*}$. In the empirical illustration in this paper, it is assumed that the target vectors of firms are in line. More specifically, it assumes that each firm sets its prices to minimize the discounted sum of squared deviations from the target vectors q^{1*} and q^{2*} through periods 1 to T. It is assumed that both firms have the same discount rate, r [7].

Using this price model for products and goods produced in our country is often difficult and sometimes impossible, because most firms in Iran set their product's price under the government order and the government fixes the ceiling price annually or semi-annually; therefore, competitive pricing does not exist for many products and goods.

We consider UF cheese pricing data from five major firms, because UF cheese is a perishable product with a short shelf-life, a top priority for the firms is to minimize the time between production and sale within each period.

Because the leader firm (firm A) claimed that in all work periods, its product has been always sold under the ceiling price, we decided to calculate the optimal pricing for each firm in four different states (according to firm A's claim) and to verify this claim based on game theory principles. Because these cheeses spoil after six months, there is no need to consider the sigma symbol in the objective function for all periods, and it is sufficient to study each period separately. Because r is constant for all five firms in each period and the objective function is considered separately for each period, r is omitted in the model. In the original model from the literature, the target sales are equivalent to the production forecasts, and remained constant in each period; however, in our model the production forecast is estimated by the OLS method in each period, and the amount is variable in each period. The constant target sales assumption in the Reported literature is one of the defects of the model that we modify in the improved model presented in this paper.

The problem is modeled in two types of objective functions for the five firms. In the first type, the objective function for each firm is minimizing the difference between sales and target sales in each work-period. These objective functions are shown by (*). The objective functions for the five firms are:

$$\min(q_t^1 - q_t^{11*})^2 \quad (4)$$

$$\min(q_t^2 - q_t^{22*})^2 \quad (5)$$

$$\min(q_t^3 - q_t^{33*})^2 \quad (6)$$

$$\min(q_t^4 - q_t^{44*})^2 \quad (7)$$

$$\min\left(q_t^5 - q_t^{55*}\right)^2 \quad (8)$$

In the second type, the objective function for each firm is minimizing the sum of differences between its sales and target sales in each period for two sequential periods. Considering that the prices of most products change annually, we assume fixed prices for each year (two sequential 6-month periods) and the objective functions for the five firms are:

$$\min[(q_t^1 - q_t^{11*})^2 + (q_{t+1}^1 - q_{t+1}^{11*})^2] \quad (9)$$

$$\min[(q_t^2 - q_t^{22*})^2 + (q_{t+1}^2 - q_{t+1}^{22*})^2] \quad (10)$$

$$\min[(q_t^3 - q_t^{33*})^2 + (q_{t+1}^3 - q_{t+1}^{33*})^2] \quad (11)$$

$$\min[(q_t^4 - q_t^{44*})^2 + (q_{t+1}^4 - q_{t+1}^{44*})^2] \quad (12)$$

$$\min[\left(q_t^5 - q_t^{55*}\right)^2 + \left(q_{t+1}^5 - q_{t+1}^{55*}\right)^2] \quad (13)$$

These objective functions are shown by (**).
where Firm i=1: A, firm **i**=2: B, firm i=3: C, firm i=4: D, firm i=5: E, p_t^i: product price of firm i in period t, q_t^i : Firm i's product sales in period t, q_t^{ij*}: Firm i anticipated production by firm j in period t.

2. ECONOMETRICS METHODOLOGY

After defining and clarifying the econometrics model that is derived from economic theories, the next step in the econometrics investigation process is estimating the parameters of the model using the available data. After estimating the parameters of the model, the econometrist should examine appropriate criteria to assess the consistency of the estimated parameters with theoretical expectations. If the selected model verifies the hypothesis or theory under investigation, the model can be used to predict future quantities of dependent variables based on known or expected future quantities of the independent variable. The estimated equations for q_t^1 ،q_t^2 ،q_t^3 ،q_t^4 and q_t^5 by Eviews software are:

$$q_t^1=0.53q_{t-1}^1-0.02431q_{t-1}^2-0.0584q_{t-1}^3-0.01788q_{t-1}^4-0.02916q_{t-1}^5-0.0029p_t^1+0.0016p_t^2+0.0019p_t^3+0.0014p_t^4+0.0023p_t^5 \quad (14)$$

$$q_t^2=-0.0984q_{t-1}^1+0.5241q_{t-1}^2-0.0411q_{t-1}^3-0.0692q_{t-1}^4-0.0529q_{t-1}^5+0.0028p_t^1-0.0078p_t^2+0.0038p_t^3+0.0034p_t^4+0.003p_t^5 \quad (15)$$

$$q_t^3=-0.0591q_{t-1}^1-0.0712q_{t-1}^2+0.48571q_{t-1}^3-0.0847q_{t-1}^4-0.0986q_{t-1}^5+0.00338p_t^1+0.0039p_t^2-0.00869p_t^3+0.0036p_t^4+0.0027p_t^5 \quad (16)$$

$$q_t^4=-0.0641q_{t-1}^1-0.0617q_{t-1}^2-0.091q_{t-1}^3+0.5163q_{t-1}^4-0.0408q_{t-1}^5+0.0025p_t^1+0.0037p_t^2+0.0029p_t^3-0.0069p_t^4+0.0017p_t^5 \quad (17)$$

$$q_t^5=-0.0533q_{t-1}^1-0.0201q_{t-1}^2-0.0417q_{t-1}^3-0.0881q_{t-1}^4+0.27134q_{t-1}^5+0.0013p_t^1+0.0017p_t^2+0.0024p_t^3+0.0021p_t^4-0.0045p_t^5 \quad (18)$$

2.1. Calculation of the Game Equilibrium Point

After estimating the q_t^1 ،q_t^2 ،q_t^3 ،q_t^4 and q_t^5 equations, we solve this problem in 4 states:

2.1.1. Static Game with Complete Information

In this state, all firms set their prices and present their product prices to the market simultaneously. This game is called static game with complete information because all players move simultaneously. It is clear that in this game, competition is based on the product pricing (UF cheese) in the five firms: because price is a continuous quantity, this game is a static game with complete information and continuous strategies.

In these types of games, it is enough to derive each firm's objective function from its own price and to set it equal to zero in order to find the critical points of the objective function. Thus, five equations are solved according to P_t^1 ،P_t^2 ،P_t^3 ،P_t^4 and P_t^5. Because we consider 18 work-periods, P_t^1 ،P_t^2 ، P_t^3 ،P_t^4 and P_t^5 are 18×1 matrices. We use MATLAB software to solve matrices and find the game equilibrium points for all 18 work-periods. If the system of equations does not have any solution or the equilibrium prices are less than zero, this game does not have any equilibrium points.

2.1.2. Dynamic Game with Complete Information with one Price Leader and Four Followers That Decide Simultaneously (Stackelberg Game)

In this state, at the beginning of each period, the leader firm (firm A) presents its product price to the market, and the followers firms B, C, D and E, price their cheese products after observing the leader firm's price. In other words, followers start a static game with complete information among themselves. Each game in each period is a dynamic game with complete information and continuous strategies, which is solved by working backward. Because each period is considered separately, we derive followers' objective functions from their prices, and set them equal to zero to find critical points. This approach yields four equations. Using these equations, P_t^2 ،P_t^3 ،P_t^4 and P_t^5 are calculated according to P_t^1:

$$A_{4\times4}P_{4\times18}+C_{4\times1}P'_{1\times18}+B_{4\times18}=0 \quad (19)$$

where P: a 4×18 matrix, rows are P_t^2 ،P_t^3 ،P_t^4 and P_t^5 and the number of columns is the number of work-periods.

P': A 1×18 matrix; this matrix is P_t^1 and the number of columns is the number of work-periods.

$$AP=-(CP'+B)$$

$$P=-A^{-1}(CP'+B)=-A^{-1}CP'-A^{-1}B \quad (20)$$

Finally, with this equation, P_t^2 ،P_t^3 ،P_t^4 and P_t^5 are calculated according to P_t^1. Variables P_t^2 ،P_t^3 ،P_t^4 and P_t^5, Calculated from P_t^1 are placed in Equation (9). Then, we derive firm 1's objective function from P_t^1, and set it equal to zero to find P_t^1. P_t^1 is placed in the original Equation (25) and p matrix to calculate P_t^2 ،P_t^3 ،P_t^4 and P_t^5.

2.1.3. Dynamic Game with Complete Information with one Price Leader and Four Followers That Decide Simultaneously Considering That Firms D And E Collude in Their Pricing In this state, the leader firm (firm A) presents its product price to the market at the beginning of each period; then, followers price their products after observing the leader firm's price. This state is similar to state 2 because pricing is set among followers simultaneously, but differs from state 2 because in this state, firms D and E collude. To account for collusion in the model, we solve for colluding players as a single player.

For example, if we assume that firms D and E have colluded in their product pricing, we replace these two firms with a firm labeled "Both", whose sale equal the sum of D's and E's sales and the target sales equals the sum of the target sales of these two firms, and price equals the average product price of firms D and E. In this state, leader firm (firm A) presents its product price to the market; after that, firms B, C and Both price their products after observing firm A's price. With the Eviews software, q_t^1 ،q_t^2 ،q_t^3 and q_t^{Both} equations are estimated:

$$q_t^1 = 0.537854q_{t-1}^1 - 0.061102q_{t-1}^3 - 0.043991q_{t-1}^4 - 0.002794p_t^1 + 0.001373p_t^2 + 0.001857p_t^3 + 0.003781p_t^4 \quad (21)$$

$$q_t^2 = -0.101504q_{t-1}^1 + 0.511151q_{t-1}^2 - 0.036003q_{t-1}^3 - 0.053794 + 0.002759p_t^1 - 0.007649p_t^2 + 0.003804p_t^3 + 0.006324p_t^4 \quad (22)$$

$$q_t^3 = -0.067847q_{t-1}^1 - 0.090239q_{t-1}^2 + 0.479661q_{t-1}^3 - 0.067391q_{t-1}^4 + 0.003259p_t^1 + 0.004016p_t^2 - 0.008617p_t^3 + 0.006312p_t^4 \quad (23)$$

$$q_t^{Both} = -0.105538q_{t-1}^1 - 0.197690q_{t-1}^3 + 0.315380q_{t-1}^4 + 0.003952p_t^1 + 0.004155p_t^2 + 0.005462p_t^3 - 0.006849p_t^4 \quad (24)$$

The continuation of solving this state's problem is similar to state 2.

2.1.4. Dynamic Game with Complete Information with one Price Leader and Four Followers who Collude in Their Product Pricing In this state, the leader firm presents its product price to the market at the beginning of each period, and followers price their products, after observing the leader firm's price. This state is similar to state 3, but differs in that all followers collude in their pricing.

We assume that firms B, C, D and E have colluded in their pricing, and replace these four firms with a firm called "All", whose sales equal the sum of B's, C's, D's and E's sales; sales target equals the sum of the target sales of these two firms; and price equals the average product price of firms B, C, D and E. In this state, firm A presents its product price to the market and then firm "All" sets its product price simultaneously after observing leader firm's price. Using the Eviews software, q_t^1 and q_t^{All} equations are estimated:

$$q_t^1 = 0.537830q_{t-1}^1 - 0.031071q_{t-1}^2 - 0.002766p_t^1 + 0.006999p_t^2 \quad (25)$$

$$q_t^{All} = -0.270770q_{t-1}^1 + 0.284101q_{t-1}^2 + 0.010162p_t^1 + 0.00711p_t^2 \quad (26)$$

The continuation of solving this state's problem is similar to state 2.

3. PREDICTING OF PRICE AND TIME SERIES APPROACH

Because price is a variable observed at discrete, equally spaced points, we can say that price is a discrete time series. We can predict price for the next period using the current price. We use MINITAB 14 software to calculate prices. In this approach, we enter the actual price of each firm and then follow this process step by step to predict a suitable model. The predicted prices for each firm are presented in Table (1).

TABLE1. Predicted prices of firms A, B, C, D and E.

t	Firm A	Firm B	Firm C	Firm D	Firm E
3	770000	800000	800000	870000	800000
4	1760000	1700000	1700000	1470000	1700000
5	1265000	1250000	1250000	1170000	1250000
6	1100000	1550000	1550000	1574000	1280000
7	1634000	1490000	1490000	1534000	1220000
8	1555714	1857143	1600000	1902857	1557143
9	1571607	1905357	1583929	1948214	1621429
10	2097393	2263039	2089121	2303333	2015272
11	2347653	2223333	2283189	2398667	2230033
12	2743212	2449697	2618107	2716848	2579153
13	2638877	2675000	2872200	2831364	2643927
14	3219842	3035664	3102230	3123776	3222016
15	3158406	3706996	3745741	3720657	3516725
16	3142113	3661215	3695993	3691213	3475445
17	3129836	3682248	3671809	3668386	3685874
18	3118074	4350959	4229305	4689065	4235689
19	3109743	4190724	4181942	4189225	4187319
20	3448872	4168493	4165937	3985418	4169411

Now, predicted price for each firm are substituted in equations 14, 15, 16, 17 and 18 and the objective function (*) for each firm is calculated. We present the calculated objective function (*) for each firm in table (2):

TABLE 2. Calculated objective function

t	Firm A	Firm B	Firm C	Firm D	Firm E
3	5418339	4117538	5037154	6731004	2726868
4	21279105	18484323	16237765	40747751	9101459
5	7256953	2537134	2135641	6746513	1613208
6	17664897	454652.7	142863.8	634779.3	4010278
7	2479184	2256731	2278400	1876827	2774981
8	12212500	62450.91	9764045	18623.49	900602
9	8025616	40991.9	8056605	52182.52	131027.8
10	11573504	5901201	5897979	2423781	980482
11	7382165	15585391	940515.2	2460252	178459.5
12	10321399	31662826	832949.8	3923803	281922.4
13	14211365	8611031	1421019	3432272	656745.5
14	15604952	14726450	3639432	11248115	673.0326
15	56513500	5206016	19959.84	5330686	3369411
16	35506870	22807.24	1330779	1182425	908272.9
17	24326077	1906977	1029371	618361.8	687176
18	84907643	292948.2	2343913	9778685	647126.9
19	38736069	7139284	3655533	210580.4	1159295
20	6430609	8907050	3970181	1737099	1891560

4. RESULTS

The optimal price for firm A with the objective functions (*) and (**) in four game states, the actual price and the predicted price by the time series approach are illustrated in Figures (1) and (2).

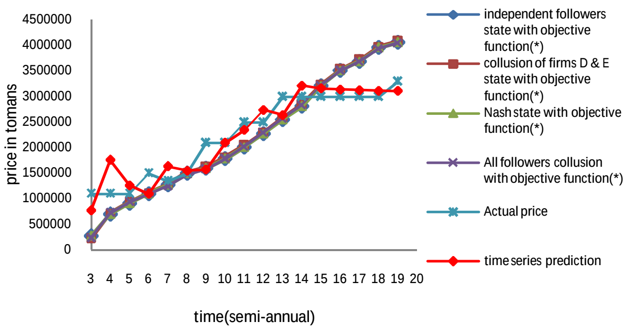

Figure 1. The optimal price of firm A with the objective function (*) in four states, the actual price and the predicted price by time series

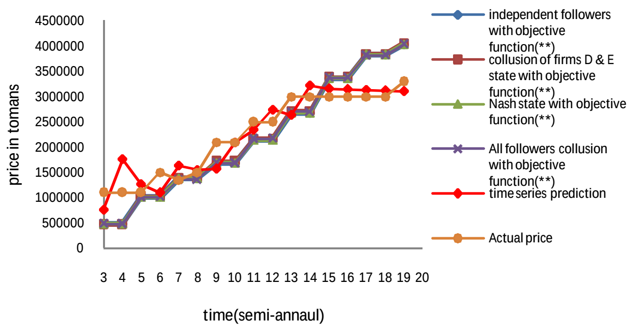

Figure 2. The optimal price of firm A with the objective function (**) in four states, the actual price and the predicted price by time series

The optimal price of firm B with the objective functions (*) and (**) in four game states, the actual price and its predicted price using the time series approach are illustrated in Figures (3) and (4).

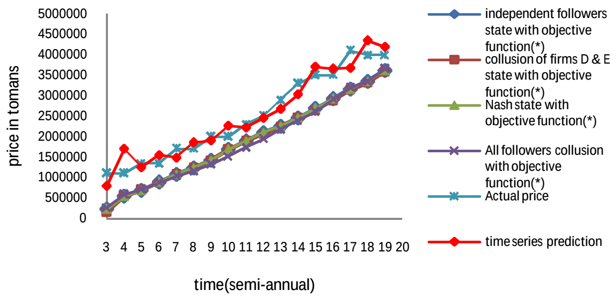

Figure 3. The optimal price of firm B with the objective function (*) in four states, its actual price and the predicted price by the time series

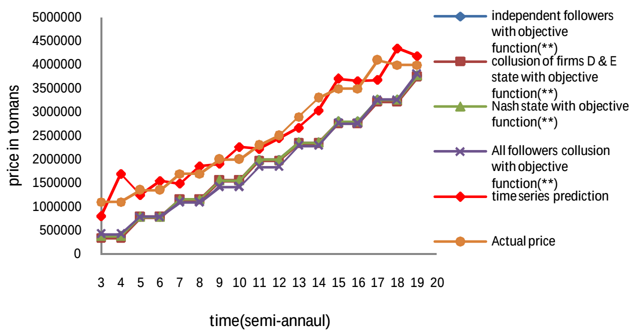

Figure 4. The optimal price of firm B with the objective function (**) in four states, its actual price and the predicted price by the time series

The trend in the optimal price of firms C, D and E is the same trend as that of the optimal price of firm B in Figures 3 and 4. In addition, all of these prices are less than their actual prices, except for firm A's diagrams which has a reverse trend from period 14. This issue is shown in Figures 1 and 2.

4.1. Comparison of the Objective Function Quantities for Four Game States In Figures 5, 6, 7 and 8, the objective function (*) for five firms in four game states are presented; in addition in Figures 9, 10, 11 and 12, the objective functions (**) for five firms in the four mentioned states are presented.

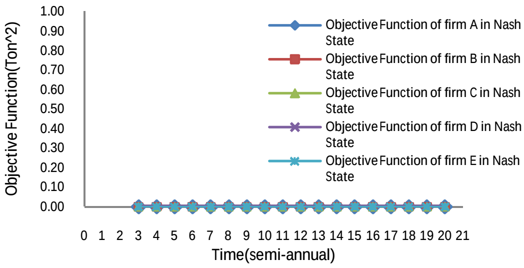

Figure 5. Comparison of the objective function (*) for five firms in state 1

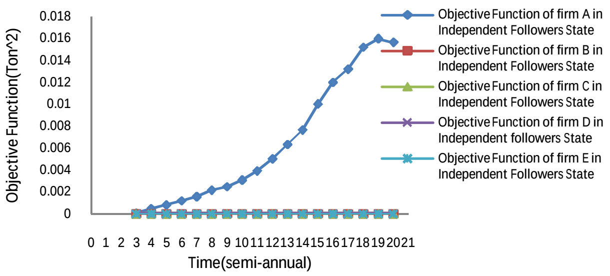

Figure 6. Comparison of the objective function (*) for five firms in state 2

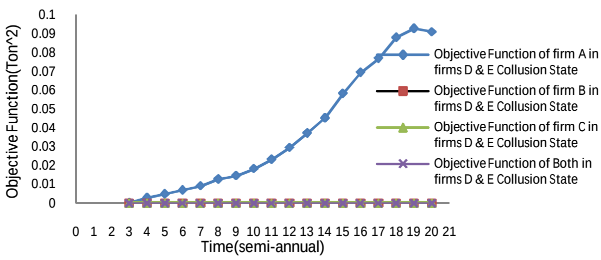

Figure 7. Comparison of the objective function (*) for firms A, B, C and Both in state 3

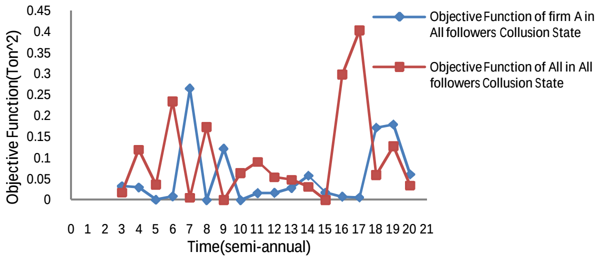

Figure 8. Comparison of the objective function (*) for firms A and All in state 4

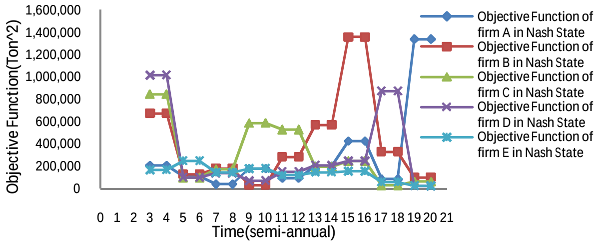

Figure 9. Comparison of the objective function (**) for five firms in state 1

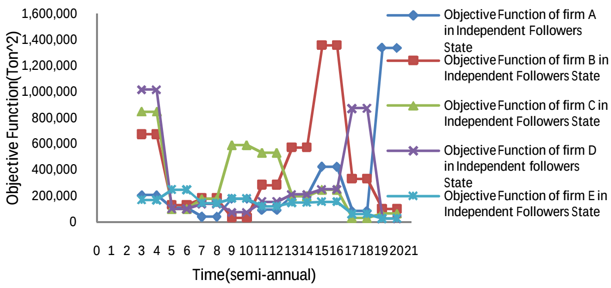

Figure 10. Comparison of the objective function (**) for five firms in state 2

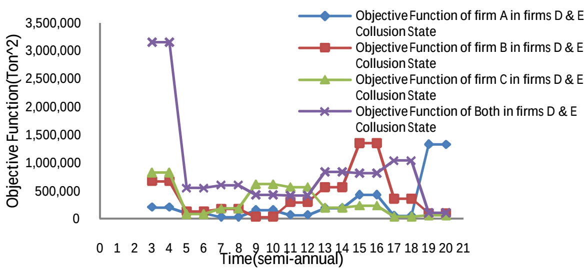

Figure 11. Comparison of the objective function (**) for firms A, B , C and Both in state 3

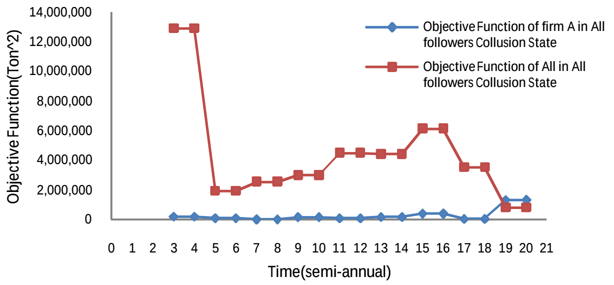

Figure 12. Comparison of the objective function (**) for firms A and All in state 4

By comparing these diagrams, we realize that the objective functions (*) and the pricing strategies in state (1) or (2) yields the best results. This comparison indicates that if players (production firms) price their products every six months, the leader firm (firm A) presents its price to the market first and followers (B, C, D and E) price their product after observing the leader firm's price, then the firms minimize the difference between each firm's actual sales and the target sales in each work-period. The diagrams show that the objective function (*) is much better than the objective function (**). Therefore, pricing in each six-month period is more suitable than annual pricing where price is assumed to be constant in each year. Moreover, in both objective functions (*) and (**), objective functions in states (1) and (2) are less than the objective functions in other states. Because the objective function is minimized, states (1) and (2) are the best pricing strategies for the objective functions (*) and (**)or all firms.

By comparing the objective functions in the time series and game theory price prediction approaches, we noticed that the objective functions in the time series approach are much greater than the objective functions in the game theory approach. The graphs of prices predicted by the time series are closer to the graphs of actual prices than the graphs of prices predicted by game theory principles. In addition, the graphs of sales predicted by the time series are closer to those of actual sales than to those of sales predicted by game theory principles.

The differences between actual prices and prices calculated by game theory are reasonable for the following reasons:

1. Considering the disutility function of "minimizing the difference between each firm's sales in each period and the target sales of the same period" is the objective function of the model.
   Because increasing income is one of the most important goals of all firms, firms price their products based on the same objective function. Therefore, the objective function in pricing models for these firms may be a utility function of "maximizing the firm's benefit" instead of a disutility function of "minimizing the difference between each firm's sales in each period and the target sales of the same period"
2. The objective function is insensitive to the sign of ($q^i_t - q^{ii*}_t$).
   Production firms prefer their sales in period *t*, (q^i_t), to be larger than their target sales in the same period,(q^{ii*}_t); thus,($q^i_t - q^{ii*}_t$) > 0 when firm *i* has sold more product than expected (target sales). However, if ($q^i_t - q^{ii*}_t$) is less than zero, then firm ***i*** prefers to minimize this quantity. Therefore, it is necessary to use the objective function min {(0, - ($q^i_t - q^{ii*}_t$))} instead of min ($q^i_t - q^{ii*}_t$)to enter the sign of ($q^i_t - q^{ii*}_t$)into the model's objective function.
3. The UF cheese market may not meet game theory assumptions.
   Game theory assumptions and rules may not be completely applicable to UF cheese competitive market in practice, and several factors of the strategic environment may substantially affect the UF cheese price. For example, firms may not have enough freedom in pricing their product, and game theories may not be completely applicable to their product pricing.
4. We assume linear sales equations.
   The assumption of linear sales equations may also cause differences between the prices predicted with game theory and actual prices.
5. The OL method is inaccurate for target sales estimation.
   We used the OL method to estimate target sales. Another method for estimating target sales would be to use the time series in MINITAB. Using the most suitable model and time series analysis to anticipate target sales may yield better results and better price estimates.

5. CONCLUSION

The optimal prices of firm A in models with objective function (*) or (**) are higher than those of other firms, but in fact, the government prevents Firm A from setting optimal prices with price ceilings. The optimal prices of five firms are similar across states 1, 2, 3 and 4 in each period. The actual prices of the followers (B, C, D and E) are more than the optimal prices in each period for all the states. The actual prices of the leader firm are greater than the optimal prices in each period for all the states, and this trend is reversed from period 14; i.e., from period 14, the actual prices are less than the optimal prices in each period. The pricing in each six-month period is more suitable than annual pricing, where price is assumed to be constant in each year. States 1 and 2 are the best pricing strategies for both objective functions (*) and (**) and for all firms if players (firms) price their products every six months and follow the state 1 pricing strategy or if all five firms present their products' prices to the market simultaneously. In this research, we presented a model with an objective function minimizing the difference between the firm's actual and target sales in each period. Because one of the important goals for production firms is maximizing income, we recommend considering this model with two objective functions:

1. Minimizing the difference between sales and production in each period, and
2. Maximizing income; solving the model and finding the optimal points that are considered in the 4 states.

We also recommend considering the model in these additional states:

- There are multiple leaders at one time.
- There is only one leader at any one time, but the leader may change over time.
- Followers do not make decisions simultaneously.
- In the Stackelberg game, two or more follower firms collude randomly.

6. REFERENCES

1. Carol, H. and Herbig, P., "Japanese Pricing Strategy", ***Journal of Consumer Marketing***, Vol. 1, No. 34, (1996), 5-17.
2. Charles, R.D., "Mathing Appropriate Pricing Strategy", ***Journal of Product Management***, Vol. 3, No. 2, (1994), 15-27.
3. Nagle, T.T., "the Strategy and Tactics of Pricing, Prentice-Hall, Englewood Cliffs, NJ", (1987).
4. Carpenter, G.S., Cooper, L.G., Hanssens, D.M. and Midgley, D.F., "Modeling Asymmetric Competition", ***Marketing Science***, Vol. 7, (1988), 393-412.
5. Hanssens, D.M., Parsons, L.J. and Schultz, R.L., Market Responce Models: Econometric and Time Series Analysis, Kluwer, Norwell, MA, (1990).
6. Rabah, A. and Stepanova, A., "Second-mover advantage and price leadership in Bertrand duopoly", ***Games and Economic Behavior,*** Vol. 55, (2006), 121-141.
7. Abhik, R., Hanssens, D.M. and Jagmohan S.R., "Competitive Pricing by a Price Leader", ***Management Science***, Vol. 40, No. 7, (1994), 809-823.
8. Marcelo, L.M, Marco, R.A., Caetano, A.L., Goldberg, M.B., Lazzarini, S.G. and César, E., "Does it pay to anticipate competitor reactions", (2006).
9. Changhyun, K., Terry, L., Mookherjee, F.R., Yao, T. and Feng, B., "Non-cooperative competition among revenue maximizing service providers with demand learning", ***European Journal of Operational Research,*** Vol. 197, (2009), 981–996.