FINITE CAPACITY QUEUEING SYSTEM WITH VACATIONS AND SERVER BREAKDOWNS

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Abstract This paper deals with finite capacity single server queuing system with vacations. Vacation starts at rate υ if the system is empty. Also, the server takes another vacation if upon his arrival to the system, finds the system empty. Customers arrive in the system in Poisson fashion at rate λ_0 during vacation, faster rate λ_f during active service and slower rate $\lambda_s \geq 0$ during the breakdown. Customers are served exponentially with the rate μ. Server breakdowns at rate b and it is immediately repaired exponentially with the rate r. We derive the explicit formulas for queue length distribution, average queue length, average number of customers in the system and average waiting time for a customer in queue, and in the system. Numerical illustrations have been cited to show that the proposed model is practically sound.

Keywords Multiple–vacation, Server breakdowns, Repair, Generating functions, Average queue length.

چکیده این مقاله در ارتباط با سیستم‌های صف تک خدمت دهنده با ظرفیت محدود و حاوی مرخصی می-باشد. در صورتی که سیستم خالی باشد، مرخصی با نرخ uشروع می‌شود. همچنین به محض ورود خدمت دهنده به سیستم، در صورتی که صف را خالی ببیند مرخصی دیگری می‌گیرد. مشتریان با توزیع پواسون وارد سیستم می‌شوند به گونه‌ای که این ورود در طی مرخصی با نرخ l_0 ؛ در طی سرویس دهی فعال با یک نرخ سریع‌تر l_f ؛ و در زمان خرابی با نرخ $l_s \geq 0$ می‌باشد. مشتریان به صورت نمایی با نرخ mسرویس دهی می‌شوند. سرویس دهنده‌ها با نرخ b خراب، و فوراً به صورت نمایی با نرخ r بازیابی می‌شوند. در این مقاله فرمول صریح برای توزیع طول صف، متوسط طول صف، متوسط تعداد مشتری در سیستم، و متوسط زمان انتظار برای یک مشتری در صف و در سرویس‌دهی استخراج می‌شود. نتایج و توصیفات عددی حاصل از مدل پیشنهادی برای نشان دادن صحت عملکرد آن ارائه شده‌اند

1. INTRODUCTION

Several researchers [1- 4] studied the queuing systems with server vacations and obtained various measures of performance. Takagi [5] studied M/G/1/N queue with server vacations and exhaustive service and obtained the distributions of the unfinished work, the virtual waiting time and the real waiting time, etc. Wang [6] proposed an N- policy M/M/1 queueing system with server breakdowns and obtained analytic closed-form solutions. Takine and Sengupta [7] obtained the queue-length distribution and waiting time distribution of a single-server queue under the provision of service interruptions. Boxma *et al.* [8] studied the length of a vacation and steady-state workload distribution, both for single and multiple vacations. Ke [9] proposed M/G/1 queuing system with server vacations, startup and breakdowns and obtained the system total expected cost function per unit time under optimal control mechanism. Zhang *et al.* [10] analyzed of an M/M/1/N queue with balking, reneging and server vacations and derived the matrix form solution of the steady-state probabilities and formulated a cost model to determine the optimal service rate. Wang *et al.* [11] made a comparative analysis between the exact results and the maximum entropy results, also illustrated through the maximum entropy results that the maximum entropy principle approach is accurate enough for practical purposes. Jain and Agrawal [12] studied the M^x/M/1 queueing system with multiple types of server breakdown under N-policy and provided the numerical results to demonstrate the effects of various parameters on the system performance characteristics. Ke and Wang [13] analyzed the operating characteristics for the heterogeneous batch arrival queue with startup and breakdown and performed a sensitivity analysis among the optimal value of N, specific values of system parameters, and the cost elements of $M^x/M/1$ queue. Srivastava and Jain [14] analyzed an optimal N-policy model for single Markovian queue with breakdown repair and state dependent arrival rate and obtained steady state for various operational characteristic and optimal value of N under a linear cost structure. Hur and Paik [15] analyzed the effect of different arrival rates on the N- policy of M/G/1 with server setup and derived the distribution function of the steady state queue length, waiting time using Laplace Stieltjes transform. Ke and Pearn [16] studied the management policy of M/M/1 queueing service system with heterogeneous arrivals under the N policy, in which the server is characterized by breakdowns and vacations and derived the distribution of the system size and mean queue length. Gray *et al.* [17, 18] studied the vacation queueing model with service breakdown under the assumption of different arrival rates and obtained formulas for queue length distribution and the mean queue length for M/M/1.

The queuing system with vacations and server breakdowns in infinite capacity system can be found in several research papers but from the practical view point this system may not always be the case .In many real life situations the finite capacity system plays the vital role such as in customized manufacturing systems, maintenance activities, and telecommunication network centers where the multi-task employees are to be deployed. In this paper we investigate finite capacity queueing system with multiple vacations and server breakdowns. The server completely stops serving customers during a vacation and start serving whenever number of customers N (≥1) in the system. Once service starts, there can be an interruption due to server breakdown, and it is sent to repair facility. As soon as the repair process completes, the server starts to serve the same interrupted customer.

2. MATHEMATICAL MODEL AND ANALYSIS

- $(0,i)$ is the state in which there are i customers in the queue and the server is on vacation, $0 \le i \le N$. Its probability is $p(0,i)$.
- $(1,i)$ is the state in which there are i customers in the system during active service, $1 \le i \le N$. Its probability is $p(1,i)$.
- $(2,i)$ is the state in which there are i customers in the system during repair process, $1 \le i \le N$. Its probability is $p(2,i)$.

The generating function for the queue length distribution is

$$F(z) = F_0(z) + F_f(z) + F_s(z) \tag{1}$$

where the partial generating functions are:

$$F_0(z) = \sum_{i=0}^{N} p(0,i)z^i, \quad F_f(z) = \sum_{i=1}^{N} p(1,i)z^i, \quad F_s(z) = \sum_{i=1}^{N} p(2,i)z^i$$

The balance equations for the queue length distribution are:

$$\lambda_0 p(0,0) = \mu p(1,1) \tag{2}$$

$$(\lambda_0 + \upsilon)p(0,i) = \lambda_0 p(0,i-1) \qquad 1 \le i < N \tag{3}$$

$$\upsilon p(0,N) = \lambda_0 p(0,N-1) \tag{4}$$

$$(\lambda_f + \mu + b)p(1,1) = \upsilon p(0,1) + \mu p(1,2) + rp(2,1) \tag{5}$$

$$\begin{aligned}(\lambda_f + \mu + b)p(1,i) &= \lambda_f p(1,i-1) \\ &+ \upsilon p(0,i) + \mu p(1,i+1) + rp(2,i)\end{aligned}, \qquad 2 \le i < N \tag{6}$$

$$(\mu + b)p(1,N) = \lambda_f p(1,N-1) + \upsilon p(0,N) + rp(2,N) \tag{7}$$

$$(\lambda_s + r)p(2,1) = bp(1,1) \tag{8}$$

$$(\lambda_s + r)p(2,i) = bp(1,i) + \lambda_s p(2,i-1), \quad 2 \le i < N \tag{9}$$

$$rp(2,N) = bp(1,N) + \lambda_s p(2,N-1) \tag{10}$$

Equation (2) gives

$$p(1,1) = \frac{\lambda_0}{\mu} p(0,0). \tag{11}$$

From Equation (4), we have:

$$p(0,N)=\frac{\lambda_0}{\upsilon}p(0,N-1) \quad (12)$$

Substituting (11) into (8), we obtain:

$$p(2,1)=\frac{b}{\lambda_s+r}p(1,1)=\frac{b\lambda_0}{\mu(\lambda_s+r)}p(0,0) \quad (13)$$

From Equation (3), we get:

$$p(0,i)=\rho_0^i p(0,0), \qquad 1\le i<N \quad (14)$$

where $\rho_0=\frac{\lambda_0}{\lambda_0+\upsilon}$

From Equations (12) and (14), we get:

$$F_0(z)=\sum_{i=0}^{N}p(0,i)z^i=\left\{\frac{1-(\rho_0 z)^N}{1-(\rho_0 z)}+\frac{\lambda_0}{\upsilon}\rho_0^{N-1}z^N\right\}p(0,0) \quad (15)$$

Multiplying Equation (6) by z^i and summing for $i=2,3,\dots,N-1$

$$\begin{aligned}&\left\{\mu+b-\frac{\mu}{z}-\lambda_f(z-1)\right\}F_f(z)=(\lambda_f+\mu+b)p(1,1)z\\&+\upsilon\left\{\frac{1-(\rho_0 z)^N}{1-(\rho_0 z)}+\frac{\lambda_0}{\upsilon}\rho_0^{N-1}z^N-1-\rho_0 z\right\}p(0,0)\\&-\frac{\mu}{z}\left\{p(1,1)z+p(1,2)z^2\right\}+rF_s(z)-rp(2,1)z.\end{aligned} \quad (16)$$

Now, we find $F_s(z)$ in terms of $F_f(z)$, then using Equation (16), we find $F_f(z)$. Multiply Equation (9) by z^i and sum for $i=2,3,\dots,N-1$

$$F_s(z)=\frac{b}{(\lambda_s+r-z\lambda_s)}F_f(z) \quad (17)$$

Substituting the (17) into (16), we get

$$\begin{aligned}&\frac{(z-1)Q(z)}{z(\lambda_s+r-\lambda_s z)}F_f(z)=(\lambda_f+\mu+b)p(1,1)z+\upsilon\left\{\frac{1-(\rho_0 z)^N}{1-(\rho_0 z)}+\right.\\&\left.\frac{\lambda_0}{\upsilon}\rho_0^{N-1}z^N-1-\rho_0 z\right\}p(0,0)-\frac{\mu}{z}\left\{p(1,1)z+p(1,2)z^2\right\}-rp(2,1)z\end{aligned} \quad (18)$$

where

$$Q(z)=\lambda_f\lambda_s z^2-(\lambda_f\lambda_s+\lambda_f r+b\lambda_s+\mu\lambda_s)z+\mu(r+\lambda_s) \quad (19)$$

In order for the queue length distribution to exist, the R.H.S. of Equation (18) must vanish when z=1. Since $p(1,1)$ and $p(2,1)$ are given, now we find $\mu p(1,2)$

$$\begin{aligned}&\mu p(1,2)=\left(\lambda_f+b-\frac{rb}{\lambda_s+r}\right)p(1,1)\\&+\upsilon\left(\frac{1-\rho_0^N}{1-\rho_0}+\frac{\lambda_0}{\upsilon}\rho_0^{N-1}-1-\rho_0\right)p(0,0)\end{aligned} \quad (20)$$

Substituting (20) into (18)

$$\begin{aligned}&\frac{(z-1)Q(z)}{z(\lambda_s+r-\lambda_s z)}F_f(z)=\mu(z-1)p(1,1)\\&+\upsilon\left\{\frac{\rho_0^{N+1}(z^N-z^2)-\rho_0^N(z^N-z)+\rho_0(z^2-1)-(z-1)}{(1-\rho_0)(1-\rho_0 z)}\right\}\\&p(0,0)+\lambda_0\rho_0^{N-1}(z^N-z)p(0,0)+\upsilon(z-1)p(0,0)\end{aligned} \quad (21)$$

or,

$$\begin{aligned}&\frac{Q(z)}{(\lambda_s+r-\lambda_s z)}F_f(z)=(\lambda_0+\upsilon)zp(0,0)+\upsilon\\&\left\{\frac{\rho_0^{N+1}\Phi(z)-\rho_0^{N+1}z^2-\rho_0^N\Phi(z)+\rho_0 z^2+\rho_0 z-z}{(1-\rho_0)(1-\rho_0 z)}\right\}p(0,0)\\&+\lambda_0\rho_0^{N-1}\Phi(z)\,p(0,0)\end{aligned} \quad (22)$$

where

$$\Phi(z)=z^N+z^{N-1}+\dots+z^2 \quad (23)$$

$$F_f(z)=\frac{(\lambda_s+r-\lambda_s z)\lambda_0}{Q(z)(1-\rho_0 z)}\left[(1-\rho_0^N)z^2-(z-1)\left\{z+\rho_0^N\Phi(z)\right\}\right]p(0,0) \quad (24)$$

For λ_s>0, discriminant Δ of the quadratic expression (19) satisfies

$$\begin{aligned}&\Delta\ge\ \lambda_s^2b^2+\lambda_s^2\mu^2+\lambda_f^2\lambda_s^2+\lambda_f^2r^2-2\lambda_f\lambda_s^2\mu-\\&2\lambda_f\lambda_s\mu r+2\lambda_f^2\lambda_s r=\lambda_s^2b^2+(\lambda_f\lambda_s+\lambda_f r-\lambda_s\mu)^2>0\end{aligned}$$

So, the equation $Q(z)=0$ has two distinct real roots Z_1 and Z_2.

In order for the steady-state queue length distribution to exist, both roots of the equation $Q(z)=0$ must be greater than 1. Since in $Q(z)$, the coefficient of z^2 is positive, the two roots of $Q(z)=0$ will be greater than 1 if $Q(1)>0$ and $Q'(1)<0$. Since $Q(1)=\mu r-\lambda_s b-\lambda_f r$, we assume that:

$$\mu r>\lambda_s b+\lambda_f r \quad \text{or} \quad \frac{\lambda_s b}{\mu r}+\frac{\lambda_f}{\mu}<1 \quad (25)$$

The above equation implies that $\mu>\lambda_f$, so if (25) holds, then

$$Q'(1)=\lambda_s(\lambda_f-\mu)-\lambda_s b-\lambda_f r<0$$

Thus, if we assume that (25) holds, then the roots Z_1 and Z_2 of $Q(z)=0$ will be greater than 1.
From (23), we have for z=1

$$\Phi(1)=N-1 \tag{26}$$

and

$$\Phi'(1)=\frac{N(N+1)}{2}-1 \tag{27}$$

Substituting Equations (15), (17) and (24) into (1), we get :

$$F(z)=\frac{\left\{\frac{1-(\rho_0 z)^N}{1-(\rho_0 z)}+\frac{\rho_0^N z^N}{1-\rho_0}\right\}Q(z)(1-\rho_0 z)+}{Q(z)} \tag{28}$$
$$\frac{(\lambda_s+r-z\lambda_s+b)\lambda_0\begin{bmatrix}(1-\rho_0^N)z^2\\ -(z-1)\left\{z+\rho_0^N\Phi(z)\right\}\end{bmatrix}}{(1-\rho_0 z)}p(0,0)$$

From (28) and the normalizing condition $F(1)=1$, we obtain:

$$p(0,0)=\frac{(\mu r-\lambda_s b-\lambda_f r)(1-\rho_0)}{\mu r-\lambda_s b-\lambda_f r+(b+r)\lambda_0(1-\rho_0^N)} \tag{29}$$

Now, assuming $\lambda_s>0$ and substituting $\alpha=1/Z_1$ and $\beta=1/Z_2$, $\therefore$ Equation (29) becomes:

$$p(0,0)=\frac{\mu(r+\lambda_s)(1-\alpha)(1-\beta)(1-\rho_0)}{\mu r-\lambda_s b-\lambda_f r+(b+r)\lambda_0(1-\rho_0^N)} \tag{30}$$

Using this in Equation (28), we obtain:

$$F(z)=R(z)\frac{(1-\alpha)(1-\beta)(1-\rho_0)}{(1-\alpha z)(1-\beta z)(1-\rho_0 z)} \tag{31}$$

where

$$R(z)=\frac{\left\{\frac{1-(\rho_0 z)^N}{1-(\rho_0 z)}+\frac{\rho_0^N z^N}{1-\rho_0}\right\}Q(z)(1-\rho_0 z)}{\mu r-\lambda_s b-\lambda_f r+(b+r)} \tag{32}$$
$$+\frac{(\lambda_s+r-z\lambda_s+b)\lambda_0\begin{bmatrix}(1-\rho_0^N)z^2\\ -(z-1)\left\{z+\rho_0^N\Phi(z)\right\}\end{bmatrix}}{\lambda_0(1-\rho_0^N)}$$

This reduces to $R(1)=1$.
In the case when there is no customer admitted in the queue during a repair process, $\lambda_s=0$. Then, Equation (19) takes the form:

$$Q(z)=\mu r(1-\rho_f z) \quad \text{where} \quad \rho_f=\frac{\lambda_f}{\mu}<1 \tag{33}$$

Substituting it into Equation (28), we get:

$$F(z)=\frac{\left\{\frac{1-(\rho_0 z)^N}{1-(\rho_0 z)}+\frac{\rho_0^N z^N}{1-\rho_0}\right\}\mu r(1-\rho_f z)(1-\rho_0 z)}{\mu r(1-\rho_f z)} \tag{34}$$
$$\frac{+(b+r)\lambda_0\begin{bmatrix}(1-\rho_0^N)z^2\\ -(z-1)\left\{z+\rho_0^N\Phi(z)\right\}\end{bmatrix}}{(1-\rho_0 z)}p(0,0)$$

Then:

$$p(0,0)=\frac{\mu r(1-\rho_f)(1-\rho_0)}{\mu r(1-\rho_f)+\lambda_0(r+b)(1-\rho_0^N)} \tag{35}$$

and

$$F(z)=R(z)\frac{(1-\rho_f)(1-\rho_0)}{(1-\rho_f z)(1-\rho_0 z)} \tag{36}$$

where

$$R(z)=\frac{\left\{\frac{1-(\rho_0 z)^N}{1-(\rho_0 z)}+\frac{\rho_0^N z^N}{1-\rho_0}\right\}\mu r(1-\rho_f z)(1-\rho_0 z)+}{\mu r-\lambda_s b-\lambda_f r+(b+r)} \tag{37}$$
$$\frac{(b+r)\lambda_0\begin{bmatrix}(1-\rho_0^N)z^2\\ -(z-1)\left\{z+\rho_0^N\Phi(z)\right\}\end{bmatrix}}{\lambda_0(1-\rho_0^N)}.$$

Equations (31) and (36) are the queue length distribution for $\lambda_s>0$ and $\lambda_s=0$ respectively.

If $\lambda_s=0$ Expression (25) becomes the necessary and sufficient condition for the queue length distribution to exist; it gives the utilization factor for M/M/1 queue which is independent from breakdown and repair rates.

For $\lambda_s>0$, the mean queue length L_q can be obtained by computing $F'(1)$ from (31) and (32)

$$L_q=\frac{\alpha}{1-\alpha}+\frac{\beta}{1-\beta}+\frac{\rho_0}{1-\rho_0}+\frac{\lambda_s\left(\lambda_f-\mu-b\right)+}{\mu r-\lambda_s b-\lambda_f r+\lambda_0} \tag{38}$$
$$\frac{\lambda_0\left\{b+r-\lambda_s\left(1-\rho_0^N\right)\right\}-\lambda_f r-\left\{\frac{\rho_0^{N+1}}{1-\rho_0}\left(\mu r-\lambda_s b-\lambda_f r\right)\right\}}{\left(b+r\right)\left(1-\rho_0^N\right)}$$

The average number of customer in the system L_s can also be obtained as:

$$L_s = L_q + \frac{\lambda_0}{\mu} + \frac{\lambda_f}{\mu} + \frac{\lambda_s}{\mu} \tag{39a}$$

Average waiting times per customer in the queue and the system are:

$$W_q = \frac{L_q}{\lambda_0} + \frac{L_q}{\lambda_f} + \frac{L_q}{\lambda_s} \tag{39b}$$

Respectively, and

$$W_s = L_s + \frac{1}{\mu} \tag{39c}$$

For $\lambda_s = 0$, the mean queue length is determined from (36) and (37) as:

$$L_q = \frac{\rho_f}{1-\rho_f} + \frac{\rho_0}{1-\rho_0} + \frac{\lambda_0 (b+r) - \mu r \rho_f - \frac{\rho_0^{N+1}}{1-\rho_0}\left\{\mu r\left(1-\rho_f\right)\right\}}{\mu r\left(1-\rho_f\right) + \lambda_0 (b+r)\left(1-\rho_0^N\right)} \tag{40}$$

And the average number of customers in the system L_s is:

$$L_s = L_q + \frac{\lambda_0}{\mu} + \frac{\lambda_f}{\mu} \tag{41a}$$

Average waiting times in the queue and in the system are:

$$W_q = \frac{L_q}{\lambda_0} + \frac{L_q}{\lambda_f} \tag{41b}$$

Respectively, and

$$W_s = L_s + \frac{1}{\mu} \tag{41c}$$

3. Special cases

If $N \to \infty$, that is when the system capacity is infinite, then our result coincide with the result obtained by Gray *et al.* [17]. The mean queue length given by (38) and (40) reduces to the following forms:

Case1: When $\lambda_s > 0$,

$$L = \frac{\alpha}{1-\alpha} + \frac{\beta}{1-\beta} + \frac{\rho_0}{1-\rho_0} + \frac{\lambda_s\left(\lambda_f - \mu - b\right) + \lambda_0\left(b + r - \lambda_s\right) - \lambda_f r}{\mu r - \lambda_s b - \lambda_f r + \lambda_0 (b+r)}$$

Case 2: When $\lambda_s = 0$,

$$L = \frac{\rho}{1-\rho} + \frac{\rho_0}{1-\rho_0} + \frac{\lambda_0 (b+r) - \mu r \rho}{\mu r (1-\rho) + \lambda_0 (b+r)}$$

4. NUMERICAL RESULTS AND INTERPRETATIONS

In this section, we provide the numerical results for various performance indices using Equations (38) and (40). For the computation purpose, we fix

For $\lambda_s > 0$

TABLE 1. N=25, λ_0 =1, λ_s=1, υ=1, r=3, b=2

	$\mu = 7$	$\mu = 7.5$	$\mu = 8$
λ_f	L_q	L_q	L_q
2	1.5000	1.4439	1.3988
3	1.7667	1.6509	1.5641
4	2.3452	2.0588	1.8667
5	4.0833	3.0693	2.5238
6	19.1667	7.2667	4.5000

TABLE 1. N=25, λ_0 =1, λ_f =2, λ_s=1, υ=1, r=3

	$b = 1$	$b = 1.5$	$b = 2$
μ	L_q	L_q	L_q
3	4.1667	5.8333	9.1667
4	2.1333	2.4444	2.8333
5	1.6667	1.8167	1.9881
6	1.4667	1.5619	1.6667
7	1.3571	1.4259	1.5000

TABLE 3. N=25, λ_0 =1, λ_f =2, λ_s=1, υ=1, b=2

	$r = 3$	$r = 3.5$	$r = 4$
μ	L_q	L_q	L_q
3	9.1667	6.8810	5.7500
4	2.8333	2.5810	2.4167
5	1.9881	1.8761	1.8000
6	1.6667	1.5976	1.5500
7	1.5000	1.4508	1.4167

For $\lambda_s = 0$

TABLE 4. N=25, λ_0 =1, υ=1, r=3, b=2

	$\mu=7$	$\mu=7.5$	$\mu=8$
λ_f	L_q	L_q	L_q
2	1.3500	1.3171	1.2899
3	1.5147	1.4505	1.4000
4	1.8333	1.6912	1.5882
5	2.5909	2.2000	1.9524
6	5.37500	3.6316	2.8182

TABLE 5. N=25, λ_0 =1, λ_f =2, υ=1, r=3

	$b=1$	$b=1.5$	$b=2$
μ	L_q	L_q	L_q
3	2.7143	2.8000	2.8750
4	1.8000	1.8571	1.9091
5	1.5128	1.5556	1.5952
6	1.3750	1.4091	1.4412
7	1.2947	1.3231	1.3500

TABLE 6. N=25, λ_0 =1, λ_f =2, υ=1, b=2

	$r=3$	$r=3.5$	$r=4$
μ	L_q	L_q	L_q
3	2.8750	2.8333	2.8000
4	1.9091	1.8800	1.8571
5	1.5952	1.5729	1.5556
6	1.4412	1.4231	1.4091
7	1.3500	1.3348	1.3231

Different system parameters are as follows:

Figure 1 displays the correlation between mean queue length (L_q) and fast arrival rate (λ_f) by varying the service rates. We also observe that the mean queue length (L_q) increases with the fast arrival rate (λ_f) whereas it decreases by increasing the service rates (μ). Figure 2 exhibits the mean queue length (L_q) by varying service rate (μ) and breakdown rate (b). It is seen that L_q decreases with the increase of service rate (μ), and L_q increases with breakdown rate (b). Figure 3 demonstrates the mean queue length (L_q) decreases with the increase of service rate (μ) and repair rate (***r***). When $\lambda_s > 0$

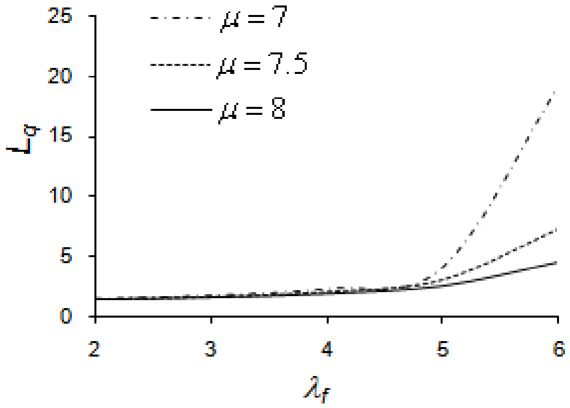

Figure 1. Fast arrival rate vs. average queue length

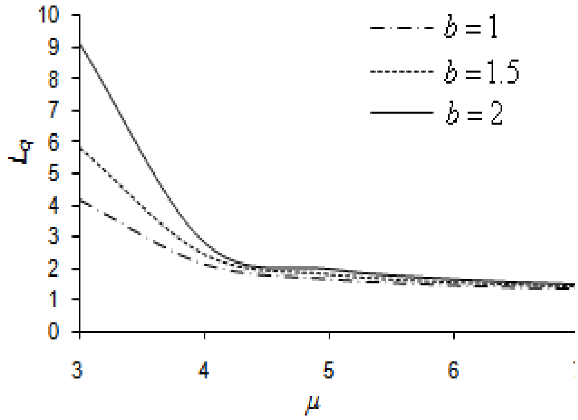

Figure 2. Service rate vs. average queue length

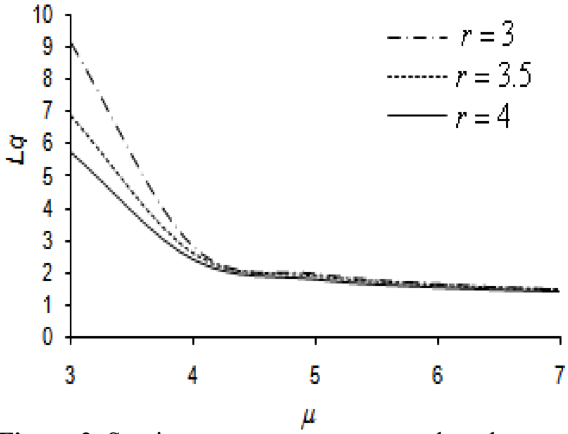

Figure 3. Service rate vs. average queue length

Figures 4, 5, 6 give the comparison of mean queue length (L_q) when no arrival during breakdown (i.e. $\lambda_s = 0$) with Figures 1, 2, 3 the mean queue length (L_q) when there may be arrivals during breakdown (i.e. λ_s >0). This comparison shows that mean queue length (L_q) in latter figures increases or decreases gradually than in former cases. When $\lambda_s = 0$,

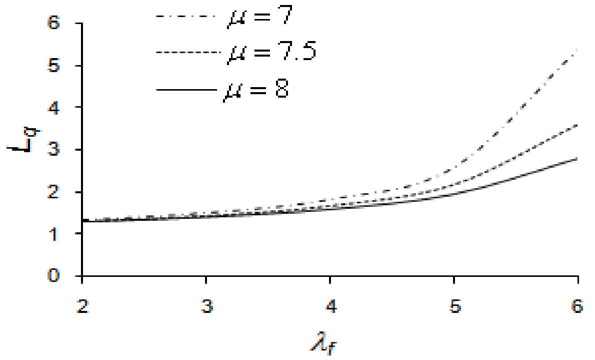

Figure 4. Fast arrival rate vs. average queue length

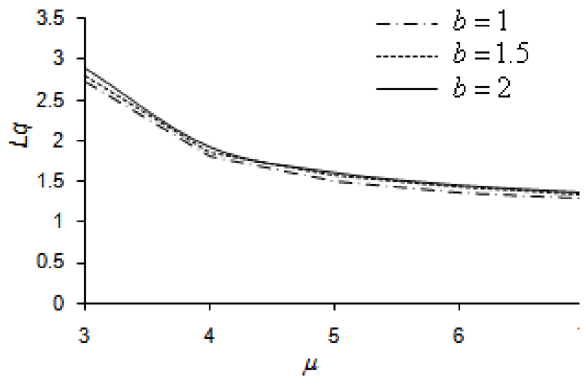

Figure 5. Service rate vs. average queue length

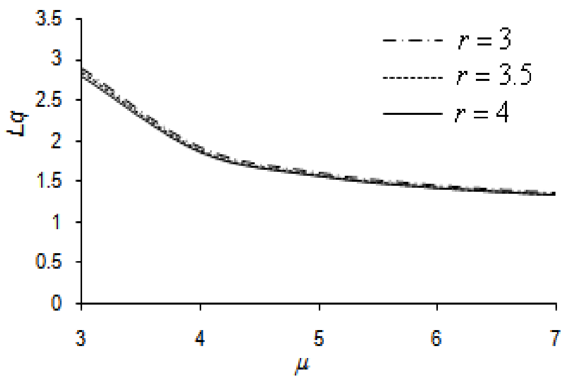

Figure 6. Service rate vs. average queue length

5. DISCUSSION

By using partial generating function method we have obtained various performance measures for a single server, finite capacity queueing system with the provision of vacations and server breakdowns. The imposition of various arrival rates to the system may have the applicability in the real life queueing system in which arrival rates can be varied so as to reduce sufficient queue length. Proposed model may have potential application in the telecommunication system, computer communication systems, machining system, etc.

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