A SIMPLE APPROACH FOR DETERMINATION OF ACTUATOR AND SENSOR LOCATIONS IN SMART STRUCTURES SUBJECTED TO THE DYNAMIC LOADS

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Abstract The present work demonstrates the successful application of a simple active vibration control procedure based on structural dynamics. Based on theories of mathematical and structural dynamics, the appropriate locations of sensor and actuator of smart structure were predicted. Also, the optimum value of actuator force which controls the structural vibrations is quickly formulated so that the first damping coordinates becomes critical. The validity and efficiency of the proposed method have been investigated by active vibration suppression of some classic structural models. The results showed the ability of suggested active control processes in suppression of the unwanted structural vibrations.

Keywords Active Vibration Control, Sensor and Actuator Locations, Smart Structures.

چکیده در این مقاله فرایند نوینی برای کنترل فعال سازه‌ها ارائه می‌گردد. با استفاده از این راهکار که بر پایه نگره‌های دینامیک سازه‌ها استوار است، می‌توان محل حسگر و عملگر و نیز نیروی لازم برای کنترل نوسانها را تعیین کرد. در اینجا، عملگر همانند یک میراگر اضافی برای سازه می‌باشد و نیروی آن به گونه‌ای رابطه سازی می‌شود که میرایی نخستین مود نوسان سازه بحرانی گردد. در نتیجه، مقدار بهینه نیروی عملگر در هر لحظه به دست می‌آید. از سوی دیگر، با استفاده از نگره‌های ریاضی و دینامیک سازه، بهترین محل نصب عملگر و حسگر بر روی سازه مشخص می‌شود. این کار راه را برای کنترل فعال سازه‌ها و از بین بردن نوسان‌های آنها در کوتاه‌ترین زمان هموار می‌سازد. برای سنجش کارایی روش پیشنهادی، چند سازه برشی با شیوه پیشنهادی کنترل می‌شوند و نتایج مقایسه می‌گردند. این تحلیلها توانایی روش پیشنهادی را برای کنترل نوسانهای سازه‌ها آشکار می‌سازد.

1. INTRODUCTION

The essential concept of smart/intelligent structures is embeding the actuators and sensors within the structures to adapt structural geometries characteristics (i.e. stiffness, damping and dimensions) in order to respond properly to the environmental or external causes [1, 2]. There are two main control strategies; passive and active/semi-active control. The passive controllers are simple to implement. These controllers remove energy from the system that do not cause any instability in structure. Tuned mass damper (TMD) system is a well-known passive control model [3-6], which has been used in several structures such as suspension bridges [7], offshore platforms [8], etc. This device consists of a mass, spring and damper that are attached to the structure which reduce the dynamic response of the structure. In active control systems, control forces are generated and applied by an external source (actuator) to the structure. These systems have a strong capacity to control wide range of vibration modes and are widely accepted as alternative to passive control systems [9]. There is a potential for introducing instability if the applied energy to the structure from the active control algorithm is not suitable and consistent with the response of the structure. The location of the actuators and the value of applied force are important factors, which highly affect the efficiency of the active control process. An extensive study has been developed on optimum locations of actuator and sensors with cost functions have been considered [10-16]. On the other hand, the structural frames are the most common case studies for verifying the ability and the efficiency of the control processes. For example, the vibrations of multi-story buildings have been controlled by different algorithms such as semi-active control process [17], dynamic fuzzy method [18], hybrid passive control [19] and multi-objective genetic algorithms [20].

In this study, a new simple active vibration control scheme is presented. According to this procedure, the equivalent actuator force was calculated. Furthermore, the most effective and suitable locations of the actuator and sensor were obtained. The validity and efficiency of the present procedure were confirmed by investigating on vibration control of some classic structures.

2. EQUIVALENT ACTUATOR FORCE

Active control of smart structures deals with determining the optimum value of the applied actuator force, the most suitable sensors and actuators locations. Different approaches, such as optimization process, have been used to achieve this goal. In this study, the structural dynamics theories were utilized to determine the optimum actuator force and proper locations of sensors and actuators. Using Newton's second law, Lagrange's [21] or Hamilton's principles [22] the dynamic equilibrium equation of a system constructed as follows:

$$[M]^n\{\ddot{D}\}^n + [C]^n\{\dot{D}\}^n + [S]^n\{D\}^n = \{P(t^n)\} \quad (1)$$

where $[M]^n$, $[C]^n$, $[S]^n$ and $\{P(t^n)\}$ are mass, damping, stiffness matrices and external forces vector at n^{th} time step, respectively. Furthermore, $\{D\}^n$ is the nodal displacement vector and dots $(\dot{})$ denotes differential with respect to time. The necessary actuators forces and the locations of the sensors and actuators are major unknown parameters in active control systems. In such systems (smart structures), the dynamic equilibrium equation is incorporated into the following equation:

$$[M]^n\{\ddot{D}\}^n + [C]^n\{\dot{D}\}^n + [S]^n\{D\}^n + \{f^a\} = \{P(t^n)\} \quad (2)$$

where, $\{f^a\}$ is the equivalent actuator force vector which is applied to the structure. Each actuator applies an equivalent force to the corresponding degree of freedom which is attached to the structure. In this study, for simplicity, it is assumed that the smart structure has only one actuator and one sensor. The active control process with more than one actuator and sensor is the goal of future investigations. There are two important questions; what is the optimum value of the actuator force? Where are the effective locations for the actuators and sensors? To answer these questions, the actuator is considered as a damper. From structural dynamics point of view, the structural vibrations damp in the shortest possible time if the structure is in critical damping condition. Hence, the critical damping theory determines the equivalent actuator force and locations of the sensor and actuator. To achieve that concept, Eq. (2) is transformed to the modal space as follows:

$$M_i\ddot{Z}_i + C_i\dot{Z}_i + S_iZ_i + \{\varphi_i\}^T\{f^a\} = \{\varphi_i\}^T\{P(t)\} \qquad i = 1,2...q \quad (3)$$

where M_i, C_i and S_i are i^{th} mass, damping and stiffness of Z_i modal coordinates, respectively. Also, q is the number of degrees of freedom and $\{\varphi_i\}$ is the i^{th} mode shape vector of free vibration of the structure. When the mode's rank in Eq. (3) increases, its effect on total dynamic response of the structure decreases. Therefore, the first mode has the largest effect on the response compared to the other vibration modes. In addition, if the actuator is attached to the structure such a way that the first mode is critically damped, the vibrations are quickly diminished. This principle is utilized to obtain the equivalent actuator force. Assume that actuator is attached to the k[th] degree of freedom; the first modal coordinates can be written as follows:

$$M_1\ddot{Z}_1 + C_1\dot{Z}_1 + S_1 Z_1 + \varphi_{k1} f_k^a = \{\varphi_1\}^T \{P(t)\} \quad (4)$$

where φ_{k1}, the k[th] element of the first modal, is shape vector and f_k^a is the equivalent actuator force attached to k[th] degree of freedom. Eq. (4) presents the first modal coordinates of the smart structure. Since the actuator acts as an additional viscous damper, Eq. (4) is transformed to the following equation:

$$M_1\ddot{Z}_1 + C_1^*\dot{Z}_1 + S_1 Z_1 = \{\varphi_1\}^T \{P(t)\} \quad (5)$$

where C_1^* is the first equivalent modal coordinates damping which is generated from the first modal coordinates damping and the equivalent damping of the actuator as follows:

$$C_1^* = C_1 + \frac{\varphi_{k1} f_k^a}{\dot{Z}_1} \quad (6)$$

The equivalent actuator force is obtained when the first equivalent modal coordinates damping (C_1^*) is equal to the critical damping i.e.: $2M_1\omega_1$, where ω_1 is the lowest natural frequency of the structure.

$$C_1^* = 2M_1\omega_1 \quad \Rightarrow \quad f_k^a = \frac{2M_1\omega_1 - C_1}{\varphi_{k1}} \dot{Z}_1 \quad (7)$$

where $\dot{Z}_1$ is the first modal velocity. Eq. (7) presents the equivalent actuator force which is required to apply to k[th] degree of freedom. This force causes the critical damping for the first modal coordinates of the structure. Since there is only one sensor in the smart structure, the first modal coordinates velocity can be calculated based on definition of the modal velocity [23]:

$$\dot{Z}_1 \approx \varphi_{1L}^{inv} \dot{D}_L \quad (8)$$

where φ_{1L}^{inv} and $\dot{D}_L$ are L[th] element in the first row of the inverse modal shape matrix and the velocity of L[th] degree of freedom which the sensor is attached to it. By substituting Eq. (8) in Eq. (7), the equivalent actuator force is obtained:

$$f_k^a \approx \frac{\varphi_{1L}^{inv}}{\varphi_{k1}} (2M_1\omega_1 - C_1) \dot{D}_L \quad (9)$$

The equivalent actuator force is applied to k[th] degree of freedom i.e. k[th] element of $\{f^a\}$. Other elements of this vector are zero; because there is only one actuator used in this structure. As a result, the actuator force from Eq. (9) will depend on the locations of sensor (L[th] degree of freedom) and actuator (k[th] degree of freedom) which should be determined in the proposed active control algorithm.

3. ACTUATOR AND SENSOR LOCATIONS

Eq. (9) represents the equivalent actuator force as a function of two parameters, φ_{1L}^{inv} and φ_{k1}. These two parameters (φ_{1L}^{inv} and φ_{k1}) depend on the locations of sensor and actuator, respectively. Choosing appropriate locations for the actuator and sensor, leads to a high performance active control process. For this purpose, following concepts are utilized to verify the actuator and sensor locations.

Based on mathematical and structural dynamics' theories, *the actuator should be attached to the degree of freedom which has the highest value in the first modal shape vector.* The effect of each degree of freedom is distinguished by its corresponding element in the mode of shape vector. The elements with high value in mode shape vector have significant effect on the selected response mode. Hence, the degree of freedom with maximum value in the first mode shape is selected for the actuator location, i.e.: $(\varphi_{k1})_{max}$.

On the other hand, *the sensor should be attached to the degree of freedom which has the highest value in the first row of the inverse modal shape matrix.* Based on the definition of modal velocity in structural dynamics theories [23], each velocity is multiplied by the corresponding element in the first row of $[\Phi]^{-1}$, which is the inverse modal shape matrix. The velocities are part of the structural response and depend on the external effects which cause vibrations. Therefore, the only available parameters for determining the sensor location are elements of the first row of the inverse modal shape matrix. From the mathematical points of view, the degrees of freedom with high values in the first row of $[\Phi]^{-1}$, have the great effect on the first modal velocity($\dot{Z}_1$). If this degree of freedom is selected for the sensor's location, $\dot{Z}_1$ is calculated with good approximation.

After determination of sensor and actuator locations using the above procedure, the actuator force is calculated from Eq. (9). In the next section, vibrations of some typical systems are actively controlled using the proposed procedure.

4. NUMERICAL EXAMPLES

To verify the validity of the proposed active control method, two numerical examples are presented. For this purpose, the suggested control process is combined with numerical dynamic analysis methods. Then, high order predictor-corrector time integration (PC-m) is utilized [24]. In this approach, accelerations of several previous time steps with high accuracy were used and the stability were compared with other time integration methods [24]. Another reason to choose this method was its simplicity. The method was experimented by few vector computational efforts. Based on numerical time integration scheme, the main steps for numerical active control process of smart structures are as follows:

1. Set n=0 and select the time step of dynamic analysis (Δt).
2. Construct the stiffness, mass and damping matrices, i.e.: $[S]$, $[M]$ and $[C]$, respectively.
3. Determine the actuator and sensor locations using proposed method, i.e. select the φ_{1L}^{inv} and φ_{k1} .
4. Start the dynamic analysis at time $t^n = n\Delta t$.
5. Compute the displacement ($\{D\}^n$) and velocity ($\{\dot{D}\}^n$) vectors at time t^n from the higher order predictor-corrector integration.
6. Choose the velocity of the sensor's location from the calculated velocity vector.
7. Calculate the equivalent actuator force from Eq. (9).
8. Compute the acceleration vector of n^{th} time step by solving the following linear system:

$$[M]\{\ddot{D}\}^n = \{P(t^n)\} - [C]\{\dot{D}\}^n - [S]\{D\}^n - \{f^a\} \tag{10}$$

9. set n=n+1
10. If $n<n_{max}$ go to 4.
11. End and print the results.

The developed procedure for finding the locations of sensor and actuator is coded in a program. This scheme is used to control vibrations of two generic structures, 2DOF system and five-story shear building subjected to harmonic and explosion loads.

4.1. System With Two Degrees Of Freedom Figure 1 shows a linear two-DOF system which idealizes a two-story frame. Since this example has been previously analyzed by Karagulle and Malgaca [25], the results validate the above program.

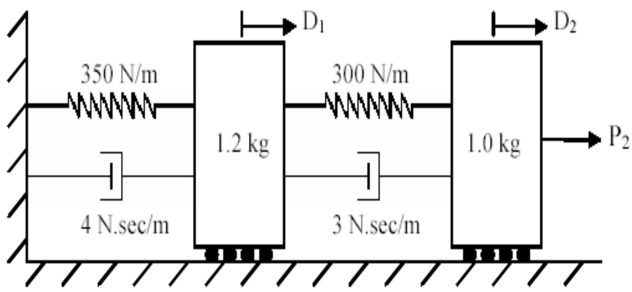

Figure 1. Two degrsee of freedom system (2-D)

Here P_2 is the external load which is defined by the following relation [25]:

$$P_2 = \begin{cases} 1/\Delta/ & t = \Delta t \\ 0 & \text{otherwise} \end{cases} \quad (11)$$

where Δt is the time step. The time step of 0.0039 second [25] is used to integrate high order predictor-corrector. Here, the PC-6 method which uses six previous accelerations is utilized for numerical dynamic analysis [24]. The procedure leads to optimum sensor and actuator locations; are summarized in Table 1. The first choice of degrees of freedom for locations of the actuator and sensor were determined. By applying the proposed method, the second degree of freedom is the optimum location for both the actuator and sensor (S2-A2), due to its highest corresponding values of $\varphi_{12}^{\text{inv}}$ =0.8167 and φ_{21} =0.8594. It is clear that the proposed method which presents these locations is quite simple, compared to active control processes with application of optimization approaches [13-15]. Other possible control choices, i.e. S1-A1, S1-A2 and S2-A1 are also considered for desired verification of the proposed case (S2-A2).

TABLE 1. The Actuator and Sensor Locations for Two Degrees of Freedom System

Degree of Freedom	$\varphi_{1L}^{\text{inv}}$	φ_{k1}	Optimum Control Case
1	0.5831	0.5113	S2-A2
2	*0.8167*	*0.8594*	

After determination of the sensor and actuator locations, the active vibration control is applied to the structure. Figures 2 and 3 depict displacement-time responses of first and second degrees of freedom for different control process, respectively. It is clear that all control methods are stable. On the other hand, the main difficulty which may happen in any active control algorithm did not appear in the proposed method. Eq. (9) presents a consistent value for the actuator force. For the control case S2-A2, the vibration damped in about 0.5 second. It is worth to note that the required time for vibration control of this system is about 1.5 seconds while ANSYS software is used [25]. Therefore, the suggested control process is more effective. If the actuator and sensor are attached to the first degree of freedom (case S1-A1), the efficiency of the control process is reduced. Also, the control case S1-A2 has lower efficiency than the two other cases. In fact S2-A1 is incapable of controlling the system. As a result, the proposed analytical algorithm presented for choosing the locations of the sensor and actuator and the actuator force has a good consistency with numerical results.

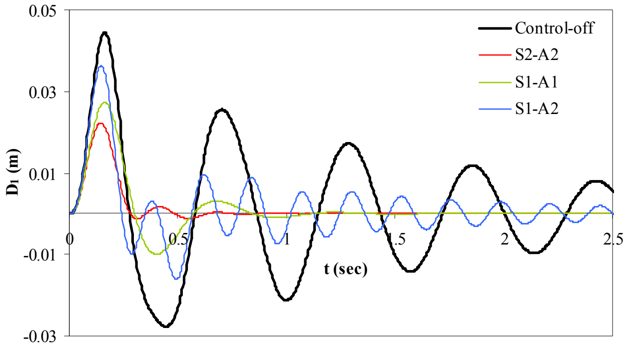

Figure 2. The displacement-time response to the first degree of freedom of 2-D system

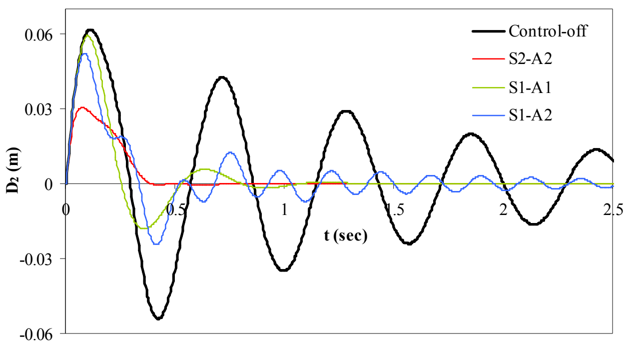

Figure 3. The displacement-time response to the second degree of freedom of 2-D system

4.2. Five-Story Shear Building A five-story shear building is shown in Figure 4. This structure is modeled by lumped mass and lateral stiffness (five horizontal degrees of freedom). A damping ratio of 5% for the first mode is assumed for constructing the Rayleigh damping matrix with two factors [23]. To control the vibration of the structure, the location of the sensor and actuator is determined based on the proposed algorithm. Table 2 presents the details which lead to the optimum locations of sensor and actuator (S4-A5), the sensor and actuator should be attached to the 4th and 5th degrees of freedom, respectively. Other possible cases such as S4-A4, S5-A5, S5-A4, S3-A4 and even S3-A5 are also investigated to demonstrate the validity of the proposed procedure.

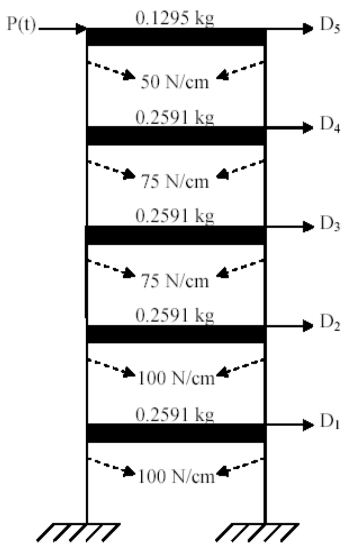

Figure 4. Five-story shear building

TABLE 2. The Actuator and Sensor Locations for Five Story Shear Building

Degree of Freedom	φ_{1L}^{inv}	φ_{k1}	Optimum Control Case
1	0.1922	0.1565	S4-A5
2	0.3673	0.2991	
3	0.5576	0.4541	
4	*0.6821*	0.5555	
5	0.3741	*0.6093*	

The formulation of the proposed control method is independent of loading condition. The structure was analyzed for two different loading conditions: harmonic and explosion loads. In the first analysis, the structure is excited by the following sinusoidal harmonic load which is applied to the top of the building;

$$\mathrm{P(t)} = 50\sin(10t) \tag{12}$$

For numerical analysis, time step of 0.001 second is considered. Furthermore, the high order predictor-corrector time integration i.e. the PC-6 algorithm is utilized for dynamic analysis [24]. The displacement-time responses of each degree of freedom have been plotted in Figures 5 to 9. Both uncontrolled and different configurations of sensor and actuators are shown and compared in these figures. Similar to the previous example, there is no numerical instability in the proposed control processes. Also, the efficiency of S4-A5 and S4-A4 cases were considerably higher than other configurations (S5-A4, S5-A5, S3-A5 and S3-A4). Moreover, Figure 9 shows that vibration of the top of this structure (5th degree of freedom i.e. D_5) is controlled by S4-A5 configuration which was more effective than S4-A4. However, these algorithms (S4-A5 and S4-A4) lead to the same results for vibration control of the other degrees of freedom (Figures 5 to 8). An additional significant result is that if both sensor and actuator were attached to the top of the frame (case S5-A5), the control efficiency was reduced. Therefore, the proposed control method is consistent with the obtained numerical results.

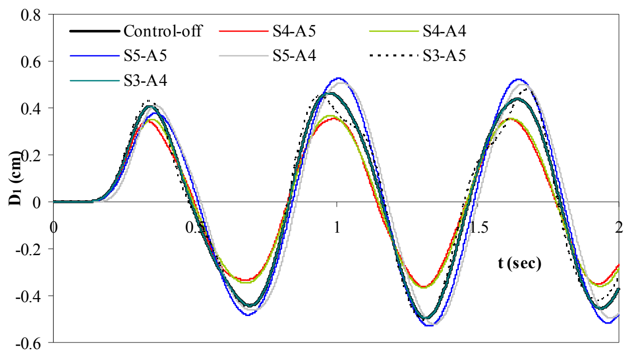

Figure 5. The displacement-time response to the first floor of shear building under the harmonic load

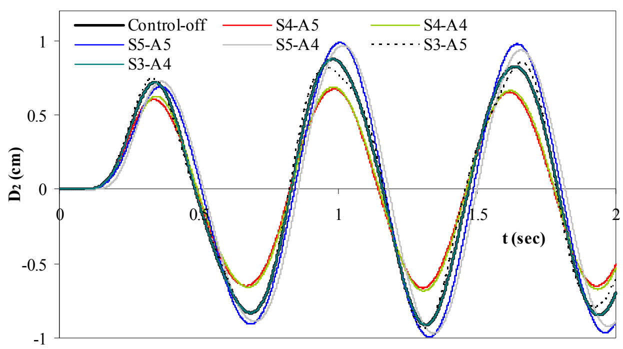

Figure 6. The displacement-time response to the second floor of shear building under the harmonic load

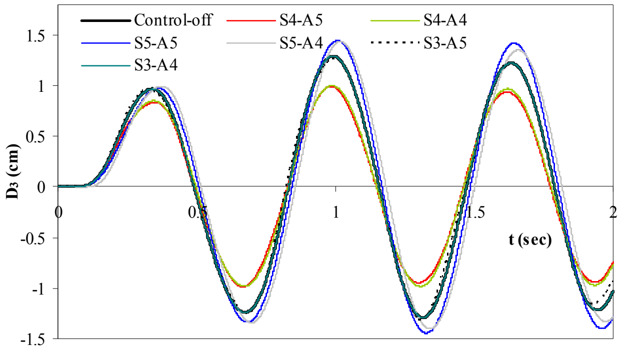

Figure 7. The displacement-time response to the third floor of shear building under the harmonic load

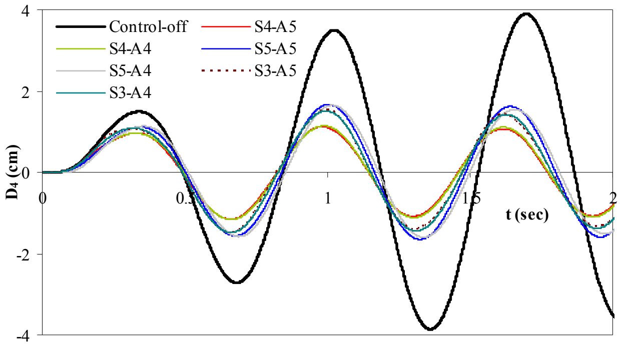

Figure 8. The displacement-time response to the forth floor of shear building under the harmonic load

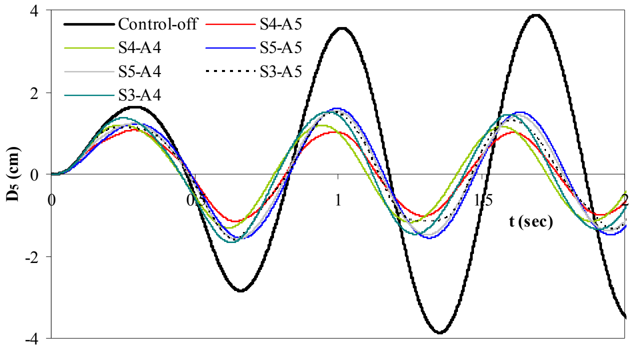

Figure 9. The displacement-time response to the fifth floor of shear building under the harmonic load

In the second stage of analysis, the structure is excited by an explosion load which was applied to the top of the building for the duration of one second:

$$P(t) = \begin{cases} 200t & 0 \le t \le 0.5 \\ 200(1-t) & 0.5 \le t \le 1 \end{cases} \quad (13)$$

The displacement-time response to first, second, third, fourth and fifth floors have been plotted in Figures 10 to 14 for different sensor actuator configurations. It is clear that the efficiency of S4-A5 and S4-A4 cases was higher than other schemes (S5-A4, S3-A5, S3-A4 and S5-A5). In the given analysis, S4-A5 and S4-A4 cases have approximately the same efficiency. Moreover, it is clear that by attaching the sensor and actuator to the top of the frame (case S5-A5), the control efficiency has reduced.

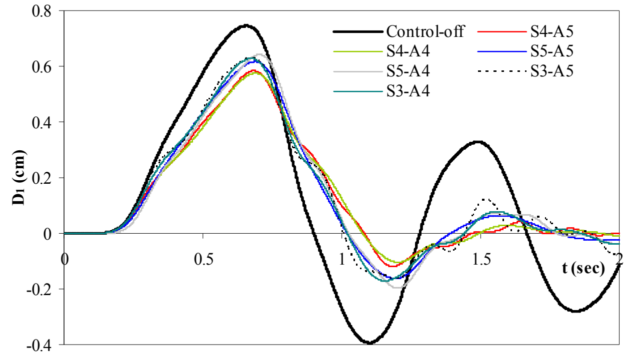

Figure 10. The displacement-time response to the first floor of shear building under the explosion load

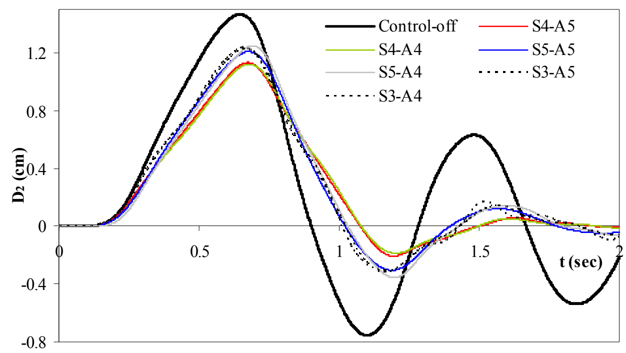

Figure 11. The displacement-time response to the second floor of shear building under the explosion load

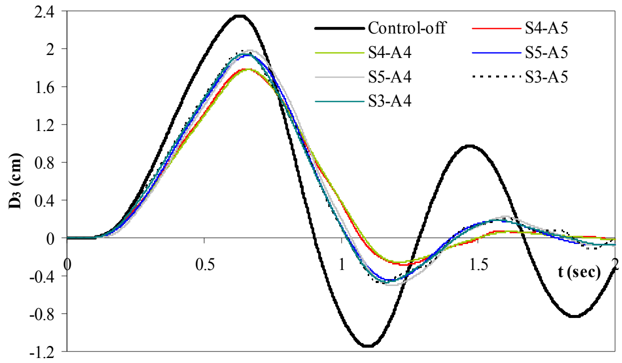

Figure 12. The displacement-time response to the third floor of shear building under the explosion load

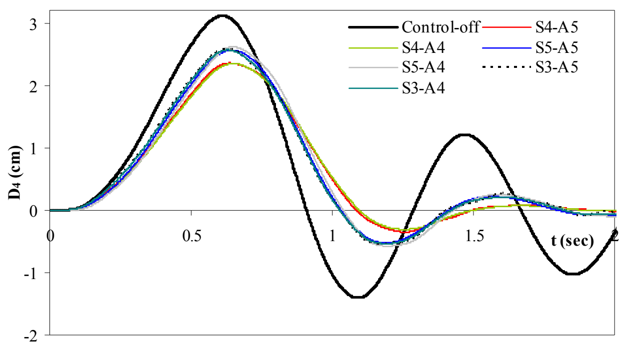

Figure 13. The displacement-time response to the forth floor of shear building under the explosion load

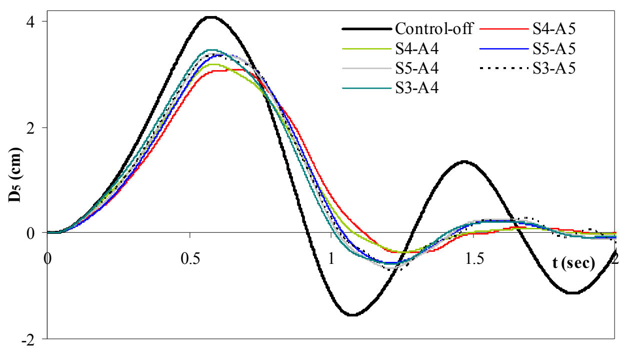

Figure 14. The displacement-time response to the fifth floor of shear building under the explosion load

These analyses clearly showed that the proposed method for structural control has good efficiency and consistency with different load conditions, harmonic and explosion loads, which were considered for this structure. As a result, the suggested control process is independent of the loading conditions.

5. CONCLUSION

A simple general purpose scheme of analysis for sensor and actuator locations is presented in this study using structural dynamic theories. Due to high effect of the first modal shape on the dynamic response, the equivalent actuator force was formulated so that the first damping coordinates is critical. Therefore, the actuator has been modeled as an additional viscous damper. Furthermore, the actuator and sensor are attached to the degrees of freedom which have the highest corresponding values in the first modal shape vector and the first row of the inverse modal shape matrix, respectively. The numerical verification of the vibrations of some classic models (known as shear buildings) has been controlled using proposed technique. These analyses showed that the suggested active control process is stable; so that it can be utilized for control systems without any concern about the instabilities in some active controllers. The presented numerical results also illustrated the efficiency and validity of the proposed simple scheme. Moreover, the vibrations are damped in few periods of time and the suggested method required less control time than the predicted values by ANSYS software. Another important point is that the suggested control process is independent of dynamic loading, so that it successfully controls the free vibration and force vibrations created by harmonic and explosion loadings.

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