NUMERICAL MODELING OF TWO-LAYERED MICROPOLAR FLUID THROUGH AN NORMAL AND STENOSED ARTERY

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Abstract In the present work a two fluid model for blood flow through abnormally constricted human artery (stenosed artery) has been developed. The model consists of a core region of suspension of all erythrocytes assumed to be micro-polar fluid so as to include the micro-structural effects in addition to the peripheral-layer viscosity effects, and a peripheral plasma layer free from cells of any kind of Newtonian fluid. This model is used to predict the effects on physiological characteristics of blood flow in normal and stenosed condition. The significance of the present model over the existing models has been pointed out by comparing the results with other theories both analytically and numerically.

Keywords Peripheral Layer, Micro-Polar Fluid, Apparent Viscosity, Stenosed Artery, Newtonian Fluid, Atherosclerosis, Resistance to Blood Flow

چکیده در این تحقیق یک مدل دو سیالی برای جریان خون از میان شریان آئورتی مسدود ارائه شده است. مدل از ناحیه هسته ای سوسپانسیون اریتروسیت ها که میکروپلار فرض شده اند تشکیل شده است تا علاوه بر تاثیرات ویسکوزیته لایه جانبی و یک لایه جانبی پلاسما که فاقد هر گونه سلول سیال نیوتونی می باشد، تاثیرات میکروساختاری را نیز در بر گیرد. این مدل به منظور پیش بینی تاثیرات بر روی خواص فیزیولوژیکی هر گونه جریان خون در شرایط طبیعی منقبض شده مورد استفاده قرار گرفته است. اهمیت مدل حاضر در برابر مدل های موجود با مقایسه نتایج با سایر تئوری های تحلیلی و عادی مورد تاکید قرار گرفته است.

1. INTRODUCTION

Blood is a suspension of red cells in plasma; a micropolar fluid may represent it. The experimental results by several mathematical models Eringen [1], Valanis [2], Aroesty [3], Thurston [4], Iida [5], have been proposed to study the various aspect of microcirculation. To explain the observed Fahraeus-Lindquist effect, Haynes [6], has considered a two-fluid model with both fluids as Newtonian fluids and with different viscosities, i e., the peripheral layer with the viscosity of plasma and the core with the viscosity equivalent to shear viscosity of blood. The authors Haynes [6], Bugliarello, et al [7], have assumed that either both layers, i.e., peripheral layer of plasma and core region, are of Newtonian fluid or both layers are of non-Newtonian fluids. This seems to be improper because it has been shown experimentally by Bugliarell et al [7], Cokelet [8], that plasma is a Newtonian fluid and core region fluid behaves like a non-Newtonian fluid. Bugliarell et al [7] have proposed that blood flow through an artery in smaller diameter consists of peripheral plasma layer which, being cell-free, is Newtonian in character and a core of red cell suspension in plasma. The effects of stenosis are much more important in microcirculation where peripheral layer thickness and viscosity effects dominate the flow characteristics. Tondon, et al [9], have investigated the effects of this peripheral layer in microcirculation, and found that the viscosity of the peripheral layer fluid is two to three times higher in the diabetic patients; these subjects are more prone to such diseases.

Shukla, et al [10], have studied the effect of stenosis on the resistance to flow through artery by considering the behaviour of blood as a power-law fluid and a Casson fluid. In these investigations the core-region and peripheral layer fluids are represented by two Newtonian fluids of different viscosities and independent of each other. Again Shukla, et al [11] have considered a two-layer model in which the peripheral plasma layer and the core are both Newtonian in character. In this analysis they have studied the influence of peripheral layer viscosity on the resistance to blood flow through a stenosed artery. Tondon, et al [12] have discussed that the blood flow in the peripheral layer is neither a Newtonian nor a non-Newtonian fluid but it is actually the suspending medium of red cells. A theoretical model for sedimentation of red cell aggregates in narrow horizontal tubes has proposed by Secomb, et al [13] in which they modelled the core region as a solid cylinder moving inside the tube. Murata [14] has proposed a sedimentation model in which he considered constant values of hematocrit and Newtonian viscosity in the circular core region, containing red cell aggregates. Murata [15] has also considered a two-layer sedimentation model with flat interface between the plasma layer and red cell layer. The experimental study of Cabel, et al [16] conducted the venous resistance for normal blood decreased by approximately 60 % when blood flow was increased 4-fold from 5 ml/(min.100 g tissue) to 20 ml/(min. 100 g tissue), and increased by approximately 70 % when blood flow was decreased 5-fold from its normal level to 1 ml/(min. 100 g tissue). For many disorders, like heart disorder, myocardial infarction, cerebrovascular disease, stroke, hypertension, Chien [17] has proposed that the agreeability and rigidity of red cells is higher than their normal value. Lerche [18] has used an empirical equation to describe smooth axisymmetric hematocrit profiles in power law form. Both hematocrit distributions have derived by assuming zero peripheral layer thickness. In this model the suspension of erythrocytes in the core region is assumed to be micropolar fluid and peripheral plasma layer is treated as Newtonian fluid.

2. ANALYSIS OF THE PROBLEM

Consider the axisymmetric flow of blood in a uniform circular tube with an axially non-symmetric but radially symmetric mild stenosis. The geometry of the stenosis, in Figure 1, is represented by

$$\frac{R(z)}{R_0} = 1 - A[L_0^{(m-1)}(z-d) - (z-d)^m], \qquad d \le z \le d + L_0$$
$$= 1, \qquad \text{otherwise}, \tag{1}$$

where R(z) is the radius of the artery with stenosis, R_o is the constant radius of the artery. L_o is the stenosis length and d indicates the stenosis location, and $m \geq 2$ is a parameter determining the stenosis shape and is referred to as stenosis shape parameter. Here axially symmetric stenosis occurs when m = 2, the parameter A is given by:

$$A = \frac{\delta}{R_0 L_0^m} \frac{m^{m/(m-1)}}{(m-1)},$$

where δ denotes the maximum height of stenosis at $z = d + L_0 / m^{1/(m-1)}$. $\delta R_0 << 1$.

The function $R_1(z)$ representing the radius of the artery in central layer, and the geometry of the stenosis in central region is given by, Figure 1

$$\frac{R_1(z)}{R_0} = \alpha, - A_1[L_0^{(m-1)}(z-d) - (z-d)^m], \qquad d \le z \le d + L_0$$
$$= \alpha, \qquad \text{otherwise}, \tag{2}$$

$$A_1 = \frac{\delta_1}{R_0 L_0^m} \frac{m^{m/(m-1)}}{(m-1)}$$

Where: δ_1 denotes the maximum bulging of interface at $z = d + L_0/m^{1/(m-1)}$ due to the presence of stenosis and α is the ratio of the central core radius to the tube radius in the unobstructed region.

3. FORMULATION OF THE PROBLEM

Consider a steady, laminar and fully developed flow of blood through a rigid circular tube of radius R. Based on the experimental results of Bugliarello, et al [7], the blood flow can be represented by a two-fluid model with core of micropolar fluid of radius R_1 and peripheral layer of plasma as a Newtonian fluid of thickness $(R–R_1)$ as shown in Figure 1. Let μ_1 be the viscosity of

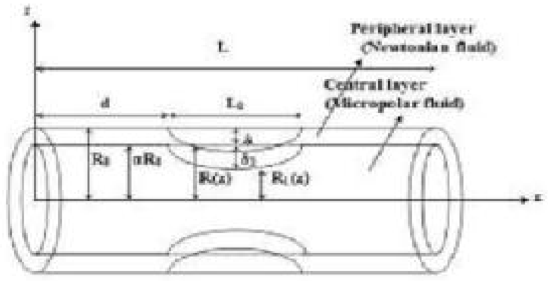

Figure 1. Geometry of stenosed artery with peripheral layer.

Newtonian fluid in the peripheral plasma layer and μ_2 be the shear viscosity of blood in the core region.

The equations for steady, one-dimensional velocity and cell rotation are given by,

$0 \le r \le R_1$

$$\left(\mu_2+\mu_R\right)\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V_2}{\partial r}\right)+2\mu_R\left(\frac{1}{r}\frac{\partial(r\omega_2)}{\partial r}\right)-\frac{\partial P}{\partial z}=0$$

$$\gamma\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(r\omega_2)}{\partial r}\right)-2\mu_R\left(\frac{\partial V_2}{\partial r}\right)-4\mu_R\omega_2=0 \qquad (3)$$

and for $R_1 \le r \le R$,

$$\mu_1\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V_1}{\partial r}\right)-\frac{\partial P}{\partial z}=0 \qquad (4)$$

$$\omega_1=-\frac{1}{2}\frac{\partial V_1}{\partial r},$$

where $\partial P/\partial z$ is the constant pressure gradient, (V_1,ω_1) and (V_2,ω_2) are the velocity and cell rotation in the region $R_1 \le r \le R$ and $0 \le r \le R_1$ respectively. $(\mu_R/2)$ is relative rotational viscosity and γ is viscosity of gradient of total rotation (4) which is assumed to be zero for Newtonian fluid.

These boundary conditions have been used to solve Equations 3 and 4:

$V_1 = 0$ at $r = R$

$$V_1 = V_2,\ \tau_1=\tau_2,\ \frac{1}{r}\frac{\partial(r\,\omega_2)}{\partial r}=0 \quad \text{at} \quad r = R_1 \qquad (5)$$

$W_2 = 0$ and V_2 is finite at $r = 0$,

where τ_1 and τ_2 are stresses at $r = R_1$.

The expression for velocities V_1, V_2 and cell rotations ω_1 and ω_2 obtained as the solution of Equation 3 and 4 with boundary condition (5) are:

For $R_1 \le r \le R$,

$$V_1=\frac{1}{4\mu_1}\left(-\frac{dp}{dz}\right)(R^2-r^2) \qquad (6)$$

$$W_1=\frac{1}{4\mu_1}\left(-\frac{dp}{dz}\right)r$$

and for $0 \le r \le R_1$

$$V_2=\frac{1}{4\mu_1}\left(-\frac{dp}{dz}\right)\left[R^2-R_1^2+\mu^1\left(R_1^2-r^2+\frac{4\mu_R R^2 M}{(\mu_2+\mu_R)}\right)\right] \qquad (7)$$

$$M=\left(\frac{I_0(l\,r/R)-I_0(l\,R_1/R)}{l^2 I_0(l\,R_1/R)}\right)$$

$$W_2=\frac{1}{4\mu_2}\left(-\frac{dp}{dz}\right)\left[r-\left(\frac{r\,l\,I_0(l\,R_1/R)-2R I_1(l\,r/R)}{l\,I_0(l\,R_1/R)}\right)\right]$$

The total flux, Q is

$Q = Q_1 + Q_2$

and Q is written as:

$$Q=\frac{\pi P}{8\mu_1}[B] \qquad (8)$$

$$B=\left[(\mu^1-1)R_1^4+R^4+\frac{8\mu_R\mu^1 R_1 R^2}{(\mu_2+\mu_R)}\left(\frac{2I_1(lR_1/R)-R_1 I_0(lR_1/R)}{\lambda^2 I_0(lR_1/R)}\right)\right]$$

Where

$$Q_1=\int_{R_1}^{R}2\pi r V_1\,dr=\frac{2\pi p}{4\mu_1}[B_1]$$

$$B_1=\left[\frac{R^4}{4}+\frac{R_1^4}{4}-\frac{R^2R_1^2}{2}\right]$$

$$Q_2=\int_{0}^{R_1}2\pi r V_2\,dr=\frac{2\pi p}{4\mu_2}[B_2]$$

$$B_2=\left[\frac{R^2R_1^2}{2}-\frac{R_1^4}{2}+\mu^1\left(\frac{R_1^4}{4}+\frac{4\mu_R R^2}{(\mu_2+\mu_R)\lambda^2}\left(\frac{2R_1I_1(lR_1/R)-R_1^2I_0(lR_1/R)}{2I_0(lR_1/R)}\right)\right)\right]$$

where $\mu^1=\frac{\mu_1}{\mu_2}$,

Since for small x the modified Bessel function can be approximated as:

$$I_0(x)=1+\frac{x^2}{4}$$

and $$I_1(x)=\frac{x}{2}+\frac{x^3}{16}$$

From Equation 8 the pressure gradient is written as follows:

$$P=\frac{8\mu_1 Q}{\pi M} \tag{9}$$

$$M=\left[(\mu^1-1)R_1^4+R^4+\frac{8\mu_R\mu^1R_1R^2}{(\mu_2+\mu_R)}\left(\frac{2I_1(lR_1/R)-R_1I_0(lR_1/R)}{\lambda^2I_0(lR_1/R)}\right)\right]$$

To determine λ, we integrate Equation 9 for the pressure P_0 and P_L are the pressure at z = 0 and z = L, respectively, where L is the length of the tube.

The resistance to flow (λ_0) is defined as follows Young [19]

$$\lambda_0=\frac{P_L-P_0}{Q} \tag{10}$$

Let λ_N be the resistance to flow for Newtonian fluid and with no stenosis, then

$$\lambda_N=\frac{8\mu_1 L}{p R_0^4} \tag{11}$$

from Equation 10 and 11 we have,

$$\lambda=\frac{\lambda_0}{\lambda_N}=\left[1-\frac{L_0}{L}+\frac{M_0}{L}\int_d^{d+L_0}\frac{dz}{A}\right] \tag{12}$$

$A = A_1 + A_2$

$$A_1=\left[(\mu^1-1)\left(\frac{R_1}{R_0}\right)^4+\left(\frac{R}{R_0}\right)^4\right]$$

$$A_2=\left[\frac{8\mu_R\mu^1}{(\mu_2+\mu_R)}\left(\frac{R_1}{R_0}\right)\left(\frac{R}{R_0}\right)^2\left(\frac{2I_1(\lambda R_1/R)}{R_0\lambda^2I_0(\lambda R_1/R)}\right)-\left(\frac{R_1}{R_0}\right)\frac{1}{\lambda^2}\right]$$

Equation 8 can be rewritten as:

$$Q=\frac{p PR^4}{8\mu_{app}}$$

Where

$$\mu_{app}=\left[\frac{\mu^1}{[1-(1-\mu^1)\alpha^4](R/R_0)^4}\right] \tag{13}$$

and can be termed as apparent viscosity of fluid flow in the tube.

The shearing stress at the (maximum height of the stenosis) can be written as:

$$t_s=\left[\frac{4\mu_2 Q(1-(d/R_0))}{p R_0^3[(1-(d/R_0))^4-(1-\mu^1)(\alpha-(d/R_0))^4]}\right] \tag{14}$$

and the shear stress for Newtonian fluid with no stenosis is as:

$$t_N=\left[\frac{4\mu_1 Q}{p R_0^3}\right] \tag{15}$$

Now the ratio of shearing stress at the wall can be written as:

$$t=\frac{t_S}{t_N}=\left[\frac{\mu^1}{[1-(1-\mu^1)\alpha^4](1-d/R_0)^3}\right] \tag{16}$$

4. RESULTS AND DISCUSSION

Using governing equations with appropriate boundary conditions the expression for resistance to flow (λ), apparent viscosity (μ_{app}) and wall shear stress (τ) have been derived and represented by Equations 12, 13 and 16 respectively. The graphs 1-8 have been plotted by taking the values of parameters based on experimental data of Ariman, et al [20] (Table 1).

The variation of resistance to flow (λ) with stenosis size (δ/R_0) for different values of stenosis shape parameter (m) is shown in Figure 2. It is evident that resistance to flow increases as stenosis size increases. Increase in the resistance to flow with stenosis shape parameter is also indicated in this figure. These results are consistent with the observations of Eringen [1] and Tandon [12]. Figure 2 shows that the resistance to flow increases with peripheral layer viscosity (μ^1) as well. Coagan [21] found that the peripheral layer viscosity of blood in diabetic patients is higher than in non-diabetic patients, resulting higher resistance to blood flow. Thus diabetic patients with higher peripheral layer viscosity are more prone to high blood pressure. Therefore, the resistance to blood flow in case of diabetic patients may be reduced by reducing viscosity of the plasma. This can be done by injecting saline water to such patients [9]; the process is called dilution in medical terms.

TABLE 1. List of Parameters used in the Paper Has Been Taken from Ariman, et al [19] Data.

Parameter	Value	
μ_R	0.98	cp
γ	12×10^{-8}	gm-cm/sec
R_0	0.2	cm
L	5	cm
α	0.95	
δ/R_0	0.02 to 0.14	
L/L_0	0.02 to 0.14	
μ^1	0.1 to 1.0	
m	2 to 9	

Figure 3 shows variation of resistance to flow (λ) with stenosis length (L_0/L) for different values of stenosis shape parameter (m). In the figure, the resistance to flow (λ) is found to decrease as stenosis length (L_0/L) increases. However, the resistance to flow (λ) is found to increase with the increase in stenosis shape parameter (Figure 2). This was also reported by Haldar [22], Siddiqui, et al [23]. The variation of apparent viscosity (μ_{app}) with stenosis size (δ/R_0) is shown in Figure 4. The apparent viscosity (μ_{app}) is seen to increase as stenosis size (δ/R_0) increases. This indeed is the case with human artery, as apparent viscosity (μ_{app}) of the blood is found to increase with stenosis growth [24] and hence this validates the present model. Apparent viscosity (μ_{app}) is symmetrically distributed in the stenotic region and its maximum lies at axially symmetric stenosis (m = 2). Figure 4 also shows that the apparent viscosity (μ_{app}) changes inversely with peripheral layer viscosity (μ^1). The symmetry of the curves may be due to the assumed symmetry in the geometrical configuration of the stenosis.

Figure 5 describes the variation of apparent viscosity (μ_{app}) with stenosis length (L_0/L) for different values of peripheral layer viscosity (μ^1). The apparent viscosity (μ_{app}) is found to increase as stenosis length (L_0/L) increases. Tandon [26] has also obtained similar results. In addition, the apparent viscosity (μ_{app}) is found to change inversely with peripheral layer viscosity (μ^1). The variation of apparent viscosity (μ_{app}) with stenosis shape parameter (m) for different values of peripheral layer viscosity (μ^1) is shown in Figure 6. The apparent viscosity (μ_{app}) is seen to increase as stenosis shape parameter (m) decreases, and is maximum at axially symmetric stenosis (m = 2) for all values of peripheral layer viscosity. It is also seen that apparent viscosity (μ_{app}) decreases as peripheral layer viscosity (μ^1) increases. The apparent viscosity (μ_{app}) is found to decrease as peripheral layer viscosity (μ^1) increases with α (Figure 7).

Variation of apparent viscosity (μ_{app}) with stenosis size (δ/R_0) for different values of stenosis

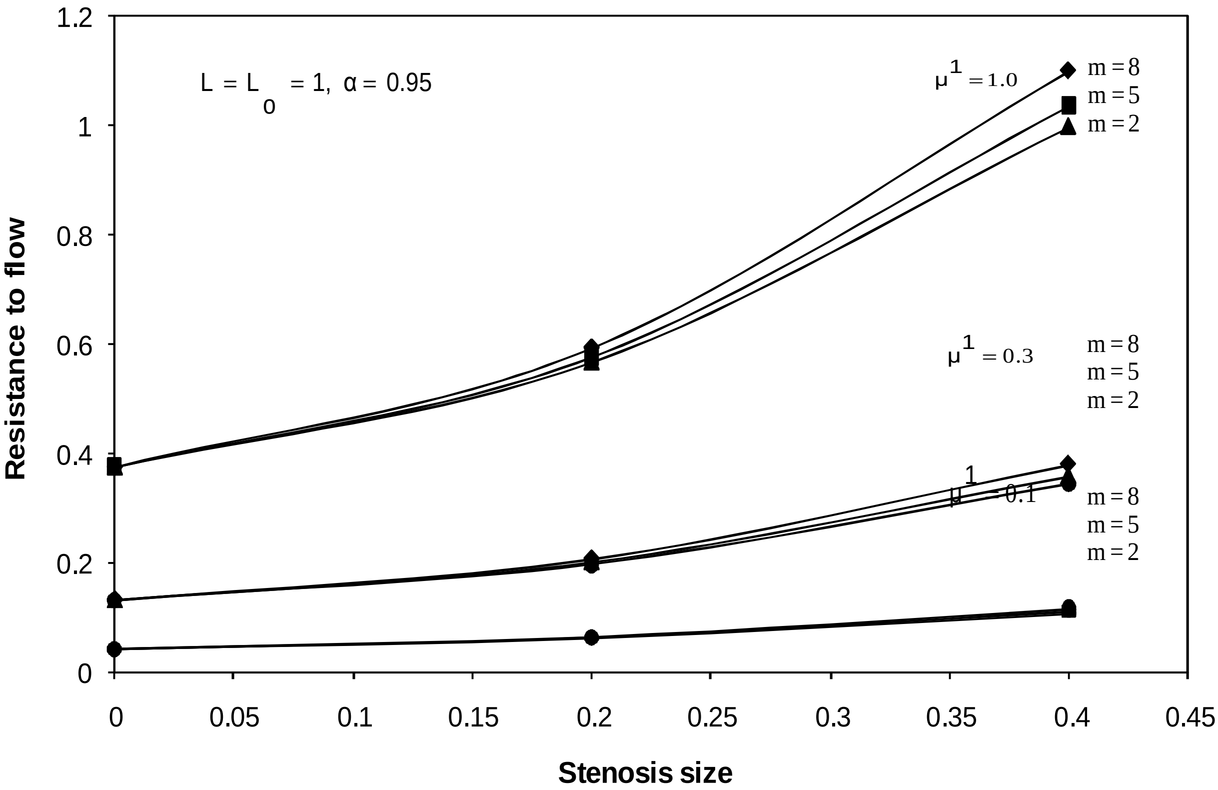

Figure 2. Variation of resistance to flow with stenosis size for different values of stenosis shape parameter.

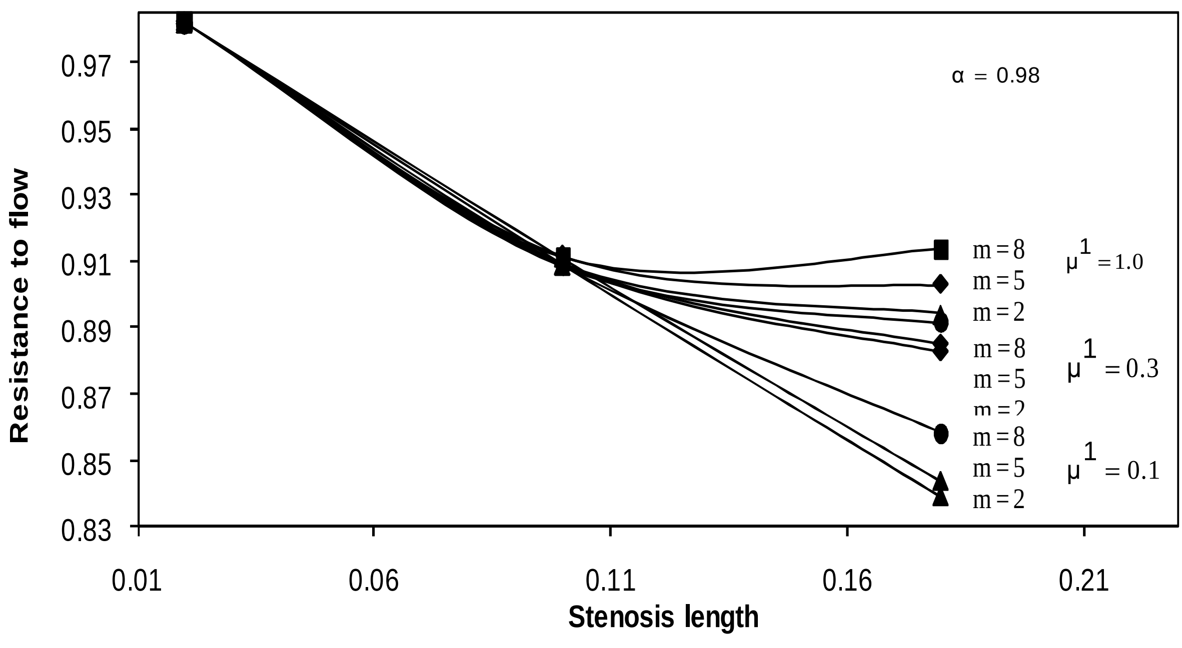

Figure 3. Variation of resistance to flow with stenosis length for different values of stenosis shape parameter.

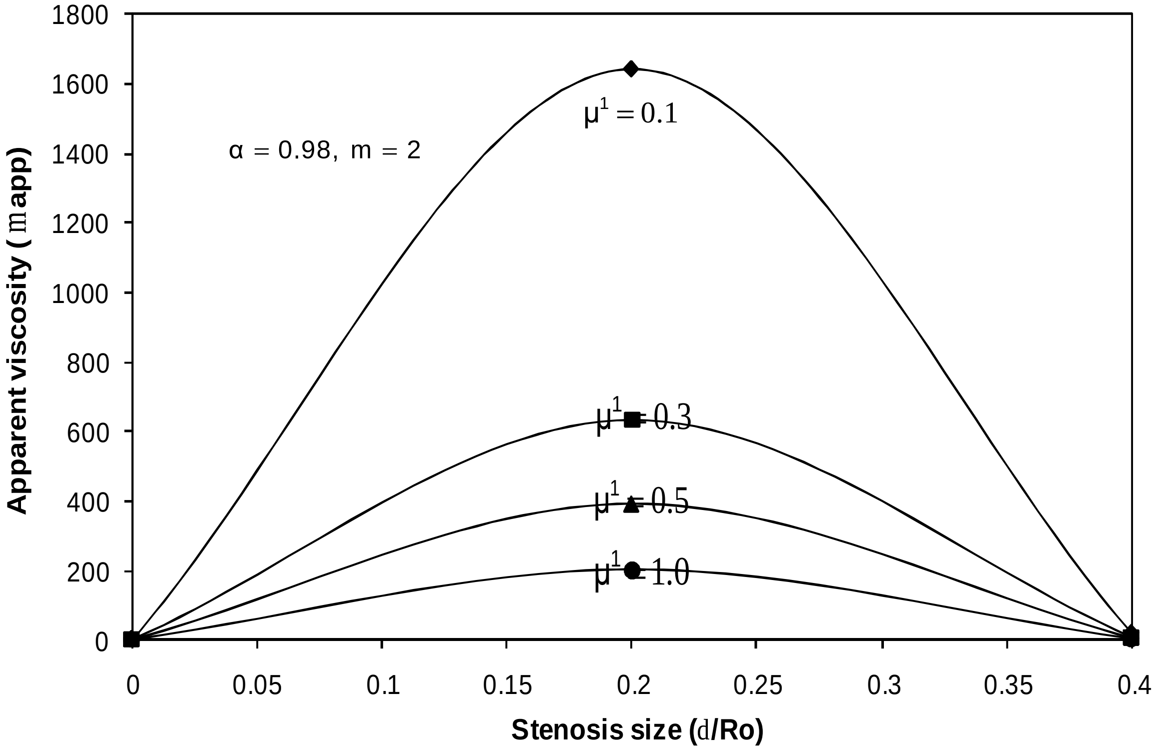

Figure 4. Variation of apparent viscosity with stenosis size for different values of peripheral layer viscosity.

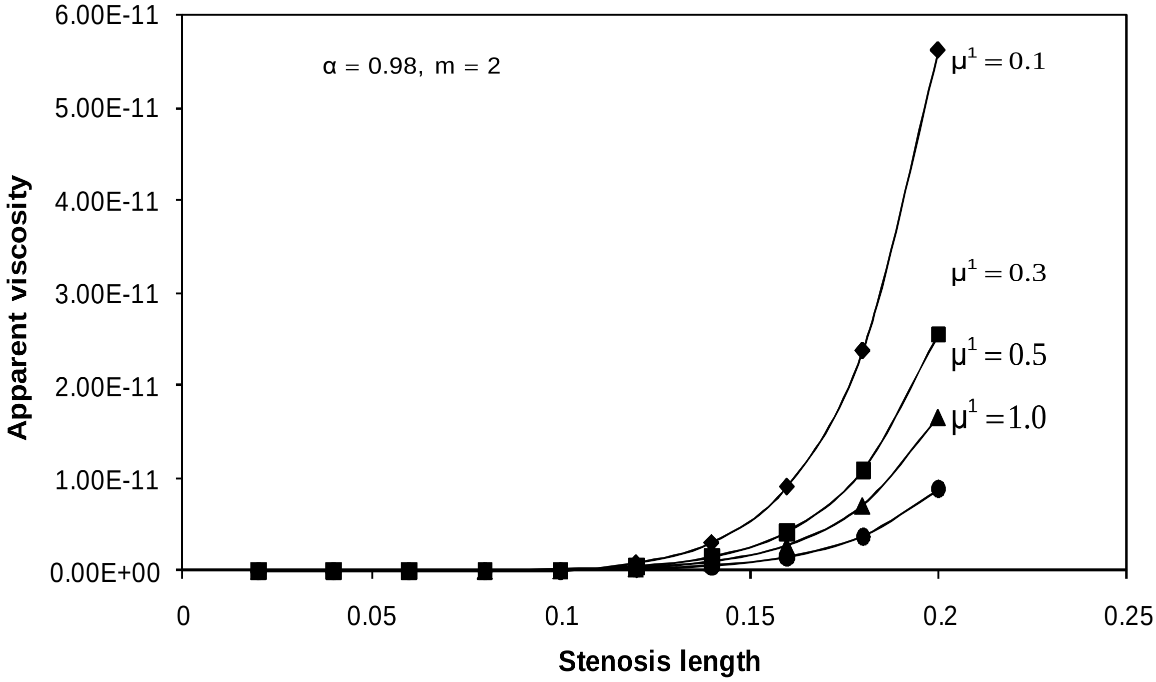

Figure 5. Variation of apparent viscosity with stenosis length for different values of peripheral layer viscosity.

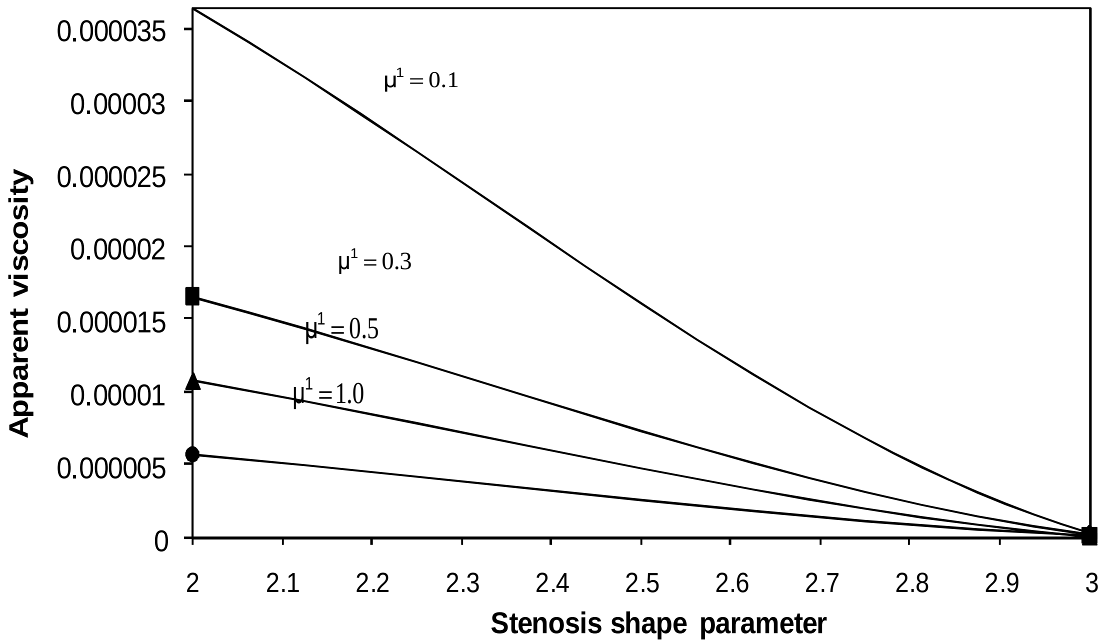

Figure 6. Variation of apparent viscosity with stenosis shape parameter for different values of peripheral layer viscosity.

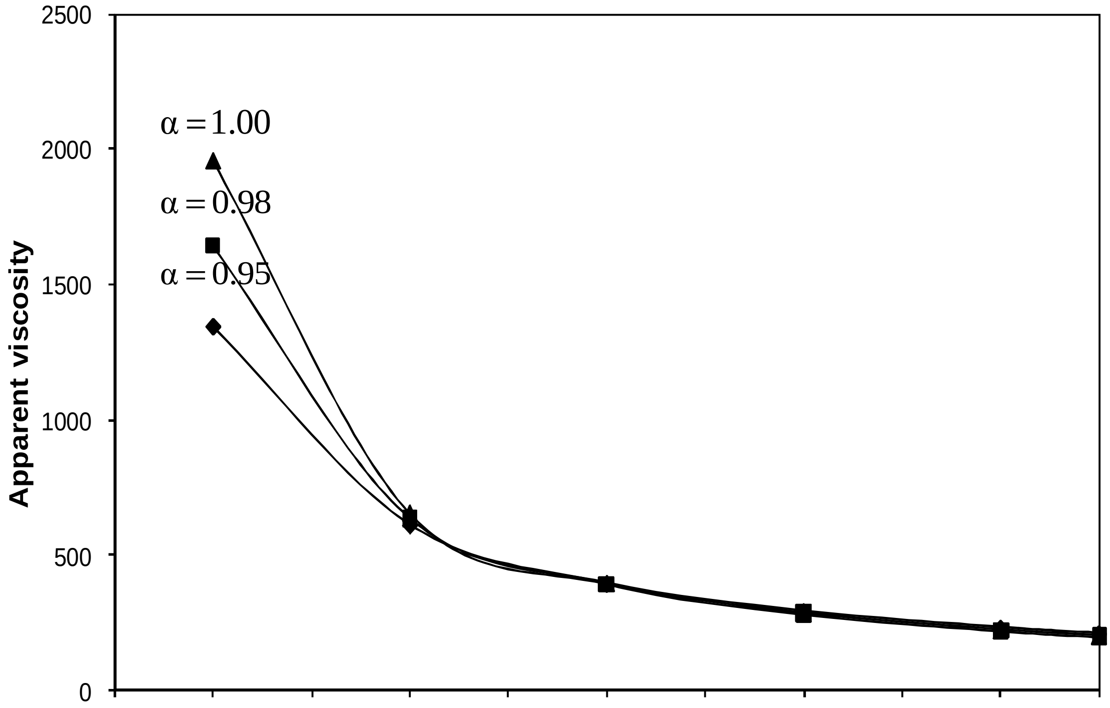

Figure 7. Variation of apparent viscosity with peripheral layer viscosity for different values of *a*.

length (L_0/L) is described in Figure 8. The apparent viscosity (μ_{app}) is found to decrease with decreasing stenosis size. But the same is not true in the absence of stenosis [24]. In normal human artery, apparent viscosity (μ_{app}) is found to decrease with the artery radius [27] and is called Fahraeus-Lindquist effect. Figure 8 also shows that the apparent viscosity (μ_{app}) increases as stenosis length (L_0/L) increases. Sirs, et al [28] observed that diabetic patients are more prone to various cardiovascular diseases due to increased apparent viscosity. The wall shear stress (τ) is found to increase with stenosis size (δ/R_0) as observed in Figure 9. Chow, et al [25] had similar observations. The wall shear stress (τ) is also found to increase with peripheral layer viscosity (μ^1).

5. CONCLUSION

The effect of peripheral layer viscosity on the blood flow in the presence of mild stenosis in the lumen of the artery has been investigated by developing a mathematical model. The resistance to flow and apparent viscosity have been found to decreases with viscosity of peripheral layer, but the same are found to increase as the size (height and length) of the stenosis increases. The model predicts increase in wall shear stress with peripheral layer viscosity. Predicted trends are found to exist in human artery and hence validate the model.

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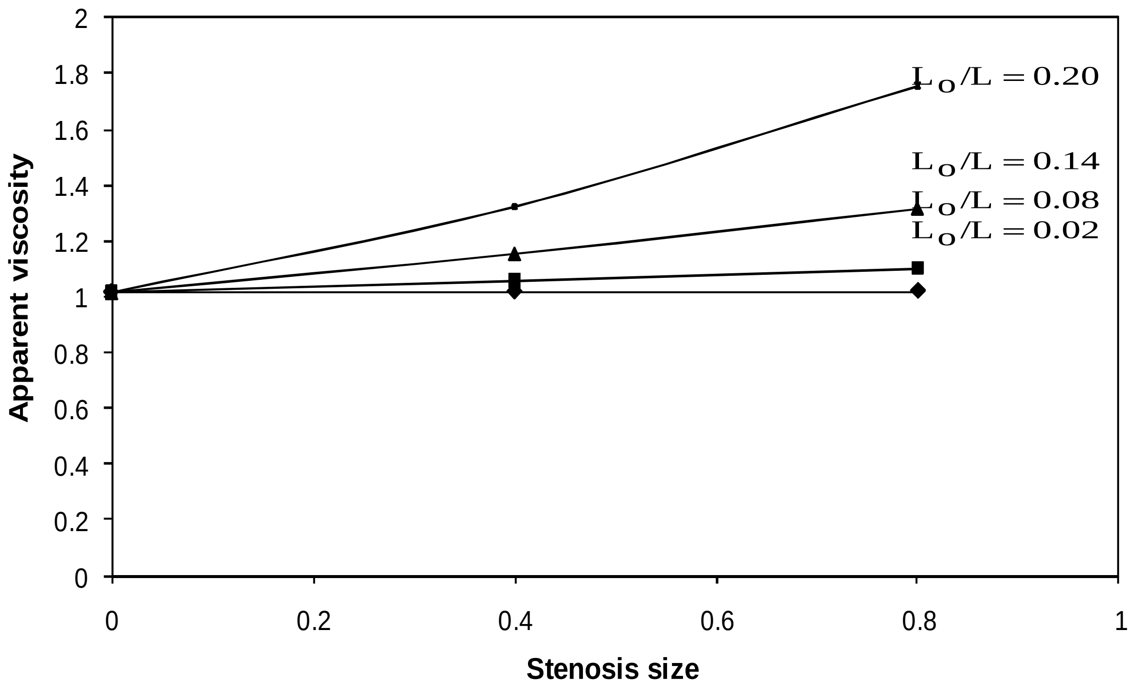

Figure 8. Variation of apparent viscosity with stenosis size for different values of stenosis length.

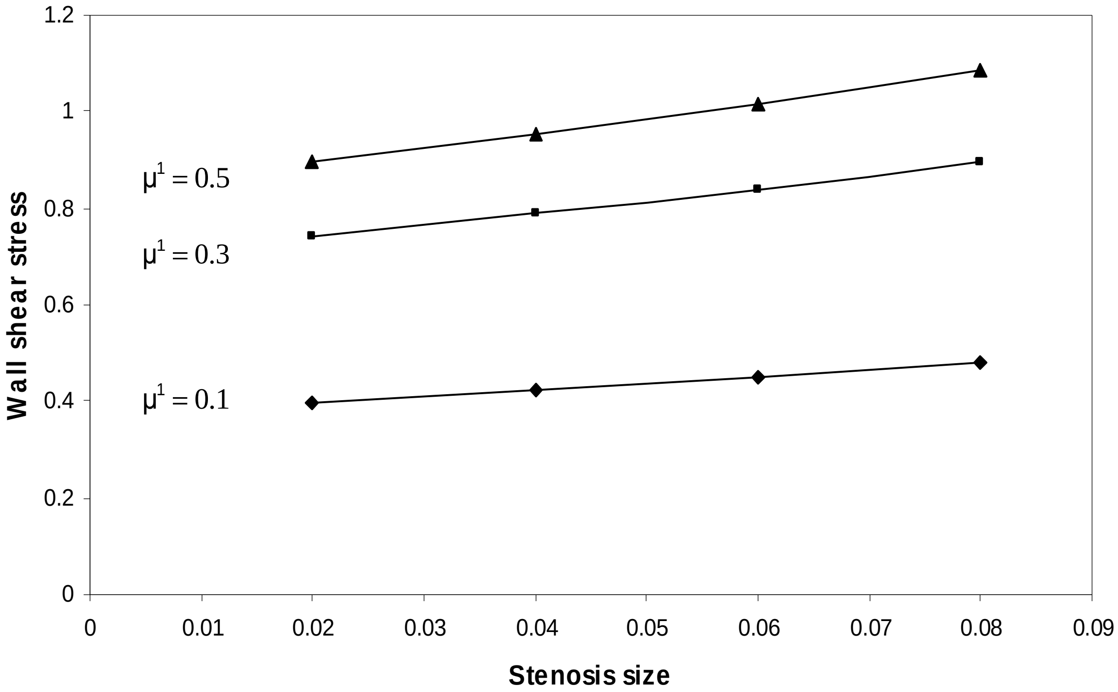

Figure 9. Variation of wall shear stress with stenosis size for different values of peripheral layer viscosity.

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