MHD FREE CONVECTIVE FLUCTUATING FLOW THROUGH A POROUS EFFECT WITH VARIABLE PERMEABILITY PARAMETER

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Abstract In the present paper, we have studied MHD free convective two dimensional unsteady viscous incompressible flows through a porous effect bounded by an infinite vertical porous plate with constant suction. The permeability of the porous medium fluctuates in time about a constant mean, and the viscosity of fluid is assumed to vary as a linear function of temperature. The flow is permitted under the influence of a uniform transverse magnetic field, whereas the induced magnetic field has little effect on the flow and can be neglected. The governing equations are solved by perturbation technique based on computer extended series solution.

Keywords Free convection, Mathematical model, Permeability parameter, Hartmann number, Magneto- hydrodynamics, Porous medium.

چکیده در مقاله حاضر، جریان های تراکم ناپذیر ویسکوز غیریکنواخت دوبعدی جابجایی آزاد MHD از میان یک منفذ مرزبندی شده با صفحه منفذدار عمودی بی نهایت با مکش ثابت مورد مطالعه قرار گرفته است. فرض شده است که نفوذپذیری نوسان های محیط متخلخل در زمان در حدود ثابت متوسط و ویسکوزیته سیال به صورت تابعی از دما تغییر می کند. جریان تحت تاثیر یک میدان مغناطیسی متقاطع یکنواخت مجاز است، جاییکه میدان مغناطیسی القاء شده اثر کمتری روی جریان دارد و می تواند چشم پوشی شود. معادلات حاکم توسط روش اختلال بر پایه حل های سری بسط یافته کامپیوتری حل شده است.

1. INTRODUCTION

The study of MHD free convective flow through a porous medium has become a principal interest in recent years because of its numerous scientific and engineering applications, viz in the fields of agricultural engineering, biomedical engineering, reservoir engineering to study the underground water resources; petroleum technology, geophysics to study the movements of natural gas, oil and water reservoirs, in chemical engineering for filtration and purification process. In recent years, the problems of MHD free convection flow through a porous medium have attracted the attention of researchers working on several geophysical problems. The study of magneto-hydrodynamics plays an important role in the engineering industries. It has resulted in an unabated exploration of new ideas and avenues in harnessing various convectional energy sources like tidal weir wind power and geothermal energy. Many researchers have worked on MHD convective fluctuating flows. Heuer et al. [1] studied the influence of production rate, permeability variation and wall spacing on solution gas drive performance. Beavers et al. [2] investigated boundary conditions at the naturally permeable wall. They suggested a boundary condition of the form $u_1 - u_2 = \frac{\sqrt{k}}{a}\frac{\partial u_1}{\partial n}$ where u1 is the fluid velocity at the interface, u2 is the Darcy velocity inside the porous medium in the stream-wise direction, $(\frac{\partial u_1}{\partial n})$ is the velocity gradient of the stream-wise component along the normal to the surface drawn into the fluid, k is the permeability of the porous medium and α the dimensionless constant known as slip parameter believed depending upon the nature of the porous material. Only, Raptis et al. [3] studied the steady free convection flow through a porous medium bounded by an infinite porous plate subject to a constant suction and variable temperature. Singh et al. [4] investigated a free convection along a vertical wall in a porous medium with periodic permeability variation. They obtained analytical expressions for the velocity and temperature using the perturbation technique and showed that the results would be useful in the design of steam displacement process in oil recovery and different geothermal systems. Moreau [5] discussed magneto hydrodynamics for various flow. Shokouhmand and Sayehvand [6] investigated viscous incompressible flow and heat transfer in a square driven cavity was examined using the SIMPLER algorithm. It is shown that flow in the cavity at low Reynolds numbers follows a symmetric pattern while at higher Reynolds numbers, a thin boundary layer formed on the walls and an inviscid core region develops. Kim [7] has considered the case of a semi-infinite moving porous plate in a porous medium in the presence of pressure gradient and constant velocity in the flow direction when the magnetic field is imposed transverse to the plate.

Kumar et al. [8] investigated the unsteady two dimensional laminar flow of a viscous incompressible electrically conducting fluid past a semi-infinite vertical porous moving plate with periodic suction in the presence of transverse magnetic field. They are solutions of governing equations obtained by Perturbation Technique.

Kumar et al. [9] studied the flow between annular spaces surrounded by rotating coaxial cylinder with coaxial cylindrical porous medium. Singh and Sharma [10] investigated the effect of periodic permeability on the free convective flow of viscous conducting fluid through highly porous medium. Mehmet et al. [11] studied a free convection flow about a cone under mixed thermal boundary conditions and a magnetic field. Emad et al. [12] investigated a viscous dissipation and Joule heating effects on MHD free convection from a vertical plate with power-law variation in surface temperature in the presence of Hall and ion-slip currents. Smolentsev and Abdou [13] studied an open-surface MHD flow over a curved wall in the 3-D thin-shear-layer approximation. Mukhopadhyay et al. [14] investigated study of MHD boundary layer flow over a heated stretching sheet with variable viscosity. Pantokratoras [15] discussed the effect of viscous dissipation in natural convection along a heated vertical plate. It is also found that the interaction between the viscous heating and buoyancy force has a strong influence on the results. Khan et al. [16] founded an exact solution for MHD flow of a generalized oldroyd-B fluid with modified Darcy’s law. It is also discussed that, MHD flow of generalised oldroyd-B fluid in a circular pipe. The flow is induced because of an oscillating pressure gradient. Xia et al. [17] investigated two descriptions mapped infinite element have generated and combined with conventional finite elements and one direction infinite element to simulate poroelasticity. Myers et al. [18] revealed the flow of a variable viscosity fluid between parallel plates with shear heating. It is also described a model for the flow of a fluid through channel with parallel plate. Makinde and Osalusi [19] founded the combined effect of magnetic field and permeable wall slip velocity on the steady flow of an electrically conducting fluid in a channel of uniform width. Kumar et al. [20] investigated performance modelling of porous medium flow and their applications. Kumar et al. [21] investigated viscous flow through co-axial cylinder in the presence of magnetic field. It is also use for solving governing equation by finite difference method. Sekhar et al. [24] discussed the flow of a conducting fluid past a circular cylinder for a range of Reynolds number from 100 to 500 and for the intermediate value of Hartmann number (M) using the finite difference method. Andersson and Dahi [22] investigated the gravity driven flow of a visco-elastic liquid film along a vertical wall. The resulting analytical expression for the film thickness reveals that the visco-elastic films grows up faster towards the down stream asymptotic state than that of the Newtonian liquid. Sadeghy et al.

[23] investigated magneto hydrodynamic (MHD) flow of viscoelastic fluids in converging/diverging channels. Zanchini [28] investigated a mixed convection with variable viscosity in a vertical annulus with uniform wall temperatures. Avedissian et al. [25] investigated free convective heat transfer in an enclosure with an internal louvered blind. It is also study the effects of Rayleigh number, enclosure aspect ratio and blind geometry on the convective heat transfer. Pantokratoras [27] investigated a study of MHD boundary layer flow over a heated stretching sheet with viscosity. The results of this work are obtained with direct numerical solution of the boundary layer equations taking a count both viscosity and Prandtl number variation across the boundary layer. Mishra et al. [26] investigated a flow and heat transfer of an MHD viscous–elastic fluid in a channel with stretching walls.

Padmavathi and Amaranath [29] have considered the problem of general non axi-symmetric Stokes flow past a porous sphere in a viscous incompressible fluid. They have also considered the flow inside the sphere governed by Brinkman's equation. Kumar and Pant [30] investigated the behaviour of steady flow of visco-elastic liquid between two porous coaxial circular cylinders, where both the cylinders are rotating with different uniform angular velocities about the common axis. Qin, et al [35] have considered a thermal instability problem in a rotating micro polar fluid. It is found that, the rotation has a stabilizing effect depending upon the values of various micro polar parameters and low values of Taylor number.

Rahimi and Jalili [31] considered a transient free convection flow around a sphere with variable surface temperature and embedded in a porous medium. Rena and Rana [32] profounded a theoretical study of the thermal instability of thin layer electrically conducting micro-polar rotating fluid, heated from below in the presence of uniform magnetic field in the porous medium. Mansour et al. [35] investigated analytical studies on MHD flow of a micro polar fluid due to heat and mass transfer through a porous medium bounded by an infinite vertical porous plate in the presence of a transverse magnetic field in slip-flow regime. Kumar et al. [33] investigated the combined wall slip and MHD steady flow of conducting viscous incompressible fluid through a channel with permeable boundaries. It is disclose that the fluid velocity is reduced by magnetic field and wall slip. Kumar et al. [34] studied a total dispersion tensor in two dimensional packed beds consisting of randomly placed parallel cylinders for porosities between 38% to 90% Pecklet numbers up to 100 and Reynolds numbers up to 20 based on the cylindrical diameter. Mahdy [42] investigated the magneto-hydrodynamic (MHD) free convection flow of a non-Newtonian power-law fluid over a vertical wavy surface with a uniform free-stream of constant velocity and temperature. Asadi et al. [37] developed rising of a single bubble in a quiescent liquid which under microgravity condition was simulated. They related to unsteady incompressible full Navier-Stokes equations which were solved using a conventional finite difference method with a structured staggered grid. Kumar et al. [38] studied the distribution of transverse velocity which was not symmetrical and for non- Newtonian fluid, large recirculation occured at upper disc in comparison with the recirculation at lower disc on increasing value of Reynolds number. Makinde et al. [39] developed a steady, axi-symmetric, magneto hydrodynamic (MHD) flow of a viscous, Newtonian, incompressible, electrically-conducting fluid through an isotropic, homogenous porous medium located in the annular zone between two concentric rotating cylinders in the presence of a radial magnetic field. Khedr et al.[40] considered a steady, laminar, MHD flow of a micro polar fluid past a stretched semi-infinite, vertical and permeable surface in the presence of temperature dependent heat generation or absorption, magnetic field and thermal radiation effects. Kumar et al. [41] studied a MHD three dimensional free convective flow of viscous incompressible fluid through a porous medium. They are solutions of governing equations obtained by Finite difference technique.

The main objective of the present paper is to investigate the effect of permeability variation on MHD free convective fluctuating flow through a porous effect bounded by vertical porous plate of infinite length with constant suction. The solutions of governing equations were obtained by perturbation technique based on extended series solution.

2. MATHEMATICAL MODEL AND ANALYSIS:

In the present investigation, we are considering the flow of viscous incompressible fluid in the magnetic effects through a porous medium bounded by an infinite vertical plate with constant suction. The plate is taken in the upward direction along the x*-axis and y*-axis which is taken normal to the plate. The induced magnetic field is neglected. The effects of magnetic field is to control the fluid flow. All the fluid properties are assumed constant expect the influences of the density variation with temperature which are considered only in the body forces terms.

Let the permeability of the porous medium fluctuate under the law:

$$K^*(t^*) = K_0^*\left(1 + ee^{iw^*t^*}\right), \tag{1}$$

where, K_0^*, w^*, t^* and e are the mean permeability of the medium, frequency of fluctuation, time and very small constant quantity, respectively.

The flow through a porous medium in the magnetic effects is governed by the following equations:

$$\frac{\partial v^*}{\partial y^*} = 0, \tag{2}$$

$$\frac{\partial u^*}{\partial t^*} + v^*\frac{\partial u^*}{\partial y^*} = g\beta(T^* - T_\infty^*) + \nu\frac{\partial^2 u^*}{\partial y^{*2}} - \frac{\nu u^*}{K^*(t^*)} - \frac{\sigma B_0^2}{\rho}u^*, \tag{3}$$

$$\frac{\partial T^*}{\partial t^*} + v^*\frac{\partial T^*}{\partial y^*} = \frac{k}{\rho C_p}\frac{\partial^2 T^*}{\partial y^{*2}}, \tag{4}$$

where u* and v* be the velocity components in the direction of x* and y* axes, $g, \beta, \nu = \frac{\mu}{\rho}, \mu, k, \rho$ and C_P are acceleration due to gravity, coefficient of volume expansion, kinematic viscosity, viscosity, thermal conductivity, density and specific heat of the fluid at constant pressure, respectively. In order to complete the formulation of the problem the boundary conditions were specified as:

$$\left.\begin{array}{l} y^* = 0;\, u^* = 0,\, T^* = T_w^* \\ y^* \to 0;\, u^* = 0,\, T^* = T_\infty^* \end{array}\right\} \tag{5}$$

where, T_w^* and T_∞^* are the temperature of the plate and the temperature of the fluid, respectively far away from the plate at constant pressure in the free stream.

Now, integrating equation (2), we get

$$v^* = -v, \tag{6}$$

where, v is a positive constant and negative sign represents that the suction is towards the plate.

The following dimensionless variables are introduced as:

$$y = \frac{y^* v}{\nu},\ t = \frac{t^* v^2}{4\nu},\ w = \frac{4w^*\nu}{v^2},\ u = \frac{u^*}{v},\ \theta = \frac{T^* - T_\infty^*}{T_w^* - T_\infty^*}.$$

Using these non-dimensional variables, the equations (3) and (4) becomes:

$$\frac{1}{4}\frac{\partial u}{\partial t} - \frac{\partial u}{\partial y} = G\theta + \frac{\partial^2 u}{\partial y^2} - \frac{u}{K_0(1 + e\,e^{iwt})} - Mu, \tag{7}$$

$$\frac{1}{4}\frac{\partial \theta}{\partial t} - \frac{\partial \theta}{\partial y} = \frac{1}{P_r}\frac{\partial^2 \theta}{\partial y^2}, \tag{8}$$

where, $G = \frac{\nu g\beta(T_w^* - T_\infty^*)}{v^2}$, Grashoff number

$P_r = \frac{\mu C_P}{k}$, Prandtl number

$K_0 = \frac{K_0^* v_0^2}{\nu^2}$, Permeability parameter and $M = \frac{sB_0^2 u}{v_0^2}$, Hartmann number.

The reduced boundary conditions are as follows:

$$\begin{aligned} y &= 0 ; u = 0 , q = 1 \\ y &\to 0 ; u = 0 , q = 0 \end{aligned} \tag{9}$$

In order to solve equations (7) and (8), we assume velocity and temperature as:

$$u(y,t) = u_0(y) + ee^{iwt}u_1(y) + ..., \tag{10}$$

$$q(y,t) = q_0(y) + ee^{iwt}q_1(y) + ..., \tag{11}$$

Substituting all these values of the equations (10) and (11) into equations (7) and (8) and equating harmonic and non-harmonic terms, we get

$$u_0'' + u_0' - \left(\frac{1}{K_0} + M\right)u_0 = -Gq_0, \tag{12}$$

$$u_1'' + u_1' - \left(\frac{iw}{4} + \frac{1}{K_0} + M\right)u_1 = -\frac{u_0}{K_0} - Gq_1, \tag{13}$$

$$q_0'' + P_r q_0' = 0, \tag{14}$$

$$q_1'' + P_r q_1' - \frac{iwP_r}{4}q_1 = 0, \tag{15}$$

where, the primes represent differentiation with respect to y.

The corresponding boundary conditions are reduced as:

$$\left.\begin{aligned} y = 0; u_0 = 0, u_1 = 0, q_0 = 1, q_1 = 0 \\ y \to \infty; u_0 = 0, u_1 = 0, q_0 = 0, q_1 = 0 \end{aligned}\right\} \tag{16}$$

Solving equations (12) to (15) by using boundary conditions (16), we get

$$u_0(y) = \frac{G}{P_R}(e^{-r_1 y} - e^{-P_r y}), \tag{17}$$

$$u_1(y) = \frac{G}{K_0 P_R}\left[\frac{4i}{w}\left(e^{-r_2 y} - e^{-r_1 y}\right) - \frac{1}{P_R - \frac{iw}{4}}\left(e^{-r_2 y} - e^{-P_r y}\right)\right], \tag{18}$$

$$q_0(y) = e^{-P_r y}, \tag{19}$$

$$q_1(y) = 0, \tag{20}$$

$$P_R = P_r^2 - P_r - \frac{1}{K_0} - M$$

where $r_1 = \frac{1}{2}\left[1 + \left(1 + \frac{4}{K_0} + 4M\right)^{\frac{1}{2}}\right]$ *and* $r_2 = .$ $\frac{1}{2}\left[1 + \left\{\left(1 + \frac{4}{K_0} + 4M\right) + iw\right\}^{\frac{1}{2}}\right]$

Thus, equations (10) and (11) become:

$$u(y,t) = \frac{G}{P_R}\left[\left(e^{-r_1 y} - e^{-P_r y}\right) + \frac{e}{K_0}\left\{\frac{4i}{w}\left(e^{-r_2 y} - e^{-r_1 y}\right) - \frac{1}{P_R - \frac{iw}{4}}\left(e^{-r_2 y} - e^{-P_r y}\right)\right\}e^{iwt}\right] \tag{21}$$

$$q(y,t) = e^{-P_r y}, \tag{22}$$

The velocity profiles can be written in terms of fluctuating parts as:

$$u(y,t) = u_0(y) + e(M_r \cos wt - M_i \sin wt), \tag{23}$$

where, $u_1(y) = M_r + iM_i$.

The fluctuating parts are given by

$$M_r = -\frac{G}{K_0}\left[\frac{e^{-r_2 y} - e^{-P_r y}}{P_R^2 + \frac{w^2}{16}}\right]$$ and

$$M_i = \frac{G}{K_0 P_R}\left[\frac{4}{w}\left(e^{-r_2 y} - e^{-r_1 y}\right) - \frac{4w\left(e^{-r_2 y} - e^{-P_r y}\right)}{w^2 + 16P_R^2}\right]$$

Thus, the expression for the transient velocity for, $wt = \frac{p}{2}$, is given by

$$u(y, p/2w) = u_0(y) - eM_i, \quad (24)$$

The expression for skin friction at the plate in terms of phase and amplitude is given

$$t = \left(\frac{\partial u}{\partial y}\right)_{y=0} = \frac{G}{P_R}(P_r - r_1) + e\,|N|\cos(wt + a), \quad (25)$$

where,

$$B_r + iB_i = \frac{G}{K_0 P_R}\left[\frac{4i(r_1 - r_2)}{w} - \frac{\left(P_R + \frac{iw}{4}\right)(P_r - r_2)}{P_R^2 + \frac{w^2}{16}}\right],$$

$$\tan a = \frac{B_i}{B_r} \quad \text{and} \quad |N| = \pm\sqrt{B_i^2 + B_r^2}$$

The expression for phase and amplitude is given by

$$|N| = -\frac{G}{K_0 P_R}\sqrt{\frac{(P_r - r_2)(P_r + r_2 - 2r_1)}{P_R^2 + \frac{w^2}{16}} + \frac{16}{w^2}(r_1 - r_2)^2},$$

$$\tan a = -\frac{P_R^2 + \frac{w2}{16}}{P_R(P_r - r_2)}\left[\frac{4(r_1 - r_2)}{w} - \frac{4w(P_r - r_2)}{w^2 + 16P_R^2}\right]$$

3. RESULTS AND DISCUSSIONS:

To discuss the physical importance of the problems like velocity profiles and skin friction coefficient, various values of parameter were chosen. The effects of various parameters on fluctuating parts Mr and Mi of velocity profiles are shown in Tables 1 and 2 and also depicted in Figures 1 and 2, respectively. Table 1 and Figure 1 clearly show that by increasing Hartmann number M and frequency ω, the fluctuating part Mr decreases. Also, the flow decelerates with an increase in Hartmann number and the frequency whereas, an increase in permeability parameter K0 and Prandtl number Pr, enhance the fluctuating part which clearly interpret that they help to accelerate the flow. On the other hand, the fluctuating part Mi increases with an increase in Hartmann number M , permeability parameter K0 and Prandtl number Pr. The obtained results also depict that an increase in ω with the condition of (y<1.5) cause an increase in Mi, but; an increase in ω with the condition of (y>1.5) decrease Mi.

Similarly, Tables 3 and 4 and Figures 3 and 4 show the variations of skin friction amplitude (|N|) and phase ($\tan a$) with the variations of Hartmann number (M), Permeability parameter (K0), Prandtl number (Pr) and frequency (ω). From Table 3 and Figure 3, it is evident that |N| decreases when M, K0 and Pr increase, whereas Table 4 and Figure 4 show that $\tan a$ diminishes when K0, Pr and ω increase. Also, an increase in Hartmann number results to an increase in the skin friction phase. The professional form the chemical industry may be interested to evaluate this limit of K, M and Gr for the fluid. They are using these values dependent on characteristics of the fluid. It is hope that present work will be helpful for understanding more complex problems involving the various physical effects which is investigated in the present problems.

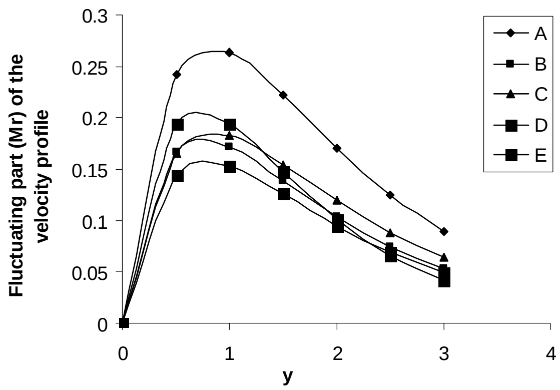

Figure 1. Fluctuating part Mr of velocity profiles at G=5 and ε = 0.2

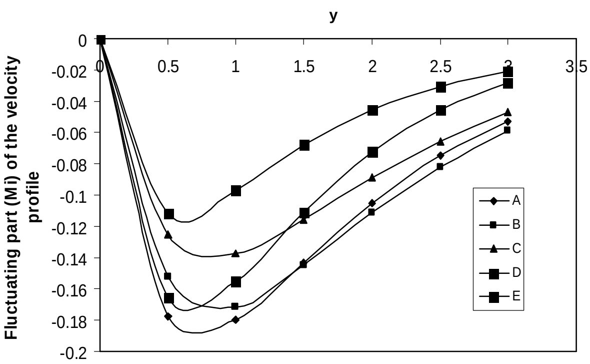

Figure 2. Fluctuating part (Mi) of velocity profiles at G=5 and ε = 0.2

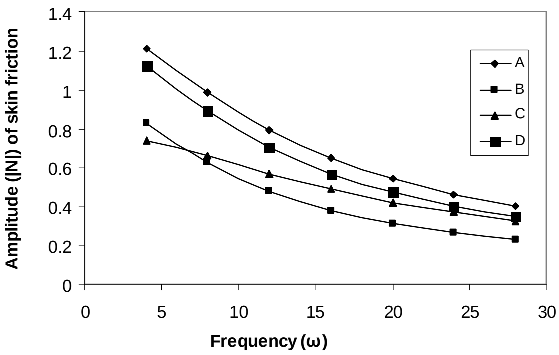

Figure 3. The amplitude of the skin friction at G=5 and ε = 0.2

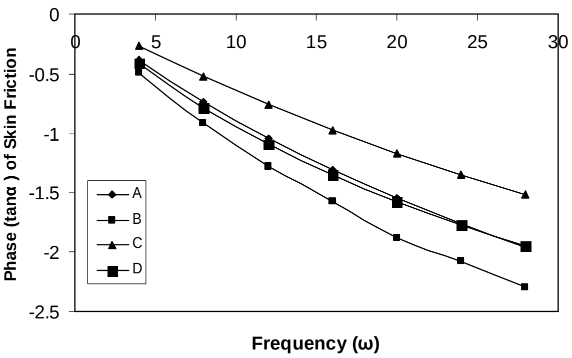

Figure 4. The phase of the skin friction at G=5 and ε = 0.2

4. CONCLUSION

The results in the present analysis demonstrate the theoretical evidence to support the field and laboratory observations showing an increase in permeability with magnetic effects of results in velocity profiles and skin friction. Tables and figures show the effects of permeability, Hartmann number and Prandtl number on the velocity and the skin friction, amplitude and phase. With regard to the applications of this analysis, they are to a steam injection process for enhancing oil recovery or to a geothermal process, chemical process. In the near future, we would be glad to compare these theoretical results with those obtained by any one in the same field.

NOMENCLATURE:

K_0 : Permeability parameter
Pr : Prandtl number
G : Grashoff number
M : Hartmann number
T : Temperature
C_P : Specific heat of the fluid
u : Velocity component along x-axis
v : Velocity component along y-axis
θ : Dimensionless temperature ratio
κ : Thermal conductivity
μ : Dynamic viscosity
ν : Kinematic viscosity
β : Coefficient of volume expansion
ρ : Density of the fluid
T_w : Temperature of the plate
T_∞ : Temperature of the fluid
t : Skin Friction
w : Frequency
ε : Mixed thermal boundary condition parameter.

Table 1. Fluctuating part (Mr) of the velocity profile for different parameters

	M	K_0	Pr	ω	Fluctuating part of velocity (Mr)						
					y=0	y=0.5	y=1.0	y=1.5	y=2.0	y=2.5	y=3.0
A	1	1	0.71	5	0	0.2419	0.2640	0.2220	0.1700	0.1248	0.0897
B	1	1	0.71	10	0	0.1666	0.1719	0.1388	0.1033	0.0745	0.0529
C	1	2	0.71	5	0	0.1651	0.1828	0.1533	0.1200	0.0886	0.0639
D	2	1	0.71	5	0	0.1437	0.1526	0.1257	0.0949	0.0690	0.0493
E	1	1	1.00	5	0	0.1945	0.1939	0.1472	0.1008	0.0657	0.0416

Table 2. Fluctuating part (Mi) of the velocity profile for different parameters

	M	K_0	Pr	ω	Fluctuating part of velocity (Mi)						
					y=0	y=0.5	y=1.0	y=1.5	y=2.0	y=2.5	y=3.0
A	1	1	0.71	5	0	-0.1774	-0.1802	-0.1431	-0.1051	-0.0749	-0.0527
B	1	1	0.71	10	0	-0.1522	-0.1714	-0.1450	-0.1114	-0.0819	-0.0589
C	1	2	0.71	5	0	-0.1255	-0.1375	-0.1160	-0.0891	-0.0655	-0.0471
D	2	1	0.71	5	0	-0.1118	-0.0973	-0.0682	-0.0457	-0.0306	-0.0207
E	1	1	1.00	5	0	-0.1660	-0.1549	-0.1112	-0.0727	-0.0456	-0.0281

Table 3. Amplitude $(|N|)$ of skin friction for different parameters

	M	K_0	Pr	**Amplitude (|N|) of skin friction**						
				ω=4	ω=8	ω=12	ω=16	ω=20	ω=24	ω=28
A	1	1	0.71	1.2094	0.9873	0.7921	0.6472	0.5418	0.4636	0.4040
B	1	2	0.71	0.8271	0.6252	0.4779	0.3800	0.3131	0.2653	0.2298
C	2	1	0.71	0.7404	0.6594	0.5689	0.4883	0.4222	0.3692	0.3265
D	1	1	1.00	1.1208	0.8904	0.7016	0.5675	0.4722	0.4026	0.3500

Table 4. Phase ($\tan a$) of skin friction for different parameters

	M	K_0	Pr	Phase ($\tan a$) of skin friction						
				ω=4	ω=8	ω=12	ω=16	Ω=20	ω=24	ω=28
A	1	1	0.71	-0.3831	-0.7333	-1.0405	-1.3095	-1.5477	-1.7616	-1.9563
B	1	2	0.71	-0.4892	-0.9156	-1.2755	-1.5779	-1.8831	-2.0792	-2.2929
C	2	1	0.71	-0.2668	-0.5214	-0.7571	-0.9725	-1.1692	-1.3494	-1.5156
D	1	1	1.00	-0.4155	-0.7828	-1.0918	-1.3529	-1.5778	-1.7756	-1.9528

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