THERMOSOLUTAL CONVECTION OF MICROPOLAR ROTATING FLUIDS SATURATING A POROUS MEDIUM

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Abstract Double-diffusive convection in a micropolar fluid layer heated and soluted from below in the presence of uniform rotation saturating a porous medium is theoretically investigated. An exact solution is obtained for a flat fluid layer contained between two free boundaries. To study the onset of convection, a linear stability analysis theory and normal mode analysis method have been used. For the case of stationary convection, the effect of various parameters like medium permeability, solute gradient, rotation and micropolar parameters (i.e. coupling, spin diffusion, micropolar heat conduction and micropolar solute parameters arising due to coupling between spin and solute fluxes) have been analyzed. The critical thermal Rayleigh numbers for various values of critical wave numbers (found by Newton Raphson method) for the onset of instability are determined numerically and depicted, graphically. The oscillatory modes were introduced due to the presence of the micropolar viscous effects, microinertia, rotation and stable solute gradient, which were non-existence in their absence. The principle of exchange of stabilities is found to hold true for the micropolar fluid saturating a porous medium heated from below in the absence of micropolar viscous effect, microinertia, rotation and stable solute gradient. An attempt was also made to obtain sufficient conditions for the non-existence of overstability.

Keywords Thermosolutal Convection, Porous Medium, Rotation Effect, Micropolar Fluids, Medium Permeability, Stable Solute Gradient, Rayleigh Number

چکیده انتقال حرارت جابجایی پخش دوگانه در یک لایه سیال میکروقطبی که از پایین تحت حرارت و حلالیت قرار گرفته و با چرخش ثابت، محیطی متخلخل را اشباع می‌کند، به طور نظری مطالعه شده است. حل دقیق برای لایه تخت سیال محدود شده بین دو مرز آزاد، به دست آمده است. از تئوری تحلیل پایداری خطی و روش تحلیل وضعیت نرمال برای مطالعه شروع جابجایی استفاده شده است. تاثیر پارامترهای مختلفی مثل قابلیت نفوذ محیط متخلخل، تغییرات حلالیت، چرخش و عوامل میکرو قطبی (کوپلینگ، نفوذ spin، هدایت حرارتی میکروقطبی و حلالیت میکروقطبی که به دلیل کوپلینگ بین شار حلالیت و چرخش افزایش می‌یابند) بر حالت جابجایی ساکن، تحلیل شده است. عدد رایلی حرارتی بحرانی و عدد موج بحرانی برای شروع ناپایداری به دست آمده و در نمودارها نشان داده شده اند. مشخص شده است که حلالیت پایدار مودهای نوسانی به دلیل حضور اثرات ویسکوز میکروقطبی، میکرواینرسی، چرخش و گرادیان به وجود می‌آیند که این مودها در حالت عدم حضور این عوامل وجود نداشتند. اصل تبادل پایداری‌ها برای سیال میکروقطبی اشباع شده در محیط متخلخل با گرمایش از زیر در غیاب اثرات ویسکوز میکروقطبی، میکرواینرسی، چرخش و گرادیان حلالیت پایدار همچنان صادق است. تلاش شد شرایط مناسب برای عدم ایجاد فوق پایداری نیز به وجود آید.

1. INTRODUCTION

A general theory of micropolar fluids has been presented by Eringen [1,2]. According to Eringen [1], a subclass of microfluids Eringen [3] which exhibit the micro-rotational effects and micro-rotational inertia is the micropolar fluid. Certain anisotropic fluids, e.g., liquid crystals which are made up of dumb bell molecules are of this type. In fact, animal blood happens to fall into this category. Other polymeric fluids and fluids containing certain additives may be represented by the mathematical model underlying micropolar fluids. Compared to the classical Newtonian fluids, micropolar fluids are characterised by two supplementary variables, i.e., the spin, responsible for the micro-rotations and the micro-inertia tensor describing the distributions of atoms and molecules inside the fluid elements in addition to the velocity vector. Liquid crystals, colloidal fluids, polymeric suspension, animal blood, etc. are few examples of micropolar fluids Labon, et al [4]. Kazakia, et al [5] and Eringen [6] extended this theory of structure continue to account for the thermal effects.

Micropolar fluids abound in engineering science and some common examples are human blood, plasma, sediments in rivers, drug suspension in pharmacology, liquid crystals, etc. The past four decades have seen an incredible interest emerge in applications of these fluid theories to numerous problems in engineering sciences ranging from biofluid mechanics of blood vessels to sediment transport in rivers and lubrication technology. It has wide applications in the developments of micropolar biomechanical flows and as such is both an engineering science one. The more applications of micropolar fluid may include lubrication theory, boundary layer theory, short waves for heat conducting fluids, hydrodynamics of multicomponent media, magnetohydrodynamics and electrohydrodynamics, biological fluid modelling, etc. Hydrodynamics of micropolar fluids has significant applications to a variety of different fields of physics and engineering (magnetohydrodynamics, tribology, etc.). We are highlighted some key areas of applications in Figures 1 and 2.

Rosenberg [7] has developed the variational formulation of the boundary value problems of the micropolar continuum and hence applied to the modelling of the bone. Micropolar fluid theory has been applied successfully by many authors, Shukla, et al [8], Sinha, et al [9,10] and Sinha, et al [11,12] ,to study various problems in lubrication.

Prakash et al [13] applied micropolar fluid theory to study theoretically the effects of solid particles in the lubrication of journal and rolling contact bearings considering cavitation. Excellent work in micropolar simulations of human joint lubrication has been reported by Sinha, et al [11]. He also presented a micropolar hip joint lubrication model. Chandra, et al [14] presented an analysis for blood flow through a narrow artery with mild stenosis by considering Eringen's simple micro fluid model for blood. Power [15] was led to use the theory of micropolar fluid for modelling the CSF (Cerebral Spinal Fluid) for low Reynolds number approximation solving linear Fredholm integral equations. A large number of references about modelling and applications aspect of micropolar fluids has been given in the book [16]. In the review paper [17], the most comprehensive discussion of applications of fluids with microstructure, micropolar fluids in particular are tabularized. Recently some work has been done on viscoelastic micropolar fluid by Eremeyev, et al [18-20]. The literature concerning applications of micropolar fluids in engineering sciences is vast and still quickly growing.

The theory of thermomicropolar convection began with Datta, et al [21] and interestingly continued by Ahmadi [22]. Labon, et al [4], Bhattacharya, et al [23], Payne, et al [24], Sharma, et al [25,26] and Rama Rao [27]. The above works give a good understanding of thermal convection in micropolar fluids. The Rayleigh-Benard instability in a horizontal thin layer of fluid heated from below is an important particular stability problem. A detailed account of Rayleigh-Benard instability in a horizontal thin layer of Newtonian fluid heated from below under varying assumptions of hydrodynamics and hydromagnetics has been given by Chandrasekhar [28]. Perez-Garcia, et al [29] have extended the effects of the microstructures in the Rayleigh-Benard instability and have found that in the absence of coupling between thermal and micropolar effects, the Principle of Exchange of Stabilities (PES) holds good. Perez-Garcia, et al [30] have shown that when coupling between thermal and micropolar effect is present, the Principle of Exchange of Stabilities (PES) may not be fulfilled and hence oscillatory motions are present in micropolar fluids. The effect of rotation on thermal convection in micropolar fluids is important in certain chemical engineering and biochemical situations. Qin, et al [31] has considered a thermal

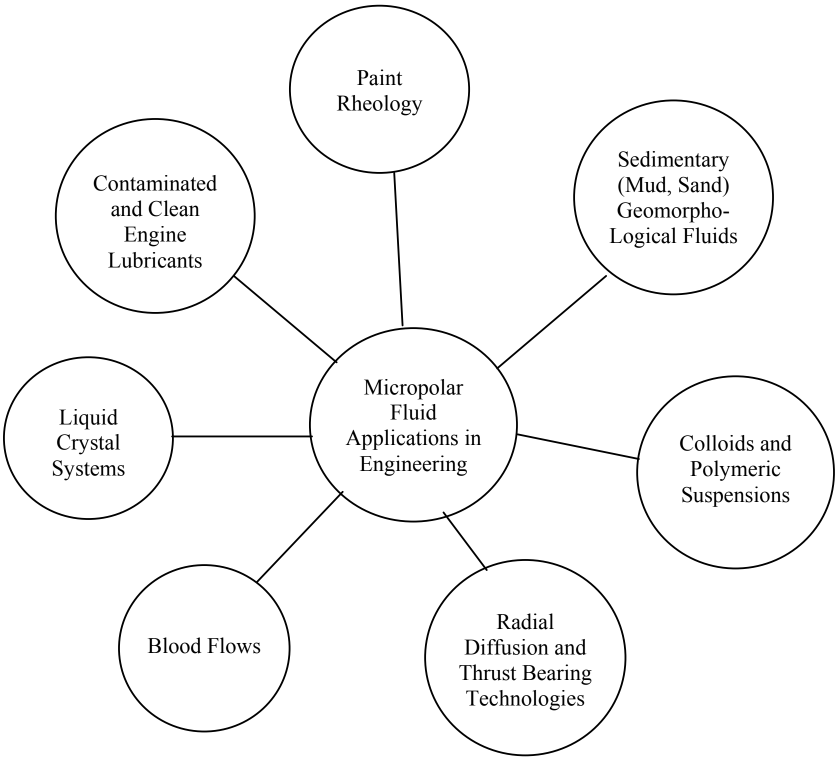


Figure 1. Micropolar fluid applied in engineering.

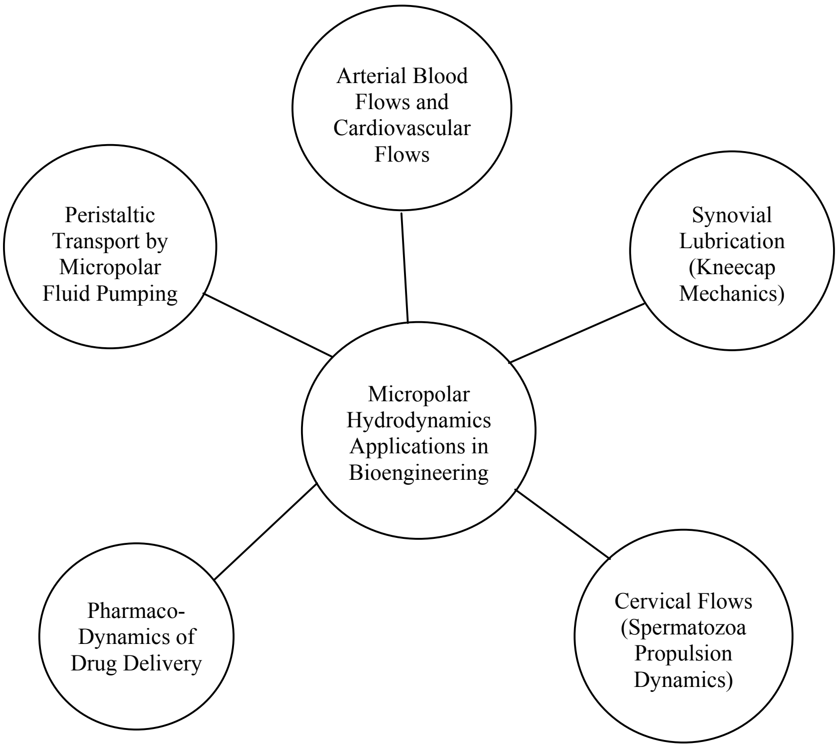


Figure 2. Micropolar hydrodynamics applied in bioengineering.

instability problem in a rotating micropolar fluid. They found that the rotation has a stabilizing effect. The effect of rotation on thermal convection in micropolar fluids has also been studied by Sharma, et al [32], whereas the numerical solution of thermal instability of a rotating micropolar fluid has been discussed by Sastry, et al [33] without taking into account the rotation effect in angular momentum equation. But we also remark that Bhattacharya, et al [34] and Qin, et al [31] have considered the effect of rotation in angular momentum equation. The medium has been considered to be non-porous in all the above studies. In recent years, there has been a lot of interest in study of the breakdown of the stability of a fluid layer subjected to a vertical temperature gradient in a porous medium and the possibility of convective flow. A porous medium is a solid with holes in it, and is characterized by the manner in which the holes are imbedded, how they are interconnected, and the description of their location, shape and interconnection. The flow in the porous medium is governed either by Darcy equation or by modified Darcy equation depending on the structure and the depth of the porous medium. A macroscopic equation describing incompressible flow of a fluid of viscosity, μ, through a macroscopically homogeneous and isotropic porous medium of permeability, k_1, is the well-known Darcy's equation, in which the usual viscous term in the equation of fluid motion is replaced by the resistance term $-(\mu/k_1)\mathbf{q}$, where **q** is the Darcian (filter) velocity of the fluid. However, to be mathematically compatible and physically consistent with Navier-Stokes equations, Brinkman [35] heuristically proposed the introduction of theterm $\frac{\mu}{\varepsilon}\nabla^2\mathbf{q}$ (now known as the Brinkman term) in addition to the Darcian term $-(\mu/k_1)\mathbf{q}$. But the main effect is through the Darcian term; the Brinkman term contributes a very little effect for flow through a porous medium. Therefore the generalized Darcy's law is proposed heuristically to govern the flow of this micropolar fluid through a porous medium.

The study of flow of fluids through porous media is of considerable interest due to its natural occurrence and importance in many problems of engineering and technology such as porous bearings, porous layer insulation consisting of solid and pores, porous rollers, etc. In addition, these flows are applicable to bio-mathematics particularly in the study of blood flow in lungs, arteries, cartilage and so on. The study of a layer of a fluid heated from below in porous media is motivated both theoretically as also by its practical applications in engineering. Among the applications in engineering disciplines, one find the food process industry, chemical process industry, solidification and centrifugal casting of metals. The stability of flow of a fluid through a porous medium taking into account the Darcy resistance was considered by Lapwood [36] and Wooding [37]. Recent studies of stellar atmosphere have shown the existence and importance of porosity in astrophysical context McDonnel [38].

The porous medium of very low permeability allows us to use the generalized Darcy's model [39] including the inertial forces. This is because for a medium of very large stable particle suspension, the permeability tends to be very small justifying the use of the generalized Darcy's model including the inertial forces. This is also because the viscous drag force is negligibly small in comparison with the Darcy's resistance due to the presence of large particle suspension. The thermoconvective instability in a micropolar fluid saturating a porous medium has been studied by Sharma, et al [40] and the effect of rotation on thermal convection in micropolar fluids in porous medium has been considered by Sharma, et al [41]. More recently, Reena, et al [42,43] have studied some of the thermal convection problems in micropolar rotating fluid saturating a porous medium. Sunil, et al [44] have studied the effect of rotation on a layer of micropolar ferromagnetic fluid heated from below saturating a porous medium. All of them found that the rotation has a stabilizing effect. A comprehensive review of the literature concerning convection in porous medium is available in the book of Nield, et al [45].

In the standard Benard problem, the instability is driven by a density difference due to variations in temperature between the upper and lower planes bounding the fluid. But if the fluid layer additionally has salt dissolved in it, then the buoyancy forces can arise not only from density differences due to variations in temperature but also from those due to variations in solute concentration, i.e., there are potentially two destabilizing sources for the density differences, the temperature field and the salt field. The phenomenon of convection arises due to the two effects such as this, is called double-diffusive convection. Thus the heat and solute are two diffusing components in thermosolutal convection phenomena. Brakke [46] explained a double diffusive instability that occurs when a solution of a slowly diffusing protein is layered over a denser solution of more rapidly diffusing sucrose. Themosolutal convection problems arise in oceanography, limnology and engineering. Examples of particular interest are provided by ponds built to trap solar heat Tabor, et al [47] and some Antarctic lakes Shirtcliffe [48]. Particularly, the case involving a temperature field and sodium chloride referred to thermohaline convection. Veronis [49] has investigated the problem of thermohaline convection in a layer of fluid heated from below and subjected to a stable salinity gradient. For three or greater field case, it is referred to as multi-component convection. There are many recent studies involving three or more fields, such as temperature and two salts such as NaCl, KCl.

Thus the study of thermosolutal convection in porous medium in a fluid is of great importance. The driving force for many studies in double-diffusive or multi-component convection has largely physical applications. O'Sullivan, et al [50] have reviewed and studied numerical techniques and their applications in geothermal reservoir simulation. The Salton sea geothermal system in southern California is specifically interesting as it involves convection of hypersaline fluids. For example, Oldenburg, et al [51] have developed a model for convection in a Darcy's porous medium, to model the Salton geothermal system, where the mechanism involves temperature, NaCl, $CaCl_2$ and KCl. Other applications include the oceans, the Earth's magma. Drainage in a mangrove system is yet another area enclosing double-diffusive flows. Solar ponds are a specifically promising means of harnessing energy from the sun by preventing convective overturning in a thermohaline system by salting from below.

The really interesting situation arises from both a geophysical and a mathematical point of view when the layer is simultaneously heated from below and salted from below. Double-diffusive convection in fluids in porous media is also of interest in geophysical systems, electrochemistry, metallurgy, chemical technology, geophysics and biomechanics, soil sciences, astrophysics, ground water hydrology. Sharma, et al [52] have studied the thermosolutal convection of micropolar fluids in hydromagnetics in porous medium. They found that Rayleigh number increases with magnetic field and solute parameter. The thermosolutal convection in a ferromagnetic fluid in porous and non-porous medium has been considered by Sunil, et al [53,54] and the double-diffusive convection in a micropolar ferromagnetic fluid has been studied by Sunil, et al [55,56]. They found the stabilizing effect of stable solute gradient.

In view of the above investigations and keeping in mind the usefulness of micropolar fluids in porous medium in various applications, the present problem deals with the thermosolutal convection of micropolar fluid saturating a porous medium in the presence of rotation. It is attempted to discuss the effect of rotation, solute gradient and how micropolar parameters affects the stability in micropolar fluid heated and soluted from below saturating a porous medium of very low permeability using generalized Darcy's model including the inertial forces. In the present analysis, for mathematical simplicity, we have not considered the effect of rotation in angular momentum equation. This simplification simplifies the analysis without changing the final conclusion. This, in the present literature appears to be unobserved phenomena. The present study can serve as a theoretical support for an experimental investigation.

2. MATHEMATICAL FORMULATION OF THE PROBLEM

The stability of an infinite, horizontal layer of thickness d of an incompressible thin micropolar fluid heated and soluted from below saturating an isotropic and homogeneous porous medium of porosity ε and permeability k_1 acted on by a uniform vertical rotation $\mathbf{\Omega}(0,0,\Omega)$ and gravity field $\mathbf{g} = (0,0,-g)$ is considered. The temperature T and solute concentration C at the bottom and top surfaces z = 0 and z = d are T_0, T_1 and C_0, C_1, respectively, and a steady adverse temperature gradient $\beta\left(=\left|\frac{dT}{dz}\right|\right)$ and a stable solute concentration gradient $\beta' \left(= \left| \frac{dC}{dz} \right| \right)$ are maintained (see Figure 3). The critical temperature gradient depends upon the bulk properties and boundary conditions of the fluid. The temperature gradient thus maintained is qualified as adverse since the fluid at the bottom will be lighter than the fluid at the top because of the thermal expansion; and this is a top-heavy arrangement, which is potentially unstable. On the other hand, the heavier salt at the lower part of the layer has exactly the opposite effect and this acts to prevent motion through convection overturning. Thus these two physical effects are competing against each other. Here, both the boundaries are taken to be free and perfect conductors of heat. Here, the porosity is defined as the fraction of the total volume of the medium that is occupied by void space. Thus 1-ε is the fraction that is occupied by solid. For an isotropic medium, the surface porosity (i.e., the fraction of void area to total area of a typical cross section) will normally be equal to ε. Here, we adopt the Boussinesq approximation [28] which implies that the density can be treated as constant everywhere except when multiplied by gravity. When the fluid flows through a porous medium, the gross effect is represented by Darcy's law.

The mathematical equations governing the motion of a micropolar fluid saturating a porous medium following Boussinesq's approximation for the above model [15,28,41,52] are as follows:

The continuity equation for an incompressible fluid is

$$\nabla.\mathbf{q} = 0 \tag{1}$$

The momentum and internal angular momentum equations for the generalized Darcy model including the inertial forces are

$$\frac{\rho_0}{\varepsilon}\left[\frac{\partial}{\partial t} + \frac{1}{\varepsilon}(\mathbf{q}.\nabla)\right]\mathbf{q} = -\nabla p + \rho g - \frac{1}{k_1}(\mu + k)\,\mathbf{q} + k(\nabla \times \mathbf{v}) + \frac{2\rho_0}{\varepsilon}(\mathbf{q} \times \mathbf{\Omega}), \tag{2}$$

and

$$\rho_0\, j\left[\frac{\partial}{\partial t} + \frac{1}{\varepsilon}(\mathbf{q}.\nabla)\right]\mathbf{v} = (\varepsilon' + \beta'')\nabla(\nabla.\mathbf{v}) + \gamma'\nabla^2\mathbf{v} + \frac{k}{\varepsilon}\nabla \times \mathbf{q} - 2k\,\mathbf{v}, \tag{3}$$

Where $\rho, \rho_0, \mathbf{q}, \mathbf{v}, \mu, k, p, \varepsilon', \beta'', \gamma', j$ and t are the fluid density, reference density, filter velocity, spin (microrotation), shear kinematic viscosity coefficient (constant), coupling viscosity coefficient or vortex viscosity, pressure, bulk spin viscosity coefficient, shear spin viscosity coefficient, micropolar coefficients of viscosity, microinertia constant and time, respectively. The effect of rotation contributes

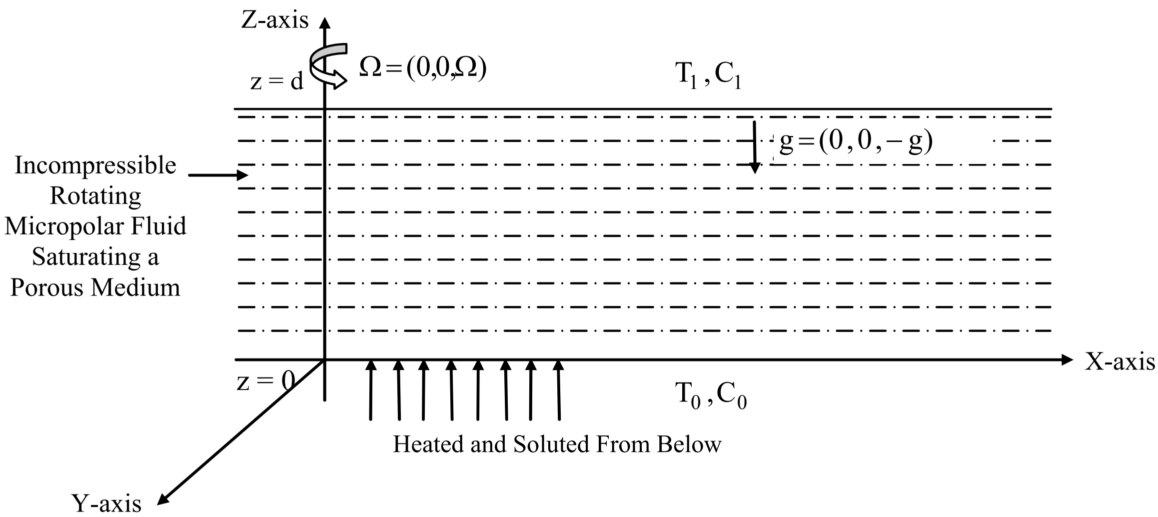


Figure 3. Geometrical configuration.

two terms: (a) centrifugal force $-\rho_0/2\, grad\left|\mathbf{\Omega}\times\mathbf{r}\right|^2$ and (b) Coriolis force $\frac{2\rho_0}{\varepsilon}(\mathbf{q}\times\mathbf{\Omega})$. In Equation 2, $p=p_f-\frac{1}{2}\rho_0\left|\mathbf{\Omega}\times\mathbf{r}\right|^2$ is the reduced pressure, whereas P_f stands for fluid pressure and $\mathbf{r}$ = (x,y,z). When the fluid flows through a porous medium, the gross effect is represented by Darcy's law. As a result, the usual viscous terms is replaced by the resistance term $-\left[\frac{\mu+k}{k_1}\right]\mathbf{q}$. When the permeability of porous material is low, then the inertial force becomes relatively insignificant as compared with the viscous drag when flow is considered. Internal energy balance equations and analogous solute equations are

$$\left[\rho_0 C_v\varepsilon+\rho_s C_s(1-\varepsilon)\right]\frac{\partial T}{\partial t}+\rho_0 C_v(\mathbf{q}.\nabla)T= K_T\nabla^2 T+\delta(\nabla\times\mathbf{v}).\nabla T \tag{4}$$

$$\left[\rho_0 C_v\varepsilon+\rho_s C_s(1-\varepsilon)\right]\frac{\partial C}{\partial t}+\rho_0 C_v(\mathbf{q}.\nabla)C= K_T'\nabla^2 C+\delta'(\nabla\times\mathbf{v}).\nabla C \tag{5}$$

and the density equation of state is given by

$$\rho=\rho_0\left[1-\alpha(T-T_0)+\alpha'(C-C_0)\right], \tag{6}$$

Where $C_v, C_s, K_T, K_T, \delta, \delta', \rho_s, \alpha, \alpha', T, C, T_0$ and C_0 are the specific heat at constant volume, heat capacity of solid (porous material matrix), thermal conductivity, solute conductivity, coefficients giving account of coupling between the spin flux with heat flux and spin flux with solute flux, density of solid matrix, thermal expansion coefficient, an analogous solvent coefficient of expansion, temperature, solute concentration, reference temperature and reference solute concentration at the lower boundary, respectively.

3. BASIC STATE AND PERTURBATION EQUATIONS

Now we are interested in studying the stability of the rest state by giving small perturbations on the rest (initial) state and examine the reactions of the perturbations on the system. The initial state is characterised by

$\mathbf{q}$ = (0, 0, 0), $\mathbf{v}$ = (0, 0, 0), p = p(z), T = T(z)

Defined as $T=-\beta z+T_0$, where $\beta=-\frac{dT}{dZ}$ is the uniform adverse temperature gradient, $C = C(z)$ defined as $C=-\beta' z+C_0$, where $\beta'=-\frac{dC}{dz}$ is the solute concentration gradient and

$\rho=\rho_0\left[1+\alpha\beta z-\alpha'\beta' z\right]$.

Now, we shall analyze the stability of the basic (initial) state by introducing the perturbations, $\mathbf{u}'(u,v,w)$, $\boldsymbol{\omega}, \rho', p', \theta$ and γ in velocity $\mathbf{q}$, spin $\mathbf{v}$, density ρ, pressure p, temperature T and solute concentration C, respectively. The change in density ρ' caused mainly by the perturbation θ and γ in temperature and solute concentration, is given by

$$\rho'=-\rho_0(\alpha\theta-\alpha'\gamma) \tag{7}$$

Then the linearized perturbation equations of the micropolar fluid become

$$\nabla.\mathbf{u}'=0 \tag{8}$$

$$\frac{\rho_0}{\varepsilon}\frac{\partial u}{\partial t}= -\frac{\partial p'}{\partial x}-\frac{1}{k_1}(\mu+k)u+k\Omega_1+\frac{2\rho_0}{\varepsilon}\Omega v, \tag{9}$$

$$\frac{\rho_0}{\varepsilon}\frac{\partial v}{\partial t}= -\frac{\partial p'}{\partial y}-\frac{1}{k_1}(\mu+k)v+k\Omega_1+\frac{2\rho_0}{\varepsilon}\Omega u, \tag{10}$$

$$\frac{\rho_0}{\varepsilon}\frac{\partial w}{\partial t}= -\frac{\partial p'}{\partial z}-\frac{1}{k_1}(\mu+k)w+k\Omega_3+g\rho_0(\alpha\theta-\alpha'\gamma), \tag{11}$$

$$\rho_0 j \frac{\partial \boldsymbol{\omega}}{\partial t} = (\varepsilon' + \beta'')\nabla(\nabla.\boldsymbol{\omega}) + \gamma'\nabla^2\boldsymbol{\omega} + \frac{k}{\varepsilon}\nabla \times \mathbf{u}' - 2k\boldsymbol{\omega}, \quad (12)$$

$$\left[\rho_0 C_v \varepsilon + \rho_s C_s (1-\varepsilon)\right]\frac{\partial \theta}{\partial t} = k_T \nabla^2 \theta - \delta(\nabla \times \boldsymbol{\omega})_z \beta + \rho_0 C_v \beta w \quad (13)$$

and

$$\left[\rho_0 C_v \varepsilon + \rho_s C_s (1-\varepsilon)\right]\frac{\partial \gamma}{\partial t} = k'_T \nabla^2 \gamma - \delta'(\nabla \times \boldsymbol{\omega})_z \beta' + \rho_0 C_v \beta' w \quad (14)$$

Where, the non-linear terms

$$(\mathbf{u}'.\nabla)\mathbf{u}', (\mathbf{u}'.\nabla)\theta, (\mathbf{u}'.\nabla)\gamma, \nabla\theta.(\nabla \times \boldsymbol{\omega}), \nabla\gamma(\nabla \times \boldsymbol{\omega})$$

and $(\mathbf{u}'.\nabla)\boldsymbol{\omega}$ in Equations 9-14 are neglected (using the first order approximations) as the perturbations applied on the system are assumed to be small, the second and higher order perturbations are negligibly small and only linear terms are retained. Also we have assumed $\mathbf{\Omega}' = (\Omega', \Omega'_2, \Omega'_3) = (\nabla \times \boldsymbol{\omega})$.

Now, it is usual to write the balance equations in a dimensionless form, scaling as

$$(x, y, z) = (x^*, y^*, z^*)d, t = \frac{\rho_0 d^2}{\mu} t^*, \theta = \beta d \theta^*,$$

$$\gamma = \beta' d y^*, \mathbf{u}' = \frac{x_T}{d}(\mathbf{u}')^*, p' = \frac{\mu x_T}{d^2}(p')^*,$$

$$\boldsymbol{\omega} = \frac{x_T}{d^2}\boldsymbol{\omega}^* \quad and \quad \mathbf{\Omega} = \frac{\mu}{\rho_0 d^2}\mathbf{\Omega}^*$$

and then removing the stars (*) for convenience, the non-dimensional form of Equations 8-14 become

$$\nabla.\mathbf{u}' = 0 \quad (15)$$

$$\frac{1}{\varepsilon}\frac{\partial \mathbf{u}'}{\partial t} = -\nabla p' - \frac{1}{P_1}(1+K)\mathbf{u}' + K(\mathbf{\Omega}') + \left(R\theta - S\gamma\frac{p_1}{q_1}\right)\hat{e}_z + \frac{2}{\varepsilon}(\mathbf{u}' \times \mathbf{\Omega}), \quad (16)$$

$$\bar{j}\frac{\partial \boldsymbol{\omega}}{\partial t} = C'_1 \nabla(\nabla.\boldsymbol{\omega}) - C'_0 \nabla \times (\nabla \times \boldsymbol{\omega}) + K\left(\frac{1}{\varepsilon}\nabla \times \mathbf{u}' - 2\boldsymbol{\omega}\right), \quad (17)$$

$$E p_1 \frac{\partial \theta}{\partial t} = \nabla^2 \theta + w - \bar{\delta}(\nabla \times \boldsymbol{\omega})_z \quad (18)$$

and

$$E q_1 \frac{\partial \gamma}{\partial t} = \nabla^2 \gamma + w - \bar{\delta}'(\nabla \times \boldsymbol{\omega})_z \quad (19)$$

Where, the new dimensionless coefficients are

$$\bar{j} = \frac{j}{d^2}, \bar{\delta} = \frac{\delta}{\rho_0 C_v d^2}, \bar{\delta}' = \frac{\delta'}{\rho_0 C_v d^2},$$

$$P_l = \frac{k_1}{d^2}, K = \frac{k}{\mu}, C'_0 = \frac{\gamma'}{\mu d^2},$$

$$C'_1 = \frac{\varepsilon' + \beta'' + \gamma'}{\mu d^2}, \ E = \varepsilon + \frac{\rho_s C_s}{\rho_0 C_v}(1-\varepsilon)$$

and $\hat{e}_z$ is a unit vector along z-axis. (20)

and the dimensionless Rayleigh number R, analogous solute number S, Prandtl number p_1 and the analogous Schmidt number q_1 are

$$R = \frac{g\alpha\beta\rho_0 d^4}{\mu x_T}, S = \frac{g\alpha'\beta'\rho_0 d^4}{\mu x_T}, \quad p_1 = \frac{\mu}{\rho_0 x_T}, \ q_1 = \frac{\mu}{\rho_0 x_T} \quad (21)$$

Where,

$$x_T = \frac{K_T}{\rho_0 C_v} \quad \text{and} \quad x'_T = \frac{K'_T}{\rho_0 C_v}$$

are thermal diffusivity and solute diffusivity. Here we consider both the boundaries to be free and perfectly heat conducting. The case of two free boundaries is a little artificial but it enables us to find analytical solutions and to make some qualitative conclusions. The dimensionless boundary conditions are

$$w = 0, \frac{\partial^2 w}{\partial z^2} = 0, \omega = 0, \ \theta = 0 = \gamma \ \text{ at z = 0 and 1.} \quad (22)$$

4. MATHEMATICAL ANALYSIS AND DISPERSION RELATION

Applying the curl operator twice to Equation 16 and taking the z-component, we get

$$\left[\frac{1}{\varepsilon}\frac{\partial}{\partial t}+\frac{1}{P_l}(1+K)\right]\nabla^2 w = R(\nabla_1^2\theta)-S(\nabla_1^2\gamma)\frac{P_1}{q_1}+K\nabla^2\Omega_3-\frac{2}{\varepsilon}\Omega\frac{\partial\zeta_z}{\partial z} \tag{23}$$

Where,

$$\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2},\nabla_1^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2},$$

$$\Omega_3=(\nabla\times\boldsymbol{\omega})_z=\left(\frac{\partial\omega_y}{\partial x}-\frac{\partial\omega_x}{\partial y}\right)$$

and $\zeta_z=(\nabla\times\mathbf{u}')_z=\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)$ is the z-component of vorticity.

Again applying curl operator once to Equations 16 and 17 and taking z-component, we get

$$\frac{1}{\varepsilon}\frac{\partial\zeta_z}{\partial t}=-\frac{1}{P_l}(1+K)\zeta_z+\frac{2}{\varepsilon}\mathbf{\Omega}\frac{\partial w}{\partial z} \tag{24}$$

and

$$\bar{j}\frac{\partial\Omega_3'}{\partial t}=C_0'\nabla^2\Omega_3'-K\left[\frac{1}{\varepsilon}\nabla^2 w+2\Omega_3'\right] \tag{25}$$

The linearized form of Equations 18 and 19 are

$$Ep_1\frac{\partial\theta}{\partial t}=\nabla^2\theta+w-\bar{\delta}\Omega_3' \tag{26}$$

and

$$Eq_1\frac{\partial\gamma}{\partial t}=\nabla^2\gamma+w-\bar{\delta}'\Omega_3' \tag{27}$$

Now, the boundary conditions are

$$w=0=\frac{\partial^2 w}{\partial z^2},\frac{\partial\zeta_z}{\partial z}=0,\zeta_z=0=\theta=\gamma=\Omega_3' \text{ at } z=0 \text{ and } 1. \tag{28}$$

$\Omega_3'=0$ at z = 0 and 1 are the boundary conditions for the spin.

In the Equation 25 for spin, the coefficient C_0' and K account for spin diffusion and coupling between vorticity and spin effects respectively.

Analyzing the disturbances into the normal modes, we assume that the solutions of Equations 23-27 are given by

$$\left[w,\Omega'_3,\zeta_z,\theta,\gamma\right]=\left[W(z),G(z),Z(z),\Theta(z),\Gamma(z)\right]\exp(ik_x x+ik_y y+\sigma t) \tag{29}$$

Where k_x, k_y are the wave numbers along with x and y directions respectively, $a=(k_x^2+k_y^2)^{1/2}$ is the resultant wave number and σ is the stability parameter which is, in general a complex constant.

Following the normal mode analysis, the linearized perturbation dimensionless equations become

$$\left[\frac{\sigma}{\varepsilon}+\frac{1}{P_l}(1+K)\right](D^2-a^2)W = -Ra^2\Theta+\frac{Sp_1}{q_1}a^2\Gamma+K(D^2-a^2)G-\frac{2}{\varepsilon}\Omega DZ \tag{30}$$

$$\left[\frac{\sigma}{\varepsilon}+\frac{(1+K)}{P_l}\right]Z=\frac{2}{\varepsilon}\Omega DW \tag{31}$$

$$\left[\bar{j}\sigma+2K-C_0'(D^2-a^2)\right]G=-K\varepsilon^{-1}(D^2-a^2)W, \tag{32}$$

$$\left[Ep_1\sigma-(D^2-a^2)\right]\Theta=W-\bar{\delta}G, \tag{33}$$

and

$$\left[Eq_1\sigma-(D^2-a^2)\right]\Gamma=W-\bar{\delta}'G, \tag{34}$$

Where, $\bar{j},K,\bar{\delta},\bar{\delta}'$ and C_0' are the non-dimensional micropolar parameters and $D\equiv\frac{d}{dz}$.

The case of two free boundaries is of little physical interest, but it is mathematically important because one can derive an exact solution, whose properties guide our analysis. Thus the exact solution of the system (30)-(34) subject to the boundary conditions

$$W = D^2W = 0,\ DZ = 0, \Theta = \Gamma = G = 0 \text{ at } z = 0 \text{ and } z=1. \quad (35)$$

is written in the form

$$W = A_1 e^{\sigma t} \sin \pi z, \Theta = B_1 e^{\sigma t} \sin \pi z, \quad G = C_1 e^{\sigma t} \sin \pi z, \Gamma = D_1 e^{\sigma t} \sin \pi z \quad (36)$$

Where, A_1, B_1, C_1 and D_1 are constants and σ is the growth rate which is, in general, a complex constant.

Substituting Equations 36 in Equations 30-34, we get following equations

$$\left[\left(\frac{\sigma}{\varepsilon} + \frac{1+K}{P_l}\right)^2 \left(\pi^2 + a^2\right) + \frac{4\pi^2\Omega^2}{\varepsilon^2}\right] A_1 - Ra^2\left(\frac{\sigma}{\varepsilon} + \overline{\frac{1+K}{P_l}}\right) B_1 + \frac{Sp_1}{q_1} a^2 \left(\frac{\sigma}{\varepsilon} + \overline{\frac{1+K}{P_l}}\right) D_1 - K\left(\frac{\sigma}{\varepsilon} + \overline{\frac{1+K}{p_l}}\right)(\pi^2 + a^2) C_1 = 0 \quad (37)$$

$$-\frac{K}{\varepsilon}(\pi^2 + a^2) A_1 + \left[\bar{j}\sigma + 2K + C_0' \ (\pi^2 + a^2)\right] C_1 = 0 \quad (38)$$

$$A_1 - (Ep_1 \sigma + \pi^2 + a^2) B_1 - \bar{\delta} C_1 = 0 \quad (39)$$

and

$$A_1 - \left[Eq_1 \sigma + (\pi^2 + a^2)\right] D_1 - \bar{\delta}' C_1 = 0 \quad (40)$$

For existence of non-trivial solutions of the above equations, the determinant of the coefficients of A_1, B_1, C_1 and D_1 in Equations 37-40 must vanish. This determinant on simplification yields

$$b'\left[\frac{\sigma}{\varepsilon} + \left(\frac{1+K}{P_l}\right)\right]^2 [Ep_1\sigma + b'][Eq_1\sigma + b'] [\bar{j}\sigma + 2K + C_0' b'] = Ra^2\left[\frac{\sigma}{\varepsilon} + \left(\frac{1+K}{P_l}\right)\right][Eq_1\sigma + b'] \left[\bar{j}\sigma + 2K + C_0' b' - \bar{\delta} K \varepsilon^{-1} b'\right] + K^2 \varepsilon^{-1} b'^2 \left[\frac{\sigma}{\varepsilon} + \left(\frac{1+K}{P_l}\right)\right][Ep_1\sigma + b'][Eq_1\sigma + b'] - \frac{Sp_1}{q_1} a^2 \left[\frac{\sigma}{\varepsilon} + \left(\frac{1+K}{P_l}\right)\right][Ep_1\sigma + b'] [\bar{j}\sigma + 2K + C'_0 b' - \bar{\delta}' K \varepsilon^{-1} b'] - 4\pi^2 \Omega^2 \varepsilon^{-2} [Ep_1\sigma + b'][Eq_1\sigma + b'][\bar{j}\sigma + 2K + C'_0 b'], \quad (41)$$

Where,

$$b' = \pi^2 + a^2.$$

In the absence of solute parameter (S=0 i.e., $\bar{\delta}' = 0$), Equation 41 reduces to

$$b'\left[\frac{\sigma}{\varepsilon} + \left(\frac{1+K}{P_l}\right)\right]^2 [Ep_1\sigma + b'][l\sigma + 2A + b'] = Ra^2\left[\frac{\sigma}{\varepsilon} + \left(\frac{1+K}{P_l}\right)\right][l\sigma + 2A + b' - \bar{\delta} A \varepsilon^{-1} b'] + KA\varepsilon^{-1} b'^2 \left[\frac{\sigma}{\varepsilon} + \left(\frac{1+K}{P_l}\right)\right][Ep_1\sigma + b'] - 4\pi^2 \Omega^2 \varepsilon^{-2} [Ep_1\sigma + b'][l\sigma + 2A + b'], \quad (42)$$

Where

$$\boldsymbol{l} = \bar{j} / C_0', A = \frac{K}{C_0'},$$

A result derived by Sharma, et al [41].

If the fluid is non-rotating (**Ω**=0), the Equation 42 reduces to

$$b'\left[\frac{\sigma}{\varepsilon} + \left(\frac{1+K}{P_l}\right)\right] [Ep_1\sigma + b'][l\sigma + 2A + b'] = Ra^2 [l\sigma + 2A + b' - \bar{\delta} A \varepsilon^{-1} A b'] + KA\varepsilon^{-1} b'^2 [Ep_1\sigma + b'] \quad (43)$$

A result derived by Sharma, et al [40].

Equation 41 is the required dispersion relation studying the effect of medium permeability, rotation and solute parameter of the system.

For simplification of further calculations, Equation 41 may also be written in the form:

$$iP_5\sigma_i^5 + P_4\sigma_i^4 - iP_3\sigma_i^3 - P_2\sigma_i^2 + iP_1\sigma_i + P_0 = 0 \quad (44)$$

Where,

$$P_5 = b\left[\frac{E^2\, p_1 q_1 I_1}{\varepsilon^2}\right],$$

$$P_4 = \left(\frac{bE}{\varepsilon^2}\right)[b\,I_1\,(p_1+q_1) + E\,p_1 q_1\,(2\varepsilon\, I_1 L_0 + L_1)],$$

$$P_3 = \frac{E^2 p_1 q_1 b}{\varepsilon}\left\{2L_0L_1 - \frac{K^2 b}{\varepsilon}\right\} + \frac{b^2 E\,(p_1+q_1)}{\varepsilon}$$
$$\left(2I_1L_0 + \frac{L_1}{\varepsilon}\right) + \frac{b^3 I_1}{\varepsilon^2} + E^2 p_1 q_1 I_1[4\Omega'^2\,\varepsilon^{-2} + bL_0^2] +$$
$$x\left(\frac{S_1 p_1}{q_1}\right)\frac{Ep_1I_1}{\varepsilon} - xR_1\left(\frac{Eq_1I_1}{\varepsilon}\right)$$

$$P_2 =$$
$$\frac{b^2E(p_1+q_1)}{\varepsilon}\left(2L_0L_1 - \frac{K^2b}{\varepsilon}\right) + \frac{b^3}{\varepsilon}\left(2I_1L_0 + \frac{L_1}{\varepsilon}\right) +$$
$$E^2 p_1 q_1\left[L_0 b\left(L_0L_1 - \frac{K^2 b}{\varepsilon}\right) + 4\Omega'^2\,\varepsilon^{-2}L_1\right] +$$
$$bE\,(p_1+q_1)\,I_1\left[bL_0^2 + 4\Omega'^2\,\varepsilon^{-2}\right] -$$
$$xR_1\left[I_1\left(\frac{b}{\varepsilon} + Ep_1L_0\right) + \frac{Eq_1L_2}{\varepsilon}\right] +$$
$$\frac{S_1p_1x}{q_1}\left[I_1\left(\frac{b}{\varepsilon} + Ep_1L_0\right) + \frac{Ep_1}{\varepsilon}L_3\right]$$

$$P_1 =$$
$$\frac{b^3}{\varepsilon}\left(2L_0L_1 - \frac{K^2b}{\varepsilon}\right) + L_0 b^2 E\,(p_1+q_1)\left(L_0L_1 - \frac{k^2b}{\varepsilon}\right)$$
$$+ L_0^2 b^3 I_1 + 4\Omega'^2\,\varepsilon^{-2} b\left\{bI_1 + E\,(p_1+q_1)L_1\right\}$$
$$+\frac{S_1p_1x}{q_1}\left\{\left(\frac{b}{\varepsilon} + Ep_1L_0\right)L_3 + bI_1L_0\right\} -$$
$$xR_1\left\{\left(\frac{b}{\varepsilon} + Eq_1L_0\right)L_2 + bI_1L_0\right\}$$

$$P_0 = b^3L_0\left(L_0L_1 - \frac{K^2b}{\varepsilon}\right) + 4\Omega'^2\,\varepsilon^{-2}b^2L_1 +$$
$$\frac{bS_1p_1x}{\varepsilon_1}L_0L_3 - bxL_0L_2R_1 \quad (45)$$

Where,

$$R_1 = \frac{R}{\pi^4},\; S_1 = \frac{S}{\pi^4},\; x = \frac{a^2}{\pi^2},\; i\sigma_i = \frac{\sigma}{\pi^2},$$
$$b = 1 + x,\; I_1 = \pi^2\bar{j}$$
$$N_3 = C_0'\pi^2\;,\; N_5 = \bar{\delta}\pi^2\;,\; N_6 = \bar{\delta}'\pi^2\;,$$
$$P_l' = \pi^2 P_l\,,\Omega' = \Omega/\pi^2\,,L_0 = \left(\frac{1+K}{P_l'}\right),$$
$$L_1 = (2K + N_3 b), L_2 = 2K + b\left(N_3 - \frac{N_5K}{\varepsilon}\right)$$

and

$$L_3 = 2K + b\left(N_3 - \frac{N_6K}{\varepsilon}\right)$$

6. THE CASE OF STATIONARY CONVECTION

Let the marginal state be stationary, so that it is characterized by putting $\sigma_i = 0$ [28]. Hence, for the stationary convection, putting $\sigma_i = 0$ in Equation 44, Rayleigh number is given by

$$R_1 =$$
$$\left[\begin{array}{l}\dfrac{S_1p_1}{q_1}\left\{2K + b\left(N_3 - \dfrac{N_6K}{\varepsilon}\right)\right\} + \dfrac{1}{x}[b\{\left(\dfrac{1+K}{P_l}\right) \\ (2K + N_3b)\{b + 4\Omega'^2\,\varepsilon^{-2}\left(\dfrac{P_l'}{1+K}\right)^2\} - \dfrac{K^2b^2}{\varepsilon}\}]\end{array}\right]$$

(46)

This leads to the marginal stability curve in stationary conditions. Equation 46 expresses the Rayleigh number R_1 as a function of dimensionless wave number, medium permeability parameter P_l (Darcy number), solute gradient parameter S_1, rotation parameter Ω', coupling parameter K (coupling between vorticity and spin effects), spin diffusion (couple stress) parameter N_3, micropolar heat conduction parameter N_5 (arises due to coupling between spin and heat fluxes) and micropolar solute parameter N_6 (arises due to coupling between spin and solute fluxes). The parameters K and N_3 measure the micropolar viscous effects and micropolar diffusion effects, respectively. The classical results in respect of Newtonian fluids can be obtained as the limiting case of present study.

Setting K = 0 and S_1 = 0 and keeping N_3 arbitrary in Equation 46, we obtained

$$R_1 = \frac{(1+x)(1+x+4\Omega'^2 \varepsilon^2 P_l'^2)}{xP_l'}, \tag{47}$$

This is the expression for the Rayleigh number of rotating micropolar fluid in porous medium.

Setting $\Omega' = 0$ in Equation 47 yields

$$R_1 = \frac{(1+x)^2}{xP_l'}, \tag{48}$$

This is the classical Rayleigh-Benard result for the Newtonian fluid case in porous medium.

To investigate the effect of medium permeability, rotation, stable solute gradient, coupling parameter, spin diffusion parameter micropolar heat conduction parameter and micropolar solute parameter, we examine the behaviour of $\frac{dR_1}{dP_l'}, \frac{dR_1}{d\Omega'}, \frac{dR_1}{dS_1}, \frac{dR_1}{dN_3}, \frac{dR_1}{dN_5}$ and $\frac{dR_1}{dN_6}$ analytically.

From Equation 46,

$$\frac{dR_1}{dP_l'} = -\frac{\left[b(bN_3+2K)\{b(1+K)^2 - 4\Omega'^2 \varepsilon^{-2} P_l'^2\}\right]}{xP_l'^2(1+K)\left[2K+b\left(N_3 - \frac{N_5K}{\varepsilon}\right)\right]} \tag{49}$$

Which is always negative if

$$4\Omega'^2 \varepsilon^{-2} < \frac{1}{P_l'^2}$$

and

$$N_3 > \frac{N_5 K}{\varepsilon} \tag{50}$$

This gives that the medium permeability has a destabilizing effect when condition (50) hold. In the absence of micropolar viscous effect (coupling parameter) and rotation, (49) yields the destabilizing effect of medium permeability on the system. Medium permeability may have a dual role in the presence of rotation. The medium permeability has a stabilizing effect if

$$4\Omega'^2 \varepsilon^{-2} > \frac{b(1+K)^2}{P_l'^2}$$

Thus, for higher values of rotation parameter Ω', the stabilizing effect of medium permeability has been predicted. Thus, there is a competition between the destabilizing role of medium permeability and stabilizing role of micro polar viscous effect and rotation, but there is complete destabilization in the above inequalities given by (50).

Thus the destabilizing behaviour of medium permeability is significantly affected by micropolar parameters, i.e., K, N_3, N_5 and rotation parameter Ω' but virtually unaffected by analogous solute gradient parameter S_1.

Equation 46 also yields

$$\frac{dR_1}{d\Omega''} = \frac{4bP_l'(bN_3+2K)\varepsilon^{-2}}{x(1+K)\left[2K+b\left(N_3 - \frac{N_5K}{\varepsilon}\right)\right]},$$

Where

$$\Omega'' = \Omega'^2 \tag{51}$$

Which is always negative if

$$N_3 > \frac{N_5K}{\varepsilon} \tag{52}$$

This implies that rotation has stabilizing effect when condition (52) holds. In the absence of micropolar viscous effect (coupling parameter, K), the rotation always has a stabilizing effect on the system.

It can easily be found from Equation 46 that

$$\frac{dR_1}{dS_1}=\frac{p_1\left\{2K+b\left(N_3-\frac{N_6K}{\varepsilon}\right)\right\}}{q_1\left[2K+b\left(N_3-\frac{N_5K}{\varepsilon}\right)\right]} \tag{53}$$

Which is always positive if

$$N_3>\max\left[\frac{N_6K}{\varepsilon},\frac{N_5K}{\varepsilon}\right] \tag{54}$$

This shows the stabilizing effect of stable solute gradient when condition (54) holds. In the absence of micro polar viscous effect, (53) yields the stabilizing effect of stable solute gradient.

Equation 46 also gives

$$\frac{dR_1}{dK}=\frac{b\left[\begin{array}{l}\frac{N_3b^2}{P_l'}\left\{\begin{array}{l}\frac{bN_5}{\varepsilon}+\frac{N_3}{(1+K)^2}(b(1+K)^2\\-4\Omega'^2\varepsilon^{-2}P_l'^2)\end{array}\right\}\\+\frac{4\Omega'^2\varepsilon^{-2}P_l'}{(1+K)^2}\left\{\begin{array}{l}2bN_3+\frac{1}{\varepsilon}(bN_5-2\varepsilon)\\\{2K^2+bN_3(2K+1)\}\end{array}\right\}\\+Kb\left(\frac{2}{P_l'}-\frac{b}{\varepsilon}\right)\left\{2K+b\left(2N_3-\frac{N_5K}{\varepsilon}\right)\right\}\\+\frac{S_1p_1}{q_1}\frac{bN_3x}{\varepsilon}(N_5-N_6)\end{array}\right]}{x\left[2K+b\left(N_3-\frac{N_5K}{\varepsilon}\right)\right]^2} \tag{55}$$

Which is always positive if

$$\frac{1}{P_l'}>\frac{b}{\varepsilon},N_3>\frac{N_5K}{\varepsilon},4\Omega'^2\varepsilon^{-2}<\frac{1}{P_l'}$$

and

$$N_5>\max\ (2\varepsilon,N_6) \tag{56}$$

This indicates that coupling parameter has a stabilizing effect when condition 56 hold. Equation 55 also yields that $(dR_1)/(dK)$ is always positive in the absence of rotation, micropolar solute parameter (coupling between spin and solute fluxes) and in a non-porous medium, thus indicating the stabilizing effect of coupling parameter.

Thus the medium permeability and porosity have a significant role in developing the condition for the stabilizing behaviour of coupling parameter.

It follows from Equation 46 that

$$\frac{dR_1}{dN_3}=\frac{b^2K\left[\begin{array}{l}b\left\{N_5b+Kb(N_5-P_l')+\frac{4\Omega'^2\varepsilon^{-2}P_l'^2N_5}{(1+K)}\right\}\\+\frac{S_1p_1}{q_1}xP_l(N_5-N_6)\end{array}\right]}{\varepsilon\,x\,P_l\left[2K+b\left(N_3-\frac{N_5K}{\varepsilon}\right)\right]^2} \tag{57}$$

Which is always negative if

$$N_5>P_l'$$

and

$$N_5>N_6$$

This implies that

$$N_5>\max(P_l',N_6) \tag{58}$$

This shows that spin diffusion has a destabilizing effect when condition (58) holds.

Equation 46 also gives

$$\frac{dR_1}{dN_5} = \frac{Kb\left[\begin{array}{l}\frac{S_1p_1}{q_1}\left\{2K+b\left(N_3-\frac{N_6K}{\varepsilon}\right)\right\}\\ +\frac{1}{x}b\left\{\left(\frac{1+K}{P_l'}\right)N_3b+\left(\frac{2K}{P_l'}\right)+K^2\left(\frac{2}{P_l'}-\frac{b}{\varepsilon}\right)\right\}\\ +4\Omega'^2\varepsilon^{-2}\frac{(bN_3+2K)P_l'}{(1+K)}\end{array}\right]}{\varepsilon\left[2K+b\left(N_3-\frac{N_5K}{\varepsilon}\right)\right]^2} \quad (59)$$

Which is always positive if

$$\frac{1}{P_l'}>\frac{b}{\varepsilon}\ , \quad N_3>\frac{N_6K}{\varepsilon} \quad (60)$$

This gives that the micropolar heat conduction has a stabilizing effect when condition (60) holds. Equation 59 also yields that $\frac{dR_1}{dN_5}$ is always positive in a non-porous medium, implying thereby the stabilizing effect of micropolar heat conduction parameter.

We can also find from Equation 36 that

$$\frac{dR_1}{dN_6}=-\frac{S_1p_1}{q_1}\frac{bK}{\varepsilon}.\left[2K+b\left(N_3-\frac{N_5K}{\varepsilon}\right)\right]^{-1}$$

Which is always negative if

$$N_3>\frac{N_5K}{\varepsilon} \quad (61)$$

Implying thereby the destabilizing effect of micropolar solute parameter under condition (61).

Now the critical thermal Rayleigh number for the onset of instability is determined numerically using Newton-Raphson method by the condition $\frac{dR_1}{dx}=0$. As a function of x, R_1 given by Equation 46 attains its minimum when

$$T_4x^4 + T_3x^3 + T_2x^2 + T_1x + T_0=0 \quad (62)$$

Where,

$$T_4 = DI, T_3 = 2DH, T_2 = \frac{S_1p_1}{q_1}(HB - IA) + C(H-2I) + 3D(H-I) - EI, T_1 = -2I(C+D+E)$$

and

$$T_0 = -H(C+D+E)$$

Where,

$$A = 2K + \left(N_3 - \frac{N_6K}{\varepsilon}\right)$$

$$B = \left(N_3 - \frac{N_6K}{\varepsilon}\right)$$

$$C = \left(\frac{1+K}{P_l'}\right)2K + 4\Omega'^2\varepsilon^{-2}\left(\frac{P_l'}{1+K}\right)N_3,$$

$$D = \left(\frac{1+K}{P_l'}\right)N_3 - \frac{K^2}{\varepsilon},$$

$$E = 4\Omega'^2\varepsilon^{-2}\left(\frac{P_l'}{1+K}\right)2K,$$

$$H = \left[2K + \left(N_3 - \frac{N_5K}{\varepsilon}\right)\right],$$

$$I = \left(N_3 - \frac{N_5K}{\varepsilon}\right),$$

With x_c determined as a solution of Equation 62, Equation 46 will give the required critical thermal Rayleigh number (R_c) for various values of critical wave numbers x_c. The critical thermal Rayleigh number (R_c), depends on medium permeability P_l', stable solute gradient S_1, rotation parameter Ω' and micropolar parameters K, N_3, N_5 and N_6. The numerical values of critical thermal Rayleigh numbers (R_c) and critical wave numbers (x_c) determined for various values of P_l', S_1, Ω' and micropolar parameters K, N_3, N_5 and N_6 are given in Tables 1-7 and values of R_c are illustrated in Figures 4-10. Here, in Figures 4-6, we have plotted the graphs for the critical thermal Rayleigh number R_c versus $P_l^{'}$ [For various values of rotation parameter Ω'], rotation parameter Ω' [for various values of $P_l^{'}$] and stable solute parameter S_1 [for various values of $P_l^{'}$], respectively, in the presence and absence of coupling parameter K. Figures 7-10 exhibit the plots of critical thermal Rayleigh number R_c versus micropolar parameters K, N_3, N_5 and N_6 for several values of $P_l^{'}$, respectively.

From Figure 4 and Table 1, one may find that as $P_l^{'}$ increases, R_c decreases for lower values of rotation parameter Ω', whereas, for sufficiently higher values of rotation parameter Ω', R_c first decreases for lower values of $P_l^{'}$ and then increases for higher values of $P_l^{'}$ and hence showing the destabilizing effect of the medium permeability for lower values of rotation parameter, whereas destabilizing or a stabilizing effect for sufficiently higher values of the rotation parameter. This behaviour can also be observed in Figure 5.

Figures 5 and 6 indicates the stabilizing behaviour of rotation parameter and stable solute parameter S_1, as the critical thermal Rayleigh number increases with the increase in rotation parameter and stable solute gradient parameter, respectively, which can also be observed from

TABLE 1. Critical Thermal Rayleigh Numbers and Wave Numbers of the Unstable Modes at Marginal Instability for the Onset of Stationary Convection for Various Values of Medium Permeability Parameter ($P_l^{'}$).

$P_l^{'}$	$K = 0.2$						$K = 0$					
	$\Omega' = 0$		$\Omega' = 10$		$\Omega' = 100$		$\Omega' = 0$		$\Omega' = 10$		$\Omega' = 100$	
	x_c	R_c	x_c	R_c	x_c	R_c	x_c	R_c	x_c	R_c	x_c	R_c
0.001	0.9900621	5827.708	0.990612	5830.656	1.043613	6118.636	1	4500	1.0008	4503.198	1.077033	4814.066
0.002	0.9891670	3187.761	0.9913644	3193.657	1.188983	3750.657	1	2500	1.003195	2506.394	1.280625	3100.624
0.003	0.9882728	2307.778	0.9932098	2316.62	1.398307	3117.169	1	1833.333	1.007174	1842.916	1.56205	2688.033
0.004	0.9873794	1867.787	0.996139	1879.568	1.647342	2901.118	1	1500	1.012719	1512.76	1.886797	2583.398
0.005	0.9864871	1603.791	1.000139	1618.507	1.920707	2843.802	1	1300	1.019804	1315.922	2.236068	2594.427
0.006	0.9855956	1427.794	1.005195	1445.434	2.209405	2861.828	1	1166.667	1.028397	1185.732	2.6	2660
0.007	0.9847051	1302.081	1.011288	1322.635	2.508163	2920.885	1	1071.429	1.038461	1093.617	2.973214	2755.204

TABLE 2. Critical Thermal Rayleigh Numbers and Wave Numbers of the Unstable Modes at Marginal Instability for the Onset of Stationary Convection for Various Values of Rotation Parameter (Ω').

Ω'	$K = 0.2$						$K = 0$					
	$P_l^{'} = 0.003$		$P_l^{'} = 0.005$		$P_l^{'} = 0.007$		$P_l^{'} = 0.003$		$P_l^{'} = 0.005$		$P_l^{'} = 0.007$	
	x_c	R_c	x_c	R_c	x_c	R_c	x_c	R_c	x_c	R_c	x_c	R_c
0	0.9882728	2307.779	0.9864871	1603.791	0.9847051	1302.081	1	1833.333	1	1300	1	1071.429
10	0.9932098	2316.62	1.000139	1618.507	1.011288	1322.635	1.007174	1842.916	1.019804	1315.922	1.038461	1093.617
30	1.031858	2386.588	1.103319	1732.981	1.203112	1479.023	1.062826	1918.418	1.16619	1438.476	1.305986	1259.653
50	1.105114	2522.916	1.285145	1948.741	1.51593	1764.521	1.16619	2064.127	1.414214	1665.685	1.720465	1557.276
70	1.2067	2720.004	1.517689	2250.795	1.890802	2156.527	1.305986	2272.524	1.720465	1980.186	2.200364	1963.19
90	1.33015	2972.082	1.781232	2628.592	2.297597	2643.558	1.47187	2536.713	2.059126	2371.651	2.711163	2467.532
100	1.398307	3117.169	1.920707	2843.802	2.508163	2920.885	1.56205	2688.034	2.236068	2594.427	2.973214	2755.204
150	1.78294	4014.848	2.662125	4165.405	3.598634	4632.786	2.059126	3619.417	3.162278	3964.911	4.317407	4539.259

TABLE 3. Critical Thermal Rayleigh Numbers And Wave Numbers of the Unstable Modes at Marginal Instability for the Onset of Stationary Convection for Various Values of Stable Solute Gradient Parameter (S_1).

S_1	$K = 0.2$						$K = 0$					
	$P_l^{'} = 0.001$		$P_l^{'} = 0.002$		$P_l^{'} = 0.003$		$P_l^{'} = 0.001$		$P_l^{'} = 0.002$		$P_l^{'} = 0.003$	
	x_c	R_c	x_c	R_c	x_c	R_c	x_c	R_c	x_c	R_c	x_c	R_c
0	0.9915109	5282.677	0.9931615	2645.675	0.9959048	1768.633	1.0008	4003.199	1.003195	2006.394	1.007174	1342.916
5	0.990612	5830.656	0.9913644	3193.657	0.9932098	2316.62	1.0008	4503.198	1.003195	2506.394	1.007174	1842.916
10	0.9897141	6378.632	0.9895712	3741.634	0.9905236	2864.6	1.0008	5003.199	1.003195	3006.394	1.007174	2342.916
15	0.9888172	6926.605	0.987782	4289.607	0.9878462	3412.574	1.0008	5503.199	1.003195	3506.395	1.007174	2842.916
20	0.9879214	7474.578	0.9859968	4837.575	0.9851778	3960.541	1.0008	6003.198	1.003195	4006.394	1.007174	3342.917
25	0.9870265	8022.547	0.9842154	5385.54	0.9825181	4508.503	1.0008	6503.198	1.003195	4506.395	1.007174	3842.916
30	0.9861325	8570.514	0.9824381	5933.499	0.9798673	5056.457	1.0008	7003.199	1.003195	5006.294	1.007174	4342.916

TABLE 4. Critical Thermal Rayleigh Numbers and Wave Numbers of the Unstable Modes at Marginal Instability for the Onset of Stationary Convection for Various Values of Coupling Parameter (K).

K	$P_l^{'} = 0.001$		$P_l^{'} = 0.002$		$P_l^{'} = 0.003$	
	x_c	R_c	x_c	R_c	x_c	R_c
0.2	0.990612	5830.656	0.9913644	3193.657	0.9932098	2316.62
0.3	0.9793898	6553.819	0.9790286	3566.964	0.9795904	2573.252
0.4	0.9651006	7315.967	0.9635771	3959.679	0.9628419	2842.776
0.5	0.9484264	8116.342	0.9457657	4371.376	0.9437879	3124.877
0.6	0.9299848	8953.981	0.9262649	4801.522	0.9231465	3419.148
0.7	0.9103344	9827.724	0.9056688	5249.49	0.9015423	3725.15
0.8	0.8899646	10736.26	0.884854	5714.579	0.8794998	4042.379

Tables 2 and 3. This shows that the stable solute parameter and rotation postpones the onset of convection. This leads to laterally onset of convection instability. Also, it is obvious from Figures 4 and 6 that only for small values of K, the onset of convection is delayed. This shows that higher values of R_c are needed for the onset of convection in the presence of K, hence justifying the stabilizing effect of the coupling parameter, which can also be observed from Figure 7 and Table 4.

Figures 7-10 represent the graphs of critical thermal Rayleigh number R_c versus micropolar parameters K, N_3, N_5 and N_6, respectively for various values of $P_l^{'}$. Figures 7 and 9 clearly show that critical thermal Rayleigh number R_c increases with increasing K and N_5, respectively, which can also be observed from Tables 4 and 6. This leads to laterally onset of convection instability, implying thereby that the coupling parameter K and the micropolar heat conduction parameter N_5 has a stabilizing effect in the presence of salinity, whereas Figures 8 and 10 indicates that the R_c decreases with increasing N_3 and N_6 respectively.

This leads to an early onset of convection implying thereby the destabiizing behaviour of micropolar spin diffusion (couple stress) parameter N_3 and micropolar solute parameter N_6 on the system. It can also observe from Tables 5 and 7, respectively.

TABLE 5. Critical Thermal Rayleigh Numbers and Wave Numbers of the Unstable Modes at Marginal Instability for the Onset of Stationary Convection for Various Values of Spin Diffusion Parameter (N_3).

N_3	$P_l^{'} = 0.001$		$P_l^{'} = 0.003$		$P_l^{'} = 0.005$	
	x_c	R_c	x_c	R_c	x_c	R_c
2	0.990612	5830.656	0.9932098	2316.62	1.000139	1618.507
4	0.9979394	5566.715	1.001887	2212.318	1.010199	1545.894
6	0.9993731	5478.702	1.003586	2177.541	1.012169	1521.688
8	0.9998848	5434.694	1.004193	2160.153	1.012872	1509.586
10	1.000124	5408.288	1.004476	2149.72	1.013201	1502.326

TABLE 6. Critical Thermal Rayleigh Numbers and Wave Numbers of the Unstable Modes at Marginal Instability for the Onset of Stationary Convection for Various Values of Micropolar Heat Conduction Parameter (N_5).

N_5	$P_l^{'} = 0.001$		$P_l^{'} = 0.002$		$P_l^{'} = 0.003$		$P_l^{'} = 0.004$		$P_l^{'} = 0.005$	
	x_c	R_c	x_c	R_c	x_c	R_c	x_c	R_c	x_c	R_c
0.08	0.9991119	5378.941	1.000676	2946.224	1.003334	2137.117	1.007089	1733.909	1.011925	1493.053
0.1	0.9987353	5398.863	1.000259	2957.136	1.002885	2145.034	1.006603	1740.332	1.011402	1498.586
0.5	0.990612	5830.658	0.9913644	3193.657	0.9932098	2316.62	0.996139	1879.568	1.000139	1618.507
1.0	0.9785781	6477.947	0.9782027	3548.205	0.9789097	2573.821	0.9806898	2088.274	0.9835296	1798.259
1.5	0.9637956	7286.212	0.9620614	3990.891	0.9614009	2894.94	0.9618039	2348.832	0.9632565	2022.662
2.0	0.945225	8323.645	0.9418267	4559.017	0.9395006	3307.005	0.9382324	2608.153	0.938007	2310.571

TABLE 7. Critical Thermal Rayleigh Numbers and Wave Numbers of the Unstable Modes at Marginal Instability for the Onset of Stationary Convection for Various Values of Micropolar Solute Parameter (N_6).

N_6	$P_l^{'} = 0.001$		$P_l^{'} = 0.003$		$P_l^{'} = 0.005$		$P_l^{'} = 0.007$	
	x_c	R_c	x_c	R_c	x_c	R_c	x_c	R_c
0.02	0.990612	5830.656	0.9932098	2316.62	1.000139	1618.507	1.011288	1322.635
0.05	0.9906682	5827.657	0.9933779	2313.62	1.000419	1615.507	1.011679	1319.633
0.10	0.9907618	5822.659	0.9936583	2308.622	1.000886	1610.507	1.012333	1314.63
0.15	0.9908553	5817.663	0.9939388	2303.624	1.001353	1605.506	1.012986	1309.627
0.20	0.990949	5812.664	0.9942193	2298.625	1.001821	1600.506	1.013641	1304.623

7. PRINCIPLE OF EXCHANGE OF STABILITIES

Here, we investigate the possibility of oscillatory modes, if any, on stability problem due to the presence of medium permeability, rotation, micropolar parameters and solute gradient. Equating the imaginary parts of Equation 44, we obtain

$$\sigma_i \left[P_5 \sigma_i^4 - P_3 \sigma_i^2 + P_1 \right] = 0$$

i.e.,

$$\sigma_i\left[b\left(\frac{E^2 p_1 q_1 I_1}{\varepsilon^2}\right)\sigma_i^4-\sigma_i^2\left\{\frac{E^2 p_1 q_1 b}{\varepsilon}\right.\right.$$

$$\left(2L_0L_1-\frac{K^2b}{\varepsilon}\right)+\frac{b^2 E(p_1+q_1)}{\varepsilon}\left(2I_1L_0+\frac{L_1}{\varepsilon}\right)+$$

$$\frac{b^3 I_1}{\varepsilon^2}+E^2 p_1 q_1 I_1 (4\Omega'^2\varepsilon^{-2}+bL_0^2)+$$

$$\left.x\left(\frac{S_1 p_1}{q_1}\right)\frac{Ep_1 I_1}{\varepsilon}-xR_1\left(\frac{Eq_1 I_1}{\varepsilon}\right)\right\}+$$

$$\frac{b^3}{\varepsilon}\left(2L_0L_1-\frac{K^2b}{\varepsilon}\right)+$$

$$L_0 b^2 E(p_1+q_1)\left(L_0L_1-\frac{K^2b}{\varepsilon}\right)+L_0^2 b^3 I_1+ \tag{63}$$

$$4\Omega'^2\varepsilon^{-2}b(bI_1+E(p_1+q_1)L_1)+$$

$$\frac{S_1p_1}{q_1}x\left[\left(\frac{b}{\varepsilon}+L_0\right)L_0+bI_1L_0\right]-$$

$$\left.xR_1\left\{\left(\frac{b}{\varepsilon}+L_0\right)L_2+bI_1L_0\right\}\right]=0$$

It is evident from Equation 63 that σ_i may be either zero or non-zero, implies that the modes may be either non-oscillatory or oscillatory.

7.1. Limiting Case In the absence of rotation $(\Omega'=0)$ and solute gradient $(S_1=0, q_1=0)$, the vanishing of determinant of the coefficients of A_1, B_1, C_1 and D_1 in Equations 37-40 gives (after simplification)

$$\sigma_i T_3\sigma_i^3-T_2\sigma_i^2+iT_1\sigma_i+T_0=0 \tag{64}$$

Where,

$$T_3=b\left[\frac{Ep_1I_1}{\varepsilon}\right],$$

$$T_2=b\left[\left(\frac{1+K}{P_l'}\right)Ep_1I_1+\frac{1}{\varepsilon}\{Ep_1(2K+N_3b)+bI_1\}\right]$$

$$T_1=b\left\{b\left((1+K)\frac{I_1}{P_l'}+\frac{(2K+N_3b)}{\varepsilon}\right)+\right.,$$

$$\left.\left(\frac{\overline{1+K}}{P_l'}(2K+N_3b)-\frac{K^2b)}{\varepsilon}\right)Ep_1\right\}-xR_1I_1$$

and

$$T_0=b^3\left[\frac{(1+K)N_3}{P_l'}-\frac{K^2}{\varepsilon}\right]+b^2\left[\left(\frac{1+K}{P_l'}\right)2K\right]-$$

$$bxR_1\left(N_3-\frac{N_5K}{\varepsilon}\right)-2xR_1K$$

Equating the imaginary parts of Equation 64, we obtain

$$\sigma_i\left[-b\left(\frac{Ep_1I_1}{\varepsilon}\right)\sigma_i^2+b\left\{b\left(1+K\frac{I_1}{P_l'}+\frac{(2K+N_3b)}{\varepsilon}\right)+\right.\right.$$

$$\left.\left.\left(\frac{1+K}{P_l'}(2K+N_3b)-\frac{K^2b}{\varepsilon}\right)Ep_1\right\}-xR_1I_1\right]=0 \tag{65}$$

It is evident from Equation 65 that σ_i may be either zero or non-zero, meaning that the modes may be either non-oscillatory or oscillatory. In the absence of micropolar viscous effect (K=0) and microinertia (I_1=0), we obtain the result as

$$\sigma_i\left[\frac{b}{\varepsilon}+\frac{Ep_1}{P_l'}\right]=0 \tag{66}$$

Here, the quantity inside the bracket is positive definite. Hence

$$\sigma_i=0 \tag{67}$$

This shows that the oscillatory modes are not possible and the principle of exchange of stabilities (PES) is satisfied for micropolar fluid heated from below, in the absence of micropolar viscous effect, microinertia, solute gradient and rotation.

Thus, we conclude that the oscillatory modes are introduced due to the presence of the micropolar viscous effect, microinertia, rotation and solute gradient, which were non-existence in their absence.

8. THE CASE OF OVERSTABILITY

In the present section, we have to find the possibility that the observed instability may really be overstability. Since σ is, in general, a complex constant, so we put $\sigma = \sigma_r + i\sigma_i$, where σ_r and σ_i are real. The marginal state is reached when $\sigma_r = 0$ If $\sigma_r = 0$ implies $\sigma_i = 0$, one says that principle of exchange of stabiities (PES) is valid otherwise we have overstability and then $\sigma = i\sigma_i$, at marginal stability. Hence it is sufficient to find conditions for which (44) will admit of solutions with σ_i real.

Equating real and imaginary parts of Equation 44 and eliminating R_1 between them, yields

$$A_3 C_1^3 + A_2 C_1^2 + A_1 C_1 + A_0 = 0 \tag{68}$$

Where,

$$C_1 = \sigma_i^2 .$$

$$A_3 = \frac{(Eq_1)^2 I_1 b}{\varepsilon^2}\left[\frac{b}{\varepsilon}\left\{I_1 + \frac{(KN_5)(Ep_1)}{\varepsilon}\right\} + (Ep_1)\left(\frac{1+K}{P_l'}\right) I_1\right] \tag{69}$$

$$\begin{aligned}
A_2 = {} & \frac{b^4}{\varepsilon^3}\left[I_1\left(I_1 + \frac{KN_5(Ep_1)}{\varepsilon}\right) + N_3 (Eq_1)^2\left(N_3 - \frac{N_5 K}{\varepsilon}\right)\right] + \\
& \frac{b^3}{\varepsilon^2}\left[(Ep_1) I_1^2 + \frac{(Eq_1)^2 I_1 K^2}{\varepsilon} + \left(\frac{2K}{\varepsilon}\right) N_3 (Eq_1)^2 + \right. \\
& (Eq_1)^2\left\{ \left(N_3 - \frac{N_5 K}{\varepsilon}\right)\frac{K}{\varepsilon}(2 - (Ep_1)K) + \right. \\
& b\left[(Ep_1)(Eq_1)^2\left\{ \frac{K}{\varepsilon^2}\left\{\left(\frac{1+K}{P_l'}\right) K - 4\Omega'^2 \varepsilon^{-2} I_1 N_5\right\} + \right.\right. \\
& \left.\left(\frac{1+K}{P_l'}\right)^3 I_1^2 + 3(Ep_1)(Eq_1)^2 \frac{K^2}{\varepsilon^2}\left(\frac{1+K}{P_l'}\right)\right\} + \frac{1}{2} N_3 \\
& \left(\frac{1+K}{P_l'}\right)((Ep_1)N_3 - I_1) + \frac{1}{2} I_1\left(\frac{1+K}{P_l'}\right)\left(N_3 - \frac{2N_5 K}{\varepsilon}\right) + \\
& \left.\left.\frac{1}{2}(Ep_1)\left(\frac{1+K}{P_l'}\right) N_3\left(N_3 - \frac{2N_5 K}{\varepsilon}\right)\right\}\right] + \\
& b^2\left[\frac{(Eq_1)^2}{\varepsilon}\left(I_1^2\left(\frac{1+K}{P_l'}\right)^2 + \frac{2K}{\varepsilon}\left\{ (Ep_1)\left(\frac{1+K}{P_l'}\right)\right.\right.\right. \\
& \left(N_3 - \frac{N_5 K}{\varepsilon}\right) + \left(\frac{1+K}{P_l'}\right)((Ep_1)N_3 - I_1) + \left(\frac{1+K}{P_l'}\right) I_1 + \\
& \left.\left.\left.\frac{K}{\varepsilon}(2 - (Ep_1)K) + (Ep_1)\left(\frac{1+K}{P_l'}\right)^2 I_1 \frac{N_5 K}{\varepsilon}\right\}\right)\right] + \\
& b\left[(Ep_1)(Eq_1)^2\left\{\frac{K}{\varepsilon^2}\left\{\left(\frac{1+K}{P_l'}\right) K - 4\Omega'^2 \varepsilon^{-2} I_1 N_5\right\} + \right.\right. \\
& \left.\left(\frac{1+K}{P_l'}\right)^3 I_1^2 + 3(Ep_1)(Eq_1)^2 \frac{k^2}{\varepsilon^2}\left(\frac{1+K}{P_l'}\right)\right\} + \\
& \left(\frac{S_1 p_1}{q_1}\right)\frac{x I_1}{\varepsilon^2}\left\{ I_1\left\{Ep_1 - Eq_1\left(1 + \frac{N_3(Ep_1)}{I_1}\right)\right\}\right\} + \\
& \left.(Ep_1)(Eq_1)\left\{\left(N_3 - \frac{N_5 K}{\varepsilon}\right) + \frac{N_6 K}{\varepsilon}\right\}\right\}\right] + \\
& 4\Omega'^2 \varepsilon^{-2} (Eq_1)^2 I_1^2 \left\{(Ep_1)\left(\frac{1+K}{P_l'}\right) - \frac{b}{\varepsilon}\right\}
\end{aligned} \tag{70}$$

and the coefficients A_1, A_0, being quite lengthy and not needed in the discussion of over stability, have not been written here.

Since σ_i is real for over stability, the three values of C_1 $(=\sigma_i^2)$ are positive. The sum of roots of Equation 68 is $-A_2/A_3$ must be positive, and if this is to be negative, then $A_3 > 0$ and $A_2 > 0$. Since $A_3 > 0$ (from Equation 69), $A_2 > 0$ gives the sufficient conditions for non-existence of overstability.

It is clear from Equation 70 that A_2 is positive if

$$N_3 > \frac{2N_5K}{\varepsilon}, Ep_1N_3 > I_1, \frac{1}{Ep_1} >$$

$$K, \frac{Ep_1}{P_l'} > \frac{b}{\varepsilon}, P_1 > q_1\left\{1 + \frac{N_3(Ep_1)}{I_1}\right\}$$

and

$$\frac{K}{P_l'} > 4\Omega'^2\varepsilon^{-2}I_1N_5$$

Which implies that

$$N_3 > \max\left\{\frac{2KN_5}{\varepsilon}, \frac{I_1}{(Ep_1)}\right\}, \frac{1}{K}(Ep_1) > P_l'\left(\frac{1+x}{\varepsilon}\right),$$

$$K_T < K'_T\left[\frac{1}{\left(1 + \frac{N_3(Ep_1)}{I_1}\right)}\right]$$

and

$$4\Omega'^2 < \frac{K\varepsilon^2}{I_1N_5P_l'}.$$

Thus, for $N_3 > \max\left\{\frac{2KN_5}{\varepsilon}, \frac{I_1}{(Ep_1)}\right\}$,

$$\frac{1}{K} > (Ep_1) > P_l'\left(\frac{1+x}{\varepsilon}\right), K_T < K_T'\left[\frac{1}{\left(1 + \frac{N_3(Ep_1)}{I_1}\right)}\right]$$

and

$$4\Omega'^2 < \frac{K\varepsilon^2}{I_1N_5P_l'},$$

Overstability can not occur and the principle of the exchange of stabilities is valid. Hence, the conditions mentioned above are the sufficient conditions for the non-existence of over stability, the violation of which does not necessarily imply the occurrence of over stability, whereas in the absence of rotation and micro polar parameters in non-porous medium, the above conditions, as expected, reduces to $K_T < K_T'$, i.e., the thermal conductivity is less than the solute conductivity.

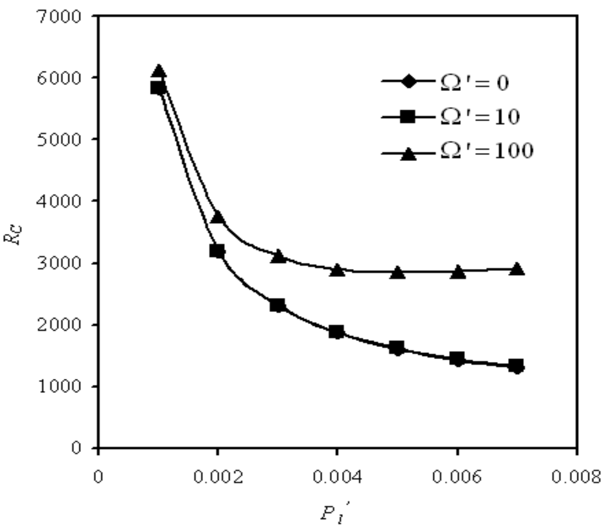
9. DISCUSSION OF RESULTS AND CONCLUSIONS

In this paper, the double diffusive convection in a micro polar fluid layer heated and soluted from below, saturating a porous medium in the presence of uniform rotation has been investigated. The effect of various parameters, like medium permeability, rotation, solute gradient, coupling parameter, spin diffusion parameter, micro polar heat conduction parameter and micro polar solute parameters on the onset of convection has been investigated. The principal conclusions from the analysis of this paper are as follows:

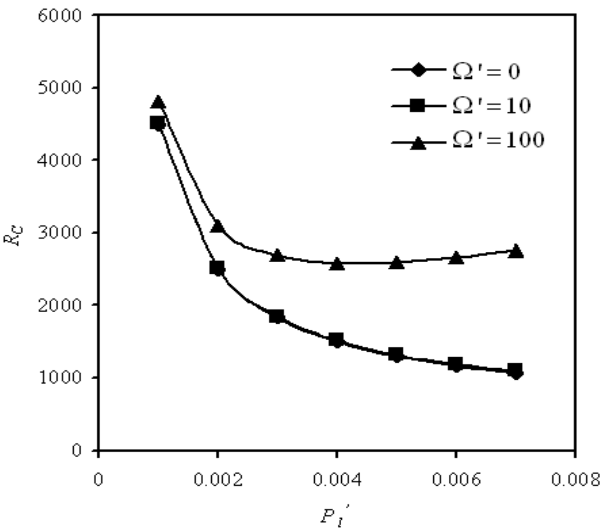
(i) The results show that for the case of stationary convection, the medium permeability has destabilizing/stabilizing effect under certain condition(s). But in the absence of micro polar viscous effect (coupling parameter) and rotation, medium permeability always has destabilizing effect. The rotation, solute gradient, micro polar heat conduction and coupling parameter has a stabilizing effect under certain condition(s), whereas the spin diffusion parameter and micro polar solute parameter has a destabilizing effect under certain condition(s). In the absence of micro polar viscous effect, the rotation and stable solute gradient always has the stabilizing effect on the system, whereas in the absence of rotation, micro polar solute parameter (coupling between spin and solute fluxes) and in a non-porous medium, coupling parameter has a stabilizing effect on the system. Here, we also observe that in a non-porous medium, the micro polar heat conduction always has a stabilizing effect.

(ii) The critical thermal Rayleigh numbers and critical wave numbers for the onset of instability are also determined numerically (using Newton-Raphson method) and the sensitiveness of the critical Rayleigh number R_c to the changes in the medium permeability parameter P_l', rotation parameter Ω', stable solute gradient parameter S_1, micro polar fluid parameters K, N_3, N_5 and N_6 is depicted graphically in Figures 4-10. The effects of governing parameters on the stability of the system are discussed below:

- Figures 4 and 5 demonstrate the influence of medium permeability parameter (Darcy number), P_l', and rotation parameter Ω' in the presence and absence of the coupling parameter K. This can also be observed from Tables 1 and 2. Figure 4 illustrates that as P_l' increases, R_c decreases for small values of Ω' whereas for the higher values of Ω', R_c decreases for lower values of P_l' and then increases for higher values of P_l', i.e., P_l' has dual role. This behaviour can also be observed form Figure 5. The physical explanations behind it are: "it is well known that the rotation introduces the vorticity into the fluid in the case of Newtonian fluid [28]. Then the fluid moves in the horizontal planes with higher velocities. Due to this motion, the velocity of the fluid perpendicular to the planes reduces, and hence delays the onset of convection. When the fluid layer is assumed to be flowing through an isotropic and homogeneous porous medium, free from rotation or small rate of rotation, then the medium permeability has destabilizing effect. As medium permeability increases, the void space increases, and a result of this, the flow quantities perpendicular to the planes will clearly be increased. Thus, increase in heat transfer is responsible for early onset of convection. Thus increasing P_l' leads to decrease in R_c implying the destabilizing effect of P_l' in the absence of rotation or for small value of rotation. In case of high rotation, the motion of the fluid prevails essentially in the horizontal planes. This motion is increased as medium permeability increases. Thus the component of the velocity perpendicular to the horizontal plane reduces, leading to the delay in the onset of convection. Hence, medium permeability has stabilizing effect in case of high rotation [44].

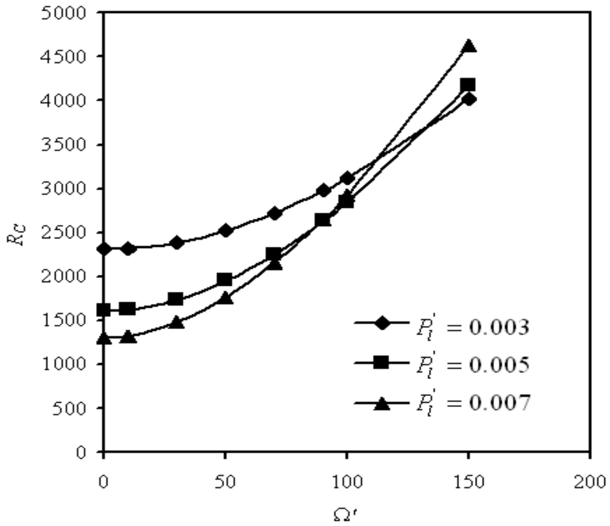


(a)

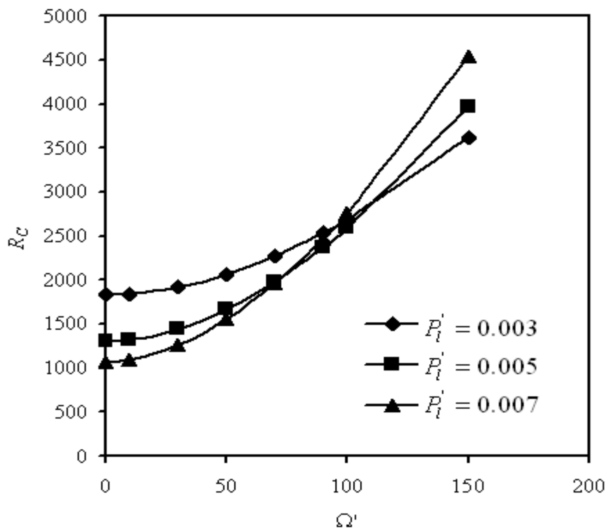


(b)

Figure 4. Marginal instability curve for variation of critical thermal Rayleigh numbers (R_c) versus medium permeability parameter (P_l') for $\varepsilon = 0.5$, $p_1 = 1$, $q_1 = 0.01$, $S_1 = 5$, $N_3 = 2$, $N_5 = 0.5$, $N_6 = 0.02$ and (a) $K = 0.2$, (b) $K = 0$.

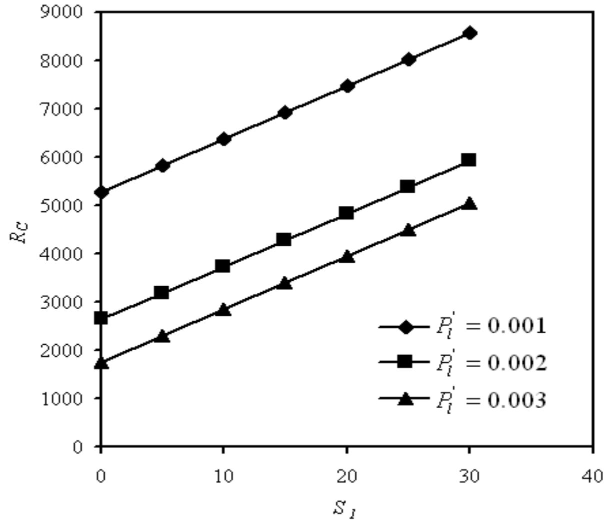


(a)

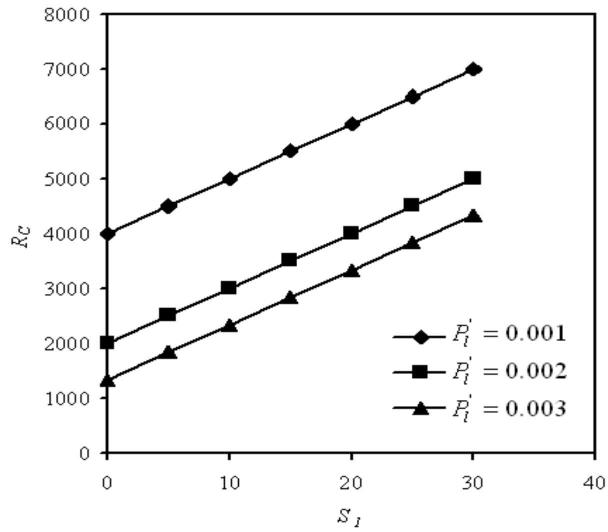


(b)

Figure 5. Marginal instability curve for variation of critical thermal Rayleigh numbers (R_c) versus rotation parameter (Ω') for $\varepsilon = 0.5$, $p_1 = 1$, $q_1 = 0.01$, $S_1 = 5$, $N_3 = 2$, $N_5 = 0.5$, $N_6 = 0.02$ and (a) $K = 0.2$, (b) $K = 0$.



(a)



(b)

Figure 6. Marginal instability curve for variation of critical thermal Rayleigh numbers (R_c) versus stable solute gradient parameter (S_1), for $\varepsilon = 0.5$, $p_1 = 1$, $q_1 = 0.01$, $\Omega' = 10$, $N_3 = 2$, $N_5 = 0.5$, $N_6 = 0.02$ and (a) $K = 0.2$, (b) $K = 0$.

- Figure 6 represents the plot of critical Rayleigh number R_c, versus stable solute gradient S_1 for various values of P_l', in the presence and absence of coupling parameter K. This figure indicates the stable solute gradient has a stabilizing effect as the critical Rayleigh number R_c increases with the increase in S_1. This shows that the stable solute gradient postpones the onset of convection. This can also be observed from the Table 3.

- Figure 7 and Table 4 clearly show R_c increases with increasing K, illustrates that the coupling parameter has a stabilizing effect. As K increases, concentration of micro elements also increases, and as a result of this, a greater part of the energy of the system is consumed by these elements in developing gyrational (twist) velocities in the fluid, and onset of convection is delayed.
- Figure 8 represents the plot of critical thermal Rayleigh number R_c versus N_3 for various values of $P_l^{'}$. This graph exhibits a destabilizing trend. In other words, N_3 destabilizes the flow in the presence of salinity. The physical reason behind it is that as N_3 increases, the couple stress of the fluid increases, which causes the micro rotation to decrease, rendering the system prone to instability. Nevertheless, the above phenomenon is true in porous or non-porous medium. This leads to the conclusion that the spin diffusion (couple stress) parameter leads to an early onset of convection in a micro polar fluid in the presence of salinity. This can also be observed in Table 5.
- Figure 9 represents the plot of critical thermal Rayleigh number R_c versus N_5 for various values of $P_l^{'}$. Figure 9 and Table 6 illustrates that as N_5 increases, R_c increases, implying thereby that micropolar heat conduction parameter has a stabilizing effect in the presence of salinity. When N_5 increases, the heat induced into the fluid due to microelements is also increased, thus inducing the heat transfer from the bottom to the top. The decrease in heat transfer is responsible for delaying the onset of convection. Thus, increasing N_5 leads to increase in R_c. In other words, N_5 stabilizes the flow.
- In Figure 10 and Table 7, we have also investigated the effect of micropolar solute parameter (N_6) (arises due to the coupling between spin and solute fluxes). Figure 10 and Table 7 illustrate that as N_6 increase, R_c decreases. In other words, N_6 destabilizes the flow. This leads to the conclusion that micropolar solute parameter leads to an early onset of convection in a micropolar fluid. Thus the system is destabilized by micropolar solute parameter N_6.

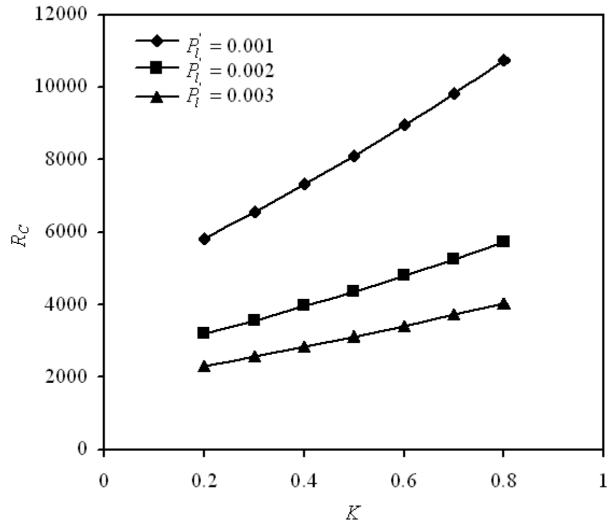


Figure 7. Marginal instability curve for variation of critical thermal Rayleigh numbers (R_c) versus coupling parameter (K) for $\varepsilon = 0.5, p_1 = 1, q_1 = 0.01, S_1 = 5, \Omega' = 10, N_3 = 2, N_5 = 0.5, N_6 = 0.02$.

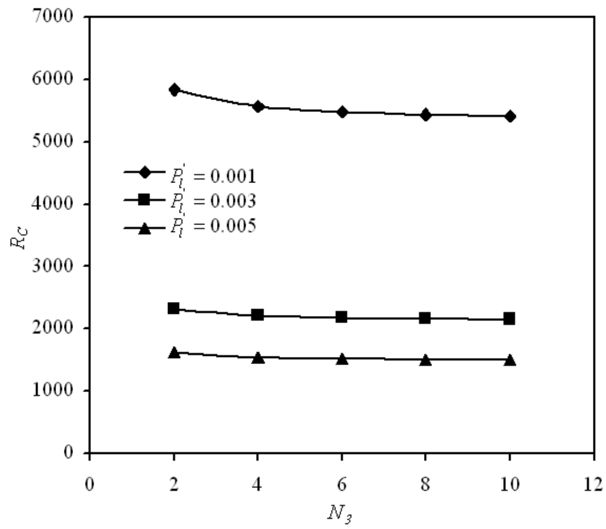


Figure 8. Marginal instability curve for variation of critical thermal Rayleigh numbers (R_c) versus micropolar spin diffusion parameter (N_3) for $\varepsilon = 0.5, p_1 = 1, q_1 = 0.01, K = 0.2, \Omega' = 10, S_1 = 5, N_5 = 0.5, N_6 = 0.02$.

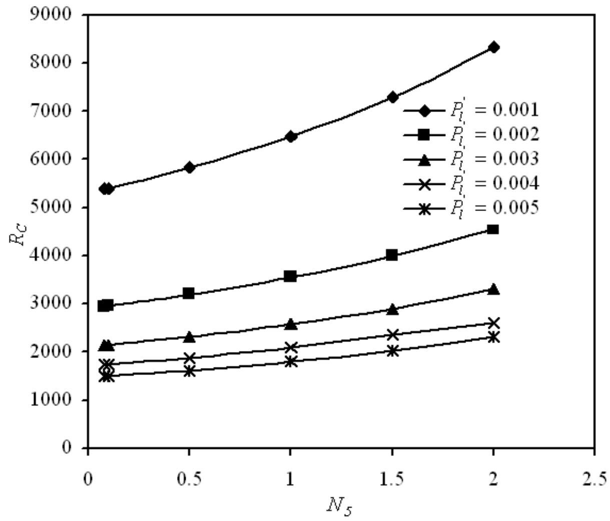


Figure 9. Marginal instability curve for variation of critical thermal Rayleigh numbers (R_c) versus micropolar heat conduction (N_5) for $\varepsilon = 0.5, K = 0.2, \Omega' = 10, p_1 = 1, q_1 = 0.01, S_1 = 5, N_3 = 2, N_6 = 0.02$.

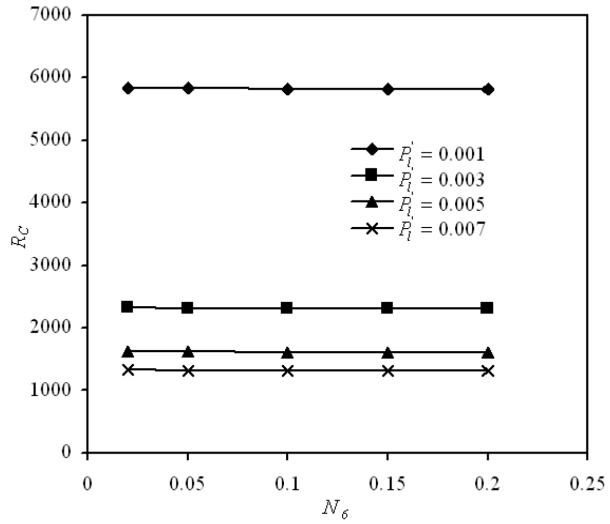


Figure 10. Marginal instability curve for variation of critical thermal Rayleigh numbers (R_c) versus micropolar solute parameter (N_6) for $\varepsilon = 0.5, K = 0.2, \Omega' = 10, p_1 = 1, q_1 = 0.01, S_1 = 5, N_3 = 2, N_5 = 0.5$.

(iii) The principle of exchange of stabilities is found to hold true for the micropolar fluid heated and soluted from below in the absence of micropolar viscous effect (coupling between vorticity and spin effect), micropolar inertia, solute gradient and rotation. Thus, oscillatory modes are introduced due to the presence of the micropolar viscous effect, microinertia, solute gradient and rotation, which were non-existence in their absence.

(iv) If

$$N_3 > \max\{\frac{2KN_5}{\varepsilon}, \frac{I_1}{(Ep_1)}\},$$

$$\frac{1}{K} > (Ep_1) > P_l'(\frac{1+x}{\varepsilon}),\ K_T < K_T'[\frac{1}{\{1+\frac{N_3(Ep_1)}{I_1}\}}]$$

and

$$4\Omega'^2 \varepsilon^{-2} < \frac{K}{I_1 N_5 P_l'},$$

Over stability can not occur and the principle of the exchange of stabilities is valid. Hence, the above conditions are the sufficient conditions for the non-existence of over stability, the violation of which does not necessarily imply the occurrence of over stability. In the absence of rotation and micropolar parameters in non-porous medium,the above conditions, as expected, reduces to $K_T < K_T'$, i.e., the thermal conductivity is less than the solute conductivity, which is in good agreement with the results obtained earlier by Sunil, et al [56].

Finally, from the above analysis, we conclude that the micro polar parameters, rotation and solute gradient have a profound effect on the onset of convection in porous medium. It is hoped that the present work will be helpful for understanding more complex problems involving the various physical effects investigated in the present problem.

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