RESEARCH NOTE

ELASTICO-VISCOUS FLOW BETWEEN TWO ROTATING DISCS OF DIFFERENT TRANSPIRATION FOR HIGH REYNOLDS NUMBERS

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Abstract The flow in an elastico-viscous fluid between two co-axial infinite rotating porous discs is considered for high cross flow Reynolds number. The discs are rotating with different angular velocity and the injection rate of the fluid at one disc is different from the suction rate of other disc. The effect of suction parameters on the velocity components have been investigated numerically and solved by iterative methods/finite difference methods and depicted graphically. This study has immense practical utility especially when the fluid is non-Newtonian. The results are applicable in the chemical industry using fluids of higher Reynolds numbers.

Keywords Elastico-Viscous, Porous Medium, Reynolds Number

چکیده جریان سیال لزج ارتجاعی بین دو دیسک متخلخل دوار هم محور بی‌نهایت بزرگ در اعـداد رینولـدز بر مبنای سرعت عرضی جریان بزرگ بررسی شده است. دیسک‌ها سرعت دورانی متفاوت دارند و نـرخ رانـش سیال از یک دیسک با نرخ مکش سیال از دیسک دیگر متفاوت است. اثرات پارامترهای مکش بر عوامل سرعت به شکل عددی و با استفاده از روش‌های تفاضل محدود و تکرار حل شده و به صورت منحنی نمایش داده شده است. این مطاله امکانات وسیع علمی برای سیال غیر نیوتنی را به دست می‌دهد. این نتایج برای صنایع شیمیایی با عدد رینولدز جریان بالا قابل استفاده است.

1. INTRODUCTION

The problem of forced flow fluid between two rotating discs is important in chemical and mechanical engineering. A large number of theoretical investigations dealing with the study of incompressible laminar flow with either injection or suction have appeared during the last few decades. Karman [1] has discussed the flow of viscous incompressible fluid under the influence of rotating disc. Following Karman [1], Lance, et al [2] have discussed the flow between two rotating discs. The problem of flow between a rotating and a stationary disc has been independently solved by Mellor, et al [3] under the assumption of similarity solutions. Narayan, et al [4] and Wilson [5] studied the same problem but they applied suction either on the stationary disc or on the rotating disc. Gaur [6] also considered the flow of a viscous incompressible fluid between two infinite porous rotating discs under the assumption that the rate of fluid injection at one disc is equal to the rate of suction at the other. Hossain, et al [7] studied the same problem in presence of transverse magnetic field. Chaudhary, et al [8] studied the flow and heat transfer of an incompressible second order fluid between two infinite porous rotating discs of infinite radius where the suction Reynolds number is assumed small.

The present paper is concerned with the flow of an incompressible second-order fluid between two infinite, porous rotating discs, where the suction

Reynolds number is assumed to be high. The constitutive equation for the incompressible second-order fluid is taken in the form of;

$$\sigma = -pI + \mu_1 A_1 + \mu_2 A_2 + \mu_3 (A_1)^2 \tag{1}$$

Where p is the stress tensor, an are the Kinematic Rivlin-Ericksen tensor; μ_1, μ_2, μ_3 are the material coefficients describing the viscosity, elasticity and cross-viscosity respectively. Equation 1 was derived by Coleman, et al [9] from simple fluids by assuming that stress is more sensitive to the recent deformation than the deformation occured in the distant past.

In this paper, it is assumed that the rate of fluid suction at one disc is different from the rate of injection at the other disc. The velocities in transverse and axial direction have been shown graphically for various values of parameters involved in the solution.

2. MATHEMATICAL FORMULATION

Consider the flow of a viscous incompressible second-order fluid between two coaxial parallel infinite porous discs. The discs are rotating with different angular velocities and the suction rate taken from the upper discs is different than the injection rate at the lower discs. We use the cylindrical polar co-ordinate $(\bar{r}, \bar{\theta}, \bar{z})$ and define the surface of the discs by planes respectively. By symmetry all the variables will be independent. Now introducing the following non-dimensional quantities.

$$r = \frac{\bar{r}}{d},\ z = \frac{\bar{z}}{d},\ u = \frac{\bar{u}d}{v_1},\ v = \frac{\overline{v_1 d}}{v_1},$$

$$w = \frac{\bar{w}d}{v_1}, p = \frac{\bar{p}}{\rho\left(v_1/d\right)^2}$$

In the governing equations for velocity subject to the boundary conditions:

$$\bar{z}=0, \bar{u}=0, \bar{v}=mR\Omega\bar{r}, \bar{w}=nw$$

$$\bar{z}=d, \bar{u}=0, \bar{v}=R\Omega\bar{r}, \bar{w}=W$$

We get the non-dimensional equations as:

$$\frac{\partial u}{\partial r}+\frac{\partial w}{\partial z}+\frac{u}{r}=0 \tag{2}$$

$$\begin{aligned}
&u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z}-\frac{v^2}{r}=-\frac{\partial p}{\partial r}+\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{\partial^2 u}{\partial z^2}-\frac{u}{r^2}+\\
&\alpha\left[u\left(\frac{\partial^3 u}{\partial r^3}+\frac{\partial^3 u}{\partial r\partial z^2}\right)+w\left(\frac{\partial^3 u}{\partial r^2\partial z}+\frac{\partial^3 u}{\partial z^3}+\frac{1}{r}\frac{\partial^2 u}{\partial r\partial z}\right)+\right.\\
&\frac{\partial^2 u}{\partial r^2}\left(9\frac{\partial u}{\partial r}+\frac{u}{r}\right)+\frac{\partial u}{\partial r}\left(\frac{\partial^2 u}{\partial z^2}+\frac{3}{r}\frac{\partial u}{\partial r}+\frac{u}{r^2}\right)+\\
&\frac{\partial u}{\partial z}\left(3\frac{\partial^2 w}{\partial r^2}-\frac{1}{r}\frac{\partial w}{\partial r}-\frac{w}{r^2}+3\frac{\partial^2 u}{\partial r\partial z}-\frac{1}{r}\frac{\partial u}{\partial z}\right)-\frac{2u}{r}\left(\frac{\partial^2 u}{\partial z^2}-\frac{2u}{r^2}\right)\\
&+2\frac{\partial v}{\partial r}\left(2\frac{\partial^2 v}{\partial r^2}+\frac{\partial^2 v}{\partial z^2}-\frac{1}{r}\frac{\partial v}{\partial r}+\frac{2v}{r^2}\right)+2\frac{\partial^2 v}{\partial r\partial z}\frac{\partial v}{\partial z}-\\
&\frac{2v}{r}\left(2\frac{\partial^2 v}{\partial r^2}+\frac{\partial^2 v}{\partial z^2}+\frac{v}{r^2}\right)+2\frac{\partial w}{\partial r}\left(\frac{\partial^2 w}{\partial r\partial z}+2\frac{\partial^2 w}{\partial z^2}+\frac{1}{r}\frac{\partial w}{\partial r}\right)+\\
&\left.4\frac{\partial w}{\partial z}\frac{\partial^2 w}{\partial r\partial z}\right]+\beta\left[\frac{\partial u}{\partial r}\left(6\frac{\partial^2 u}{\partial r^2}+\frac{2}{r}\frac{\partial u}{\partial r}+\frac{2u}{r^2}\right)+\right.\\
&\frac{\partial u}{\partial z}\left(2\frac{\partial^2 u}{\partial r\partial z}-\frac{1}{r}\frac{\partial u}{\partial z}+2\frac{\partial^2 w}{\partial r^2}\right)-\frac{2u}{r}\left(\frac{\partial^2 u}{\partial z^2}+\frac{2u}{r^2}\right)+\\
&\frac{\partial v}{\partial r}\left(2\frac{\partial^2 v}{\partial r^2}+\frac{\partial^2 v}{\partial z^2}-\frac{2}{r}\frac{\partial v}{\partial r}+\frac{4v}{r^2}\right)+\frac{\partial v}{\partial z}\left(\frac{\partial^2 u}{\partial r\partial z}-\frac{2}{r}\frac{\partial v}{\partial z}\right)\\
&-\frac{v}{r}\left(2\frac{\partial^2 v}{\partial r^2}+\frac{\partial^2 v}{\partial z^2}+\frac{2v}{r^2}\right)+\frac{\partial w}{\partial r}\left(2\frac{\partial^2 u}{\partial r\partial z}+2\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}\right)+\\
&\left.2\frac{\partial^2 w}{\partial r\partial z}\frac{\partial w}{\partial z}\right]
\end{aligned} \tag{3}$$

$$\begin{aligned}
&u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z}+\frac{uv}{r}=\frac{\partial^2 v}{\partial r^2}+\frac{1}{r}\frac{\partial v}{\partial r}-\frac{v}{r^2}+\frac{\partial^2 v}{\partial z^2}+\\
&\alpha\left[u\frac{\partial^3 u}{\partial r^3}+w\left(\frac{\partial^3 v}{\partial r^2\partial z}+\frac{1}{r}\frac{\partial^2 u}{\partial r\partial z}+\frac{\partial^3 v}{\partial z^3}\right)+\frac{\partial^2 u}{\partial r^2}\right.\\
&\left(\frac{\partial v}{\partial r}-\frac{v}{r}\right)+\frac{\partial u}{\partial r}\left(2\frac{\partial^2 v}{\partial r^2}+\frac{3}{r}\frac{\partial v}{\partial r}-2\frac{\partial^2 u}{\partial z^2}-\frac{3v}{r^2}\right)+
\end{aligned}$$

$$\frac{\partial^2 u}{\partial z^2}\left(\frac{\partial v}{\partial z}-\frac{v}{r}\right)+2\frac{\partial u}{\partial z}\left(\frac{\partial^2 v}{\partial r\partial z}+\frac{1}{r}\frac{\partial v}{\partial z}\right)+$$
$$\frac{\partial v}{\partial z}\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}-\frac{w}{r^2}+\frac{\partial^2 w}{\partial z^2}\right)+2\frac{\partial w}{\partial r}\frac{\partial^2 v}{\partial r\partial z}+$$
$$\left.\frac{u}{r}\left(4\frac{\partial^2 v}{\partial r^2}+\frac{\partial^2 v}{\partial z^2}\right)\right]+\beta\left[\frac{\partial^2 u}{\partial r^2}\left(\frac{\partial v}{\partial r}-\frac{v}{r}\right)+\right.$$
$$\frac{\partial u}{\partial r}\left(2\frac{\partial^2 v}{\partial r^2}+\frac{3}{r}\frac{\partial v}{\partial r}-\frac{3v}{r^2}\right)+2\frac{\partial u}{\partial z}\left(\frac{\partial^2 v}{\partial r\partial z}+\frac{1}{r}\frac{\partial v}{\partial z}\right)+$$
$$\frac{2u}{r}\frac{\partial^2 v}{\partial r^2}+\frac{\partial v}{\partial r}\left(\frac{\partial^2 u}{\partial z^2}+\frac{u}{r}\right)+2\frac{\partial^2 v}{\partial z^2}\left(\frac{\partial w}{\partial z}+\frac{u}{r}\right)+\frac{\partial v}{\partial z}$$
$$\left.\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}+\frac{\partial^2 w}{\partial z^2}\right)+2\frac{\partial^2 v}{\partial r\partial z}\frac{\partial w}{\partial z}-\frac{v}{r}\left(\frac{\partial^2 u}{\partial z^2}-\frac{u}{r^2}\right)\right]\tag{4}$$

$$u\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z}=-\frac{\partial p}{\partial z}+\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}+\frac{\partial^2 w}{\partial z^2}+$$
$$\alpha\left[u\left(\frac{\partial^3 u}{\partial r^3}+\frac{\partial^3 w}{\partial r\partial z^2}\right)+w\left(\frac{\partial^3 w}{\partial z^3}+\frac{\partial^3 w}{\partial r^2\partial z}+\frac{1}{r}\frac{\partial^2 w}{\partial r\partial z}\right)+\right.$$
$$\frac{\partial u}{\partial r}\left(3\frac{\partial^2 u}{\partial r\partial z}+\frac{1}{r}\frac{\partial w}{\partial z}+\frac{1}{r}\frac{\partial w}{\partial r}\right)+2\frac{\partial u}{\partial z}\left(2\frac{\partial^2 u}{\partial z^2}+\frac{2u}{r^2}+\frac{\partial^2 w}{\partial r\partial z}\right)+$$
$$2\frac{\partial v}{\partial z}\left(\frac{\partial^2 v}{\partial r^2}+2\frac{\partial^2 v}{\partial z^2}\right)+2\frac{\partial^2 v}{\partial r\partial z}\left(\frac{\partial v}{\partial r}-\frac{v}{r}\right)+$$
$$\frac{\partial w}{\partial r}\left(\frac{\partial^2 u}{\partial r^2}+3\frac{\partial^2 u}{\partial z^2}+4\frac{\partial^2 w}{\partial r\partial z}+\frac{3}{r}\frac{\partial w}{\partial z}\right)+$$
$$\left.\frac{\partial w}{\partial z}\left(\frac{\partial^2 w}{\partial r^2}+10\frac{\partial^2 w}{\partial z^2}\right)-\frac{u}{r}\left(\frac{\partial^2 u}{\partial r\partial z}+\frac{\partial^2 w}{\partial r^2}\right)\right]+$$
$$\beta\left[2\frac{\partial u}{\partial z}\left(\frac{\partial^2 u}{\partial z^2}+\frac{u}{r^2}+\frac{\partial^2 w}{\partial r\partial z}+\frac{1}{r}\frac{\partial w}{\partial z}\right)+\frac{\partial v}{\partial z}\left(\frac{\partial^2 v}{\partial r^2}+2\frac{\partial^2 v}{\partial z^2}\right)+\right.$$
$$\frac{\partial^2 v}{\partial r\partial z}\left(\frac{\partial v}{\partial r}-\frac{v}{r}\right)+2\frac{\partial w}{\partial r}\left(\frac{\partial^2 u}{\partial z^2}+\frac{u}{r^2}+\frac{\partial^2 w}{\partial r\partial z}\right)+$$
$$\left.2\frac{\partial w}{\partial z}\left(4\frac{\partial^2 w}{\partial z^2}+\frac{1}{r}\frac{\partial w}{\partial r}\right)-\frac{2u}{r}\left(\frac{\partial^2 u}{\partial r\partial z}+\frac{\partial^2 w}{\partial r^2}\right)\right]\tag{5}$$

Subject to the boundary conditions:

$$z=0,\ u=0,\ v=m\lambda r R,\ w=nR$$

$$z=1,\ u=0,\ v=\lambda r R,\ w=R$$

Where

$\lambda=\dfrac{d^2\Omega}{v_1}$, is a dimensionless rotational parameter,

$R=\dfrac{Wd}{v_1}$, is a suction Reynolds number,

$$\alpha=\frac{\mu_2}{\rho d^2},\quad \beta=\frac{\mu_3}{\rho d^2}$$

α (the elastico-viscous parameter) is negative since $\mu_2 < 0$. Further the constitutive relation (1) is valid for flow at low shear rates such that $|\alpha| << 1$.

3. METHOD OF SOLUTION

To find the solution of Equations 2 to 5, we introduce the following variables

$$\left\{u=-\left(\frac{rR}{2}\right)F'(z),\ v=rR\lambda G(z),\ w=RF(z),\ P=p(z)+\frac{A}{2}\lambda^2 r^2\right\}\tag{6}$$

Then the equation of continuity (2) is identically satisfied with the Equations 3 to 5 are transformed to

$$\begin{aligned}&2RF'''-2R^2FF''+R^2F'^2-4R^2\lambda^2G^2+4A\lambda^2+\\&\alpha\left[2R^2FF^{IV}-2R^2F''^2\right]+\\&\beta\left[2R^2F'F'''-R^2F''^2+4R^2\lambda^2G^2\right]=0\end{aligned}\tag{7}$$

$$\begin{aligned}&G''-RFG'+RF'G+\alpha\left[RFG'''-RF''G'\right]+\\&\beta\left[RF'G''-RF''G\right]=0\end{aligned}\tag{8}$$

$$RF'' - R^2FF' - P' +$$
$$\alpha\left[11R^2F'F'' + R^2FF''' + r^2R^2F''F''' + 4r^2R^2\lambda^2 G'G''\right]$$
$$+\beta\left[7R^2F'F'' + \frac{1}{2}r^2R^2F''F''' + 2r^2R^2G'G''\right] = 0 \tag{9}$$

The corresponding boundary conditions are

$$z = 0, F' = 0, G = m, F = n,$$

$$z = 1, F' = 0, G = 1, F = 1.$$

Equations 7 and 8 are highly non-linear in F and G which are functions of z. Divide z [0,1] into hundred equal parts of 0.001 length each. The finite-difference approximations scheme for the first, second, third and fourth order derivatives has been used to discretize the differential Equations 7 and 8 to obtain

$$2R\frac{F_{i+2} - 3F_{i+1} + 3F_i - F_{i-1}}{h^3} - 2RF_i\frac{F_{i+1} - 2F_i + F_{i-1}}{h^2} +$$
$$R\left(\frac{F_{i+1} - F_{i-1}}{2h}\right)^2 - 4R^2\lambda^2G_i^2 + 4A\lambda^2 +$$
$$\alpha\left[2RF_i\frac{F_{i+3} - 4F_{i+2} + 6F_{i+1} - 4F_i + F_{i-1}}{h^4} - \right.$$
$$\left. 2R^2\left(\frac{F_{i+1} - 2F_i + F_{i-1}}{h^2}\right)^2\right] +$$
$$\beta\left[2R\frac{F_{i+1} - F_{i-1}}{2h}\frac{F_{i+2} - F_{i+1} + F_i - F_{i-1}}{h^3} - \right.$$
$$\left. R^2\left(\frac{F_{i+1} - 2F_i + F_{i-1}}{h^2}\right)^2 + 4R\lambda\left(\frac{G_{i+1} - G_{i-1}}{2h}\right)^2\right] \tag{10}$$

$$\frac{G_{i+1} - G_i + G_{i+1}}{h^2} + R\frac{F_{i+1} - F_{i-1}}{2h}G_i - RF_i$$
$$\frac{G_{i+1} - G_{i-1}}{2h} + \alpha\left[RF_i\frac{G_{i+2} - 3G_{i+1} + 3G_i - G_{i-1}}{h^3}\right.$$
$$\left. - R\frac{F_{i+1} - 2F_i + F_{i-1}}{h^2}\frac{G_{i+1} - G_{i-1}}{2h}\right] +$$
$$\beta\left[R\frac{F_{i+1} - F_{i-1}}{2h}\frac{G_{i+1} - 2G_i + G_{i-1}}{h^2}\right.$$
$$\left. - RG_i\frac{F_{i+1} - F_i + F_{i-1}}{h^2}\right] = 0 \tag{11}$$

The Equations 10 and 11 have been solved numerically using iterative scheme.

4. RESULTS AND DISCUSSIONS

Figures 1-6 represent the behavior of axial velocity F with z for different values of visco-elastic parameters α, β for fixed Reynolds number. It has been observed and concluded that for larger values of Reynolds numbers, the axial velocity has negative sign i.e. liquid comes out of disc. The study is of great importance that the chemical industry may be interested to find the limit of Reynolds number for the fluid being used for such set-ups. These values depend on the characteristics of the fluid. The results corresponding to Newtonian fluid can be deduced from the above results by setting $\alpha = \beta = 0$

Figures 1-2 depict that for $\alpha = \beta = 0$ and $\alpha = -0.01$, $\beta = 0.01$ the results are in good agreement with the results of Chaudhary, et al [8], but for $\alpha = -0.05$, $\beta = 0.05$ recirculation occurs at the end points in the lower and upper discs. From point z = 0.4 to 0.9 the set of values shows the normal study of circulation and attains maximum value at z=0.7. Hence, it is concluded that for non-Newtonian fluid large recirculation occurs at lower disc in comparison to the recirculation at upper disc. The behavior of the fluid is turbulent as higher value of Reynolds number is considered. It is also noticed that the behavior of the fluid in the practical purpose is to be seen by the technocrats and engineers in the industry.

Figure 3 represents that for $\alpha = -0.05$, $\beta = 0.05$ recirculation occurs at the end point in lower disc and attains maximum value at z = 0.05. From z = 0.15 to z = 0.1 the set of values depicts normal behavior of circulation and attains the maximum value at point z = 0.9. For $\alpha = -0.01$, $\beta = 0.01$ the curve depicts normal behavior of circulation which has maxima at point z = 0.8 and minima at point z = 0.05.

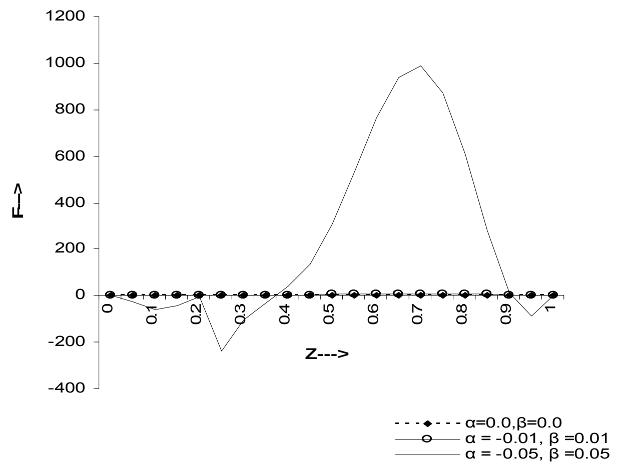

Figure 1. For m = 2, n = 2, λ = 1, Re = 1.

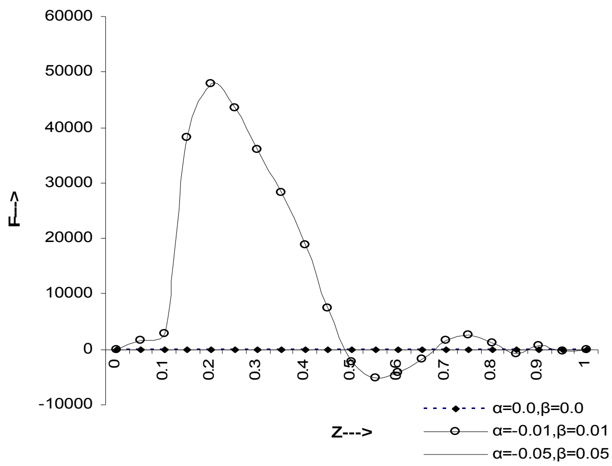

Figure 4. For m = 0.5, n = 2, λ = 1, Re = 3.

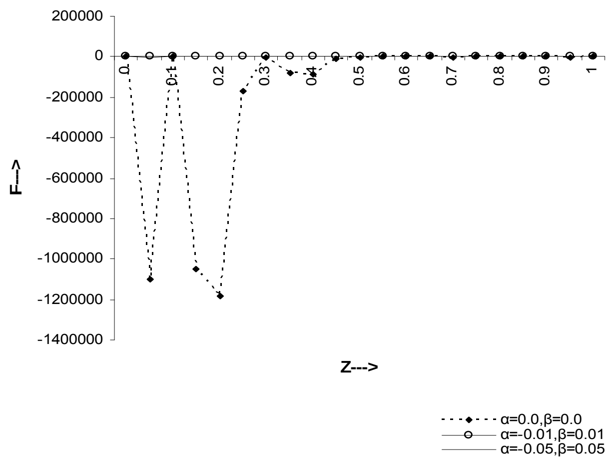

Figure 2. For m = 2, n = 2, λ = 1, Re = 3.

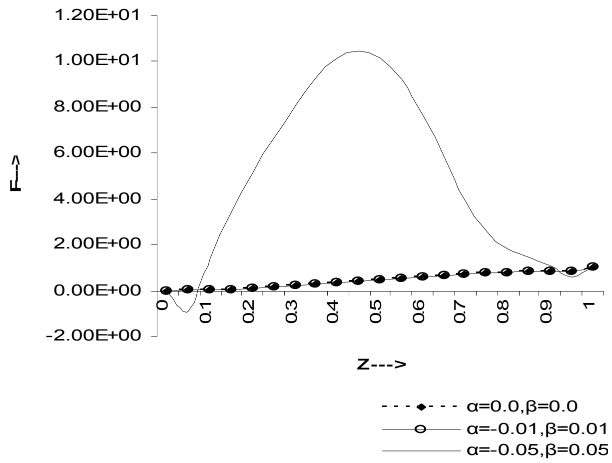

Figure 5. For m = 0, n = 0, λ = 1, Re = 1.

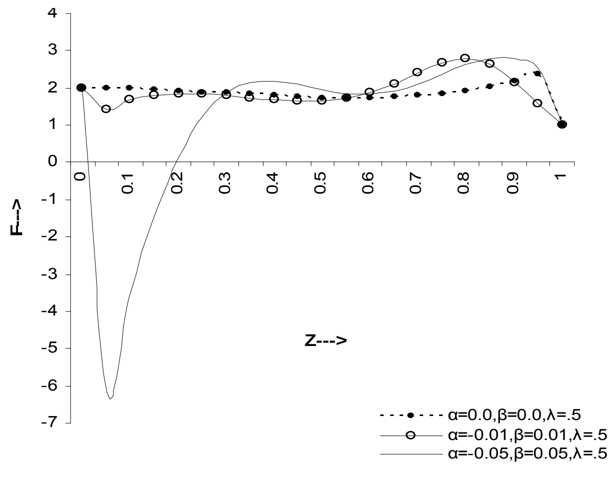

Figure 3. For m = 2, n = 2, λ = 0.5, Re = 1.

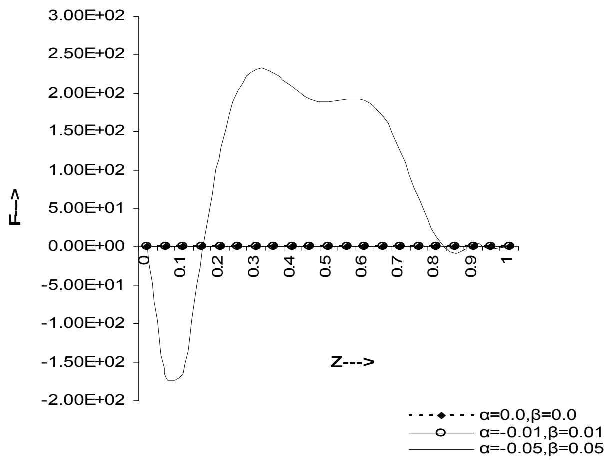

Figure 6. For m = 0, n = 0, λ = 1, Re = 3.

Figure 4 depicts that the axial velocity F with z shows normal circulation curve and attains maximum value at point z = 0.25 accepts from point z =0.5 to 0.7, where recirculation occurs.

Figure 5 depicts recirculation for α = -0.05, β = 0.05 at end point of lower disc and attains maximum value at point z = 0.5 and minimum value at z = 0.95. It is almost symmetrical about z = 0.5 except end points in the lower and upper discs.

Figure 6 shows that for α = -0.05, β = 0.05 at the end point of the lower disc recirculation occurs and is maximum at z =0.35, but for other values of α and β, the set of values represents normal behavior of circulation.

The comparison of different values of Reynolds number has been shown graphically in Figure 7.

Figures 8-11 represent the behavior of transverse velocity G with z for different values of m, n and λ and visco-elastic parameters α, β for fixed Reynolds numbers Re = 1 and Re = 3. It is observed that recirculation takes place for different values of z for different parameters.

5. CONCLUSION

Hence it is concluded that the distribution of transverse velocity is not symmetrical and for non-Newtonian fluid, large recirculation occurs at upper disc in comparison to the recirculation at lower disc on increasing the value of Reynolds number.

The comparison of different values of Reynolds number has been shown graphically in Figure 12.

The present study could be of much interest to bio-engineers involved in design and construction of artificial organs, e.g. artificial dialysis.

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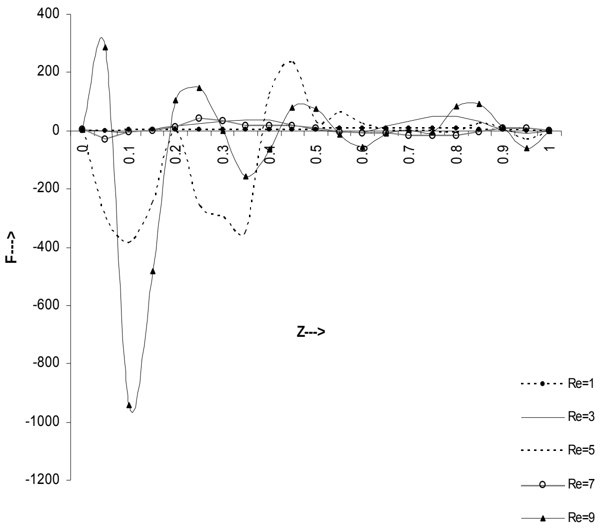

Figure 7. For m = 2, n = 2, λ = 1, α = -0.01, β = 0.01.

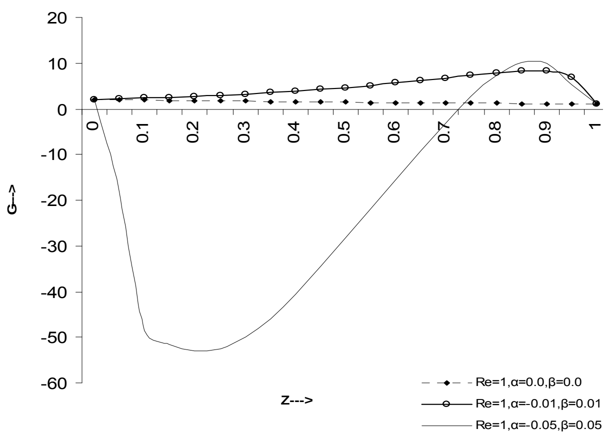

Figure 8. For m = 2, n = 2, λ = 0.5, Re = 1.

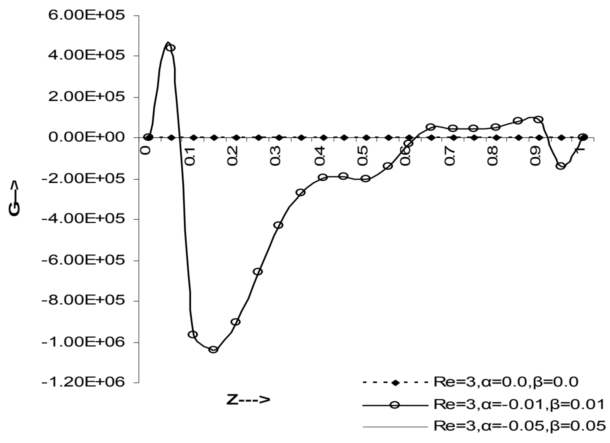

Figure 9. For m = 0.5, n = 2, λ = 1, Re = 3.

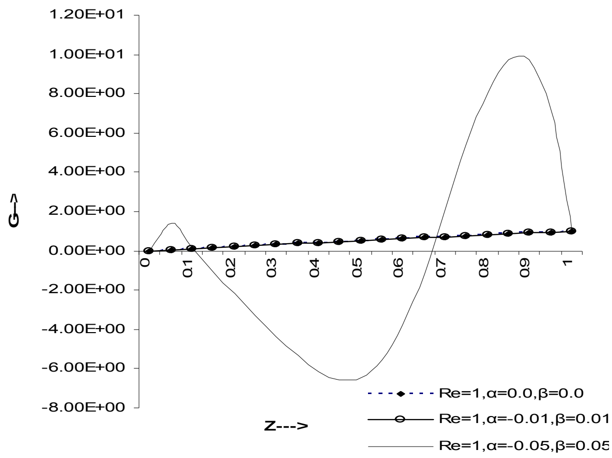

Figure 10. For m = 0, n = 0, $\lambda = 1$, Re = 1.

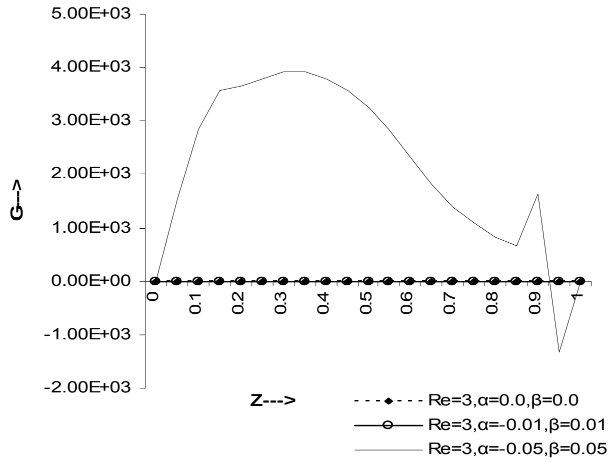

Figure 11. For m = 0, n = 0, $\lambda = 1$, Re = 3.

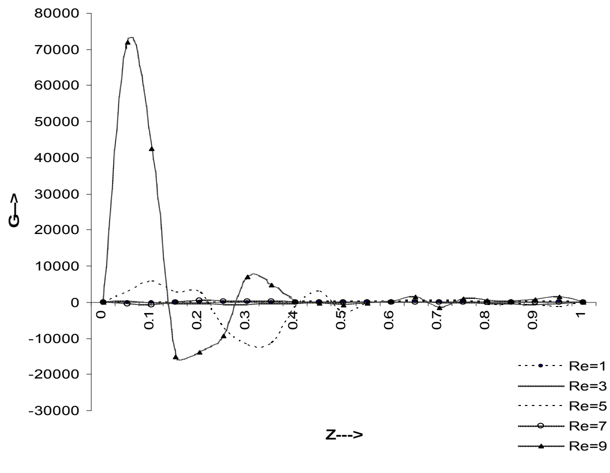

Figure 12. For m = 2, n = 2, $\lambda = 1$, $\alpha = -0.01$, $\beta = 0.01$.

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