

OPTIMIZATION OF THE TMD PARAMETERS TO SUPPRESS THE VERTICAL VIBRATIONS OF SUSPENSION BRIDGES SUBJECTED TO EARTHQUAKE EXCITATIONS

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Abstract Flexible structures (such as long-span suspension bridges which undergo wind or earthquake excitations) exhibit complex dynamic behavior. Due to high cost of strengthening such structures, as well as the technological advances in recent years, much attention has been given to using innovative means of enhancing structural functionality and safety against natural hazards. Among the methods used to mitigate the excessive vibration of structures, the energy absorber systems are more promising. The present paper discusses parametric studies of the TMD system to find the optimal values of its parameters, to suppress the vertical responses of suspension bridges subjected to earthquakes' vertical accelerations. Thomas Suspension Bridge located in Los Angeles, U.S.A., is chosen as a case study, and the vertical acceleration records of 18 major worldwide earthquakes are used in numerical studies. The analysis is performed in time domain. The results of the numerical studies show that the proposed system is capable of reducing the maximum vertical displacement of the bridge to a considerably low value.

Keywords Passive Control, Suspension Bridge, TMD Control, Vertical Vibration

چکیده پل‌های معلق به دلیل دهانه بلندی که دارند در مقابل بارهای دینامیکی نظیر باد و زلزله بسیار انعطاف پذیر هستند و به این دلیل عموماً در برابر این نیروها دچار تغییر شکل‌های بزرگی می‌شوند. برای کاهش این تغییر مکان‌ها، در سال‌های اخیر روش‌های مختلفی ابداع شده است که از جمله آن‌ها می‌توان به جرم هم‌ساز شده یا TMD (Tuned Mass Damper) اشاره نمود. سیستم TMD از سال‌ها قبل در سازه‌های مختلف استفاده شده است. در مقاله حاضر سعی شده تا مقادیر بهینه پارامترهای مختلف سیستم، با در نظر گرفتن حداکثر کاهش در تغییر شکل‌های پل‌های معلق در اثر نیروهای زلزله و با استفاده از روش سعی و خطا تعیین گردند. به عنوان یک مثال عددی پل معلق توماس واقع در لوس آنجلس آمریکا انتخاب شده و تحت اثر ۱۸ زلزله مختلف تحلیل شده است. نتیجه مطالعه نشان می‌دهد که با استفاده از این سیستم، تغییر مکان‌های حداکثر پل را تا حد قابل توجهی می‌توان کاهش داد.

1. INTRODUCTION

Since suspension bridges have long spans, their slender and flexible structures, are highly vulnerable to forces applied by support excitation during earthquake. Vibration control of these structures is being considered as an important issue in structural engineering. In recent years, many different structural control systems such as Tuned Mass Damper (TMD), Active Tuned Mass Damper (ATMD), and Base Isolation systems are widely used to mitigate the structural responses under

dynamic loadings [1-4].

The passive TMD control method is proved to be economically advantageous, due to its low cost maintenance. Warburton, et al [5] performed a study to find the optimum absorbing parameters for simple systems. Application of TMD systems in suspension bridges have been studied by many researchers. Lin, et al [6] studied the effect of TMD system in the reduction of torsional and vertical responses of suspension bridges subjected to wind loading. They used a TMD system with two degrees of freedom, vertical and torsional.

They obtained a TMD mass ratio of 2 % for getting a reduction of 25 % and 33 % in vertical and torsional responses of the bridges, respectively.

Yozo Fojino [7] published a paper on vibration control and monitoring of long-span bridges, in particular, emphasizing on cable-supported bridges recently developed in Japan. The primary stress is placed on the vibration due to motion-dependent forces such as wind-induced aerodynamic forces and its control. Implementation of passive and active control in the long-span bridges in Japan was described. At the end of the paper, the importance and usefulness of vibration monitoring of long-span bridges was discussed with a real example.

Larsena, et al [8] studied design aspects of tuned mass dampers for Great Belt East Bridge. The Great Belt East Bridge includes two approach bridges with steel girders designed as multi-span beams. The design of these continuous long span girders led to a flexible structures characterized by closely spaced eigen frequencies for vertical vibration modes. Wind tunnel tests including section and full-bridge aeroelastic models have confirmed that the approach bridge structures are prone to vortex-shedding excitation at wind speeds encountered regularly at the bridge site. The paper discusses design aspects of tuned mass dampers (TMDs) for controlling wind-induced vibrations.

Gu, et al [9] published a paper on controlling wind-induced vibrations of long-span bridges by semi-active lever-type TMD. A new semi-active (SA) lever-type TMD with an adjustable frequency and the corresponding control strategy are primarily developed. A case study of the Yichang Bridge, a suspension bridge with a main span of 960 m, shows that the SA lever-type TMD device is much superior to passive TMD in control efficiency and robustness.

As reported above, many research studies have been conducted on controlling the response of long-span cable supported bridges. However, challenges are still going on this topic for getting more findings.

Jung, et al [10] published a paper on hybrid seismic protection of cable-stayed bridges. They proposed a hybrid control strategy combining passive (i.e. base isolation) and semi-active control systems for seismic protection. A clipped-optimal control algorithm is used to determine the control

action for semi-active dampers. However, it is shown that this hybrid system is quite effective compared to that of the passive control strategy.

Park, et al [11] conducted a study on fuzzy supervisory control (FSC) technique for the seismic response of cable-stayed bridges. The proposed technique is a hybrid control method, which adopts a hierarchical structure consisting of several sub-controllers and a fuzzy supervisor. Simulation results showed that both linear quadratic Gaussian (LQG) and FSC control systems can significantly reduce the seismic forces transferred to the towers, simultaneously keeping tensions in the stay cables within a recommended range of allowable values.

In the present study, optimization of the TMD different parameters for reducing the vertical displacement of suspension bridges subjected to earthquake vertical excitations is investigated using trial and error method. The analysis is performed in time domain. Thomas Suspension Bridge (Los Angeles, U.S.A.) is chosen for the numerical studies. The optimal values of the TMD parameters are obtained under effect of 18 worldwide major earthquakes.

2. EQUATION OF MOTION OF THE SYSTEM

The motion equation of the system is written using finite element method and energy principles. The system is subjected to vertical acceleration of earthquake transmitted to the bridge deck through the piers. The bridge deck is considered as a simply supported beam between the towers, connected by several hangers to the main cables (Figure 1). The end connections of the bridge girders to the towers are assumed to be hinged. In Finite element model of the bridge, two noded beam elements with 2 degrees of freedom at each node, vertical displacement and bending rotation, are considered (Figure 2). As shown in Figure 3 the bridge element consists of the bridge girder, at least two vertical hangers, and the main cables [12]. The stiffness matrix of each finite element can be derived through calculation of its potential energy and using the Hamilton's principle. The total stiffness matrix of the bridge, then, can be

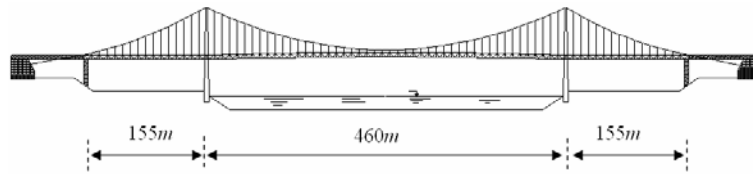


Figure 1. Thomas suspension bridge model.

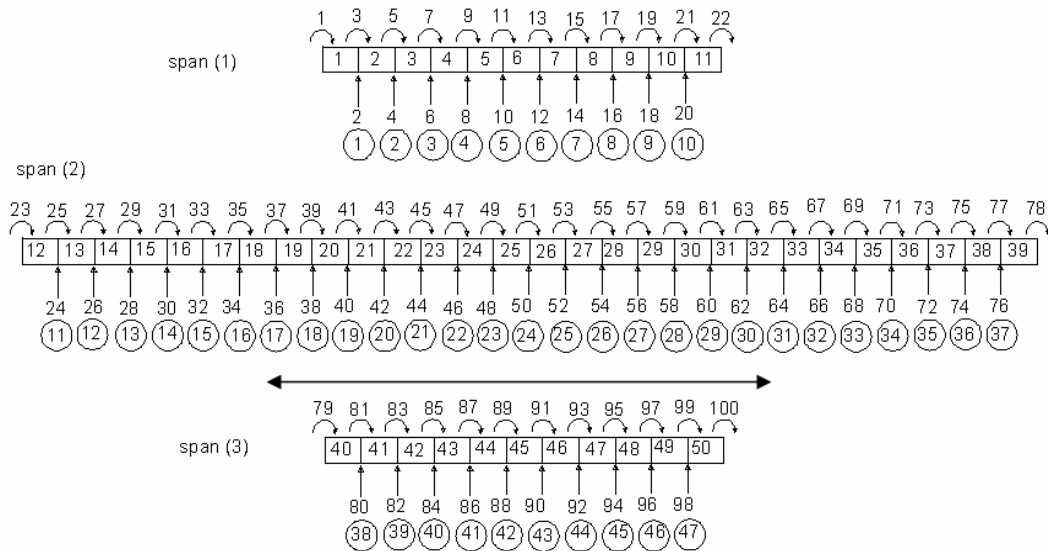


Figure 2. Finite elements of the bridge (11, 28, 11 elements in each span) and the related DOFs (bending rotation and vertical displacement).

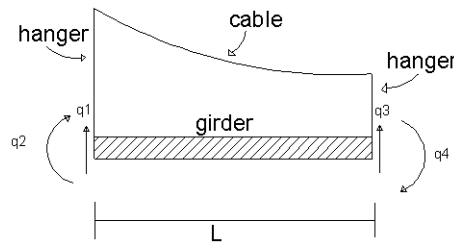


Figure 3. Finite element of the bridge deck containing the girder, main cables and at least two hangers.

obtained by assembling the stiffness matrices of all elements [12].

Since the main objective of this paper is to evaluate the optimal values of the TMD parameters, therefore, detail procedure of the finite-element modeling of the bridge is not presented here. Static condensation of the system stiffness matrix is

performed resulting in elimination of the bending rotational degrees of freedom [13]. By considering the elements' mass equally concentrated on the end nodes, a diagonal $n \times n$ lumped mass matrix will be obtained where n is the total number of vertical degrees of freedom. Finally, the equation of motion of the bridge (without TMD) can be

$$\{B\} (2n \times 1) = \begin{Bmatrix} \{0\}_{n \times 1} \\ \{1\}_{n \times 1} \end{Bmatrix} \quad (7)$$

in which [I] is the identity matrix of order n; [0] and {0} are zero n × n matrix and n × 1 vector, respectively. It should be noted that by replacing n with (n+1), the equation 5 represents the motion of a system with one TMD in state-space form, when the TMD characteristics k_T, m_T and c_T are added, appropriately, to the structural properties of the initial system [k], [m], and [c], respectively. Similarly, when p TMDs are installed, n should be replaced with (n+p), and appropriate modification in the structural properties of the initial system should be made.

3. NUMERICAL STUDY

Vincent Thomas Suspension Bridge is chosen for numerical study (Figure 1). The bridge is situated in Los Angeles, California, U.S.A., and has three spans: central span of 460 m and two side spans of 155 m. The weight of the decks and two main cables are 52438 N/m and 12390 N/m, respectively. The cable cross-section area is 780 cm² and the initial tension in the cable due to dead load is 30038*10³ N per cable [12]. Frequencies and mode shapes of the first 6 modes of the bridge are given in Figure 4. The earthquake acceleration records of 18 worldwide major earthquakes have been used to investigate the optimum values of the TMD parameters for getting the maximum reduction on bridge responses. These earthquakes are selected such that a variety range of peak ground accelerations (PGA), frequency content levels, and distance to fault rupture (near field and far field effects) can be included in the study. More details of some earthquake records are provided in Table 1. In finite-element model of the bridge, for each side spans 11 elements and for middle span 28 elements are considered [12]. Therefore, there would be 51 nodes in total length of the bridge span (Figure 2). In this part of the study, three TMD systems are considered for the bridge, one at the center of each span (node numbers 5, 24, and 42 on Figure 2, respectively) with a mass ratio (ratio of all TMDs mass to bridge total mass) of

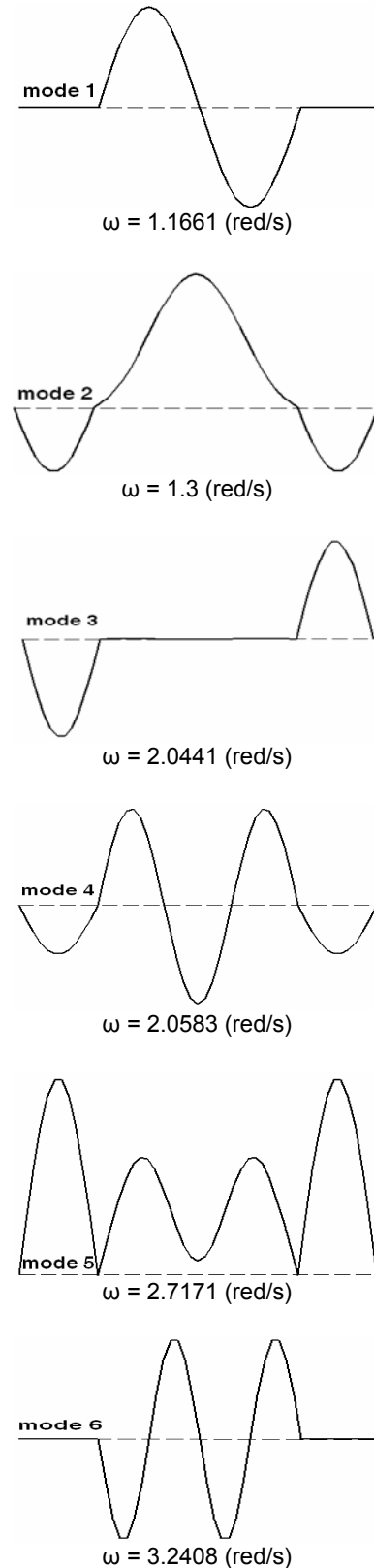


Figure 4. Frequencies and mode shapes of the first 6 modes of the example bridge.

TABLE 1. The Most Important Earthquake Records Considered in this Study.

No.	Earthquake	Country	Station	Year	Magnitude				PGA Vertical Copm.	Hypocentral Distance (km)	Closest Distance to Fault Rupture (km)
					M _b	M _l	M _s	M _w			
1	Imperial Valley	U.S.A.	El Centro Array # 6	1979 (October)	6.5	6.6	6.9	-	1.655	-	1.0
2	Northridge	U.S.A.	Newhall-Fire Sta	1994 (January)	6.7	6.6	6.7	-	0.548	-	7.1
3	Loma Prieta	U.S.A.	Corralitos	1989 (October)	6.9	-	7.1	-	0.455	-	5.1
4	Cape Mendocino	U.S.A.	Petrolia	1992 (April)	7.1	-	7.1	-	0.168	-	9.5
5	Northridge	U.S.A.	Santa Monica City Hall	1994 (January)	6.7	6.6	6.7	-	0.230	-	27.6
6	Loma Prieta	U.S.A.	Hollister-South and Pine	1989 (October)	6.9	-	7.1	-	0.197	-	28.8
7	El Centro	U.S.A.	Imperial Valley (Array Sta. 9)	1940 (May)	7.0	-	7.2	6.9	0.205	12.0	-
8	Kobe	Japan	Takatori	1995 (January)	-	-	-	6.9	0.272	22.2	1.5
9	Bam	Iran	Bam	2003 (December)	-	-	-	6.5	1.008	10.2	-
10	Sarein	Iran	Kariq	1997 (February)	-	-	-	6.1	0.221	60.1	-
11	Zanjiran	Iran	Zanjiran	1994 (June)	-	-	-	6.1	1.003	12.2	-
12	Roodbar	Iran	Abbar	1990 (June)	7.3	-	7.7	7.7	0.547	35.0	-

about 4 %; and TMD damping ratio of about 20 % for each TMD. As the maximum vertical displacement of the bridge along the span occurs at the centre of each span, therefore, TMDs are considered at these nodes. The frequency of the TMD system is tuned to the frequency of the first mode of the bridge. Therefore, TMD characteristics

can be written as (Figure 5):

$$K_T = M_T \times (\omega_1)^2 \quad (8)$$

$$C_T = 2 \times \xi_T \times (M_T \times K_T)^{\left(\frac{1}{2}\right)} \quad (9)$$

Figure 6a shows the comparison between controlled and uncontrolled vertical responses of the example bridge subjected to the Roodbar earthquake, for which the time history of vertical acceleration is shown in Figure 6c. The figure indicates a considerable reduction in the maximum deflection of the bridge. In the figure, the maximum vertical responses of the bridge at each node along the bridge span are compared. For more comparison, time histories of the controlled and uncontrolled responses of the bridge at the middle point of the center span (node No. 24) are compared in Figure 6b. Here also a considerable reduction in bridge response is seen.

Since the effectiveness of the TMD system highly depends on its properties such as mass, stiffness, and so on, therefore in rest of the study, a sensitivity analysis is conducted to optimize these parameters for getting maximum reduction in bridge responses.

3.1. Optimum Mass Ratio of the TMD System In order to investigate the optimum values of the TMD characteristics, the bridge is analyzed under vertical component of 18 worldwide major earthquake accelerations, from which some of the most important ones are: El Centro 1940, Northridge 1994, Kobe 1995, Roodbar (Iran) 1990, and Bam (Iran) 2003. These earthquakes are selected such that a variety range of earthquake PGA and frequency content levels and distance to fault rupture can be included in the study (Table 1). Subsequently, the obtained results could be more reliable in practice. By applying the

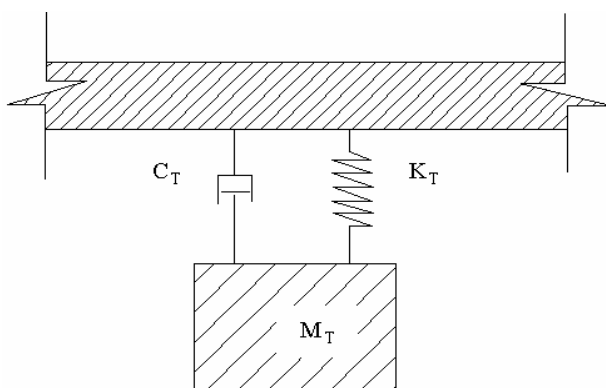
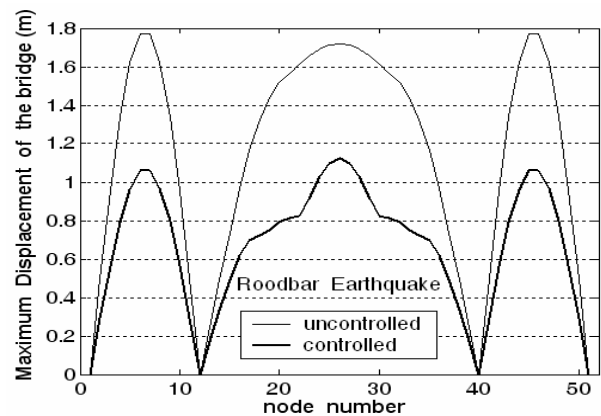
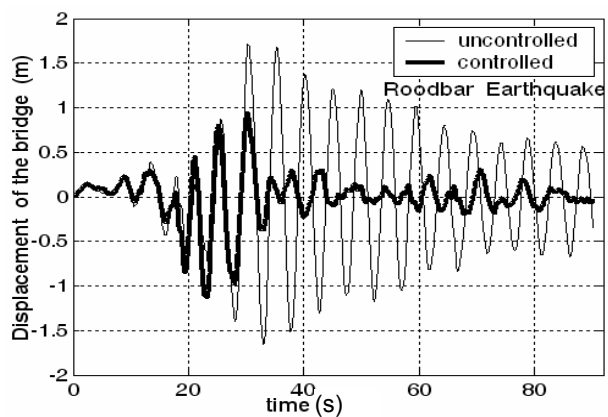


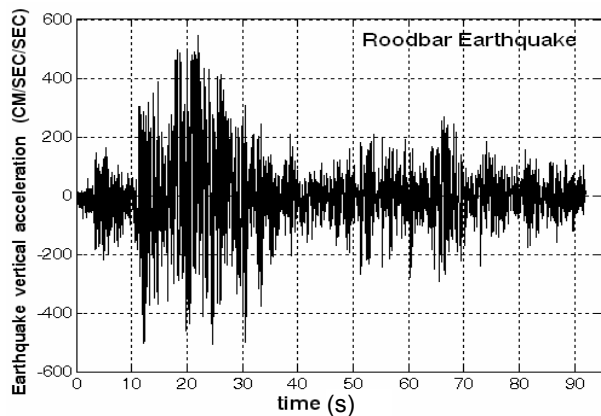
Figure 5. TMD system used in the study.



(a)



(b)



(c)

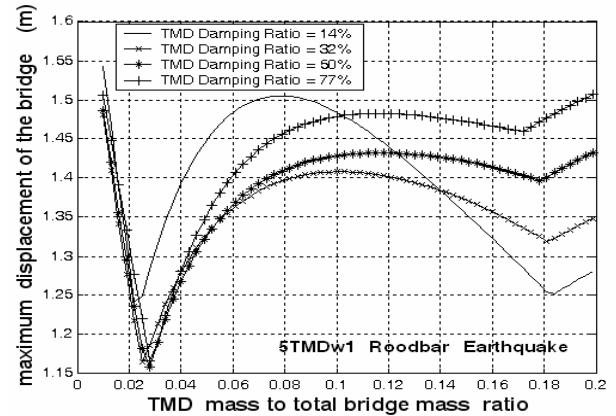
Figure 6. The graphs show: (a) controlled and uncontrolled maximum displacements of the bridge along the span ($\xi_T = 20\%$, $M_T = 4\%$ and 3TMD is used), (b) time histories of the controlled and uncontrolled responses of the bridge at the middle of the center span and (c) time history of the vertical component of the Roodbar earthquake acceleration.

vertical component of the earthquake accelerations on the bridge piers, the maximum responses of the bridge with and without TMD control system are calculated. The highest values of bridge responses have been observed under Roodbar, Zanjiran and El Centro earthquakes and will be presented hereafter.

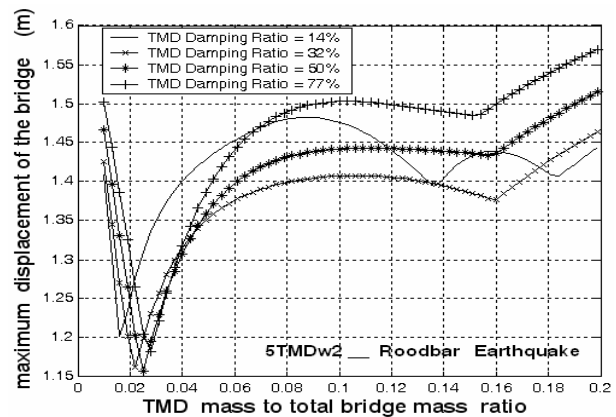
In this part of the study, the effect of TMD mass ratio (The ratio of all TMDs masses to the bridge total mass) on the reduction of bridge vertical response is investigated using trial and error method. In total 5 TMDs are used, one in the middle of center span (node No. 24) and 2 in each side spans (node numbers 4,7,41 and 44). The frequencies of the TMDs are tuned to the frequencies of the bridge first, second, and third modes (three cases). The results are shown in Figures 7 to 9. Figure 7a shows the results for Roodbar earthquake when the TMDs are tuned to the frequency of the bridge first mode (case 1). Figures 7b,c show that of the Roodbar earthquake for tuning to the frequencies of the bridge second and third modes, respectively (cases 2 and 3). As well, Figures 8 and 9 show the same results obtained for Zanjiran and El Centro Earthquakes, respectively.

All these figures are plotted for different values of the TMD damping ratios. It can be seen from the figures that the optimum values of the mass ratios for Roodbar, Zanjiran and El Centro earthquakes are about 3 %, 5 % and 4 %, respectively. The results of the study for other earthquake records, which are not shown here, indicate similar ratios. Therefore, an optimal mass ratio of 4 % seems to be a reasonable value. From the figures it is seen that, in each case, after reaching the mass ratio to an optimal value, the controlled response of the bridge increases with the increase in the value of mass ratio. This is due to the fact that by increasing the mass ratio, the total mass of the bridge (including the TMD mass) will increase such that after which the TMD system instead of reducing the bridge response, will increase it and therefore becomes ineffective in controlling the bridge vibration.

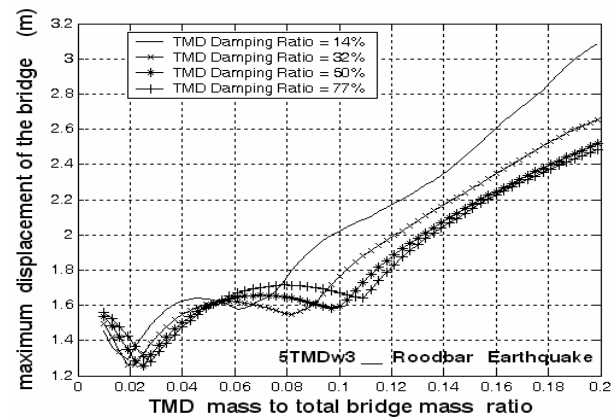
3.2. Optimum Bridge Mode to which the TMD is to be Tuned Which bridge mode does give the best result when the TMD frequency is tuned to? Answer of this question depends on



(a)

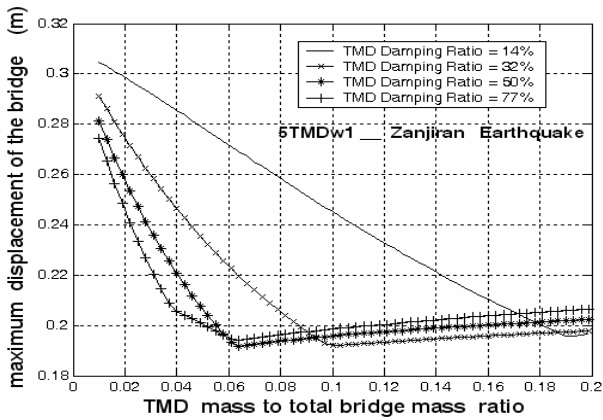


(b)

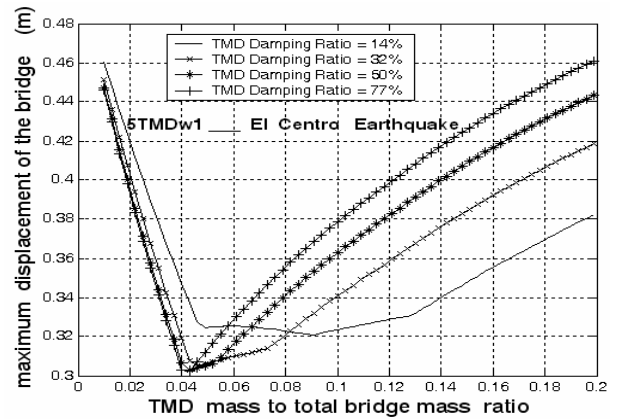


(c)

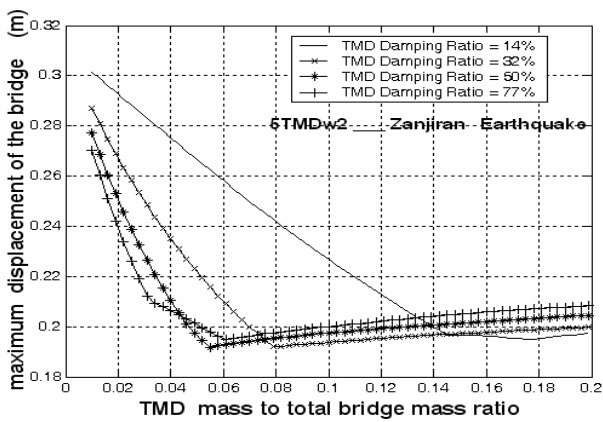
Figure 7. Effect of the TMD mass ratio on reduction of the bridge maximum displacement for different TMD damping ratio under Roodbar earthquake: (a) tuned to the frequency of the bridge first mode (Case 1), (b) tuned to the frequency of the bridge second mode (Case 2) and (c) tuned to the frequency of the bridge third mode (Case 3).



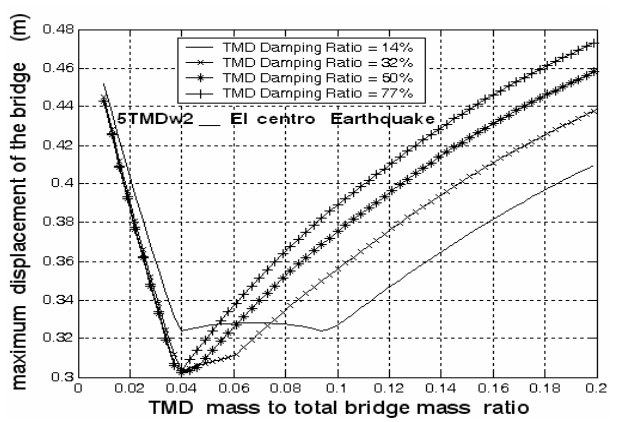
(a)



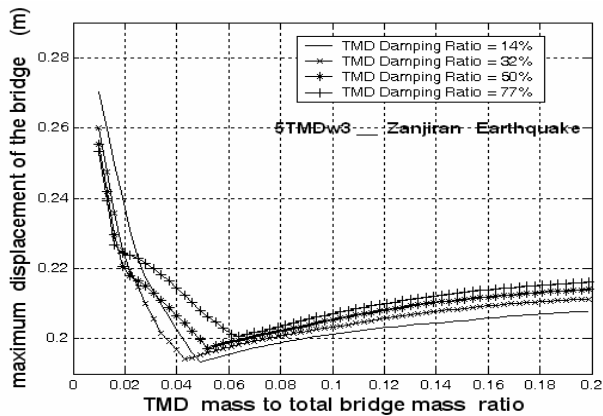
(a)



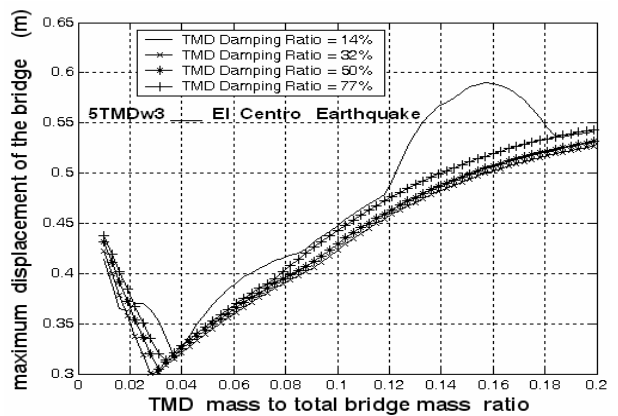
(b)



(b)



(c)



(c)

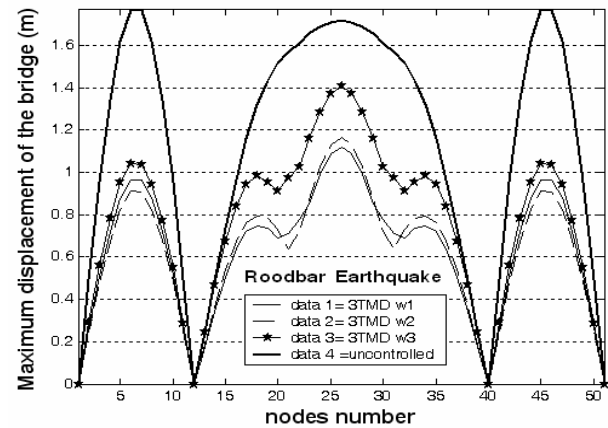
Figure 8. Effect of the TMD mass ratio on reduction of the bridge maximum displacement for different TMD damping ratio under Zanjiran earthquake: (a) tuned to the frequency of the bridge first mode (Case 1), (b) tuned to the frequency of the bridge second mode (Case 2) and (c) tuned to the frequency of the bridge third mode (Case 3).

Figure 9. Effect of the TMD mass ratio on reduction of the bridge maximum displacement for different TMD damping ratio under el Centro earthquake: (a) tuned to the frequency of the bridge first mode (Case 1), (b) tuned to the frequency of the bridge second mode (Case 2) and (c) Tuned to the frequency of the bridge third mode (Case 3).

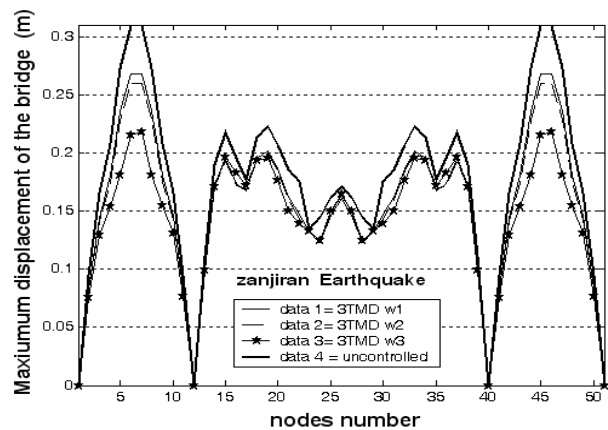
modal properties of the bridge and frequency content of the earthquake. One bridge vibrational mode in which the frequency lies in the range of frequency content of the input earthquake may be considered as the dominant mode of the bridge to which the TMD system is to be tuned. This mode may be found using a mode by mode analysis and comparing the results of the uncontrolled responses obtained in each mode. Since, in this study, the bridge is analyzed in state space, therefore in order to recognize the dominant mode to which tuning the TMD system gives the best result, a trial and error method is used. For this purpose, the TMD was tuned to the frequencies of the bridge modes 1 to 3 and the structure is subjected to 18 earthquake records. The results of the controlled and uncontrolled maximum responses of the bridge along total span for Roodbar, Zanjiran and El Centro earthquakes are shown in Figure 10. The analyses are performed in two cases using 3 and 5 TMDs (the arrangements are mentioned earlier), but for briefness, only the results of the 3 TMDs are presented. Figure 10a shows that the maximum uncontrolled response of the bridge under Roodbar earthquake is about 1.8 m and tuning of the TMD system to the bridge first mode gives the most reduction in maximum deflection of the structure. The results obtained from Zanjiran and El Centro earthquakes both indicate that tuning to the bridge third mode is more effective in comparison with other two modes. Since the bridge maximum response under Roodbar record is considerably higher than the other earthquakes, hence, the first mode of the bridge, which provides the most reduction under this earthquake, is chosen as the optimal mode, in rest of the study, to which the TMD is to be tuned.

3.3. Optimum Value of TMD Damping Ratio

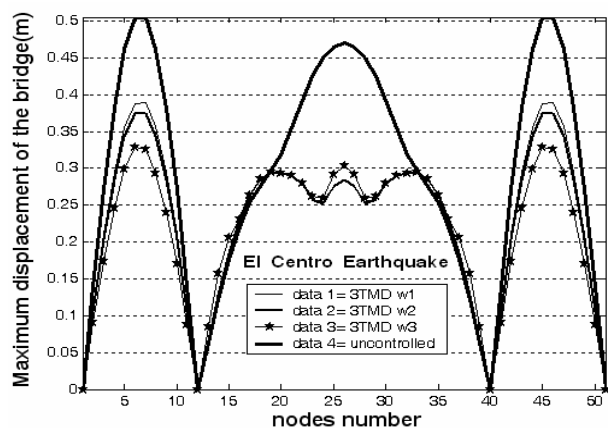
Ratio Figure 11 shows the relationship between TMD damping ratio (ξ_T) and the controlled maximum displacement of the bridge. Results obtained from Roodbar, El Centro, Zanjiran, Lucern, and Garmkhan earthquakes are shown in the figure for comparison. The figures show that initially the maximum deflection decreases when the value of ξ_T increases, but there is some certain value of ξ_T after which increase in the TMD damping ratio has no positive effect on the bridge response.



(a)

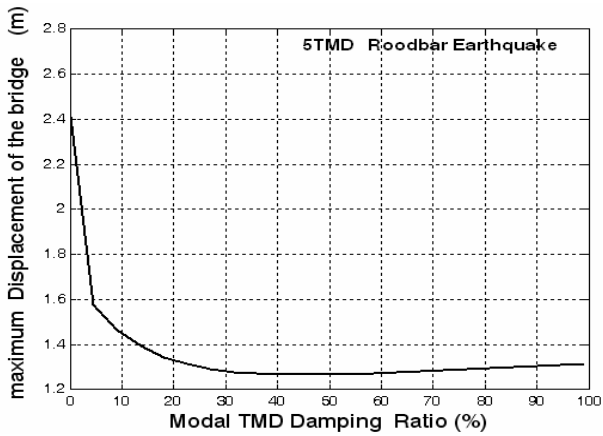


(b)

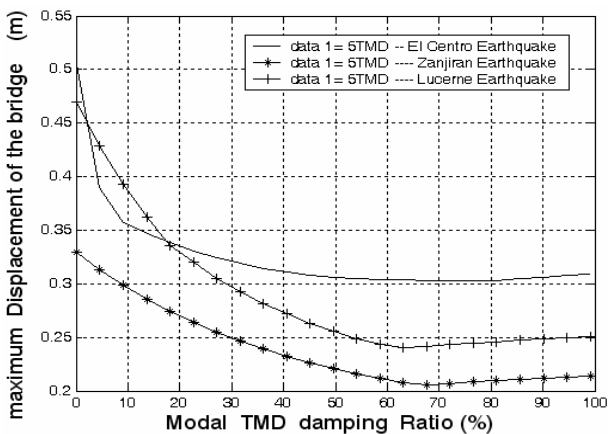


(c)

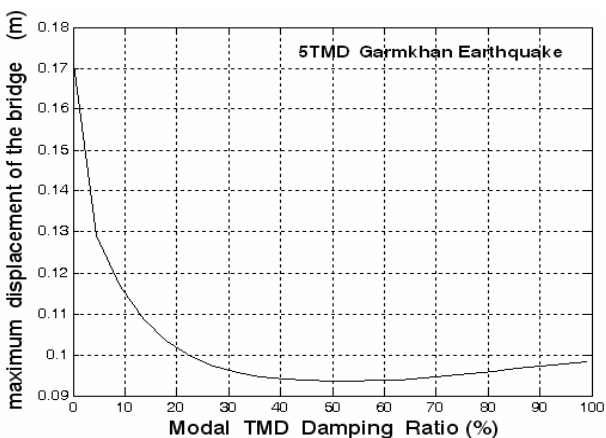
Figure 10. Effect of the tuning mode on reduction of the bridge maximum response for 3 TMDs: (a) under Roodbar earthquake, (b) under Zanjiran earthquake and (c) under el Centro earthquake.



(a)



(b)



(c)

Figure 11. Effect of TMD damping ratio on bridge maximum displacement reduction: (a) under Roodbar earthquake, (b) under el Centro, Zanjiran, Lucerne earthquakes and (c) under Garmkhan earthquake.

The optimum values of ξ_T obtained from figures are respectively about 40 %, 60 %, 67 %, 63 %, and 50 % for the above mentioned earthquakes. In general, the average value of optimal TMD damping ratio is obtained about 60.4 % (by considering 18 earthquakes), which is difficult to provide in practice. But, by referring to the figures, it is seen that before reaching to the optimum value of the ξ_T , most of the curves almost are horizontal, and increase in the value of ξ_T does not have much effect in decreasing the bridge response. Therefore, from economical and practical point of view a TMD damping ratio equal to 34 % can be suggested. A comparison of the results obtained by applying the suggested value and the optimum value obtained earlier (i.e. $\xi_T=60.4\%$) consolidates this finding (Figure 12). No major difference can be found between the results. Figure 12 is plotted for El Centro earthquake and 5TMD systems have been considered in the analysis.

3.4. Effect of Tuning Frequency In the earlier sections it was recommended to use the frequency of the first mode for tuning the TMD system. Further analyses indicate that the best result can be obtained when TMD system is tuned to a percentage of this frequency. Figure 13 expresses the relationship between the controlled response of the bridge and the frequency ratio (the ratio of frequency of TMD to that of the bridge first mode, $\frac{\omega_T}{\omega_1}$). It can be seen that tuning to 90%

of the bridge first mode frequency yields better result in comparison to the whole value. The figure shows the response of the bridge for Roodbar Earthquake. Similar results are obtained for other earthquake records, which for brevity, are not presented in this paper.

4. CONCLUSIONS

The effect of passive TMD control system on seismic behavior of the suspension bridges is discussed. Thomas Suspension Bridge in Los Angeles, U.S.A., is chosen as a case study. 18 worldwide major earthquakes are used in the analyses, and the optimum values of different parameters of the TMD system are obtained using

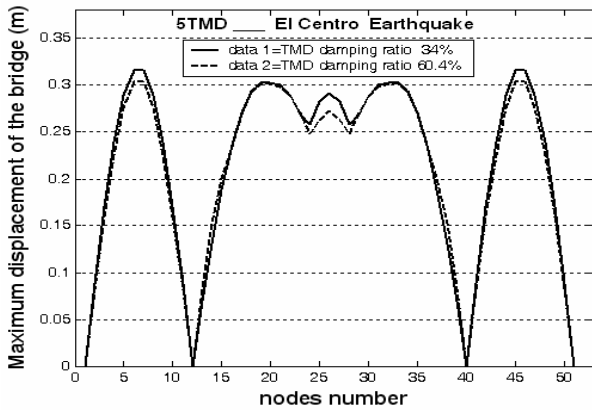


Figure 12. Comparison of the bridge controlled responses along the bridge span for 60.4 % and 34 % TMD damping ratios.

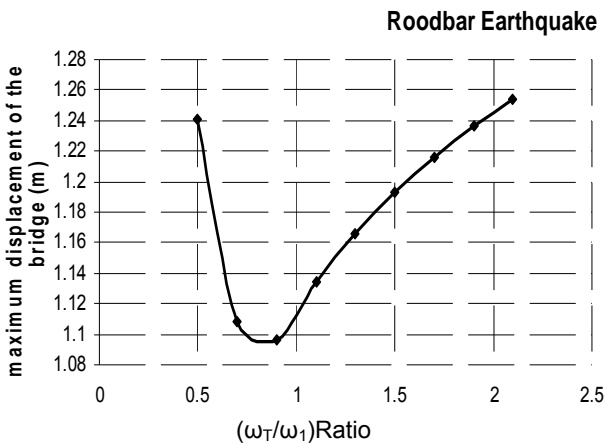


Figure 13. Effect of tuning frequency on reduction of the bridge maximum displacement.

trial and error method. The results of the numerical study indicate that:

1. The appropriate mass of the TMD is equal to 4 % of the total mass of the bridge.
2. The best mode for tuning TMD system is the bridges' first mode of vibration, and it is recommended to use 90 % of the frequency of this mode for tuning.
3. The optimum value suggested for TMD damping ratio is about 34 %.
4. It is recommended to install at least one TMD in each span of the bridge.

5. REFERENCES

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