THERMAL CONVECTION OF ROTATING MICROPOLAR FLUID IN HYDROMAGNETICS SATURATING A POROUS MEDIUM

Reena*

Department of Mathematics, Shri K.K. Jain (P.G.) College
Khatauli, Distt. Muzaffarnagar-251001, Uttar Pradesh, India
reena_math@rediffmail.com

U.S. Rana

Department of Mathematics, D. A. V. (P.G.) College
Dehradun-248001, Uttarakhand, India
drusrana@yahoo.co.in

*Corresponding Author

(Received: October 9, 2007 - Accepted in Revised Form: May 9, 2008)

Abstract This paper deals with the theoretical investigation of the thermal instability of a thin layer of electrically conducting micropolar rotating fluid, heated from below in the presence of uniform vertical magnetic field in porous medium. A dispersion relation is obtained for a flat fluid layer, contained between two free boundaries using a linear stability analysis theory, and normal mode analysis method. The principle of Exchange of Stabilities (PES) is found to hold true for the micropolar fluid saturating a porous medium, heated from below in the absence of magnetic field, rotation and coupling between thermal and micropolar effects. It is also found that PES is valid in the presence of rotation and magnetic field under certain conditions. The oscillatory modes are introduced due to the presence of magnetic field and rotation, which were non-existence in their absence. The presence of coupling between thermal and micropolar effects may also introduce oscillatory modes. For the case of stationary convection, the effect of various parameters like medium permeability, rotation, magnetic field (in the presence and absence of micropolar heat conduction parameter), coupling parameter, micropolar coefficient and micropolar heat conduction parameter has been analyzed and results are depicted graphically. The sufficient conditions for the non-existence of overstability are also obtained. In this paper, an attempt is also made to apply the variational principle for the present problem and found that the said principle can be established for the present problem in the absence of coupling between spin and heat flux.

Keywords Thermal Convection, Medium Permeability, Rayleigh Numbers, Porous Medium, Micropolar Fluids, Hydromagnetics, Rotation Effect

چکیده در این مقاله، تئوری ناپایداری دما در یک لایه نازک سیال دوار میکروپولار رسانا، در یک محیط متخلخل واقع در میدان مغناطیسی یکسان عمودی که از پایین حرارت داده شده بررسی گردید. رابطه پراکندگی برای یک لایه سیال مسطح، بین دو مرز آزاد، با استفاده از تئوری ثابت آنالیز خطی و روش آنالیز مد نرمال بدست آمد. اصل تبادل پایداری (PES)، در شرایط خاص، با وجود دوار بودن سیال و میدان مغناطیسی، در شرایط خاص معتبر می‌باشد. حالت نوسانی اعمال شده و با چرخش سیال در میدان مغناطیسی دوران، که در عدم حضور عوامل فوق چنین حالتی وجود نداشت. در حضور دو عامل میکروپولار و حرارت عوامل دیگری نظیر مود نوسانی نیز موثر بوده است. در حضور جابجایی پایدار پارامترهای مختلفی نظیر نفوذ محیط، میزان چرخش سیال در میدان مغناطیسی مورد بررسی قرار گرفت. وضمنا پارامترهای کاپلینگ و ضریب میکروپولار بر پارامتر حرارت هدایتی مورد تجزیه تحلیل قرار گرفت و نتایج آن بصورت نمودار بیان گردید. در شرایط لازم و عدم حضور در شرایط کاملا پایدار نیز بدست آمده است. در این مقاله کاربرد تغیرات اصولی برای حضور هرگونه مساله ای را مورد بررسی قرار داده و بر اساس اصول طرح شده روابطی را می توان در عدم حضور کاپلینگ میان اسپین و شار حرارتی دست یافت.

1. INTRODUCTION

A general theory of micropolar fluids has been presented by Eringen [1-3]. These fluids have such internal structures in which coupling between the spin of each particle and the macroscopic velocity field is taken into account. Compared to the classical Newtonian fluids, micropolar fluids are characterised by two supplementary variables, i.e., the spin, responsible for the micro-rotations and the micro-inertia tensor describing the distributions of atoms and molecules inside the fluid elements in addition to the velocity vector. Liquid crystals, colloidal fluids, polymeric suspension, animal blood, etc. are few examples of micropolar fluids. Kazakia, et al [4] and Eringen [5] extended this theory of structure continue to account for the thermal effects.

Micropolar fluids instability has become an important field of research now a days. The theory of thermomicropolar convection began with Datta, et al [6] and interestingly continued by Ahmadi [7]. Labon, et al [8], Bhattacharya, et al [9], Payne, et al [10] and Sharma, et al [11]. The above works give a good understanding of thermal convection in micropolar fluids. The numerical solution of thermal instability of a rotating micropolar fluid has been discussed by Sastry, et al [12]. A detailed account of thermal convection in a horizontal thin layer of Newtonian fluid heated from below, under varying assumptions of hydromagnetics, has been given by Chandrasekhar [13].

The effect of rotation on thermal convection in micropolar fluids is important in certain chemical engineering and biochemical situations. Qin, et al [14] have considered a thermal instability problem in a rotating micropolar fluid. They found that depending upon the values of various micropolar parameters and low values of Taylor number, the rotation has a stabilizing effect. The effect of rotation on thermal convection in micropolar fluids has also been studied by Sharma, et al [15], whereas the effect of rotation on thermal convection in micropolar fluids in porous medium has been considered by Sharma, et al [16]. The effects of magnetic field on the micropolar fluids heated from below have been studied by Sharma, et al [17], they also have studied the effects of magnetic field on the micropolar fluids heated from below in porous medium [18]. They found that in the presence of various coupling parameters, magnetic field has a stabilizing effect whereas the permeability has destabilizing effect on stationary convection. The thermosolutal convection of micropolar fluids in hydromagnetics in porous medium has been studied by Sharma, et al [19].

The physical properties of comets, meteorites and interplanetary dust strongly suggest the importance of porosity in astrophysical context (McDonnel [20]). Keeping in mind the importance and applications in geophysics, astrophysics and biomechanics, the effect of uniform magnetic field on the thermal convection in micropolar rotating fluid in porous medium in the presence and absence of micropolar heat conduction parameter has been considered in the present paper. It is hoped that the present study can serve as a theoretical support to an experimental investigation.

2. MATHEMATICAL FORMULATION OF THE PROBLEM

Here, we consider an infinite, horizontal layer of thickness d of an incompressible, electrically conducting thin micropolar rotating fluid heated from below saturating a porous medium (See Figure 1).

The temperature T at the bottom and top surfaces $z = 0$ and $z = d$ are T_0 and T_1 respectively and a uniform temperature gradient $\beta\left(=\left|\frac{dT}{dz}\right|\right)$ is maintained. Both the boundaries are taken to be free and perfect conductors of heat. The gravity field $\mathbf{g} = (0, 0, -g)$ and uniform vertical magnetic field intensity $\mathbf{H} = (0, 0, H)$ pervade the system. The whole system is assumed to be acted on by a uniform rotation $\mathbf{\Omega}\,(0, 0, \Omega)$ along the vertical axis, which is taken as z-axis. This fluid layer is assumed to be flowing through an isotropic and homogeneous porous medium of porosity ε and the medium permeability k_1. Also assumed that the external couples and heat sources are not present. The mathematical formulation of the motion of micropolar rotating fluids saturating a porous medium for the above model are as follows [11,16,18]:

The continuity equation for an incompressible fluid is

$$\nabla.\mathbf{q} = 0 \qquad (1)$$

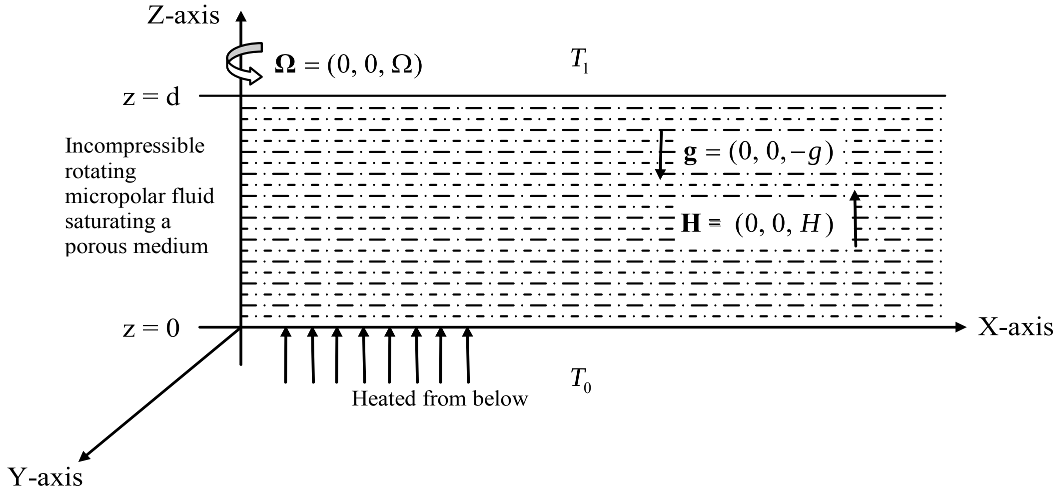


Figure 1. Geometrical configuration.

The momentum and internal angular momentum equations for the Darcy model are

$$\frac{\rho_0}{\varepsilon}\left[\frac{\partial \mathbf{q}}{\partial t} + \frac{1}{\varepsilon}(\mathbf{q}.\nabla)\mathbf{q}\right] = -\nabla p - \rho g \hat{e}_z - \frac{1}{k_1}(\mu + k)\,\mathbf{q} + k\nabla\times\mathbf{v} + \frac{2\rho_0}{\varepsilon}(\mathbf{q}\times\mathbf{\Omega}) + \frac{1}{4\pi}(\nabla\times\mathbf{H})\times\mathbf{H}\,, \qquad (2)$$

$$\rho_0 j\left[\frac{\partial \mathbf{v}}{\partial t} + \frac{1}{\varepsilon}(\mathbf{q}\,.\,\nabla)\,\mathbf{v}\right] = (\varepsilon' + \beta')\,\nabla(\nabla.\mathbf{v}) + \gamma'\nabla^2\mathbf{v} + \frac{k}{\varepsilon}\nabla\times\mathbf{q} - 2k\mathbf{v} \qquad (3)$$

The temperature equation is

$$\left[\rho_0 C_v \varepsilon + \rho_s C_s (1-\varepsilon)\right]\frac{\partial T}{\partial t} + \rho_0 C_v (\mathbf{q}.\nabla)\,T = k_T \nabla^2 T + \delta'(\nabla\times\mathbf{v}).\nabla T \qquad (4)$$

The density equation of state is given by

$$\rho = \rho_0\left[1 - \alpha(T - T_0)\right] \qquad (5)$$

Where $\mathbf{q}, \mathbf{v}, t, T, \alpha, \rho, \rho_0, \rho_s, p, \mu, \hat{e}_z, \eta$ and j denote respectively, filter velocity, spin (micro rotation), time, temperature, coefficient of thermal expansion, fluid density, reference density, density of solid matrix, pressure, coefficient of viscosity, unit vector in z-direction, electrical resistivity and microinertia constant. The parameters $\varepsilon', \beta', \gamma'$ and k stand for the micropolar coefficients of viscosity, C_υ, C_s, k_T and δ' are specific heat at constant volume, heat capacity of solid matrix, thermal conductivity and coefficient giving account of coupling between the spin flux and heat flux respectively and $\mathbf{r} = (x, y, z)$. The effect of rotation contributes two terms:

(a) centrifugal force $-\frac{\rho_0}{2}\,grad\,|\mathbf{\Omega}\times\mathbf{r}\,|^2$ and

(b) Coriolis force $\frac{2\rho_0}{\varepsilon}(\mathbf{q}\times\mathbf{\Omega})$.

In Equation 2, $p = p_f - \frac{1}{2}\rho_0\,|\,\mathbf{\Omega}\times\mathbf{r}|^2$ is the reduced pressure, whereas p_f stands for fluid pressure [21]. When the fluid flows through a porous medium the gross effect is represented by Darcy's law. As a result, the usual viscous term is replaced by the resistance term $-\left[\frac{\mu + k}{k_1}\right]\mathbf{q}$, where μ, k, k_1 and $\mathbf{q}$ denote respectively the viscosity coefficient, micropolar heat conduction coefficient, medium permeability and the filter velocity.

The Maxwell's equation yields

$$\varepsilon\frac{\partial \mathbf{H}}{\partial t} = \nabla\times(\mathbf{q}\times\mathbf{H}) + \varepsilon\eta\nabla^2\mathbf{H} \qquad (6)$$

and

$$\nabla . \mathbf{H} = 0 \tag{7}$$

Let us now consider the stability of the system in the usual way by giving small perturbations on the initial (rest) state and seeing the reaction of the perturbations on the system. The initial state is $\mathbf{q} = 0$, $\mathbf{v} = 0$, $p = p(z)$, $\rho = \rho(z)$ defined as $\rho = \rho_0 \left[1 + \alpha\beta z\right]$, and $T = T(z)$ defined as $T = -\beta z + T_0$, where $\beta = -\dfrac{dT}{dz}$ is the uniform adverse temperature gradient.

3. PERTURBATION EQUATIONS

Let $\mathbf{u}\,(u_x, u_y, u_z)$, $\boldsymbol{\omega}, \delta p, \delta\rho, \theta$ and $\mathbf{h}\,(h_x, h_y, h_z)$ denote respectively the perturbations in velocity $\mathbf{q}$, spin $\mathbf{v}$, pressure p, density ρ, temperature T and magnetic field $\mathbf{H}$.

The change in density $\delta\rho$, caused mainly by the perturbation θ in temperature is given by

$$\delta\rho = -\rho_0 \alpha\theta \tag{8}$$

Then the Equations 1 to 7 yield the linearized perturbation equations

$$\nabla . \mathbf{u} = 0\,, \tag{9}$$

$$\frac{\rho_0}{\varepsilon}\frac{\partial \mathbf{u}}{\partial t} = -\nabla\delta p - \frac{1}{k_1}\left(\mu + k\right)\mathbf{u} + g\alpha\rho_0\theta\hat{e}_z + k\nabla\times\boldsymbol{\omega} + \frac{2\rho_0}{\varepsilon}(\mathbf{u}\times\boldsymbol{\Omega}) + \frac{1}{4\pi}(\nabla\times\mathbf{h})\times\mathbf{H}\,, \tag{10}$$

$$\rho_0 j\frac{\partial\boldsymbol{\omega}}{\partial t} = (\varepsilon' + \beta')\,\nabla(\nabla.\boldsymbol{\omega}) + \gamma'\nabla^2\boldsymbol{\omega} + \frac{k}{\varepsilon}\nabla\times\mathbf{u} - 2k\boldsymbol{\omega}\,, \tag{11}$$

$$\left[\rho_0 C_v \varepsilon + \rho_s C_s (1-\varepsilon)\right]\frac{\partial\theta}{\partial t} = k_T\nabla^2\theta - \delta'(\nabla\times\boldsymbol{\omega})_z\beta + \rho_0 C_v u_z \beta\,, \tag{12}$$

$$\varepsilon\frac{\partial\mathbf{h}}{\partial t} = \nabla\times(\mathbf{u}\times\mathbf{H}) + \varepsilon\eta\nabla^2\mathbf{h}\,, \tag{13}$$

and

$$\nabla . \mathbf{h} = 0 \tag{14}$$

Where, the non-linear terms $(\mathbf{u}.\nabla)\mathbf{u}, (\mathbf{u}.\nabla)\theta$, $\nabla\theta.(\nabla\times\boldsymbol{\omega})$ and $(\mathbf{u}.\nabla)\boldsymbol{\omega}$ in Equations 9-14 are neglected (using the first order approximations) as the perturbations applied on the system are assumed to be small, the second and higher order perturbations are negligibly small and only linear terms are retained.

Now, it is usual to write the balance equations in a dimensionless form, scaling as

$$(x, y, z) = (x^*, y^*, z^*)\,d,\;\; t = \frac{\rho_0 d^2}{\mu}t^*,\;\; \theta = \beta d\theta^*,$$

$$\mathbf{u} = \frac{x_T}{d}\mathbf{u}^*,\;\; p = \frac{\mu x_T}{d^2}p^*,\;\; \boldsymbol{\omega} = \frac{x_T}{d^2}\boldsymbol{\omega}^*,\;\; \boldsymbol{\Omega} = \frac{\mu}{\rho_0 d^2}\boldsymbol{\Omega}^*,$$

$$\mathbf{h} = \sqrt{\left(\frac{\mu\, x_T}{d^2}\right)}\,.\,\mathbf{h}^*$$

and then removing the stars (*) for convenience, the non-dimensional form of Equations 9-14 become

$$\nabla . \mathbf{u} = 0 \tag{15}$$

$$\frac{1}{\varepsilon}\frac{\partial\mathbf{u}}{\partial t} = -\nabla\delta p - \frac{1}{P_l}(1+K)\,\mathbf{u} + R\theta\hat{e}_z + K\nabla\times\boldsymbol{\omega} + \frac{2}{\varepsilon}(\mathbf{u}\times\boldsymbol{\Omega}) + \frac{1}{4\pi}(\nabla\times\mathbf{h})\times\mathbf{H}\,, \tag{16}$$

$$\bar{j}\frac{\partial\boldsymbol{\omega}}{\partial t} = C_1\nabla(\nabla.\boldsymbol{\omega}) - C_0\nabla\times(\nabla\times\boldsymbol{\omega}) + K\left(\frac{1}{\varepsilon}\nabla\times u - 2\boldsymbol{\omega}\right), \tag{17}$$

$$Ep_1\frac{\partial\theta}{\partial t} = \nabla^2\theta + u_z - \bar{\delta}(\nabla\times\boldsymbol{\omega})_z\,, \tag{18}$$

$$\varepsilon\frac{\partial\mathbf{h}}{\partial t} = \nabla\times(\mathbf{u}\times\mathbf{H}) + \frac{\varepsilon}{p_2}\nabla^2\mathbf{h}\,, \tag{19}$$

and

$$\nabla . \mathbf{h} = 0 \tag{20}$$

Where the following new non-dimensional

parameters are introduced

$$\bar{j} = \frac{j}{d^2},\ \bar{\delta} = \frac{\delta'}{\rho_0 C_v d^2},\ P_l = \frac{k_1}{d^2},\ K = \frac{k}{\mu},\ C_0 = \frac{\gamma'}{\mu d^2},$$

$$C_1 = \frac{\varepsilon' + \beta' + \gamma'}{\mu d^2},\ E = \varepsilon + \frac{\rho_s C_s}{\rho_0 C_v}(1-\varepsilon)$$

$$\left.\begin{array}{l} \text{and } R = \dfrac{g_0 \alpha \beta \rho_0 d^4}{\mu x_T} \text{ is the dimensionless} \\ \text{Rayleigh number,} \\ p_1 = \dfrac{\mu}{\rho_0 x_T} \text{ is the dimensionless Prandtl number,} \\ p_2 = \dfrac{\mu}{\rho_0 \eta} \text{ is the dimensionless magnetic Prandtl} \\ \text{number.} \end{array}\right\} \quad (21)$$

and we have put $x_T = \dfrac{k_T}{\rho_0\ C_v}$ for thermal diffusivity.

Here, we consider the boundary conditions for both free boundaries and perfectly heat conducting. On a free surface, shear stress is zero and the velocity normal to the surface is zero. For micro-rotation boundary conditions, we assume the micro-rotation to be zero on the surface. Thus, the dimensionless boundary conditions are

$$u_z = 0, \frac{\partial^2 u_z}{\partial z^2} = 0, \boldsymbol{\omega} = 0, \theta = 0 \text{ at } z = 0 \text{ and } z = 1. \quad (22)$$

4. DISPERSION RELATION

Applying the curl operator twice to Equation 16 and taking the z-component, we get

$$\frac{1}{\varepsilon}\frac{\partial}{\partial t}\nabla^2 u_z = R\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\theta - \frac{1}{P_l}(1+K)\nabla^2 u_z$$
$$+ K\nabla^2\Omega_z' - \frac{2}{\varepsilon}\Omega\frac{\partial \zeta_z}{\partial z} + \frac{H}{4\pi}\frac{\partial}{\partial z}(\nabla^2 h_z) \quad (23)$$

Where,

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2},\ \Omega_z' = (\nabla \times \boldsymbol{\omega})_z = \left(\frac{\partial \omega_y}{\partial x} - \frac{\partial \omega_x}{\partial y}\right)$$

and

$\zeta_z = (\nabla \times \mathbf{u})_z = \left(\dfrac{\partial u_y}{\partial x} - \dfrac{\partial u_x}{\partial y}\right)$ is the z-component of vorticity.

Applying the curl operator once to Equations 16, 17 and 19 and taking z-component, we get

$$\frac{1}{\varepsilon}\frac{\partial \zeta_z}{\partial t} = -\frac{1}{P_l}(1+K)\zeta_z + \frac{2}{\varepsilon}\Omega\frac{\partial u_z}{\partial z} + \frac{H}{4\pi}\frac{\partial \xi_z}{\partial z} \quad (24)$$

$$\bar{j}\frac{\partial \Omega_z'}{\partial t} = C_0\nabla^2\Omega_z' - K\left[\frac{1}{\varepsilon}\nabla^2 u_z + 2\Omega_z'\right] \quad (25)$$

$$\varepsilon\frac{\partial \xi_z}{\partial t} = H\frac{\partial \zeta_z}{\partial z} + \frac{\varepsilon}{p_2}\nabla^2\xi_z \quad (26)$$

Where, $\xi_z = (\nabla \times \mathbf{h})_z = \left(\dfrac{\partial h_y}{\partial x} - \dfrac{\partial h_x}{\partial y}\right)$ is the z-component of current density.

The linearized form of Equation 18 is

$$Ep_1\frac{\partial \theta}{\partial t} = \nabla^2\theta + u_z - \bar{\delta}\,\Omega_z' \quad (27)$$

and the z-component of Equation 19 is

$$\varepsilon\frac{\partial h_z}{\partial t} = H\frac{\partial u_z}{\partial z} + \frac{\varepsilon}{p_2}\nabla^2 h_z \quad (28)$$

If the medium adjoining the fluid is electrically non-conducting, then the boundary conditions are

$$u_z = 0,\ \frac{\partial^2 u_z}{\partial z} = 0, \frac{\partial \zeta_z}{\partial z} = 0,\ \xi_z = 0, \frac{\partial h_z}{\partial z} = 0,$$
$$\Omega_z' = 0 \text{ and } \theta = 0 \text{ at } z = 0 \text{ and } z = 1. \quad (29)$$

In Equation 25 for spin, the coefficients C_0 and K account for spin diffusion and coupling between vorticity and spin effects respectively.

Analyzing the disturbances into normal modes, we assume that the perturbation quantities are of the form

$$\left[u_z, \zeta_z, \Omega_z', \xi_z, \theta, h_z\right] =$$
$$\left[U(z), Z(z), G(z), X(z), \Theta(z), B(z)\right]\exp(ik_x x + ik_y y + \sigma t),$$
(30)

Where, σ is the stability parameter which is, in general, a complex constant and $k=\sqrt{k_x^2+k_y^2}$ is the wave number.

Following the normal mode analysis, the linearized perturbation dimensionless equations are

$$(D^2-k^2)\left[\frac{\sigma}{\varepsilon}+\frac{1}{P_l}(1+K)\right]U=-Rk^2\Theta+$$

$$K(D^2-k^2)G-\frac{2}{\varepsilon}\Omega Dz+\frac{H}{4\pi}(D^2-k^2)\,DB\,, \tag{31}$$

$$\left[\frac{1}{\varepsilon}\sigma+\frac{1}{P_l}(1+K)\right]Z=\frac{2}{\varepsilon}\Omega DU+\frac{H}{4\pi}DX\,, \tag{32}$$

$$\left[l\sigma+2A-(D^2-k^2)\right]G=-A\varepsilon^{-1}(D^2-k^2)U\,, \tag{33}$$

$$\left[Ep_1\sigma-(D^2-k^2)\right]\Theta=U-\overline{\delta}G \tag{34}$$

$$\left[\sigma-\frac{1}{p_2}(D^2-k^2)\right]X=\varepsilon^{-1}H\;DZ\,, \tag{35}$$

and

$$\left[\sigma-\frac{1}{p_2}(D^2-k^2)\right]B=\varepsilon^{-1}H\;DU \tag{36}$$

Where,

$$l=\overline{j}\frac{A}{K},\; A=\frac{K}{C_0}$$

and

$$D\equiv\frac{d}{dz}.$$

Here, the micropolar coefficient A is the ratio between the micropolar viscous effects and micropolar diffusion ones.

Now, for finding the dispersion relation, diminishing Θ, Z, B and G from Equations 31-36, we obtain

$$(D^2-k^2)\left[\sigma\varepsilon^{-1}+\frac{1}{P_l}(1+K)\right]$$

$$\left[Ep_1\sigma-(D^2-k^2)\right]\left[l\sigma+2A-(D^2-k^2)\right]$$

$$\times\left[\sigma-\frac{1}{p_2}(D^2-k^2)\right]$$

$$\left\{\left[\sigma\varepsilon^{-1}+\frac{1}{P_l}(1+K)\right]\left[\sigma-\frac{1}{p_2}(D^2-k^2)\right]-\frac{H^2\varepsilon^{-1}}{4\pi}D^2\right\}U$$

$$=-Rk^2$$

$$\left\{\left[\sigma\varepsilon^{-1}+\frac{1}{P_l}(1+K)\right]\left[\sigma-\frac{1}{p_2}(D^2-k^2)\right]-\frac{H^2\varepsilon^{-1}}{4\pi}D^2\right\}$$

$$\times\left[l\sigma+2A-(D^2-k^2)+\overline{\delta}\varepsilon^{-1}A(D^2-k^2)\right]$$

$$\left[\sigma-\frac{1}{p_2}(D^2-k^2)\right]U$$

$$-kA\varepsilon^{-1}(D^2-k^2)^2\left[Ep_1\sigma-(D^2-k^2)\right]$$

$$\left[\sigma-\frac{1}{p_2}(D^2-k^2)\right]$$

$$\times\left\{\left[\sigma\varepsilon^{-1}+\frac{1}{P_l}(1+K)\right]\left[\sigma-\frac{1}{p_2}(D^2-k^2)\right]-\frac{H^2\varepsilon^{-1}}{4\pi}D^2\right\}U$$

$$-4\Omega^2\varepsilon^{-2}\left[Ep_1\sigma-(D^2-k^2)\right]\left[l\sigma+2A-(D^2-k^2)\right]$$

$$\left[\sigma-\frac{1}{p_2}(D^2-k^2)\right]^2D^2U$$

$$+\frac{H^2\varepsilon^{-1}}{4\pi}(D^2-k^2)\left[Ep_1\sigma-(D^2-k^2)\right]\left[l\sigma+2A-(D^2-k^2)\right]$$

$$\times\left\{\left[\sigma\varepsilon^{-1}+\frac{1}{P_l}(1+K)\right]\left[\sigma-\frac{1}{p_2}(D^2-k^2)\right]\right.$$

$$\left.-\frac{H^2\varepsilon^{-1}}{4\pi}D^2\right\}D^2U \tag{37}$$

The boundary conditions (29) transform to

$$U=D^2U=0,\; DZ=0,\; \Theta=0,\; X=0,\; G=0,\; DB=0 \tag{38}$$

at $z=0$ and $z=1$

Using (38), Equations 31-36 give

$$D^2\Theta=0,\; D^2G=0,\; D^3Z=0,\; D^2X=0,\; D^3B=0 \tag{39}$$

at $z=0$ and $z=1$.

Differentiating (31) twice with respect to z and using (39), it can be shown that $D^4U=0$, i.e., all even order derivative of U vanish on the boundaries.

Now, solutions of Equations 31-36 must be sought which satisfy the boundary conditions (38) and (39).

The proper solutions for U belonging to the lowest mode, satisfying boundary conditions (38) and (39) is given by

$$U = U_0 \sin \pi z \tag{40}$$

Where U_0 is a constant.

Substituting (40) in (37), we get

$$\begin{aligned}
&Rk^2 \left\{\left[\sigma\varepsilon^{-1} + \frac{1}{P_l}(1+K)\right]\left[\sigma + \frac{b}{p_2}\right] + \frac{H^2\pi\varepsilon^{-1}}{4}\right\} \\
&\left[l\sigma + 2A + b - \bar{\delta}\varepsilon^{-1}Ab\right] \times \left[\sigma + \frac{b}{p_2}\right] \\
&= b\left[\sigma\varepsilon^{-1} + \frac{1}{P_l}(1+K)\right]\left[Ep_1\sigma + b\right]\left[l\sigma + 2A + b\right] \\
&\times \left[\sigma + \frac{b}{p_2}\right]\left\{\left[\sigma\varepsilon^{-1} + \frac{1}{P_l}(1+K)\right]\left[\sigma + \frac{b}{p_2}\right] + \frac{H^2\pi\varepsilon^{-1}}{4}\right\} \\
&-KA\varepsilon^{-1}b^2\left[Ep_1\sigma + b\right]\left[\sigma + \frac{b}{p_2}\right] \\
&\left\{\left[\sigma\varepsilon^{-1} + \frac{1}{P_l}(1+K)\right]\left[\sigma + \frac{b}{p_2}\right] + \frac{H^2\pi\varepsilon^{-1}}{4}\right\} \\
&+ 4\Omega^2\pi^2\varepsilon^{-2}\left[Ep_1\sigma + b\right]\left[l\sigma + 2A + b\right]\left[\sigma + \frac{b}{p_2}\right]^2 \\
&+ \frac{H^2\varepsilon^{-1}\pi}{4} b\left[Ep_1\sigma + b\right]\left[l\sigma + 2A + b\right] \\
&\times \left\{\left[\sigma\varepsilon^{-1} + \frac{1}{P_l}(1+K)\right]\left[\sigma + \frac{b}{p_2}\right] + \frac{H^2\pi\varepsilon^{-1}}{4}\right\}
\end{aligned} \tag{41}$$

Where,

$$b = \pi^2 + k^2 .$$

Equation 41 is the required dispersion relation studying the effect of medium permeability, rotation and magnetic field of the system.

In the absence of rotation $(\Omega = 0)$, Equation 41 reduces to

$$\begin{aligned}
&Rk^2\left[\sigma + \frac{b}{p_2}\right]\left[l\sigma + 2A + b - \bar{\delta}\varepsilon^{-1}Ab\right] = \\
&\left[\sigma + \frac{b}{p_2}\right]\left[Ep_1\sigma + b\right] \\
&\times\left[l\sigma + 2A + b\right]\left[\varepsilon^{-1}\sigma b + \frac{1}{P_l}(1+K)\, b\right] - KA\varepsilon^{-1}b^2 \\
&\times\left[Ep_1\sigma + b\right]\left[\sigma + \frac{b}{p_2}\right] \\
&+ \frac{H^2\pi\varepsilon^{-1}}{4}(b^2 + Ep_1\sigma b)\,(l\sigma + 2A + b)
\end{aligned} \tag{42}$$

A result derived by [18].

In the absence of magnetic field $(H = 0)$, Equation 41 reduces to

$$\begin{aligned}
&b\left[\sigma\varepsilon^{-1} + \frac{1}{P_l}(1+K)\right]^2\left[Ep_1\sigma + b\right]\left[l\sigma + 2A + b\right] = \\
&Rk^2\left[\sigma\varepsilon^{-1} + \frac{1}{P_l}(1+K)\right]\left[l\sigma + 2A + b - \bar{\delta}\varepsilon^{-1}Ab\right] \\
&+ KA\varepsilon^{-1}b^2\left[Ep_1\sigma + b\right]\left[\sigma\varepsilon^{-1} + \frac{1}{P_l}(1+K)\right] \\
&- 4\Omega^2\pi^2\varepsilon^{-2}\left[Ep_1\sigma + b\right]\left[l\sigma + 2A + b\right]
\end{aligned}$$

A result derived by [Sharma, et al [16], Equation 31].

In the absence of both magnetic field and rotation, Equation 41 reduces to

$$\begin{aligned}
&b\left[\varepsilon^{-1}b + \frac{1}{P_l}(1+K)\right]\left[Ep_1\sigma + b\right]\left[l\sigma + 2A + b\right] = \\
&Rk^2\left[l\sigma + 2A + b - \bar{\delta}\varepsilon^{-1}Ab\right] + KA\varepsilon^{-1}b^2\left[Ep_1\sigma + b\right]
\end{aligned}$$

a result derived by [Sharma, et al [11], Equation 28].

5. STABILITY OF THE SYSTEM AND OSCILLATORY MODES

Here, we examine the possibility of the oscillatory modes, if any, on stability problem due to the presence of rotation, magnetic field, medium permeability and micropolar parameters.

Multiplying Equation 31 by U^*, the complex conjugate of U and integrating w.r.t. z between the limits $z=0$ to $z=1$ and making use of Equations 32 to 36 with the help of boundary conditions (38) and (39), we get

$$\left[\frac{\sigma}{\varepsilon}+\frac{1}{P_l}(1+K)\right]I_1 - Rk^2\left\{\left(Ep_1\sigma^* I_2+I_3\right)+\overline{\delta}\int_0^1 G^*\Theta dz\right\} + \left[\varepsilon^{-1}\sigma^* + \frac{1}{P_l}(1+K)\right]I_4 + \frac{\varepsilon}{4\pi}\left(\sigma I_5 + \frac{1}{p_2}I_6\right) - \frac{K\varepsilon}{A}\left\{(l\sigma^* + 2A)\,I_7 + I_8\right\} + \frac{\varepsilon}{4\pi}\left(\sigma^* I_9 + \frac{1}{p_2}I_{10}\right) = 0 \quad (43)$$

Where,

$$\left.\begin{aligned} & I_1=\int_0^1\left(|DU|^2+k^2|U|^2\right)dz, I_2=\int_0^1|\Theta|^2\,dz, \\ & I_3=\int_0^1(|D\Theta|^2+k^2|\Theta|^2)\,dz,\ I_4=\int_0^1|Z|^2\,dz, \\ & I_5=\int_0^1|X|^2\,dz, \\ & I_6=\int_0^1\left[|DX|^2+k^2|X|^2\right]dz, \\ & I_7=\int_0^1|G|^2\,dz, \\ & I_8=\int_0^1\left[|DG|^2+k^2|G|^2\right]dz, \\ & I_9=\int_0^1\left[|DB|^2+k^2|B|^2\right]dz \\ & \text{and} \\ & I_{10}=\int_0^1\left[|D^2B|^2+2k^2|DB|^2+k^4|B|^2\right]dz \end{aligned}\right\} \quad (44)$$

The integrals I_1 to I_{10} are all positive definite. Putting $\sigma=\sigma_r+i\sigma_i$ and equating the real and imaginary parts of Equation 43, we obtain

$$\left[\frac{1}{\varepsilon}(I_1+I_4)+\frac{\varepsilon}{4\pi}(I_5+I_9)-Rk^2Ep_1I_2-\frac{K\varepsilon}{A}\,l\,I_7\right]\sigma_r = -\left[\frac{(1+K)}{P_l}(I_1+I_4)-Rk^2I_3+\frac{\varepsilon}{4\pi}\frac{1}{p_2}(I_6+I_{10})-\frac{K\varepsilon}{A}(2AI_7+I_8)\right] \quad (45)$$

and

$$\left[\frac{1}{\varepsilon}(I_1-I_4)+Rk^2Ep_1I_2+\frac{\varepsilon}{4\pi}(I_5-I_9)+\frac{K\varepsilon l}{A}I_7\right]\sigma_i = 0 \quad (46)$$

Where, for the sake of convenience, we have taken $\overline{\delta}=0$, i.e., absence of coupling between spin and heat flux.

It is evident from Equation 45 that σ_r is positive or negative. The system is, therefore, stable or unstable. It is clear from Equation 46 that σ_i may be zero or non-zero, meaning that modes may be non-oscillatory or oscillatory. In the absence of rotation and magnetic field, Equation 46 reduces to

$$\left[\frac{1}{\varepsilon}I_1+Rk^2Ep_1I_2+\frac{K\varepsilon l}{A}I_7\right]\sigma_i = 0 \quad (47)$$

and the terms in the bracket are positive definite. Thus $\sigma_i=0$ which means that oscillatory modes are not allowed and the principle of exchange of stabilities (PES) is satisfied for micropolar rotating fluids heated from below saturating a porous medium in the absence of rotation, magnetic field and coupling between spin and heat flux ($\overline{\delta}=0$). Thus, oscillatory modes are introduced due to the presence of rotation and magnetic field which were non-existence in their absence. The presence of coupling between spin and heat flux may also introduce oscillatory modes.

5.1. Effect of Magnetic Field

In the absence of rotation ($\Omega=0$) Equation 46 become

$$\left[\frac{1}{\varepsilon}I_1-\frac{\varepsilon}{4\pi}I_9+Rk^2Ep_1I_2+\frac{\varepsilon}{4\pi}I_5+\frac{K\varepsilon l}{A}I_7\right]\sigma_i = 0 \quad (48)$$

Equation 48 must be satisfied at the marginal state since $\sigma_r=0$. Now, for the principle of exchange of stabilities (PES) to be valid at the marginal state, we must have $\sigma_i=0$, which implies that the terms in the bracket of Equation 48 must be positive definite. To prove this, Equation 36 and $\sigma_i=0$ yield

$$-\frac{1}{p_2}(D^2-k^2)\,B \;=\; \varepsilon^{-1} H\; DU \tag{49}$$

Multiplying both sides of Equation 49 by B^* the complex conjugate of B, integrating the resulting equation over the vertical range of z and separating the real parts of both the sides of equations obtained, yield

$$-\frac{1}{p_2}\int_0^1 B^*(D^2-k^2)\,B\,dz \;=\mathrm{Re}\,\frac{H}{\varepsilon}\int_0^1 B^* DU\;dz$$

or

$$\frac{1}{p_2}I_9 \;=\; \frac{H}{\varepsilon}\,\mathrm{Re}\int_0^1 B^* DU\;dz \tag{50}$$

Now,

$$\mathrm{Re}\int_0^1 B^*\;DUdz \;\le\; \left|\int_0^1 B^* DUdz\right|$$

$$\le\; \int_0^1 |B^*|\,|DU|\,dz$$

$$=\; \int_0^1 |B|\,|DU|\,dz$$

$$\le\; \sqrt{\int_0^1 |B|^2}\;\sqrt{\int_0^1 |DU|^2\;dz}$$

$$\le\; \sqrt{\int_0^1 \left[|DB|^2+k^2|B|^2\right]dz}\;\sqrt{\int_0^1 |DU|^2\;dz}$$

$$\le\; \sqrt{I_9}\;\sqrt{\int_0^1 |DU|^2\;dz} \tag{51}$$

Equations 50 and 51 give

$$\frac{1}{p_2}I_9 \;\le\; \frac{H}{\varepsilon}\sqrt{I_9}\;\sqrt{\int_0^1 |DU|^2\;dz}$$

Which implies

$$\frac{1}{p_2}I_9 \le \frac{H^2}{\varepsilon^2}\;p_2\int_0^1 |DU|^2\;dz \tag{52}$$

Also,

$$I_1 \;=\; \int_0^1\left[|DU|^2+k^2|U|^2\right]dz \;\ge\; \int_0^1 |DU|^2\;dz \tag{53}$$

Making use of Equations 52 and 53 in Equation 48, we get

$$\left[\frac{1}{\varepsilon}\left(I_1-\frac{\varepsilon^2}{4\pi}I_9\right)+Rk^2Ep_1I_2+\frac{\varepsilon}{4\pi}I_5+\frac{K\varepsilon l}{A}I_7\right]$$

$$\ge\left[\frac{1}{\varepsilon}\left(1-\frac{H^2p_2^2}{4\pi}\right)\int_0^1 |DU|^2\;dz+\right.$$

$$\left.Rk^2Ep_1I_2+\frac{\varepsilon}{4\pi}I_5+\frac{K\varepsilon l}{A}I_7\right]$$

But if $\frac{H^2p_2^2}{4\pi}<1$, then terms in the bracket of Equation 48 are positive definite, which implies that $\sigma_i=0$ and hence the necessary conditions for the validity of the principle of exchange of stabilities (PES) in thermal instability of micropolar fluids in porous medium in the presence of magnetic field is

$$\left(\frac{H\;p_2}{\sqrt{4\pi}}\right)^2 \;<\; 1$$

or

$$\left(\frac{H\mu}{\rho_0\eta\sqrt{4\pi}}\right)^2<1,$$

Where

$$p_2=\frac{\mu}{\rho_0\eta}$$

or

$$H \;<\; \frac{2\rho_0\eta}{\mu}\sqrt{\pi}$$

Thus, in the absence of coupling between spin and heat flux (i.e., $\overline{\delta}=0$), rotation $(\Omega=0)$ and in the presence of magnetic field ($H\ne 0$), the necessary conditions for existing PES, i.e., non-oscillatory modes, i.e., non-existence of overstability at the marginal state is

$$H \;<\; \left(\frac{2\rho_0\eta}{\mu}\right)\sqrt{\pi} \tag{54}$$

5.2. Combined Effect of Rotation and

Magnetic Field We may write Equation 46 as

$$\left[\frac{1}{\varepsilon}\left\{I_1-\left(I_4+\frac{\varepsilon^2}{4\pi}I_9\right)\right\}+Rk^2Ep_1I_2+\right.$$
$$\left.\frac{\varepsilon}{4\pi}I_5+\frac{K\varepsilon l}{A}I_7\right]\sigma_i \;=\; 0 \tag{55}$$

Equation 32 and $\sigma_i=0$ yields

$$\left(\frac{1+K}{P_l}\right)Z \;=\; \frac{2}{\varepsilon}\Omega\, DU+\frac{H}{4\pi}DX \tag{56}$$

Multiplying both sides of Equation 56 by Z^* the complex conjugate of Z, integrating the resulting equation over the vertical range of z and separating the real parts of both the sides of equation so obtained, we get

$$\left(\frac{1+K}{P_l}\right)\int_0^1 Z^*Z\,dz \;=$$
$$\mathrm{Re}\left(\frac{2\Omega}{\varepsilon}\int_0^1 Z^*DUdz+\frac{H}{4\pi}\int_0^1 Z^*DX\,dz\right)$$

or

$$\left(\frac{1+K}{P_l}\right)\int_0^1 |Z|^2\,dz \;\le\; \frac{2}{\varepsilon}\Omega\left|\int_0^1 Z^*DU\,dz\right|$$
$$\left[\because \mathrm{Re}\,(x+y)\;\le\;|x+y|\le|x|+|y|\,\right]$$
$$\le\;\frac{2\,\Omega}{\varepsilon}\int_0^1 |Z^*|\,|DU|\,dz$$
$$=\;\frac{2\,\Omega}{\varepsilon}\int_0^1 |Z|\,|DU|\,dz$$
$$\le\;\frac{2\,\Omega}{\varepsilon}\sqrt{\int_0^1 |Z|^2\,dz}\;\sqrt{\int_0^1 |DU|^2\,dz}$$

i.e., $$I_4\le\frac{4\Omega^2P_l^{\,2}}{\varepsilon^2(1+K)^2}\int_0^1 |DU|^2\,dz \tag{57}$$

Equations 52 and 57 together gives

$$\left(I_4+\frac{\varepsilon^2}{4\pi}I_9\right)\le\left(\frac{4\Omega^2P_l^{\,2}}{\varepsilon^2(1+K)^2}+\frac{H^2p_2^2}{4\pi}\right)\int_0^1 |DU|^2\,dz \tag{58}$$

Also,

$$I_1 \;=\; \int_0^1\left[|DU|^2+k^2\,|U|^2\right]dz\;\ge\;\int_0^1 |DU|^2\,dz \tag{59}$$

Equations 58 and 59 together gives

$$\left[I_1-\left(I_4+\frac{\varepsilon^2}{4\pi}I_9\right)\right]\ge$$
$$\left[1-\left\{\frac{4\Omega^2P_l^{\,2}}{\varepsilon^2(1+K)^2}+\frac{H^2p_2^2}{4\pi}\right\}\right]\int_0^1 |DU|^2\,dz \tag{60}$$

Thus the terms within the bracket of Equation 55 are positive definite if

$$\left\{\frac{4\Omega^2P_l^{\,2}}{\varepsilon^2(1+K)^2}+\frac{H^2p_2^2}{4\pi}\right\}\;<\;1$$

or

$$H\;<\;\frac{2\rho_0\eta}{\mu}\sqrt{\pi\left\{1-\frac{4\Omega^2P_l^{\,2}}{\varepsilon^2(1+K)^2}\right\}} \tag{61}$$

Where,

$$P_l\;=\;\frac{k_1}{d^2}$$

Which implies that $\sigma_i=0$ and hence Equation 61 is the necessary condition for the validity of the principle of exchange of stabilities (PES) in thermal instability of micropolar fluids in porous medium in the presence of magnetic field and rotation both.

5.3. Effect of Rotation Similarly, under the effect of only rotation ($\Omega\ne 0,\ H=0$) the condition (61) reduces to

$$\Omega\;<\;\frac{\varepsilon d^2(1+K)}{2k_1} \tag{62}$$

for the validity of PES.

6. THE CASE OF OVERSTABILITY

The present section is devoted to find the possibility that the observed instability may really be overstability. Let us put $\sigma=\sigma_r+i\sigma_i$, where

σ_r, σ_i are real, it being remembered that σ is in general, a complex constant. The marginal state is reached when $\sigma_r = 0$ implies $\sigma_i = 0$, one says that principle of exchange of stability (PES) is valid otherwise we have overstability and then $\sigma = i\sigma_i$ at marginal stability. Thus putting $\sigma = i\,\sigma_i$ in Equation 41, equating real and imaginary parts of Equation 41 and eliminating R between them, we obtain

$$f_4\, C_1^4 + f_3 C_1^3 + f_2 C_1^2 + f_1 C_1 + f_0 = 0 \qquad (63)$$

Where,

$$C_1 = \sigma_i^2$$

$$f_4 = lb\left(\frac{1}{\varepsilon}\right)^2\left[(Ep_1\bar{\delta}\varepsilon^{-1}A + l)\,\frac{1}{\varepsilon}\,b + Ep_1 l\left(\frac{1+K}{P_l}\right)\right] \qquad (64)$$

$$\begin{aligned}
f_3 = {} & b^4\left[\left(\frac{1}{\varepsilon}\right)^3\left\{\frac{2l}{p_2^2}(l + Ep_1\bar{\delta}\varepsilon^{-1}A) + (1-\bar{\delta}\varepsilon^{-1}A)\right\}\right] \\
& + b^3\left[\left(\frac{1}{\varepsilon}\right)^2\left\{Ep_1\,(1-\bar{\delta}\varepsilon^{-1}A)\left(\frac{1+K}{P_l} - KA\varepsilon^{-1}\right)\right.\right. \\
& \left. + \frac{2l^2 Ep_1}{p_2}\left(\frac{1+K}{P_l}\right) + A\varepsilon^{-1}l\left(K - \frac{1+K}{P_l}\bar{\delta}\right)\right\} \\
& \left. + \left(\frac{1}{\varepsilon}\right)^3\left\{2A(1-\bar{\delta}\varepsilon^{-1}A) + 2A\right\}\right] \\
& + b^2\left[\frac{1}{\varepsilon}l(l + Ep_1\bar{\delta}\varepsilon^{-1}A)\left(\frac{1+K}{P_l}\right)^2 + \left(\frac{1}{\varepsilon}\right)^2\right. \\
& \left\{Ep_1.2A\left(\frac{1+K}{P_l}\right) + \left(\frac{1+K}{P_l}\right)Ep_1.2A(1-\bar{\delta}\varepsilon^{-1}A)\right\} \\
& \left. + \left(\frac{1}{\varepsilon}\right)^2(2A^2)\left\{\frac{1}{\varepsilon}(2-KEp_1)\right\}\right] \\
& + \frac{bl}{\varepsilon}\left[\frac{H^2\pi\varepsilon^{-2}b}{4}\left\{\frac{EP_1 l}{p_2} - 7\,(l + Ep_1\bar{\delta}\varepsilon^{-1}A)\right\}\right. \\
& + (l + Ep_1\bar{\delta}\varepsilon^{-1}A)\,\pi\varepsilon^{-2}\left\{H^2 b - 4\Omega^2\pi\right\} \\
& \left. + \left(\frac{1+K}{P_l}\right)l^2 Ep_1\,\frac{\pi\varepsilon^{-2}}{2}\left\{8\Omega^2\pi - b\,H^2\right\}\right. \\
& \left. + b\left[\left(\frac{1}{\varepsilon}\right)^2\left(\frac{1+K}{K}\right)Ep_1.4A^2 + \left(\frac{1+K}{P_l}\right)^3.\,Ep_1 l^2\right]\right]
\end{aligned} \qquad (65)$$

The other coefficients being quite lengthy and not needed in the discussion of overstability, has not been written here.

Since σ_i is real for overstability, the true values of $C_1\ (=\sigma_i^2)$ are positive. The sum of roots of Equation 63 is $-f_3/f_4$ and if this is to be negative, then $f_3 > 0$ (because $f_4 > 0$).

It is clear from (65) that f_3 is always positive if

$$0 < \bar{\delta} < \frac{\varepsilon}{A},\ \bar{\delta} < \left(\frac{K}{1+K}\right)P_l,\ \left(\frac{K}{1+K}\right)P_l < \frac{\varepsilon}{A},$$
$$KEp_1 < 2,\ Ep_1 l > 7\,p_2\,(l + Ep_1\bar{\delta}\varepsilon^{-1}A)$$

and

$$4\Omega^2\pi < (\pi^2 + k^2)\,H^2 < 8\Omega^2\pi.$$

Which implies that

$$\left.\begin{aligned}
& KEp_1 < 2,\quad 0 < \bar{\delta} < \min\left\{\frac{\varepsilon}{A}, \frac{\varepsilon l}{A}\left(\frac{1}{7p_2} - \frac{1}{Ep_1}\right)\right\} \\
& \text{and} \\
& 4\Omega^2\pi < (\pi^2 + k^2)\,H^2 < 8\Omega^2\pi
\end{aligned}\right\} \qquad (66)$$

Thus for the above conditions, overstability can not occur and the principle of exchange of stabilities is valid. Hence the above conditions are the sufficient conditions for the non-existence of overstability, the violation of which does not necessarily imply the occurrence of overstability.

6.1. Particular Cases

1. In the absence of magnetic field condition (66) reduces to

$$0 < \bar{\delta} < \frac{\varepsilon}{A},\ KEp_1 < 2 \text{ and } \Omega < \frac{(1+K)\,\varepsilon\sqrt{b}}{2\pi P_l} \qquad (67)$$

2. In the absence of coupling between spin and heat flux ($\bar{\delta} = 0$), magnetic field ($\mathbf{H} = 0$), rotation ($\mathbf{\Omega} = 0$) and for very-very large medium

permeability ($P_l \to \infty$), it is clear from (65) that f_3 is always positive if $KEp_1 < 2$ (Sharma, et al [11]) is automatically satisfied. Then the only possible solution of Equation 63 are those with $\sigma_i = 0$ and so overstable solutions will not exist. Thus presence of magnetic field, rotation, permeability and coupling between spin and heat flux may bring overstability in the system

3. Effect of magnetic field and rotation together

(a) In the absence of coupling between spin and heat flux ($\bar{\delta} = 0$) and medium permeability very-very large ($P_l \to \infty$): The sufficient conditions for the non-existence of overstability become

$$KEp_1 < 2,\ H^2 > \frac{8\Omega^2\pi}{(\pi^2 + k^2)} \text{ and } Ep_1 > 5p_2 \qquad (68a)$$

(b) In the presence of coupling between spin and heat flux ($\bar{\delta} \neq 0$) and medium permeability very-very large ($P_l \to \infty$), the sufficient conditions for non-existence of overstability become

$$0 < \bar{\delta} < \frac{\varepsilon}{A},\ KEp_1 < 2,$$

$$\frac{Ep_1 l}{p_2} > 5\ (l + Ep_1\bar{\delta}\varepsilon^{-1}A) \text{ and } H^2 > \left(\frac{8\Omega^2\pi}{\pi^2 + k^2}\right)$$

Which implies that

$$0 < \bar{\delta} < \min\left[\frac{\varepsilon}{A}, \frac{\varepsilon l}{A}\left(\frac{1}{5p_2} - \frac{1}{Ep_1}\right)\right],\ KEp_1 < 2$$

and

$$H^2 > \left(\frac{8\Omega^2\pi}{\pi^2 + k^2}\right) \qquad (68b)$$

7.STATIONARY CONVECTION

When the instability sets in as stationary convection and coupling between spin and heat flux is present ($\bar{\delta} \neq 0$), the marginal state will be characterized by $\sigma_i = 0$. Hence putting $\sigma = 0$ in Equation 41, then the Rayleigh number is given by

$$R = \frac{1}{k^2}$$
$$\left[\left\{(2A+b)\left(\frac{1+K}{P_l}\right).b - KA\varepsilon^{-1}b^2 + (2A+b)\frac{H^2\pi\varepsilon^{-1}}{4}p_2\right\}\right.$$
$$\times\left\{\left(\frac{1+K}{P_l}\right)\frac{b}{p_2} + \frac{H^2\pi\varepsilon^{-1}}{4}\right\}\frac{b^2}{p_2} + 4\Omega^2\varepsilon^{-2}\pi^2\frac{b^3}{p_2^2}(2A+b)\Big]$$
$$\times\left[\left\{\left(\frac{1+K}{P_l}\right)\frac{b}{p_2} + \frac{H^2\pi\varepsilon^{-1}}{4}\right\}\frac{b}{p_2}\times(2A+b-\bar{\delta}\varepsilon^{-1}Ab)\right]^{-1}$$
$$= \frac{1}{k^2}\left[\left\{b^2\left(\frac{1+K}{P_l}\right)(2A+b) - KA\varepsilon^{-1}b^3 + \right.\right.$$
$$\left.\frac{H^2\pi\varepsilon^{-1}}{4}p_2 b\,(2A+b)\right\}\times\left\{2A+b(1-\bar{\delta}\varepsilon^{-1}A)\right\}^{-1}$$
$$+ 4\Omega^2\varepsilon^{-2}\pi^2 b^2\,(2A+b)\times$$
$$\left[\left\{\left(\frac{1+K}{P_l}\right)b + \frac{H^2\pi\varepsilon^{-1}}{4}p_2\right\}\left\{2A+b(1-\bar{\delta}\varepsilon^{-1}A)\right\}\right]^{-1} \qquad (69)$$

In the absence of rotation $(\Omega = 0)$, Equation 69 becomes

$$R = \frac{\left[\frac{1+K}{P_l} - KA\varepsilon^{-1}\right]b^3 + \left[\frac{2A(1+K)}{P_l}\right]b^2 + \frac{\pi H^2 p_2 b\varepsilon^{-1}(2A+b)}{4}}{k^2\left[2A+b(1-\bar{\delta}\varepsilon^{-1}A)\right]}$$

Which is in good agreement with the result obtained by [R. C. Sharma, et al [18], Equation 44].

In the absence of magnetic field $(H = 0)$, Equation 69 becomes

$$R = \frac{b^2\left(\frac{1+K}{P_l}\right)\left[\frac{(1+K)}{P_l} - kA\varepsilon^{-1}\right] + b^2\left[\frac{2A(1+K)^2}{P_l^2} + 4\Omega^2\varepsilon^{-2}\pi^2\right] + b\,[8\Omega^2\varepsilon^{-2}\pi^2 A]}{\frac{k^2(1+K)}{P_l}\left[b\,(1-\bar{\delta}\varepsilon^{-1}A) + 2A\right]}$$

A result derived by [Sharma, et al [16], Equation 37].

In the absence of both magnetic field $(H = 0)$ and rotation $(\Omega = 0)$, Equation 69 becomes

$$R = \frac{\left[\left\{\left(\frac{1+K}{P_l}\right) - KA\varepsilon^{-1}\right\}b^3 + 2A\left(\frac{1+K}{P_l}\right)b^2\right]}{R^2\ \left[2A+b(1-\bar{\delta}\varepsilon^{-1}A)\right]}$$

a result derived by Sharma, et al [11] (Equation 39).

Before we investigate the effects of various parameters and a discussion of the results depicted by figures, we first make assumptions that the micropolar parameters K, A and $\overline{\delta}$ are non-negative [1,21].

To investigate the effect of medium permeability (P_l), rotation (Ω), magnetic field (H), coupling parameter (K), micropolar coefficient $A\,(=K/C_0)$ and micropolar heat conduction parameter $(\overline{\delta})$, we examine the behaviour of $\frac{dR}{dP_l}, \frac{dR}{d\Omega}, \frac{dR}{dH}, \frac{dR}{dK}, \frac{dR}{dA}$ and $\frac{dR}{d\overline{\delta}}$ analytically. Equation 69 gives

$$\frac{dR}{dP_l} = -\frac{1}{k^2\{2A+b(1-\overline{\delta}\varepsilon^{-1}A)\}}\times \left[b^2\left(\frac{1+K}{P_l^{\,2}}\right)(2A+b)\left\{1-\frac{4\Omega^2\varepsilon^{-2}\pi^2 b}{\left(\frac{1+K}{P_l}b+\frac{H^2\pi^2\varepsilon^{-1}p_2}{4}\right)^2}\right\}\right]$$

Which is always negative if

$$\frac{4\Omega^2\varepsilon^{-2}\pi^2 b}{\left(\frac{1+K}{P_l}b+\frac{H^2\pi^2\varepsilon^{-1}p_2}{4}\right)^2} < 1 \quad \text{and } \overline{\delta} < \frac{\varepsilon}{A}$$

Which implies that

$$\overline{\delta} < \frac{\varepsilon}{A}, \quad P_l < \frac{b\varepsilon(1+K)}{\left(2\Omega\sqrt{b}-\frac{H^2 p_2}{4}\right)\pi}$$

or

$$H^2 > \frac{8\Omega\sqrt{b}}{p_2} \tag{70}$$

This shows that the medium permeability has a destabilizing effect when condition (70) holds. In the absence of micropolar heat conduction parameter ($\overline{\delta}=0$) and rotation ($\Omega=0$), the medium permeability always has a destabilizing effect on the system for stationary convection in porous medium. From condition (70), it is clear that this phenomenon also exists in the absence of magnetic field.

Equation 69 also gives

$$\frac{dR}{d\Omega} = \frac{8\Omega\varepsilon^{-2}\pi^2 b^2(2A+b)}{k^2\left[\left\{\left(\frac{1+K}{P_l}\right)b+\frac{H^2\pi\varepsilon^{-1}}{4}p_2\right\}\left\{2A+b(1-\overline{\delta}\varepsilon^{-1}A)\right\}\right]}$$

Which is always positive if

$$\overline{\delta} < \frac{\varepsilon}{A} \tag{71}$$

This shows that the rotation has a stabilizing effect when condition (71) holds. In the absence of micropolar heat conduction parameter ($\overline{\delta}$), the rotation always has a stabilizing effect on the system. It is clear from (71) that stabilizing effect of rotation remains unaffected in the presence of magnetic field.

Equation 69 also yields

$$\frac{dR}{dH} = \frac{\frac{2H\pi\varepsilon^{-1}}{4}p_2 b(2A+b)\left\{1-\frac{4\Omega^2\varepsilon^{-2}\pi^2 b}{\left\{\left(\frac{1+K}{P_l}\right)b+\frac{H^2\pi\varepsilon^{-1}}{4}p_2\right\}}\right\}}{k^2\left\{2A+b\,(1-\overline{\delta}\varepsilon^{-1}A)\right\}}$$

Which is always positive if

$$\frac{4\Omega^2\varepsilon^{-2}\pi^2 b}{\left\{\left(\frac{1+K}{P_l}\right)b+\frac{H^2\pi\varepsilon^{-1}}{4}p_2\right\}} < 1 \text{ and } \overline{\delta} < \frac{\varepsilon}{A}$$

Which implies that

$$H^2 > \frac{8\Omega\sqrt{b}}{p_2} \text{ and } \overline{\delta} < \frac{\varepsilon}{A} \tag{72}$$

This shows that the magnetic field has a stabilizing effect when conditions (72) hold. In the absence of micropolar heat conduction parameter and rotation, the magnetic field always has a stabilizing effect, on the system.

It can easily be derived from Equation 69 that

$$\frac{dR}{dK} = \frac{1}{k^2\lambda\,\{2A + b\,(1-\bar{\delta}\varepsilon^{-1}A)\}}\times$$

$$\left(Ab^2\left(\frac{2}{P_l}-\frac{b}{\varepsilon}\right)+\frac{b^3}{P_l}\left\{1-\frac{4\Omega^2\varepsilon^{-2}\pi^2(2A+b)}{\left\{\left(\frac{1+K}{P_l}\right)b+\frac{H^2\pi\varepsilon^{-1}}{4}p_2\right\}^2}\right\}\right)$$

Which is always positive if

$$\frac{1}{P_l} > \frac{b}{\varepsilon},\quad \bar{\delta} < \frac{\varepsilon}{A}$$

and

$$K > \frac{\pi P_l}{b\varepsilon}\left[2\Omega\,\sqrt{(2A+b)} - \frac{H^2 p_2}{4}\right] - 1$$

or

$$K > \frac{\pi P_l}{b\varepsilon}\left[2\Omega\,\sqrt{(2A+b)}\right] \tag{73}$$

This shows that coupling parameter has a stabilizing effect when conditions (73) hold. In the absence of rotation and in a non porous medium, (73) yields that $\frac{dR}{dK}$ is always positive, thereby the stabilizing effect of coupling parameter. It is clear from (73) that stabilizing behaviour of K is independent of presence of magnetic field.

It follows from Equation 69 that

$$\frac{dR}{dA} = \left[b^4\varepsilon^{-1}\left(\frac{1+K}{P_l}\right)\left\{\bar{\delta}-\left(\frac{K}{1+K}\right)P_l\right\}+\right.$$

$$\left. b^3\bar{\delta}\,\pi\,\varepsilon^{-2}\frac{H^2 p_2}{4}+\frac{4\Omega^2\varepsilon^{-3}\pi^2 b^4\bar{\delta}}{\left\{\left(\frac{1+K}{P_l}\right)b+\frac{H^2\pi\varepsilon^{-1}}{4}p_2\right\}}\right]$$

$$\Big/\, k^2\left[2A + b\,(1-\bar{\delta}\varepsilon^{-1}A)\right]^2$$

Which is always positive if

$$\bar{\delta} > P_l \tag{74}$$

This shows that the micropolar coefficient A has a stabilizing effect when condition (74) holds. Thus the stabilizing behaviour of micropolar coefficient is virtually unaffected by magnetization parameter (H) but is significantly affected by micropolar heat conduction parameter $(\bar{\delta})$. The stabilizing effect of micropolar coefficient (A) also implies the destabilizing effect of spin diffusion (couple stress, $C_0 = K/A$) parameter which has been found earlier by many authors for micropolar ferromagnetic fluids [21-23].

Equation 69 also yields

$$\frac{dR}{d\bar{\delta}} =$$

$$b\varepsilon^{-1}A\left[b^3\left\{\frac{1+K}{P_l}-KA\varepsilon^{-1}\right\}+b^2\left\{2A\left(\frac{1+K}{P_l}\right)+\frac{H^2\pi\varepsilon^{-1}}{4}p_2\right\}+\right.$$

$$\left. b\frac{H^2\pi\varepsilon^{-1}}{4}p_2.2A+\frac{4\Omega^2\varepsilon^{-2}\pi^2 b^2(2A+b)}{\left\{\left(\frac{1+K}{P}\right).b+\frac{H^2\pi\varepsilon^{-1}}{4}p_2\right\}}\right]$$

$$\Big/\, k^2\;(2A + b\,(1-\bar{\delta}\varepsilon^{-1}A)^2$$

Which is always positive if

$$\frac{1}{P_l} > \frac{A}{\varepsilon} \tag{75}$$

This shows that the micropolar heat conduction parameter $(\bar{\delta})$ has a stabilizing effect when condition (75) holds. Here we also observe that in a non-porous medium, $\frac{dR}{d\bar{\delta}}$ is always positive, implying thereby the stabilizing effect of micropolar heat conduction parameter. Thus, we have seen that the stabilizing behaviour of micropolar heat conduction is virtually unaffected by effect of magnetic field.

In article 9, the dispersion relation (69) is also analyzed numerically. Marginal instability curves have been plotted for stationary condition by giving some numerical values to the dimensionless parameters to depict the stability characteristics in the presence and absence of micropolar heat coupling parameter $(\bar{\delta})$.

8. VARIATIONAL PRINCIPLE

A Variational Principle can be established for the present problem following Chandrasekhar [13]. Let one of the characteristic values be σ_i and let the corresponding solutions be denoted by a subscript 'i', then from Equations 15-20 after using (30), we have

$$-k^2 L_i = \left[\frac{\sigma_i}{\varepsilon} + \frac{1}{P_l}(1+K)\right] DU_i - KDG_i + \frac{2}{\varepsilon} Z_i \Omega - \frac{H}{4\pi}(D^2 - k^2)\, B \tag{76}$$

and

$$-DL_i = \left[\frac{\sigma_i}{\varepsilon} + \frac{1}{P_l}(1+K)\right] U_i - KG_i - R\Theta_i \tag{77}$$

Where L is the form of $F(z)$ in δp and $D \equiv \frac{\partial}{\partial z}$.

Let σ_j be a characteristic value different from σ_i, and let subscript 'j' distinguishes the corresponding solutions. We multiply Equations 76 and 77 respectively by DU_j and U_j and integrate them with respect to z from $z=0$ to $z=1$ using the boundary conditions

$U = D^2U = 0,\ DZ = 0,\ G = 0,\ \Theta = 0, X = 0, DB = 0$ at $z=0$ and $z=1$ (78)

Further, using Equations 32-36, putting $i=j$ and suppressing the subscript, yields

$$\begin{aligned}
&-\frac{\sigma}{\varepsilon}\int_0^1 \left[U^2 + \frac{1}{k^2}(DU)^2\right] dz - \frac{(1+K)}{P_l}\int_0^1 \left[U^2 + \frac{1}{k^2}(DU)^2\right] dz \\
&+ \frac{K}{k^2 A\varepsilon^{-1}}\int_0^1 (l\sigma + 2A)G^2 dz + \frac{K}{A\varepsilon^{-1}}\int_0^1 \left[G^2 + \frac{1}{k^2}(DG)^2\right] dz \\
&+ R\int_0^1 Ep_1\sigma\Theta^2 dz + R\int_0^1 \left[\left(D\Theta^2\right) + k^2\Theta^2\right] dz \\
&- \frac{1}{k^2}\int_0^1 \left[\frac{\sigma}{\varepsilon} + \frac{1}{P_l}(1+K)\right] Z^2 dz \\
&+ \bar{\delta}\int_0^1 R\Theta G dz - \frac{\varepsilon}{4\pi k^2}\int_0^1 \sigma X^2 dz \\
&- \frac{\varepsilon}{4\pi k^2 p_2}\int_0^1 [(DX)^2 + k^2 X^2] dz \\
&- \frac{\varepsilon\sigma}{4\pi k^2}\int_0^1 \left[(DB)^2 + k^2 B^2\right] dz \\
&- \frac{\varepsilon}{4\pi k^2}\frac{1}{p_2}\int_0^1 [D^2B)^2 + 2k^2(DB)^2 + k^4B^2]\, dz = 0
\end{aligned} \tag{79}$$

Now we will show below that Equation 79 provides a basis for the variational formulation of the problem.

Let

$$\left.\begin{aligned}
J_1 &= \int_0^1 \frac{1}{\varepsilon}\left[U^2 + \frac{1}{k^2}(DU)^2\right] dz, \\
J_2 &= \int_0^1 \left(\frac{1+K}{P_l}\right)\left[U^2 + \frac{1}{k^2}(DU)^2\right] dz, \\
J_3 &= \int_0^1 \frac{K}{k^2 A\varepsilon^{-1}} lG^2\, dz, \\
J_4 &= \int_0^1 \frac{K}{k^2 A\varepsilon^{-1}} 2A\, G^2\, dz, \\
J_5 &= \int_0^1 \frac{K}{A\varepsilon^{-1}}\left[G^2 + \frac{1}{k^2}(DG)^2\right] dz, \\
J_6 &= \int_0^1 REp_1\Theta^2 dz, \\
J_7 &= \int_0^1 R\left[(D\Theta)^2 + k^2\Theta^2\right] dz, \\
J_8 &= \int_0^1 \frac{1}{k^2\varepsilon} Z^2\, dz, \\
J_9 &= \int_0^1 \frac{(1+K)}{k^2 P_l} Z^2\, dz, \\
J_{10} &= \int_0^1 \bar{\delta} R\Theta G\, dz, \\
J_{11} &= \frac{\varepsilon}{4\pi k^2}\int_0^1 X^2 dz, \\
J_{12} &= \frac{\varepsilon}{4\pi k^2}\frac{1}{p_2}\int_0^1 \left[(DX)^2 + k^2 X^2\right] dz, \\
J_{13} &= \frac{\varepsilon}{4\pi k^2}\int_0^1 \left[(DB)^2 + k^2 B^2\right] dz, \\
J_{14} &= \frac{\varepsilon}{4\pi k^2}\frac{1}{p_2}\int_0^1 \left[(D^2B)^2 + 2k^2(DB)^2 + k^4 B^2\right] dz
\end{aligned}\right\}$$

(80)

With the help of (80), Equation 79 can be written as

$$-\sigma[J_1 - J_3 - J_6 + J_8 + J_{11} + J_{13}] - J_2 + J_4 + J_5 + J_7 - J_9 + J_{10} - J_{12} - J_{14} = 0 \quad (81)$$

Let us now consider the variations $\delta\sigma$ in σ caused by the first order small variations $\delta U, \delta\Theta, \delta G, \delta X$ and δB in U, Θ, G, Z, X and B respectively. Further, we assume that $\delta U, \delta\Theta, \delta G, \delta Z$ and δB satisfy the boundary conditions (78). The changes in U, Θ, G, Z, X and B lead to the corresponding changes in $J_i^{'}s$, denotes by $\delta J_i^{'}s$. We can analyse these changes with the help of Equation 81 which gives,

$$-\delta\sigma[J_1 - J_3 - J_6 + J_8 + J_{11} + J_{13}] - \sigma[\delta J_1 - \delta J_3 - \delta J_6 + \delta J_8 + \delta J_{11} + \delta J_{13}] - \delta J_2 + \delta J_4 + \delta J_5 + \delta J_7 - \delta J_9 + \delta J_{10} - \delta J_{12} - \delta J_{14} = 0 \quad (82)$$

We, now use the expressions for $J_i^{'}s$ given by (80) to evaluate $\delta J_i^{'}s$. Integrating Equation 80 by parts a suitable number of times and using (78), we find

$$\left.\begin{aligned}
\frac{\delta J_1}{2} &= -\int_0^1 \frac{1}{\varepsilon k^2}\,\delta U\,(D^2 - k^2)\,U\,dz, \\
\frac{\delta J_2}{2} &= -\int_0^1 \left(\frac{1+K}{P_l}\right)\frac{1}{k^2}\,\delta U\,(D^2 - k^2)\,U\,dz, \\
\frac{\delta J_3}{2} &= \int_0^1 \frac{K\,l}{k^2 A\varepsilon^{-1}}(\delta G)G\,dz, \\
\frac{\delta J_4}{2} &= \int_0^1 \frac{2KA}{k^2 A\varepsilon^{-1}}(\delta G)\,G\,dz, \\
\frac{\delta J_5}{2} &= -\int_0^1 \frac{K}{k^2 A\varepsilon^{-1}}\,\delta G\,(D^2 - k^2)\,G\,dz, \\
\frac{\delta J_6}{2} &= \int_0^1 REp_1(\delta\Theta)\,\Theta dz, \\
\frac{\delta J_7}{2} &= -\int_0^1 R\delta\Theta\,(D^2 - k^2)\,\Theta\,dz, \\
\frac{\delta J_8}{2} &= \int_0^1 \frac{1}{k^2\varepsilon}(\delta Z)\,Z\,dz, \\
\frac{\delta J_9}{2} &= \int_0^1 \left(\frac{1+K}{k^2 P_l}\right) Z\,\delta Z\,dz, \\
\frac{\delta J_{10}}{2} &= \int_0^1 \frac{\bar{\delta}R}{2}\{G\delta\Theta + \Theta\delta G\}\,dz, \\
\frac{\delta J_{11}}{2} &= \int_0^1 \frac{\varepsilon}{4\pi k^2}(\delta X)\,X\,dz,
\end{aligned}\right\}$$

$$\left.\begin{aligned}
\frac{\delta J_{12}}{2} &= -\int_0^1 \frac{\varepsilon}{4\pi k^2}\frac{1}{p_2}[\delta X\,(D^2 - k^2)\,X\,]\,dz, \\
\frac{\delta J_{13}}{2} &= -\int_0^1 \frac{\varepsilon}{4\pi k^2}[\delta B\,(D^2 - k^2)\,B\,]\,dz, \\
\frac{\delta J_{14}}{2} &= \int_0^1 \frac{\varepsilon}{4\pi k^2}\frac{1}{p_2}[\delta B\,(D^2 - k^2)^2\,B\,]\,dz
\end{aligned}\right\} \quad (83)$$

Combining Equations 82 and 83 and using Equations 32 to 36 in it and rearranging the terms, we get

$$-\frac{\delta\sigma}{2}[J_1 - J_3 - J_6 - J_8 + J_{11} + J_{13}] + \delta\sigma(J_8 - J_3 + J_{13}) + R\int_0^1 [U\delta\Theta - \Theta\delta U]\,dz + \int_0^1 \frac{\bar{\delta}R}{2}\{\Theta\delta G - G\delta\Theta\}\,dz = 0 \quad (84)$$

Further, taking first order perturbations in Equation 34, integrating and simplifying, we get

$$R\int_0^1 [U\delta\Theta - \Theta\delta U]\,dz = -\delta\sigma J_6 + \int_0^1 R\bar{\delta}\,\{\Theta\delta G - G\delta\Theta\}\,dz \quad (85)$$

using Equation 85, Equation 84 reduces to

$$-\frac{\delta\sigma}{2}[J_1 + J_3 + J_6 - J_8 + J_{11} - J_{13}] + \bar{\delta}\int_0^1 \frac{R}{2}(\Theta\delta G - G\delta\Theta)\,dz = 0 \quad (86)$$

in the absence of coupling between spin and heat flux ($\bar{\delta} = 0$), we get

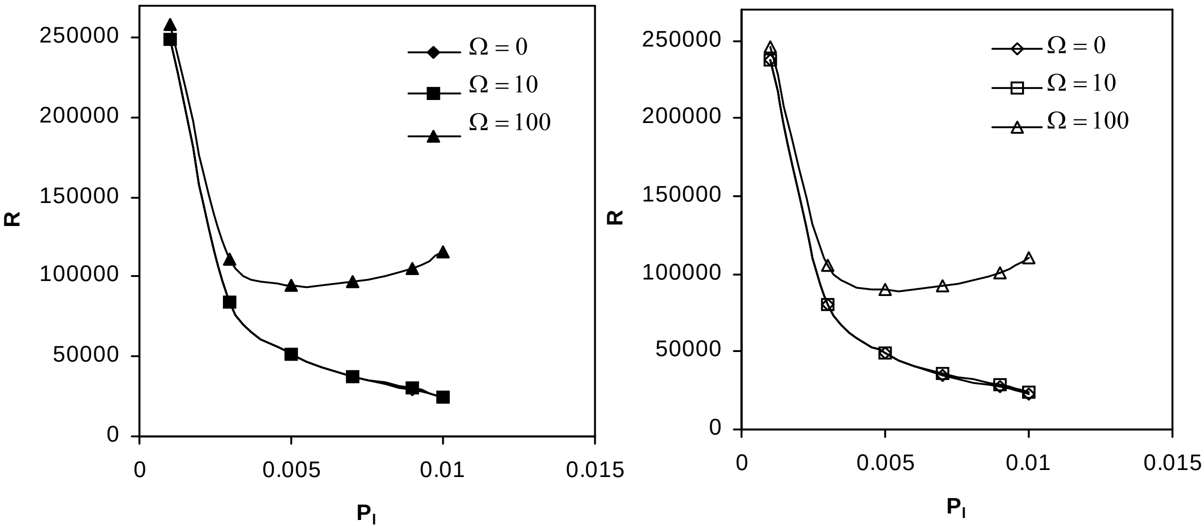
$$\delta\,\sigma = 0$$

Because the quantity within [ ] on the L.H.S. of Equation 86 can not vanish. Therefore Equation 79 provides a basis for variational formulation of the problem under investigation.

9. NUMERICAL COMPUTATION

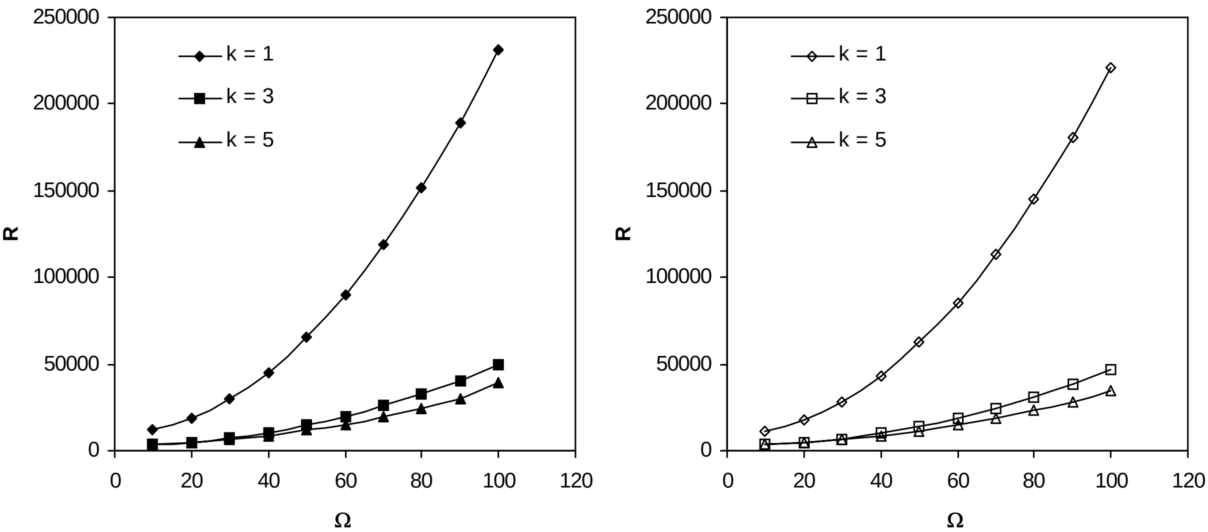
Here in this section, we have plotted the variation of thermal Rayleigh number R with P_l, Ω, H and micropolar parameters K, A and $\bar{\delta}$ in Figures 2-7.

Figures 2-4 represent the plots of thermal Rayleigh number R versus P_l for various values



(a) (b)

Figure 2. Marginal instability curve for variation of rayleigh number R versus P_l for $\varepsilon = 0.5, k = 1,$ $A = 0.5,\ p_2 = 4,\ p_1 = 2, E = 1, H = 5G,$ (i) $\overline{\delta} = 0.05$, (ii) $\overline{\delta} = 0$.

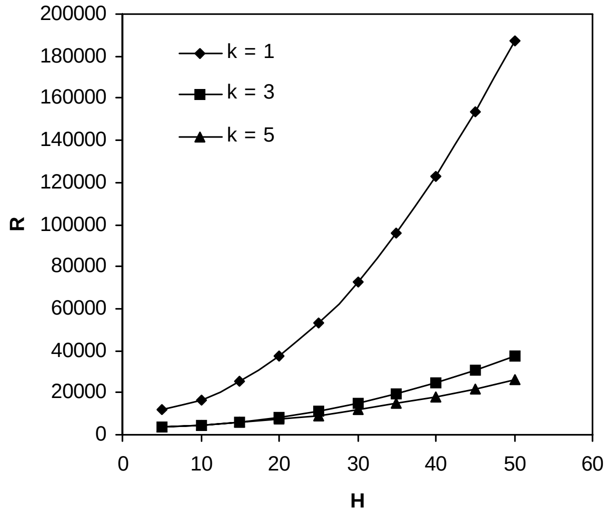


(a) (b)

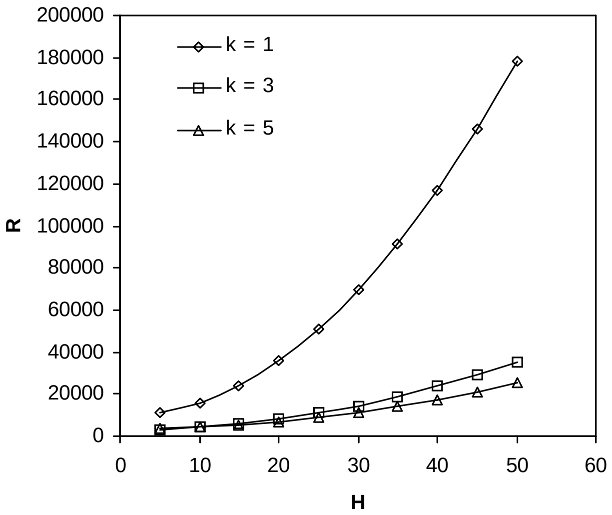
Figure 3. Marginal instability curve for variation of Rayleigh number R versus Ω for $\varepsilon = 0.5, K = 1,$, $A = 0.5,\ P_l = 0.03,\ p_2 = 4,\ p_1 = 2, E = 1, H = 5G$, (i) $\overline{\delta} = 0.05$, (ii) $\overline{\delta} = 0$.

of Ω, R versus Ω for various values of wave number k and R versus H for various values of k, respectively, in the presence and absence of micropolar heat conduction parameter $\overline{\delta}$.

Figure 2 depicts that as P_l increases, R decreases for small values of Ω, whereas for the



(a)



(b)

Figure 4. Marginal instability curve for variation of Rayleigh number R versus H for $\varepsilon = 0.5, K = 1, \quad A = 0.5, \ \Omega = 10$ riv min^{-1}, $p_2 = 4$, $p_1 = 2, E = 1, \quad P_l = 0.03$, (i) $\overline{\delta} = 0.05$, (ii) $\overline{\delta} = 0$.

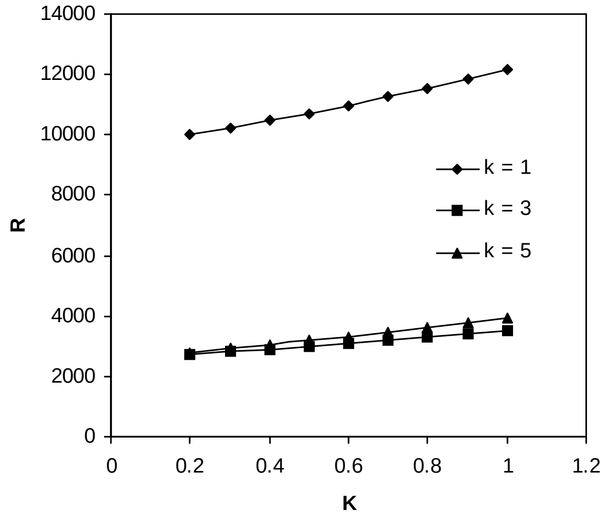


Figure 5. Marginal instability curve for variation of Rayleigh number R versus K for $\varepsilon = 0.5, H = 5G, \quad A = 0.5$, $\Omega = 10$ riv min^{-1}, $p_2 = 4$, $p_1 = 2, E = 1, P_l = 0.03$ and $\overline{\delta} = 0.05$.

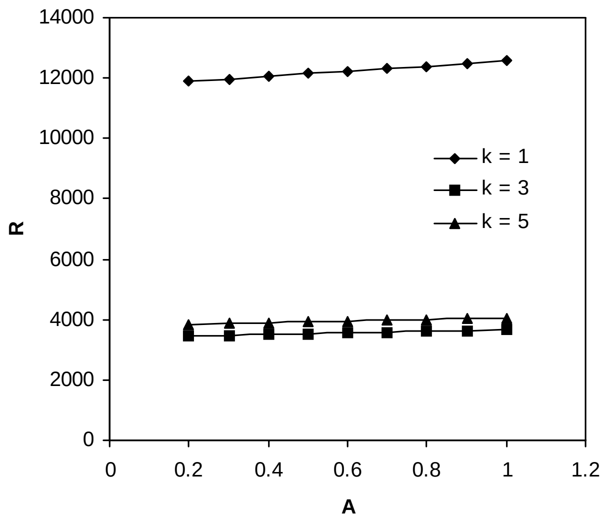


Figure 6. Marginal instability curve for variation of Rayleigh number R versus A for $\varepsilon = 0.5, K = 1, H = 5G, \quad \Omega = 10$ riv min^{-1}, $p_2 = 4$, $p_1 = 2, E = 1, P_l = 0.03$ and $\overline{\delta} = 0.05$.

higher values of Ω, R decreases for lower value of P_l and then increases for higher values of P_l, in the presence and absence of $\overline{\delta}$, depicting the destabilizing effect of the medium permeability for lower values of the rotation parameter Ω, whereas for sufficiently higher values of rotation parameter, the medium permeability may have a destabilizing or a stabilizing effect, which are in good agreement with the result obtained earlier [21,24-27].

Figure 3 and 4 represent the plot of thermal

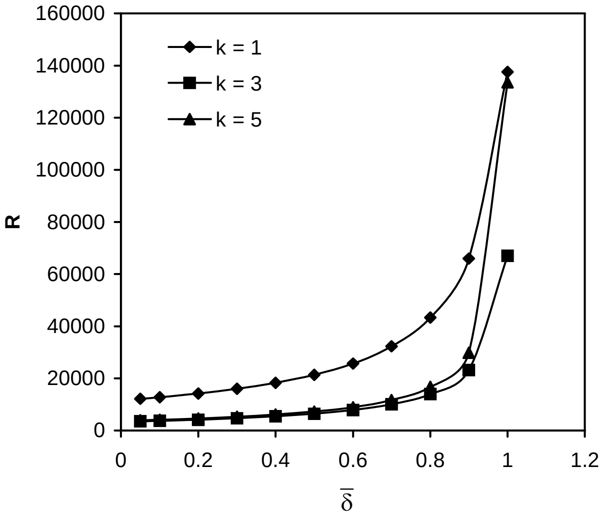


Figure 7. Marginal instability curve for variation of R versus $\bar{\delta}$ for $\varepsilon = 0.5, K = 1, A = 0.5, H = 5G, \quad \Omega = 10$ riv min^{-1}, $p_2 = 4$, $p_1 = 2, E = 1, P_l = 0.03$.

Rayleigh number R versus Ω and R versus magnetic field H respectively, for various values of the wave number k, in the presence and absence of micropolar heat conduction parameter $\bar{\delta}$. These figures indicate that the Rayleigh number R increases with the increase in rotation parameter (Ω) and magnetic field (H), respectively, in the presence and absence of $\bar{\delta}$, indicating that the onset of instability is delayed by the presence of the rotation and magnetic field, depicting the stabilizing effect of rotation and magnetic field on the stationary convection, which are in good agreement with the results obtained earlier [15-19]. The Figures 2-4 also indicate that higher values of R are needed for the onset of convection in the presence of $\bar{\delta}$, hence justifying the stabilizing behaviour of micropolar heat conduction parameter $\bar{\delta}$. This can also be observed in Figure 7. It is also noted from Figures 2-4 that the Rayleigh number in the absence of micropolar heat conduction parameters is less than the Rayleigh number for stationary convection (i.e., in the presence of micropolar heat conduction parameter) for a fixed wave number.

Figures 5-7 represent the plots of thermal Rayleigh R versus coupling parameter K, micropolar coefficient (A) and micropolar heat conduction parameter $\bar{\delta}$ for various values of k. These figures indicate the value of R increases with the increase in the value of K, A and $\bar{\delta}$, respectively, depicting the stabilizing effect of coupling parameter, micropolar coefficient and micropolar heat conduction parameter, respectively. The stabilizing behaviour of micropolar coefficient A also implies the destabilizing effect of spin diffusion (couple stress, $C_0 = K/A$) parameter. The stabilizing behaviour of coupling parameter (K), micropolar heat conduction parameter $(\bar{\delta})$ and destabilizing behaviour of spin diffusion parameter (C_0) has been depicted earlier by many authors [21-23].

10. DISCUSSION OF RESULTS

In order to investigate our results, we must review the results and its physical applications.

It is well known that the rotation introduces vorticity into the fluid in case of Newtonian fluid [13]. Then the fluid moves in the horizontal planes with higher velocities. On account of this motion, the velocity of the fluid perpendicular to the planes reduces, and hence delay the onset of convection implying stabilization behaviour of rotation as in Figure 3. When the fluid layer is assumed to be flowing through an isotropic and homogeneous porous medium, free from rotation or small rate of rotation, then the medium permeability has a destabilizing effect. As medium permeability increases, the void space increases and as a result of this, the flow quantities perpendicular to the planes will clearly be increased. Thus, increase in heat transfer is responsible for early onset of convection, thus increasing P_l lead to decrease in R implying the destabilizing effect of P_l in the absence of rotation or for small value of rotation (Figure 2). In case of high rotation, the motion of the fluid prevails essentially in the horizontal planes. This motion increased as medium permeability increases. Thus the component of the velocity perpendicular to the horizontal planes reduces, leading to delay in the onset of convection. Hence, the destabilization effect of medium permeability changes to stabilizing effect in case of high rotation, implying the destabilizing or stabilizing effect of medium permeability in case of high rotation (Figure 2).

When the magnetic field permeating the medium is considerably large, it induces viscosity into the field, and the magnetic lines are distorted by the convection, these magnetic lines hinder the growth of disturbances, leading to the delay in the onset of instability. However, the viscosity produced by the magnetic field lessens the rotation of the fluid particles; thus controlling the stabilizing effect of K [28]. Similar phenomenon has been noticed, when the system is subjected to both rotation and magnetic field [13].

When the fluid is heated from below $(R > 0)$, the onset of instability is delayed as K, A or $\bar{\delta}$ increases or it is hastened as spin diffusion $C_0 (= K/A)$ increases. The mechanism underlying this physical phenomenon can be understood by having a probe into the nature of micropolar fluids.

The increase in K indicates the increasing concentration of microelements and as a result of this, a greater part of the energy of the system is consumed by these elements in developing gyration velocities in the fluid, leading to delay on the onset of convection, implying thereby the stabilizing effect of K (Figure 5).

As value of micropolar coefficient A increases, the spin diffusion, i.e., couple stress (C_0) decreases which causes the microrotation to increase and hence makes the system more stable, depicting the stabilizing effect of A (Figure 6) or destabilizing effect of spin diffusion parameter (C_0). Also when $\bar{\delta}$ increases, the heat induced into the fluid due to micro elements is also increased, thus reducing the heat transfer from the bottom to the top. The decrease in the heat transfer is responsible for delaying the onset of instability. Thus increasing $\bar{\delta}$ leads to increase in R. In other words, $\bar{\delta}$ stabilizes the flow (Figure 7).

Nevertheless, the above phenomenon is true whether the magnetic field is present or not. Thus the discussion concludes that the stabilizing effect of microrotation is controlled by the presence of magnetic field, which is in good agreement with the results obtained by K. V. Rama Rao [28] in a non-porous medium.

11. CONCLUSIONS

In this paper, thermal convection in a thin electrically conducting micropolar rotating fluid layer, heated from below in the presence of uniform vertical magnetic field, in a saturated isotropic and homogeneous porous medium has been investigated. The behaviour of various parameters such as medium permeability, rotation, magnetic field, coupling parameter, all micropolar parameters (spin diffusion parameter and micropolar heat conduction parameter) on the onset of convection has been analysed analytically and numerically and the results have been depicted graphically in the Figures 2-7. The results show that for the case of stationary convection, the medium permeability has destabilizing/stabilizing effect under certain condition(s), whereas rotation, magnetic field and micropolar parameters (K, A, and $\bar{\delta}$) have a stabilizing effect under certain condition(s). In the absence of micropolar heat conduction parameter, rotation and magnetic field always has a stabilizing effect on the system, whereas in the absence of micropolar heat conduction parameter and rotation, medium permeability always has destabilizing effect.

The principle of exchange of stabilities (PES) is found to hold true for the present problem in the absence of magnetic field, rotation and micropolar heat conduction parameter, whereas in the presence of rotation and magnetic field, PES is valid under certain conditions. The oscillatory modes are introduced due to the presence of magnetic field and rotation, which were non-existence in their absence. The presence of micropolar heat conduction parameter may also introduce the oscillatory modes.

In addition, conditions $KEp_1 < 2$, $0 < \bar{\delta} < \min\left\{\frac{\varepsilon}{A}, \frac{\varepsilon l}{A}\left(\frac{1}{7p_2} - \frac{1}{Ep_1}\right)\right\}$, and $4\ \Omega^2\pi < (\pi^2+k^2)\ H^2 < 8\Omega^2\pi$, are the sufficient conditions for the non-existence of overstability, violation of which does not necessarily imply the occurrence of overstability. For the medium permeability, very very large ($P_1 \to \infty$) and in the absence of micropolar heat conduction parameter ($\bar{\delta}$), rotation and magnetic field, the expected condition reduce to $KEp_1 < 2$, which is in good agreement with the results obtained earlier [Sharma, et al

[11]]. Thus, presence of coupling between spin and heat flux, magnetic field, rotation and medium permeability may bring overstability in the system.

An attempt is also made to apply the variational principle for the present problem and found that a variational principle can be established for the present problem in the absence of coupling between spin and heat flux.

Thus, from the above analysis, we conclude that the micropolar parameters, rotation, magnetic field and permeability have a deep effect on the onset of convection in porous medium. It is hoped that present work will be helpful for understanding more complex problems involving the various physical effects investigated in the present problem.

12. ACKNOWLEDGEMENT

The authors are thankful to U.G.C. for awarding the Teacher Fellowship No. F.1-15 (T.F-15)/2006 (NRCB), dated 20.2.2006. The authors are thankful to Prof. Sunil, Department of Applied Science, National Institute of Technology, Hamirpur, H.P.-177 005 (India), for his valuable suggestions and moral support. Authors are also thankful to both the referees for their valuable comments.

13. REFERENCES

1. Eringen, A. C., "Simple Microfluids", ***Int. J. Engg. Sci***., Vol. 2 (1964), 205-217.
2. Eringen, A. C., "Theory of Anisotropic Micropolar Fluids", ***Int. J. Engg. Sci***., Vol. 18, (1980), 5.
3. Eringen, A. C., "Theory of Thermomicrofluids", ***J. Math. Mech***., Vol. 16, (1966), 1-18.
4. Kazakia, Y. and Ariman, T., "Heat Conducting Micropolar Fluids", ***Rheol. Acta***, Vol. 10, (1971), 319.
5. Eringen, A.C., "Theory of Thermomicro Fluids", ***J. Math. Anal. Appl***., Vol. 38, (1972), 480.
6. Datta, A. B. and Sastry, V. U. K., "Thermal Instability of a Horizontal Layer of Micropolar Fluid Heated from Below", ***Int. J. Engg. Sci***., Vol. 14, (1976), 631-637.
7. Ahmadi, G., "Stability of a Micropolar Fluid Layer Heated from Below", ***Int. J. Engg. Sci***., Vol. 14, (1976), 81-89.
8. Labon, G. and Perez-Garcia, C., "Convective Instability of a Micropolar Fluid Layer by the Method of Energy", ***Int. J. Engg. Sci***., Vol. 19, (1981), 1321-1329.
9. Bhattacharya, S. P. and Jena, S. K., "On the Stability of a Hot Layer of Micropolar Fluid", ***Int. J. Engg. Sci***., Vol. 21, (1983), 1019-1024.
10. Payane, L. E. and Straughan, B., "Critical Rayleigh Numbers for Oscillatory and Non-Linear Convection in an Isotropic Thermomicropolar Fluid", ***Int. J. Engg. Sci***., Vol. 27, (1989), 827-836.
11. Sharma, R. C. and Gupta, U., "Thermal Convection in Micropolar Fluids in Porous Medium", ***Int. J. Engg. Sci.***, Vol. 33, No. 11, (1995), 1887-1892.
12. Sastry, V. U. K. and Rama Mohan Rao, V., "Numerical Solution of Thermal Instability of a Rotating Micropolar Fluid Layer", ***Int. J. Engg. Sci***., Vol. 21, (1983), 449-461.
13. Chandrasekhar, S., "Hydrodynamic and Hydromagnetic Stability", Dover Publication, Inc., 31 East 2nd Street Mineola, New York, U.S.A., (1961).
14. Qin, Y. and Kaloni, P.N., "A Thermal Instability Problem in a Rotating Micropolar Fluid", ***Int. J. Engg. Sci***., Vol. 30, (1992), 1117-1126.
15. Sharma, R.C. and Kumar, P., "Effect of Rotation on Thermal Convection in Micropolar Fluids", ***Int. J. Engg. Sci***., Vol. 32, (1994), 545-551.
16. Sharma, R. C. and Kumar, P., "Effect of Rotation on Thermal Convection in Micropolar Fluids in Porous Medium", ***Ind. J. Pure Appl. Math***., Vol. 29, (1998), 95-104.
17. Sharma, R.C. and Kumar, P., "On Micropolar Fluids Heated from Below in Hydromagnetics", ***J. Non Equilibria Thermodyn***, Vol. 20, (1995), 150-159.
18. Sharma, R. C. and Kumar, P., "On Micropolar Fluids Heated from Below in Hydromagnetics in Porous Medium", ***Czech. J. Phys***., Vol. 47, (1997), 637-647.
19. Sharma, V. and Sharma, S., "Thermosolutal Convection of Micropolar Fluids in Hydromagnetics in Porous Medium", ***Indian J. Pure and Appl. Math***., Vol. 31, No. 10, (2000), 1353-1367.
20. McDonnel J. A. M., "Cosmic Dust", John Wiley and Sons, Toronto, Canada, (1978).
21. Sunil Sharma, A., Bharti, P. K. and Shandil, R. G., "Effect of Rotation on a Layer of Micropolar Ferromagnetic Fluid Heated from Below Saturating a Porous Medium", ***Int. J. Engg. Sci***., Vol. 44, (2006), 683-698.
22. Abraham, A., "Rayleigh-Benard Convection in a Micropolar Ferromagnetic Fluid", ***Int. J. Engg. Sci***., Vol. 40, (2002), 449-460.
23. Sunil Sharma, A., Bharti, P. K. and Shandil, R. G., "Marginal Stability of Micropolar Ferromagnetic Fluid Saturating a Porous Medium", ***J. Geophys. Engg.***, Vol. 3, (2006), 338-347.
24. Sunil, Divya Sharma, V., "Effect of Dust Particles on Rotating Ferromagnetic Fluid Heated from Below Saturating a Porous Medium", ***J. of Colloid. Interface***, Vol. 291, (2005), 152-161.
25. Sunil Sharma, A., Shandil, R. G., Gupta, U., "Effect of Magnetic Field Dependent Viscosity and Rotation on Ferroconvection Saturating a Porous Medium in the Presence of Dust Particles", ***Int. Comm. Heat and Mass Transfer***, Vol. 32, (2005), 1387-1399.

26. Sunil Divya Sharma, R. C., “Effect of Magnetic Field Dependent Viscosity on a Rotating Ferromagnetic Fluid Heated and Soluted from Below Saturating a Porous Medium”, ***J. of Porous Media***, Vol. 8, No. 6, (2005), 569-588.
27. Sunil Divya Sharma, R. C., “ Effect of Rotation on Ferromagnetic Fluid Heated and Soluted from Below Saturating a Porous Medium”, ***J. Geophys. Engg***., Vol. 1, (2004), 116-127.
28. Rama Rao, K. V., “Thermal Instability in a Micropolar Fluid Layer Subject to a Magnetic Field”, ***Int. J. Engg. Sci***., Vol. 18, (1980), 741-750.