SHEAR-WAVE DYNAMIC BEHAVIOR USING TWO DIFFERENT ORIENTATIONS

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Abstract For laterally complex media, it may be more suitable to take a different orientation of the displacement vector of Shear-waves. This may change the sign of several imaginary reflections and conversion coefficients to be used in reservoir characterization and AVO (Amplitude Versus Offset) analysis or modeling. In this new convention the positive direction of the displacement vector of reflected Shear-waves is chosen to the left of ray tangent (in the direction of wave propagation). Therefore, the definition of the displacement vector of shear-waves can be used properly even for very complicated media. Finally the shear-wave dynamic behavior of a reservoir zone can be illustrated for laterally varying structures in terms of the amplitude variation and phase behavior using this new orientation.

Keywords Shear-Wave, Dynamic Behavior, Two Different Orientations

چکیده   در محیط های ساختمانی پیچیده دارای تغییرات جانبی، بهتر است به منظور جابجائی ذرات، محیط توسط امواج برشی در جهتی غیر از جهت متداول انتخاب شود. این تغییر جهت، علامت ضرایب موهومی امواج بازتابی و تبدیلی را تغییر می‌دهد که در تعیین خصوصیات مخزن و تحلیل مدل های AVO بسیار موثر است. جهت مثبت بردار جابجائی امواج برشی در این تدوین بگونه ای انتخاب شده که در صفحه انتشار موج در سمت چپ پرتو واقع شود. رفتار دینامیکی موج برشی در ناحیه مخزنی با تغییرات ساختمانی جانبی موجب تغییرات دامنه و فاز موج می‌شود که با تدوین جدید بهترمطابقت دارد. از این رو این تدوین برای محیط های ساختمانی پیچیده متناسب تر است.

1. INTRODUCTION

The crucial importance of shear waves as well as P to S conversion waves and their applications in seismology led to the investigation of the dynamic properties (i.e. reflection, transmission, conversion coefficients, amplitude ratios and phase behavior) in terms of angles of incidence.

Since 1919 the complex plane wave reflection and transmission coefficients given by Zoeppritz were sometimes erroneously used due to different notations and sign orientations. Some of these

errors are reported by Hales and Roberts [1], other errors have been found in reference [2].

The sixteen explicit plane wave reflection and transmission coefficients of P and SV-waves between two homogeneous media are given by Cerveny and Ravindra [3]. These coefficients are rewritten by Cerveny et al. [4] with a different notation for plane boundaries and applied by Rectora, et al. [6], and Marra, et al. [7].

In the definition of the equations given by Cerveny and Ravindra [3], the orientation of the displacement vector of the SV- wave is chosen in

such a way that its projection on the x-axis is positive (Figure 1a). Its implication is that the displacement vector of SV-wave may change sign at certain points of the ray and (-1) multiplier must be introduced. In this research paper the Fuchs' convention [5], Cerveny's convention [3,4] and modified versions [2] are compared computationally.

2. GOVERNING EQUATIONS

In order to determine the general solutions of P-SV reflection and transmission coefficients suitable for inhomogeneous media with planar boundaries, the sixteen explicit coefficients (Figure 1a) given by Cerveny (1971) can be written as follows.
For downgoing P-waves;

$$R_{11} = -1 + 2P_1$$
$$\left(\alpha_2 \beta_2 P_2 X^2 + \beta_1 \alpha_2 \rho_1 \rho_2 P_4 + q^2 P^2 P_2 P_3 P_4 \right)/D$$
$$R_{12} = -2\alpha_1 PP_1 \left(q P_3 P_4 Y + \alpha_2 \beta_2 XZ \right)/D$$
$$R_{13} = 2\alpha_1 \rho_1 P_1 \left(\beta_2 P_2 X + \beta_1 P_4 Y \right)/D$$
$$R_{14} = -2\alpha_1 \rho_1 PP_1 \left(q P_2 P_3 - \beta_1 \alpha_2 Z \right)/D$$

$$(1a)$$

For downgoing SV-waves;

$$R_{21} = -2\beta_1 PP_2 \left(q P_3 P_4 Y + \beta_2 \alpha_2 XZ \right)/D$$
$$R_{22} = 1 - 2P_2$$
$$\left(\alpha_2 \beta_2 P_1 X^2 + \beta_2 \alpha_1 \rho_1 \rho_2 P_3 + q^2 P^2 P_1 P_3 P_4 \right)/D \quad (1b)$$
$$R_{23} = 2\beta_1 \rho_1 PP_2 \left(q P_1 P_4 - \beta_2 \alpha_1 Z \right)/D$$
$$R_{24} = 2\beta_1 \rho_1 P_2 \left(\alpha_1 P_3 Y + \alpha_2 P_1 X \right)/D$$

For upgoing P-waves;

$$R_{31} = 2\alpha_2 \rho_2 P_3 \left(\beta_2 P_2 X + \beta_1 P_4 Y \right)/D$$
$$R_{32} = 2\alpha_2 \rho_2 P_3 P \left(q P_1 P_4 - \alpha_2 \beta_2 Z \right)/D$$

$$R_{33} = -1 + 2P_3$$
$$\left(\alpha_1 \beta_1 P_4 Y^2 + \alpha_1 \beta_2 \rho_1 \rho_2 P_2 + q^2 P^2 P_1 P_2 P_4 \right)/D \quad (1c)$$
$$R_{34} = 2\alpha_2 PP_3 \left(q P_1 P_2 X + \alpha_1 \beta_1 YZ \right)/D$$

And for upgoing SV-waves;

$$R_{41} = 2\beta_2 \rho_2 PP_4 \left(q P_2 P_3 - \alpha_2 \beta_1 Z \right)/D$$
$$R_{42} = 2\beta_2 \rho_2 P_4 \left(\alpha_2 P_1 X + \alpha_1 P_3 Y \right)/D$$
$$R_{43} = 2\beta_2 PP_4 \left(q P_1 P_2 X + \alpha_1 \beta_1 YZ \right)/D \quad (1d)$$
$$R_{44} = 1 - 2P_4$$
$$\left(\alpha_1 \beta_1 P_3 Y^2 + \alpha_2 \beta_1 \rho_1 \rho_2 P_1 + q^2 P_1 P_2 P_3 \right)/D$$

Where,

$$D = \rho_1 \rho_2 \left(\beta_1 \alpha_2 P_1 P_4 + \alpha_1 \beta_2 P_2 P_3 + \alpha_1 \beta_1 (P_3 P_4 Y^2 + \alpha_2 \beta_2 P^2 Z^2) + P_1 P_2 (q^2 P^2 P_3 P_4 + \alpha_2 \beta_2 X^2) \right)$$

$$q = 2(\rho_2 \beta_2^2 - \rho_1 \beta_1^2), \ P = Sin\theta_j / V_j, \ X = (\rho_2 - qP^2),$$

$$Y = (\rho_1 + qP^2), \ Z = (\rho_2 - \rho_1 - qP^2),$$

$$P_1 = (1 - \alpha_1^2 P^2)^{1/2} = Cos\theta_1, \ P_2 = (1 - \beta_1^2 P^2)^{1/2} =$$

$$Cos\theta_2, \ P_3 = (1 - \alpha_2^2 P^2)^{1/2} = Cos\theta_3,$$

$$P_4 = (1 - \beta_2^2 P^2)^{1/2} = Cos\theta_4$$

Equations 1 can be modified for the new sign convention according to Figure 1b as the followong eight explicit forms;
For downgoing P-waves;

$$R_{11} = [\rho_1 \rho_2 (\alpha_2 \beta_1 P_1 P_4 - \beta_2 \alpha_1 P_2 P_3) - \alpha_1 \beta_1 (P_3 P_4 Y^2 + \alpha_2 \beta_2 P^2 Z^2) + P_1 P_2 (q^2 P^2 P_3 P_4 + \alpha_2 \beta_2 X^2)]/D$$

$$R_{12} = 2\alpha_1 PP_1 \left(q P_3 P_4 Y + \alpha_2 \beta_2 XZ \right)/D \quad (2a)$$

$$R_{13} = 2\alpha_1 \rho_1 P_1 \left(\beta_2 P_2 X + \beta_1 P_4 Y \right)/D$$

$$R_{14} = -2\alpha_1 \rho_1 PP_1 \left(q P_2 P_3 - \beta_1 \alpha_2 Z \right)/D$$

For downgoing SV-waves;

$$R_{21} = -2\beta_1 PP_2\left(q P_3 P_4 Y + \beta_2 \alpha_2 XZ\right)/D$$

$$R_{22} = [\rho_1 \rho_2(\alpha_1 \beta_2 P_2 P_3 - \beta_1 \alpha_2 P_1 P_4) -$$

$$\alpha_1 \beta_1 (P_3 P_4 Y^2 + \alpha_2 \beta_2 P^2 Z^2) +$$

$$P_1 P_2 (q^2 P^2 P_3 P_4 + \alpha_2 \beta_2 X^2)]/D \qquad (2b)$$

$$R_{23} = -2\beta_1 \rho_1 PP_2\left(\beta_2 \alpha_1 Z - q P_1 P_4\right)/D$$

$$R_{24} = 2\beta_1 \rho_1 P_2\left(\alpha_1 P_3 Y + \alpha_2 P_1 X\right)/D$$

Therefore, for a laterally inhomogeneous medium with planar boundaries (Figure 1c), the Equations 2 can be used instead of corresponding R-coefficients in the Equations 1.

For $P > 1/\alpha_j$, all P_j in Equations 1 and 2 must be replaced by $P_j = \mp i (P^2 V_j^2 - 1)^{1/2}$, respectively for j = 1 to 4, i.e. minus sign must be used with Equations 1 and plus sign with Equations 2.

Where; $V_1 = \alpha_1$, $V_2 = \beta_1$, $V_3 = \alpha_2$, $V_4 = \beta_2$ and $i = \sqrt{-1}$ are used for each interface between two media ($V_1 = \alpha_1$, $V_3 = \alpha_2$ are P-wave and $V_2 = \beta_1$, $V_4 = \beta_2$ are S-wave velocities) and ρ is the density. Note that the sign convention for Cerveny's notations and Alaskari's notations are chosen according to Figures 1a and 1b, respectively.

If the receiver is located on the surface of the earth, it is necessary to consider both the incident and the interference waves (generated at the free surface) and the reflection coeffcients:

$$R_{11} = [-(1-2\beta_1^2 P^2)^2 + 4P^2 P_1 P_2 \beta_1^3/\alpha_1]/B$$

$$R_{12} = -4\beta_1 PP_1\left(1-2\beta_1^2 P^2\right)/B$$

$$R_{22} = [4\rho^2 P_1 P_2 \beta_1^3/\alpha_1 - \left(1-2\beta_1^2 P^2\right)^2]/B$$

$$R_{21} = [4PP_2 \beta_1^2/\alpha_1\left(1-2\beta_1^2 P^2\right)]/B$$

A common situation in exploration seismology is that the receiver is located on the surface of the earth. Therefore the Hilbert transformation of the source function must be slightly modified according to new convention. Beside for the incident P to S waves, the generated waves at the free surface must also be considered, which interference occurs. To obtain the interferences, it will be better to introduce the so called conversion vector $\vec{q}$ as follow (Figure 1c);

For incident P-wave;

$$\vec{q} = \hat{t} + R_{11}\hat{e}_1 + R_{12}\hat{e}_2$$

For incident SV-wave;

$$\vec{q} = \hat{t} + R_{21}\hat{e}_1 + R_{22}\hat{e}_2$$

The system of local ray coordinates is chosen as shown in Figure 1c. For the implication of the above conversion vector in the source term see refrence [8]. The components of the conversion vector corresponding to the local coordinate system are called the conversion coefficients (i.e. q_y, q_z) and can be written as follows;

For incident P-wave;

$$q_y = 4\beta_1 P_1 P_2 P/B \qquad q_Z = -2P_1(1-2\beta_1^2 P^2)/B \qquad (3)$$
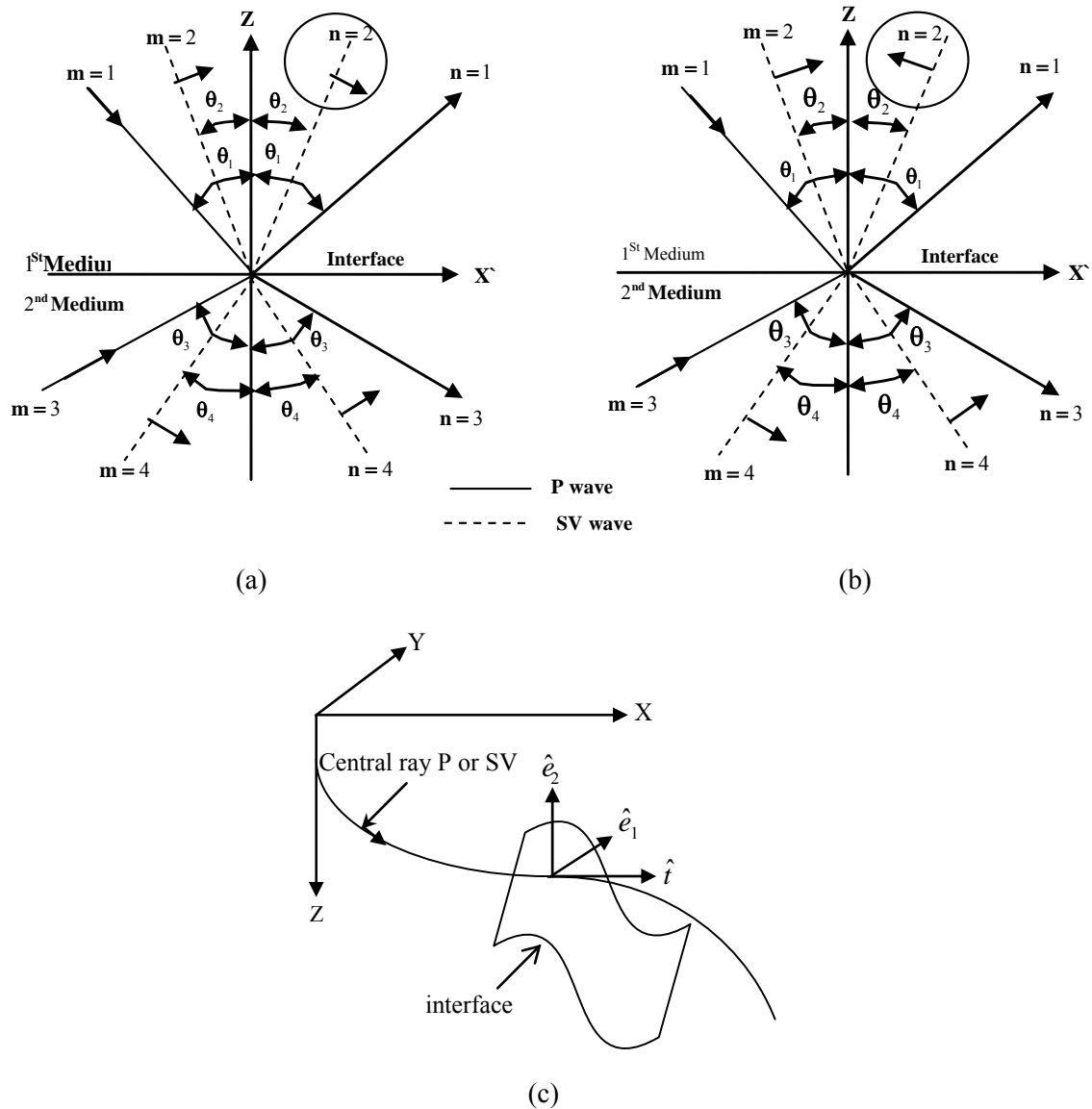
For incident SV-wave;

$$q_y = 2P_2(1-2\beta_1^2 P^2)/B \qquad q_Z = 4\beta_1^2 P_1 P_2 P/\alpha_1 B \qquad (4)$$

where B is called Rayleigh function and is given by;

$$B = (1-2\beta_1^2 P^2) + (4\beta_1^3 P^2 P_1 P_2)/\alpha_1 \qquad (5)$$

When the wave impinges on the free surface against the positive direction of the x-axis, the signs of the conversion coefficients q_y for incident P-wave and q_z for the incident SV-wave and the sign of R_{12}, R_{14}, R_{21}, and R_{23} must be changed in Equations 2. It is also necessary to mention that for waves incident from the lower medium(upgoing waves), all input parameters α_j, β_j and ρ_j, must be reversed. In order to apply conversion vector $\vec{q}$, at the free surface, the angle of emergence must be found at the earth's surface by using the phase

(a)

(b)

(c)

Figure 1. Sign convention of Reflection and Transmission Coefficients for incident P and S-waves, the changes are high-lighted by circles. (a) Standard sign convention, (b) Modified sign convention (c) 3D-ray representation of central-ray at the curved interface.

matching method. $\theta_j = \mathrm{Sin}^{-1}(V_0 P_j)$, where $j = 3$ for P-waves and $j = 4$ for SV-waves, i.e. $\theta_{3,4} = \mathrm{Sin}^{-1}(V_0 P_{3,4})$.

Note that in the definition of Equations 1 the positive direction of the displacement vector of the SV-wave is chosen such that its projection on the x-axis is positive (Figure 1a). The implication of this is that the displacement vector of SV-wave may change signs at certain points of the ray and (-1) multiplier must be introduced. Different situations are justified in laterally inhomogeneous media. Therefore, it may be more suitable to take a different orientation of the displacement vector of SV-waves, which may change the sign of shear-

wave reflection coefficients (Figure 1b). For example, in definition of the modified Equations 2 the particle motion was chosen on the displacement vector of the SV-wave to the left of the tangent to the ray (in direction of propagation) as positive. This convention is more suitable for applying the R-coefficients in laterally as well as vertically varying media [2].

This is properly defined and remains well defined even for very complicated rays in laterally varying media. The modulus of the R-coefficients in both cases are of course, the same (see Figure 2b), but the shear-wave reflection coefficients have different signs convention. Note that according to governing Equations 2, the above mentioned R-coefficients must be used very carefully(Figure 1c).

3. SYNOPSIS

To check the signs of the R-Coefficients and the sensitivity of the dynamic properties of seismic body waves, three different Algorithms were tested [2]. Three model examples (Table 1) are illustrated

in this paper to verify the conversion coefficients (Figures 2a and 3). In these examples the R-coefficient of P and SV incident waves on an elastic discontinuous medium were calculated on CDC 6600 (Mainframe) for all real angles of incidence, which varied from 0 to 90 degrees, in one degree increment except near the critical angle, where the coefficients were computed at 0.25 degree increments.
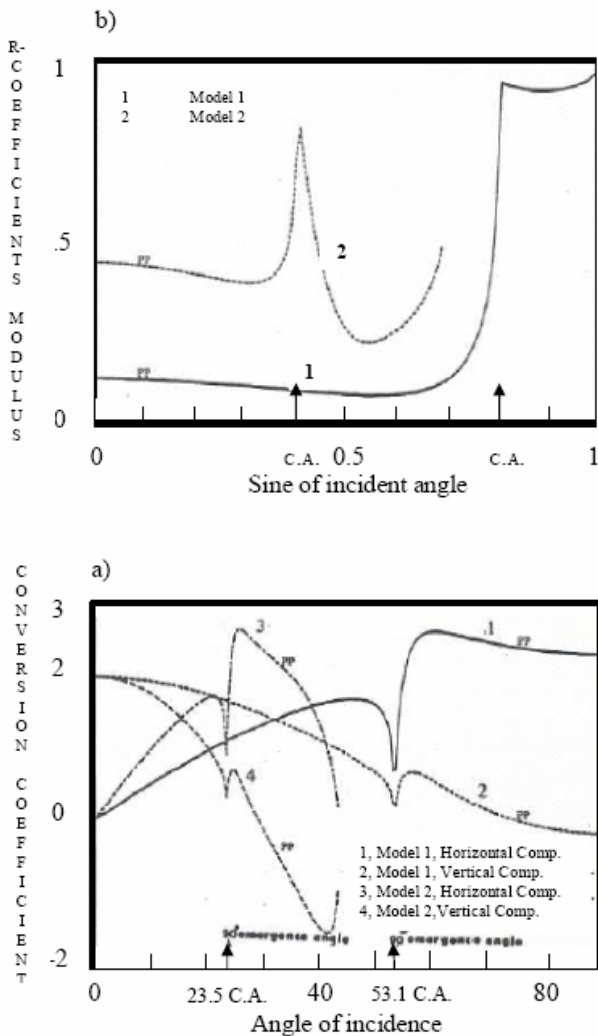
In this investigation both real and imaginary coefficients were considered in the computations. The modulus of the reflected P-wave for models 1 and 2 are compared in Figure 2b. In Figure 2a the surface conversion coefficients are computed for the models 1 and 2 (see Table 1).

These types of coefficients should be replaced by tangential vectors in order to correct the effect of wave interference for the receivers located on the earth's surface. The curves in Figures 2a and 3a are plotted for the real angles of incidence whose reflection and transmission coefficients are real and that portion of plots whose reflection and transmission coefficients become imaginary are omitted.
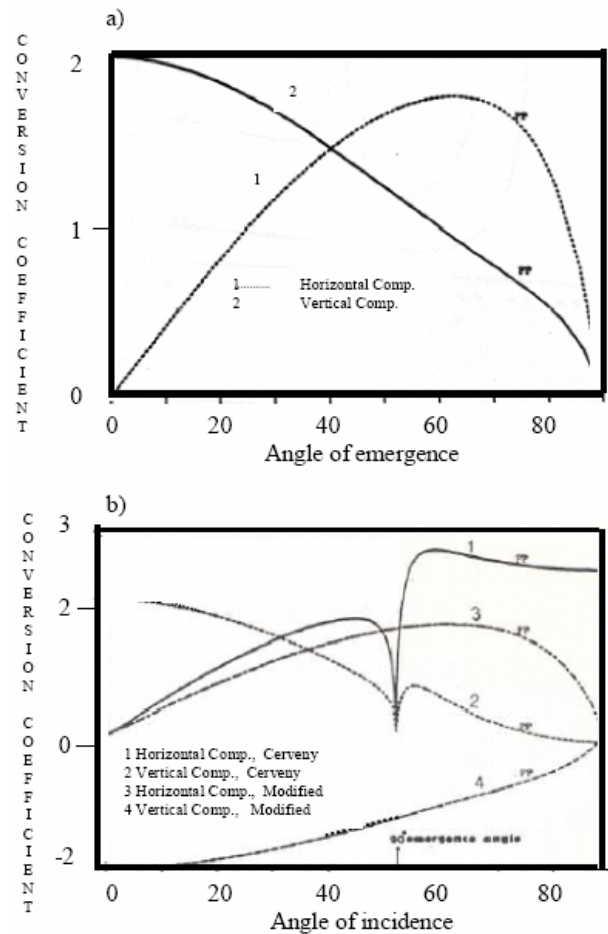
Figure 3a shows that for model (3) the

TABLE 1. **Test Models.**

Mathematical Models (Single Layer Over a Half Space)		
(1) Low Velocity Contrast (M1-Cerveny, 1971)	(2) High Velocity Contrast (M2-Cerveny, 1971)	(3) Average Crust Model (M3-Alaskari, 1983)
$VP_1 = 6.4km/sec$ $VP_2 = 8.0km/sec$ $\dfrac{VP}{VS} = 1.732$ $\rho_1 = 3.0g/cm^3$ $\rho_2 = 3.3g/cm^3$ Average crust = 30km $f_0 = 10HZ$ Zero-phase	$VP_1 = 2.0km/sec$ $VP_2 = 5.0km/sec$ $\dfrac{VP}{VS} = 1.732$ $\rho_1 = 3.0g/cm^3$ $\rho_2 = 4.0g/cm^3$ Depth of reflector = 1.0km $f_0 = 50HZ$ Zero-phase	$VP_1 = 6.5km/sec$ $VP_2 = 8.2km/sec$ $\dfrac{VP}{VS} = 1.732$ $\rho_1 = 3.0g/cm^3$ $\rho_2 = 3.3g/cm^3$ Average crust = 30km $f_0 = 10HZ$ Zero-phase

Figure 2. Coefficients computed for the models 1 and 2 given in Table 1, (a) Surface conversion coefficients, (b) Modulus of the P-wave reflection.

Figure 3. The vertical and horizontal components of conversion coeficients computed for the model 3 given in Table 1. For angles of emergence only, For angles of incidence compared to angles of emergence.
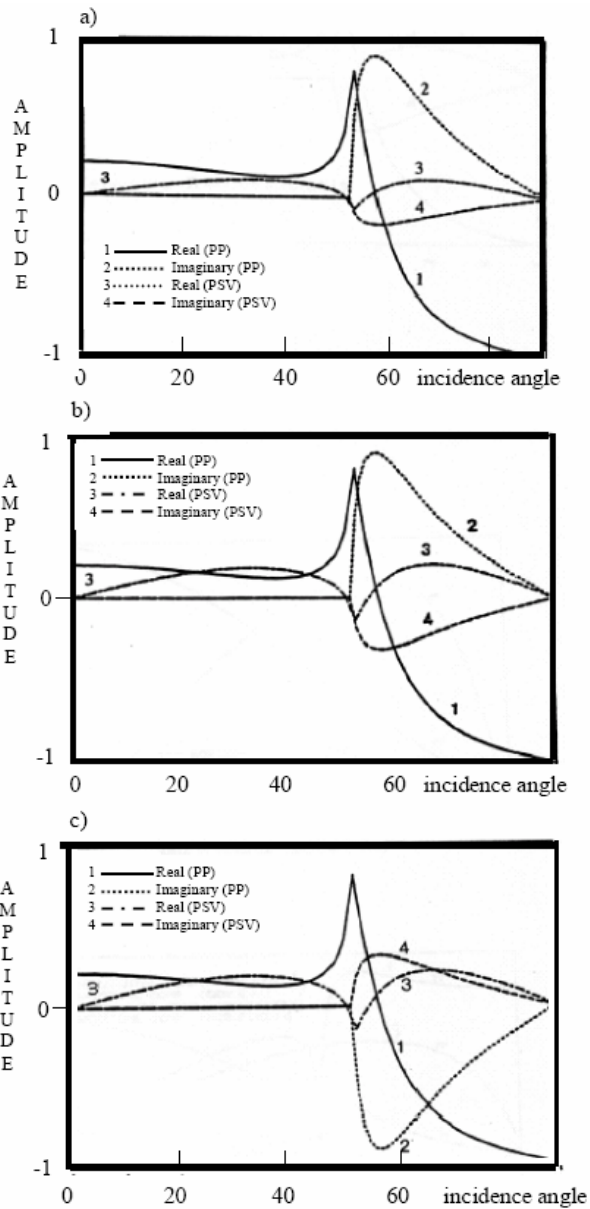
horizontal and vertical components of the P-wave conversion coefficients are zero at critical angle for the Cerveny's conventions and the horizontal and vertical components of the P-wave conversion coefficients are not zero for modified version. The computed conversion coefficients for the same model (model 3) shown in Figure 3b are plotted for the emergence angles. In Figure 3-a the arrow represents the 90 degree emergence angle for incoming waves corresponding to the critical angle. As seen in Figure 3a the P-wave conversion coefficients using the modified version is not
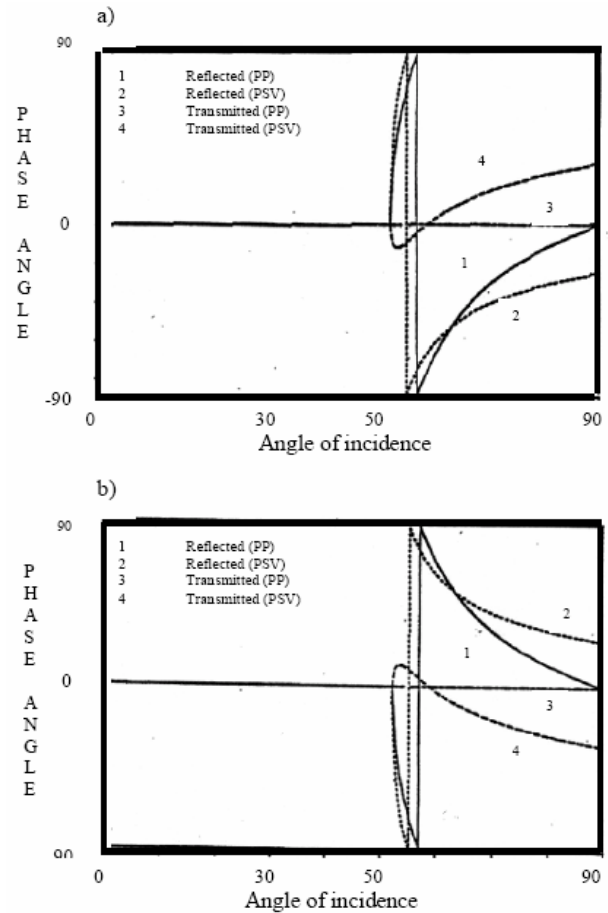
singular at the critical angle (52.4).

For three different approaches given in this paper the reflection coefficients were computed to check the signs and magnitudes of P to P and P to SV converted waves in Figure 4 for the same model (model-3). The complex reflectivity of Fuchs [5] (Figure 4a) and the Cerveny's (Figure 4b) are compared with the modified version, (Figure 4c). As it can be seen in Figure 4 the modulus of the reflection coefficients using the three different methods are the same. It means that the results of the modified version is not going to change the physical concept of the problem, but introduces a new phase change in the complex media.

Figure 4. R-coefficients computed for the model 3 given in Table 1 (a) Fuchs',(b) Cerveny's, (c) Modified.

Figure 5. The phase angles of reflected and transmitted waves for model 3. (a) Cerveny's formula, (b) The modified formula.

The phase angles of reflected and transmitted waves are computed for the model 3 using standard and modified fotmulae in Figures 5 and 6. As it can be seen from Figures 5 and 6 the phases for all generated waves at the subcritical angles are zero. The P to P, P to SV and SV to P, SV to SV reflected waves have rapid phase changes at the critical 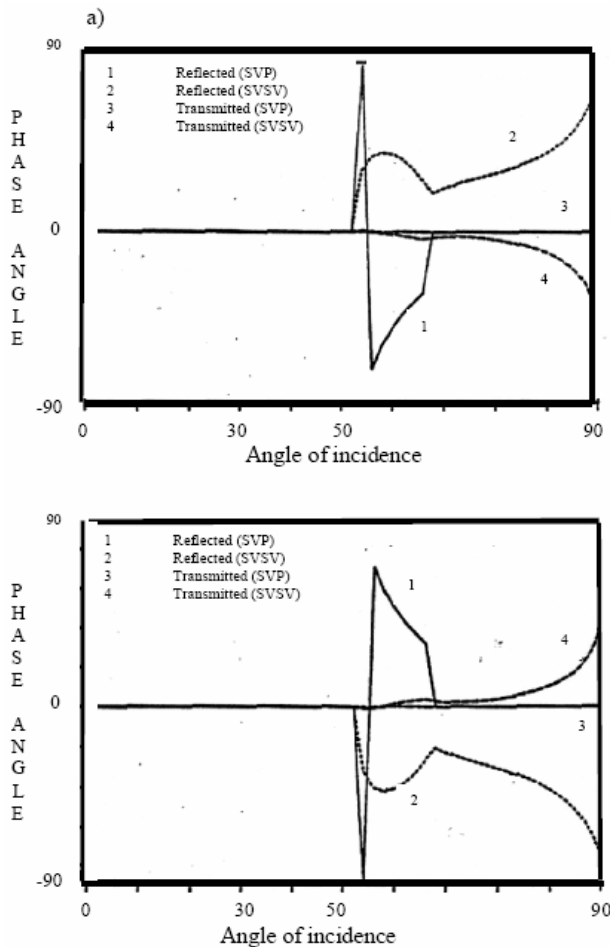angles. The P to P and SV to P transmitted waves have no phase changes for all angles of incidence, but the P to SV and SV to SV transmitted waves have phase changes in the post-critical angles.

In Figure 5b and 6b all coefficients have opposite signs for all angles of incidence compared to Figures 5a and 6a, except for the P to P and SV to P transmitted waves.

4. CONCLUSION

Based on several investigations [2], it can be concluded that the amplitude ratios and phase properties are relatively insensitive to changes in

Figure 6. The reflected phase angle versus angle of incidence for model 3. (a) Cerveny's formula, (b) The modified formula.

and transmission coefficients are, of course, the same. Beyond the first critical angles, the amplitudes of the generated waves are complex, and all waves undergo phase changes. The phase changes near the critical angles are often large and rapid.

The significance of a phase change in the generated wave is to change the arrival time of a peak or trough of the wave. The amount of delay or advance in reservoir characterization can be interpreted as negative or positive phase change respectively. This type of shear wave application would include the correction of observed travel times by the appropriate delay or advance time, determined from the phase angle in order to locate the depth of reflectors more precisely for laterally varying structures. The verification of the new sign convention against the field data will be illustrated in the future comming paper.

density ratio, but changes in the velocity ratio may cause significant changes in amplitude ratios. At the critical angle, the amplitude ratios of reflected waves change rapidly and approach its maximum. Beyond the critical angle small changes in velocity ratio (VP/VS) may have effect on the amplitude ratio. Therefore, it is more suitable to apply the modified R-coefficients given in Equation 2 for reservoir characterization, AVO analysis and AVO modeling.

In conclusion for the two different orientations mentioned in this paper, the modulus of reflection

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