TECHNICAL NOTE

INVESTIGATION ON THE EFFECT OF DIFFERENT PARAMETERS IN WHEELED MOBILE ROBOT ERROR

M. H .Korayem*, P. Peydaie and V. Azimirad

Department of Mechanical Engineering, Robotic Research Laboratory
Iran University of Science and Technology, Tehran, Iran
hkorayem@iust.ac.ir

*Corresponding Author

(Received: October 18, 2005 – Accepted in Revised Form: May 31, 2007)

Abstract This article has focused on evaluation and identification of effective parameters in positioning performance with an odometry approach of an omni-directional mobile robot. Although there has been research in this field, but in this paper, a new approach has been proposed for mobile robot in positioning performance. With respect to experimental investigations of different parameters in omni-directional mobile robots error, the effects of velocity, length of path, type of wheels and other parameters are analyzed with Gaussian error function estimation. A new approach for determining better characteristics in omni-directional mobile robots error is presented. With respect to arrangement of effective parameters by using statistical process control we have reached a model equation for assumed process (odometry errors). The results show statistical process control as a new trend for reduction of mobile robot error.

Keywords Mobile Robot, Experiments, Positioning Performance, Statistical Analysis

چکیده این مقاله در زمینه ارزیابی و شناخت پارامترهای تاثیرگذار در عملکرد مکان یابی (Positioning) به روش ادومتری (Odometry) و کیفیت ربات متحرک همه جهته (Omni-directional) می‌باشد. اگرچه در این زمینه تحقیقات کمی صورت گرفته است، اما در این مقاله روندی برای بررسی پارامترهای موثر و تحلیل آماری آن جهت بهبود دادن کیفیت دقت ربات متحرک ارائه شده است. در خاتمه با استفاده از کنترل فرآیند آماری (Statistical Process Control) به اثرات پارامترهای مختلف ربات متحرک نظیر سرعت ربات متحرک، مسافت طی شونده توسط ربات متحرک همه جهته و همچنین حرکت در راستای چرخ ربات متحرک پرداخته شده است.

1. INTRODUCTION

Exact knowledge of the positioning is an important issue in mobile robot navigation. In this regard researchers have developed different approaches for solving these problems. In relation to their efforts, they have of course reached to different techniques in mobile robot positioning which have been classified in the following classes:

1. Odometry
2. Inertial navigation
3. Magnetic compasses
4. Active beacon
5. Global positioning system (GPS)
6. Model matching
7. Landmark navigation

The above classifications were only the navigation systems of mobile robots. So for evaluating the performance of a system beside its theoretical assessment, experimental assessment is necessary. This research has tried to analyze Mechatronics Research Laboratory (MRL) mobile robot experimental data with a new trend for performing a better quality for its positioning errors. UMBmark which has been proposed by Borenstein, et al. is a general procedure just for assessment of errors with a definition of the limits for a mobile robot errors but it doesn't emphasize on the improvement of them. There is no consensus about a significant mobile robot test, especially for improving the errors of the mobile robot positioning, with odometry navigation system [1].

Although researches have been conducted to determining mobile robot error but they have been designed for special types with respect to their navigation systems [2]. There are several reasons for these facts. Differences in mobile robots with respect to their applications in different environments have made the situation challenging. It seems impossible to propose an exact formula with desired precision for error in such systems because there are uncertainties in parameters and nonlinearities in mobile robot's performance [3-4]. So statistical analyzes is the essence of such analyses in this field [5]. We have proposed a statistical process control for analyzing mobile robot behavior which can be considered for different types of mobile robots especially for improvement in performance.

This paper has been organized into three major Sections. In Section 2, the characteristics of the mentioned mobile robots are described. In Section 3 we have focused on properties of the designed experiments and in Section 4 it has pointed to the impacts of different parameters in mobile robot error. Finally, the new approach for improving mobile robot error with respect to statistical process control is presented.

2. INTRODUCING THE MRL MOBILE ROBOT

The mobile robot contains three main components:

1. Mechanical system
2. Hardware system
3. Software system

First the mechanical characteristics of the mobile robot will be introduced [6].

2.1. Mechanical Characteristic The MRL mobile robot is a three wheel triangular robot with omni-directional wheels. The chassis is a triangular base with 120 degree differences between its wheels directions which have been placed in an assumed circle. Figure 1 shows the details of the mobile robot mechanical configuration. Mechanical characteristics of the MRL mobile robot have been listed in Table 1.

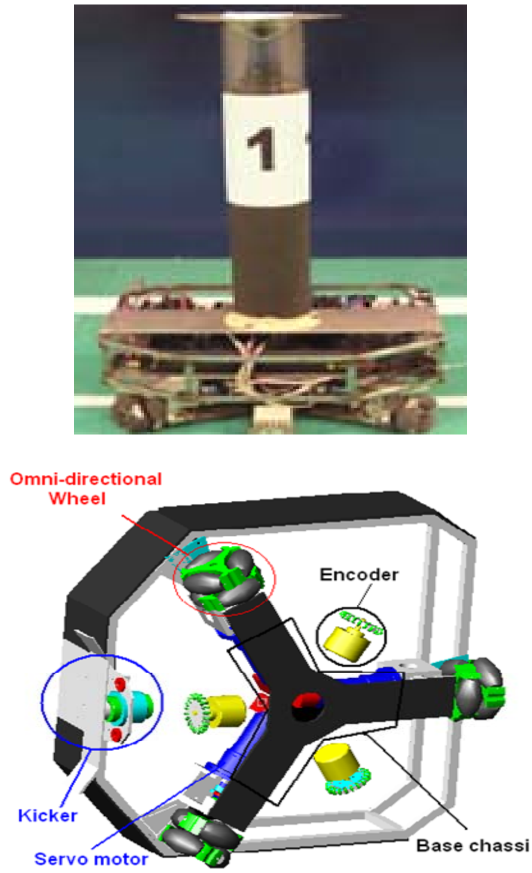


Figure 1. MRL mobile robot base chassis and its components.

2.2. Hardware System A multi-layered hardware system is implemented to control the navigation of the mobile robot. The control system is composed of motor drivers which are connected to a master board via a bus mechanism. Atmega 128 and Atmega 8535 microcontrollers are used for the master and drivers respectively. Encoders are used for feedback loops in the control system of robot. Communications between laptop and mobile robot master boards are accomplished by RS-232 port. The motor's speeds are measured and

TABLE 1. Mechanical Characteristics of MRL Mobile Robot.

3 Wheels Robot	Number of Wheel
Omni-Directional	Plat Form
2 m/s	Maximum Speed
3 m/s^2	Maximum Acceleration
12 Kg	Weight
4 Faulhaber DC 80 Watt	Wheels and Kicker Motors

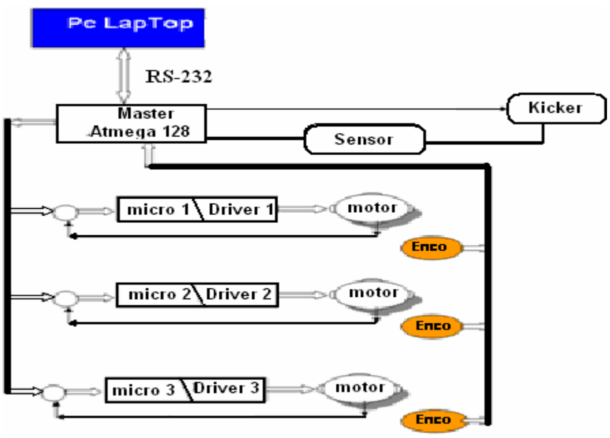


Figure 2. Hardware blocks of MRL mobile robot.

controlled by using shaft encoders. A special mechanism has been devised for encoders which have been used to measure the displacements in the perpendicular direction of wheels. Figure 2 indicates the use of an encoder mechanism in the mobile robot base.

2.3. Software System The software system is a programming in C++ Builder with graphical user interface (GUI) which has been developed for users. The main advantage of the program is its graphical presentation of designed path and maneuverability of drawing different paths. Graphical data on program canvas which is processed in the computer is sent to the mobile robot master boards via RS-232 port. Although there are parameters on the command window which a user can specify for designing a path with assigning X, Y and ϕ values. X and Y are symbols for the distance to forward and Y is the distraction from the straight forward path. ϕ is the rotation angle of the mobile robot about its center. Beside the main window in right side path report window presents all the X, Y values from the encoders and other details about the characteristics of the path. The square path has been illustrated for the UMBmark test as shown in Figure 3.

3. THE DESIGNED EXPERIMENT

In this Section, it has been decided to design experiment instructions for devising a new approach to investigate the errors and the impacts of other parameters in mobile robot tests. The designed experiments have advantages which are considered very useful for our test. The first is the path of the test and the second is the type of mobile robot motion. The first advantage of a straight line path is the simplicity of it without turning in the direction of the mobile robot path. The second is the motion of the mobile robot which is a simple motion along the straight path without rotation around its center. We can say there is some homogeneity (uniformity) in its motion while it is passing the path.

3.1. The Path of Experiment The Path of the test is composed of a simple straight line and with related kinematics of the mobile robot. It has been programmed to move along a specified path with respect to the length of the path.

3.2. Repetition of the Experiment Repetition of the experiment has to achieve a better precision and acceptable tolerance processing the valuable data. S and E have been assigned to the starting and ending points for the mobile robot in forward motion as shown in Figure 4. The result achieved from the UMBmark instruction, as it is clear, represent only the rough estimation of the error and no solution have introduced for improving the error.

4. STATISTICAL ANALYSES OF DIFFERENT PARAMETERS IN MOBILE ROBOT ERROR

In this section, parameters which have impact on the performance of mobile robot error have been analyzed. Classification of the parameters is based on experimental tests and theoretical analysis of motion with respect to the kinematics and kinetics of the mobile robots [7-8]. The effective parameters have been enumerated in the list below:

1. Velocity of the mobile robot
2. Length of the path
3. Direction of the mobile robot motion (Different wheel direction)
4. Material and type of the mobile robot wheel
5. Control system of mobile robot

As a preface for this discussion normal distribution have been pointed to. There are many types of distribution in the theory of probability distribution. Normal distribution function is usually used for scientific events such as follows:

$$\mu = \frac{1}{n}\sum_{i=1}^{n} E_i \tag{4-1}$$

$$E_i = x_a - x_t \tag{4-2}$$

$$p(x) = \frac{1}{\delta\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\delta^2}} \tag{4-3}$$

where μ is the average of error values. p(x) is the normal density function.

4.1. Effect of Velocity on Mobile Robot Error

By analyzing the acquired data from different experiments and plotting the probability distribution function, the differences can be discovered by referring to the graphs. Difference in velocities has resulted in difference probability of density curves as shown in Figure 5. In previous researches there were no reports on the impact of velocity in mobile robot error.

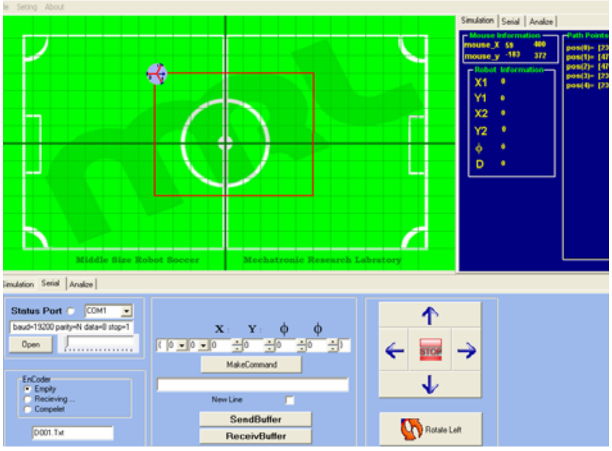


Figure 3. GUI program for testing of MRL mobile robot.

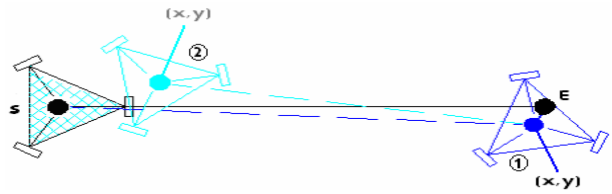


Figure 4. Path of experiment for MRL mobile robot.

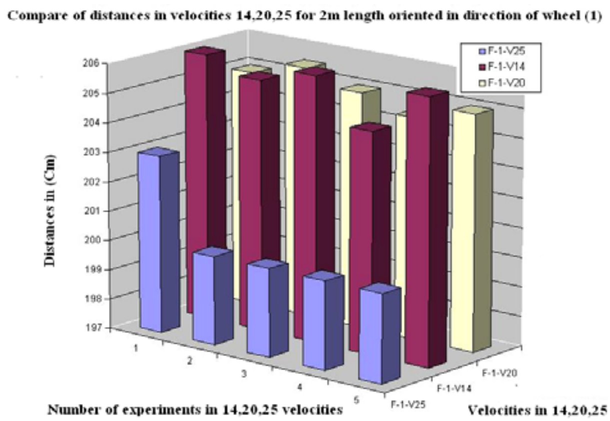


Figure 5. 3D graphical representation of velocities effect on the mobile robot error.

4.2. Effect of Path Length on Mobile Robot Error

In this part of the experiments

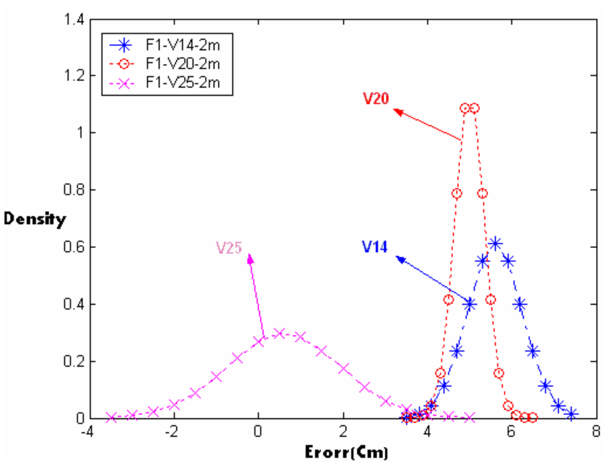


Figure 6. Statistical illustration of different velocities by density function.

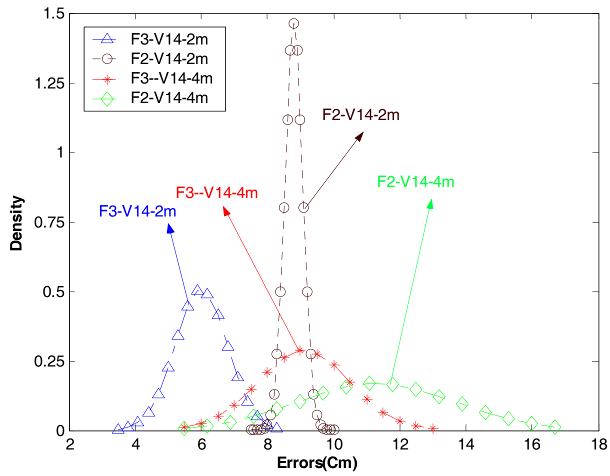


Figure 8. Statistical illustration of length of path in different directions by density function.

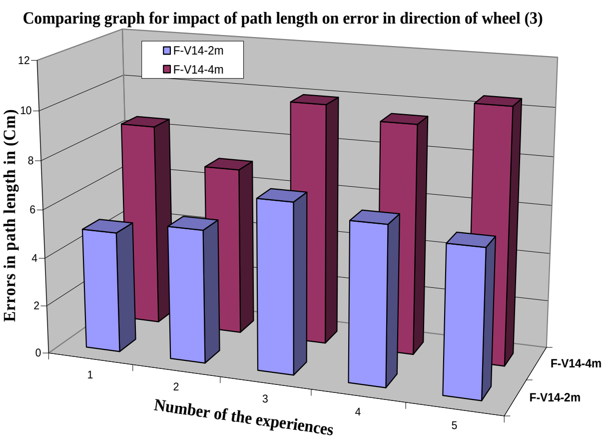


Figure 7. 3D graphical representation of path length impact on the mobile robot error.

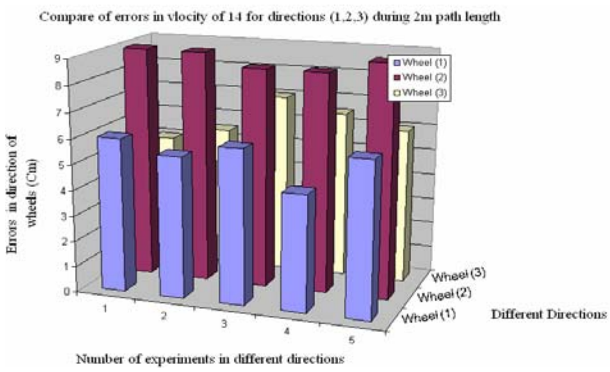


Figure 9. 3D graphical representation of direction impact on mobile robot error.

two different path lengths 2m, 4m have been considered. By variation of path length with the same velocity, differences between results are significant. Figure 6 indicates the errors of two different path lengths. Probability density of mobile robot error in each direction has been compared as shown in Figure 7. The notation of the graph for assigning the curve's property has been described in the following format: F2-V14-2m means that motion of the mobile robot is in the direction of wheel 2 (F2) with velocity of 14 (V14) and path length of 2 meters (2m). Velocity in the above format has been reported as a percentage of the rated velocity of the servo motor.

4.3. Effect of Direction on Mobile Robot Error The effect of direction on mobile robot error, which has been investigated in the current research has been illustrated in Figure 9. The differences in results show that the direction of motion has an impact on error. Figures 6 and 9 show the differences between directions of motion. From the graphs it is obvious that wheel 1 and wheel 3 have similar effects in the performance of

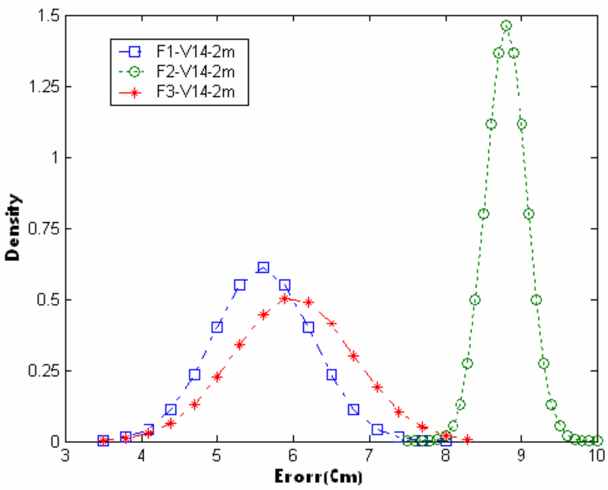


Figure 10. Statistical illustration of direction impact by density function.

(a)

(b)

Figure 11. Different types of wheels (a) Handmade wheel (b) Kornylack-corp wheels.

mobile robot.

4.4. Effect of Type and Material of Mobile Robot Wheel Two types of wheels have been selected, Figure 11-a shows the wheel which has been made by primary equipment, Figure 11-b shows the second wheel which was provided from Kornylack-corp. Indeed the causes on friction coefficient and geometry of the wheels can be found. These sorts of problems will cause nonsystematic errors in mobile robot motion.

4.5. Effect of Control System on Mobile Robot Errors A good control system can improve mobile robot error which depends on various and complicated parameters such as unsynchronized rotation of the wheels, geometrical considerations of wheels and encoder's defects (sliding, misalignment, etc). Previous instances are common problems in mobile robots. These problems are usually manufacturing defects and depend on precision of the tools. For example control systems with wheel velocity feedback can synchronize the rotation of wheels especially in straight motion or in rotation of the robot over its gravity center. Figure 12 is an illustration of unsynchronized wheels velocity effects on robot rotation during 360 degree rotation over its center. Point 1 is initial position and point 2 is its correspondent position after 360 degree of rotation as shown in Figure 12.

Figure 13 is a 3D graph for comparing errors in control systems with wheel velocity as a feedback. Various velocities have been tested and plotted versus durations of 4m in a straight path. Figure 13 shows the related curves for both wheels and position and velocity as a feedback by statistical analysis of different control systems.

5. ANALYSIS OF MOBILE ROBOT ERROR TEST USING STATISTICAL PROCESS CONTROL

The basic tool for accomplishing this theory is the statistical process control which provides a scientific instruction for improvement of mobile robot performance. Table 2 is an arrangement of effective parameters which is called the spread

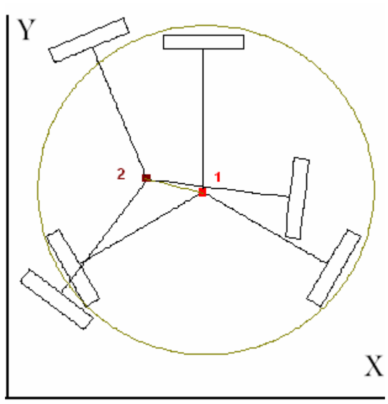


Figure 12. Rotation of mobile robot in testing unsynchronized wheel.

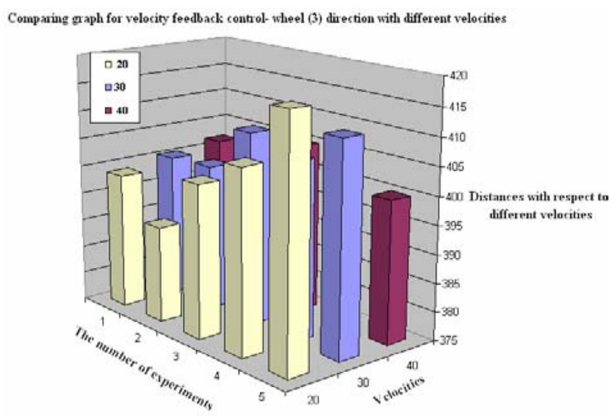


Figure 13. 3D graphical representation of direction impact on mobile robot error.

sheet method for analyzing situations [11]. The effective parameters have been divided into two levels quantities. As is listed in Table 2, the obtained data from the experiments has been processed and written in their corresponding blocks. The arrangement of effective parameters will not affect the result of the survey but for developing the assumed model for the process limits of quantity levels are important. Details and other specific definitions have been pointed to in the references.

By referring to relations in the following lines, first some prerequisite notations will be defined:

$$E(A) = \bar{Y}_{A+} - \bar{Y}_{A-} \quad (5\text{-}1)$$

$$E(AB) = \frac{1}{2}\left[(\bar{Y}_{A+} - \bar{Y}_{A-})_{B+} - (\bar{Y}_{A+} - \bar{Y}_{A-})_{B-} \right] \quad (5\text{-}2)$$

$$S^2 = \frac{\Sigma (X_i - \bar{X})^2}{N-1} \quad (5\text{-}3)$$

where E(A) is the effect of parameter A and the right term is subtraction of the average errors in low level values of parameter A from the average error in high level value. To illustrate the mutual impacts, both A and B, E(AB) are considered as a combinational impact of both parameters A and B simultaneously. S is standard deviation of its relevant experiment in Equation 5-3.

Table 3 has been obtained from pointed relations in the preceding lines. The next step is to compute the effects of effective parameters. The columns which are specified by A, B, C and their combinations are the effective parameters in these experiments. Rows 1 to 8 are the numbers of treatments which have been conducted with regard to the related values of the effective parameters A, B and C. Indices in Table 3 which have been assigned by plus sign (+) or minus (-) specify high levels and low levels of the relevant parameters, respectively. E in Table 3 stands for effects of the parameters with respect to the relevant columns of the table. $\bar{S}_+^2$ and $\bar{S}_-^2$ are the square averages of standard deviations of related signs of a parameter. Although other effective parameters can be added to this discussion, this depends on the researcher's attitude toward the significant impacts of parameters and whether or not they are effective. The final word is the preceding experiments in Section 4 which can be used as criteria for decision making about the effective parameters. Table 3 is a corner stone for the paces toward the decision limit (DL). Now with respect to the paratho effect graph which has been based on following data the effectiveness of the parameter can be determined.

Figure 15 is the paratho effect graph which shows the effect values in a bar graph. The meaningful effects according to the paratho effects

TABLE 2. Parameters Combinations with Different Levels with Spread Sheet Approach.

	Velocity of Mobile Robot in centimeters per second(A)							
	Low level (-) Velocity(14)				High level (+) Velocity(20)			
	Distance in meters (C)				Distance in meters (C)			
	Low Level (-) 2m		High Level (+) 4m		Low Level (-) 2m		High Level (+) 4m	
	Error	Average	Error	Average	Error	Average	Error	Average
Direction of Wheel (2) (-) (B)	9		15		6		11	
	9		11.5		6		10	
	8.5	8.8	11	11.3	6	5.3	11	9.6
	8.5		10.5		4		11	
	9		8.5		4.5		5	
Direction of Wheel (3) (+) (B)	5		8.5		7		7.5	
	5.5		7		5		10.5	
	7	6	10	9.1	5.5	6.2	13	10.5
	6.5		9.5		6.5		12.5	
	6		10.5		7		9	

graph are specified. Decision limits is a criteria for assigning the meaningful effects as follows:

$$S_e = \sqrt{\left(\frac{\Sigma S_i^2}{K}\right)} = \sqrt{\frac{0.075 + 0.95 + \ldots + 5.375}{8}} = \sqrt{\frac{22.15}{8}} = \sqrt{2.76} = 1.66 \quad (5\text{-}4)$$

$$S_{eff} = S_e\sqrt{\left(\frac{4}{N}\right)} = 1.66 \times \sqrt{\frac{4}{40}} = 1.66 \times \sqrt{0.1} = 0.52 \quad (5\text{-}5)$$

where S_i is standard deviation for combinations of different experiments with regard to the effective parameters quantity level, S_{eff} is effect deviation which is computed with consideration to the number of experiments and N related to number of

TABLE 3. Processed Data of Mobile Robot Error Related to Effective Parameters.

Experiments	A	B	C	AB	AC	BC	ABC	$\overline{Y}$	S^2
1	-	-	-	+	+	+	-	8.8	0.075
2	+	-	-	-	-	+	+	5.3	0.95
3	-	+	-	-	+	-	+	6	0.625
4	+	+	-	+	-	-	-	6.2	0.825
5	-	-	+	+	-	-	+	11.3	5.575
6	+	-	+	-	+	-	-	9.6	6.8
7	-	+	+	-	-	+	-	9.1	1.925
8	+	+	+	+	+	+	+	10.5	5.375
$\sum Y_+$	31.6	30.9	40.5	36.8	34.9	33.7	33.1		
$\sum Y_-$	35.2	35	26.3	30	31.9	33.1	33.7		
$\overline{Y}_+$	7.9	8.75	6.57	7.5	7.97	8.27	8.27		
$\overline{Y}_-$	8.8	8.75	6.57	7.5	7.97	8.27	8.42		
E	-0.9	-0.8	3.55	1.7	0.75	0.15	-0.15		
$\overline{S}_+^2$	3.48	2.18	4.91	2.96	3.21	2.08	3.13		
$\overline{S}_-^2$	2.05	3.35	0.61	2.57	2.31	3.45	2.40		
F	1.69	1.53	8.04	1.15	1.38	1.65	1.30		

effective parameters. Decision limits can be achieved by using the following equation.

$$DL = \pm t S_{eff} \quad (5\text{-}6)$$

where t is computed by considering both degrees of freedom (DOF) and the number of experiments repeated from its correspondent table.

DOF = (repeatance of the experiments-1) × (number of experiments combinations) = (5-1) × (8) = 32

By the above interpretation decision limits (DL) can be computed as below:

$$DL = \pm t \times S_{eff} = \pm 2.03 \times 0.52 = \pm 1.05 \quad (5\text{-}6a)$$

In fact decision limits (DL) are compared to parameters effects and whether they are greater than DL.

As shown in Figure 16, the effect of C factor and mutual effect of AB are out of the decision limit. By considering these effects as meaningful effects of the following process a linear model for the process can be derived.

A model can be proposed for improvement of the error with respect to meaningful effects and

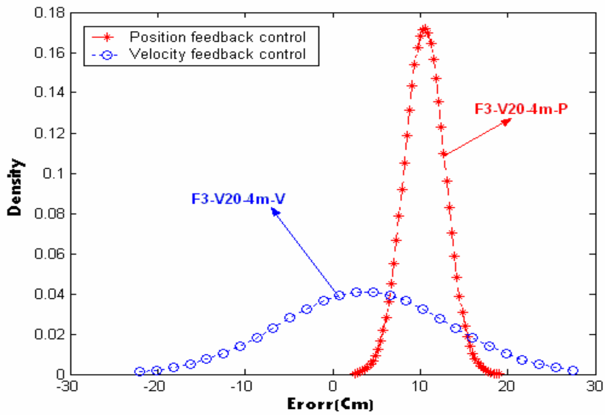


Figure 14. Statistical illustration of control system in mobile robot error.

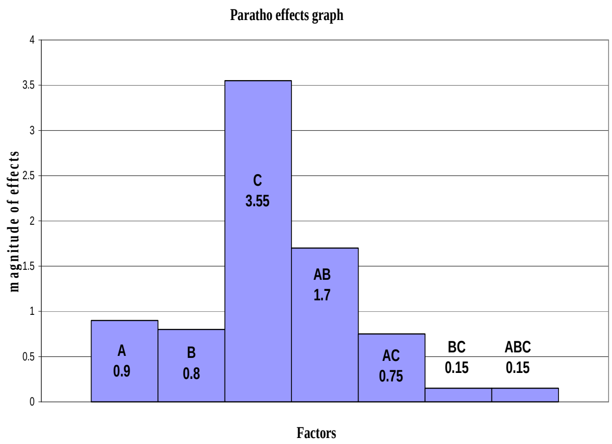


Figure 15. Paratho effects graph with related effects on mobile robot positioning error.

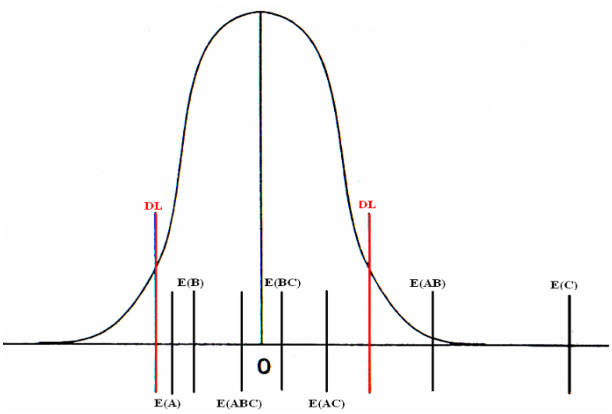


Figure 15. Decision limit graph for recognition of meaningful effects.

hierarchical rule. In relation to hierarchical rule all meaningful effects in the model equation must be considered and if the mutual effects of their combinations are greater than the decision limit value then effects of each parameter will be consiedered in the predicted model. Equations 5-7 indicate the derived model for positioning error of mobile robot.

$$\hat{Y} = \bar{\bar{Y}} + \frac{E(A)}{2}A + \frac{E(B)}{2}B + \frac{E(C)}{2}C + \frac{E(AB)}{2}AB \quad (5\text{-}7)$$

It is seen in the above relation, effects of A and B are considered because of hierarchical rule. In Equations 5-7 E(A), E(B) and E(AB) are effects of related parameters. A, B and AB are encoded levels of the parameters which can contain plus (+1) or minus (-1) sign in computations. $\bar{\bar{Y}}$ is the average of all treatments. Now with previous explanations the average of errors for better performance are computed:

$$\hat{Y} = 8.35 + \frac{-0.9}{2}A + \frac{-0.8}{2}B + \frac{3.55}{2}C + \frac{1.7}{2}AB \quad (5\text{-}7a)$$

The coefficients of Equations 5-7 can be obtained according to the paratho effects and level of the parameters:

$A = +1, \quad B = -1, \quad C = -1, \quad AB = (+1) \times (-1)$

So the new value for $\hat{Y}$ is obtained by replacement of acquired data from Equations 5-7a,

$$\hat{Y} = 5.67$$

It should be noted that the following procedures are restricted by our considerations toward effective parameters and the linearity assumption between the levels of parameters. For the sake of brevity, each individual robot can show a different performance in the positioning system which stems from the control system, manufacturing tools, measurments devices and etc.

5.1. Comparing the Means of Two Normal Distributions (T-TEST)

It is often useful to

have some measure of performance of a particular algorithm, control mechanism, etc. If, for example, two different control programs produced two different means of a particular result, it is necessary to decide whether there is a significant difference between these two means, in order to determine whether one of the two programs produces better results than the other. To establish whether the two means μ_1, μ_2 are significantly different, the value T is computed as follows [12-13]:

$$T = \frac{\mu_1 - \mu_2}{\sqrt{(n_1-1)\sigma_1^2 + (n_2-1)\sigma_2^2}} \times \sqrt{\frac{n_1 n_2 (n_1 + n_2 - 2)}{n_1 + n_2}} \qquad (5\text{-}8)$$

where n_1, n_2 is the number of data points in experiment 1 and experiment 2 respectively, μ_1 and δ_1 mean and standard deviation of experiment 1, and μ_2 and δ_2 mean and standard deviation of experiment 2. The test is conducted as follows: the value of t_α is determined from the related table with $K = n_1 + n_2 - 2$. If the inequality $|T| > t_\alpha$ holds, the null hypothesis H_0 is rejected, meaning that the two means differ significantly. The result of the T-Test also depend on the value of t_α. It is common to take the values for the 5 % error level (P = % 5). The following expression is pursued to find the results:

$$K = n_1 + n_2 - 2 = 5 + 5 - 2 = 8$$

Experiments with changes in their parameters have been compared with each other in Table 5. The format in which Table 5 has been arranged are similar to those is Section 4-2. At the last raw of the Table 4, V stands for the control system with wheel velocity as feedback and P is for position control system.

In another work an experimental test to analysis the localization error of a mobile robot named Sweeper was designed as shown in Figure 17. In order to increase the friction of the wheel with the ground, a layer of PVC was used for each wheel. A gearbox was designed for transmission of torque from stepper motors to the wheels. This gearbox has a transmission ratio of 4:1. [14].

To do this test and determine the exact error of robot numerically, the robot was programmed to move along a straight line path with a length of 1 meter and analyze the error at the motion axis of the robot and the axis perpendicular to motion axis (y) as shown in Figure 18. The test is replicated on Tile and Asphalt. The result is shown in Tables 5 and 6 also the graphical results are shown in Figures 19 and 21 and normal distribution are shown in Figures 20 and 21.

$$n_1 = 16 \quad n_2 = 16 \quad \mu_1 = 1.08 \quad \mu_2 = 4.50 \quad \sigma_1 = 0.05$$

$$\sigma_2 = 0.58 \quad k = n_1 + n_2 - 2 = 30$$

Using Equations 5-7, T = 23.44 and the related t_α

TABLE 4. T-Test results for Mobile Robot Errors.

Experiment (1)	Experiment (2)	μ_1	δ_1	μ_2	δ_2	T	$\lvert T\rvert > t_\alpha$
3-V20-2m	2-V20-2m	6.2	0.908	5.3	0.974	8.776	√
2-V20-4m	2-V20-2m	9.6	2.607	6.2	0.974	2.731	√
1-V14-2m	3-V14-2m	5.6	0.651	6	0.790	0.807	×
3-V20-4m	3-V14-4m	10.5	2.318	9.1	1.387	1.158	×
3-V20-4m(V)	3-V20-4m(P)	3.6	10.511	10.5	2.318	-1.433	×

TABLE 5. Location Error of Sweeper Mobile Robot at X Axis (The Motion Axis of Robot).

No. of Experiment	Error on Tile (cm)	Error on Asphalt (cm)
1	1.5	4.85
2	1.3	5.6
3	1	3.35
4	0.7	4.25
5	0.9	4.8
6	0.8	3.5
7	1.1	3.4
8	1.3	5.7
9	1	4.4
10	1.3	5.2
11	1.4	4
12	1	4.5
13	1.1	5
14	0.9	4.2
15	1.2	3.9
16	0.8	5.4

from table is 2.04 [13]. So $|T| > t_\alpha$ and there is a significant position error between paths from asphalt to tile; there is a 95 % probability that the accuracy on tile at X axis is 3.42 centimeters more than the accuracy on asphalt.

The results at Y axis analyzed below as done for X axis:

$$n_1 = 16 \quad n_2 = 16 \quad \mu_1 = 2.16 \quad \mu_2 = 10.35 \quad \sigma_1 = 0.49$$
$$\sigma_2 = 4.74 \quad k = n_1 + n_2 - 2 = 30$$

Using Equations 5-7, T = 6.86 and the related t_α from related table is 2.04 [13]. So $|T| > t_\alpha$ and there is a significant position error between paths from

Figure 17. The sweeper mobile robot.

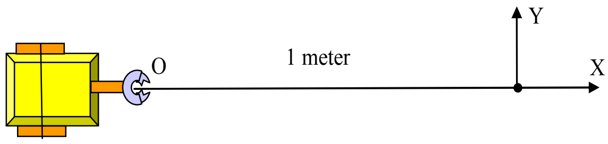
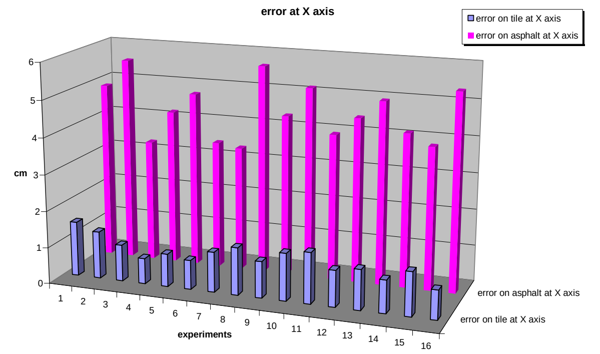


Figure 18. The X and Y axis.

asphalt to tile; there is a 95 % probability that the accuracy on tile at Y axis is 8.19 centimeters more than the accuracy on asphalt.

5.2. Determining the Association Between Two Variables (X^2-TEST)

Mean, standard deviation, T-test and many other statistical analysis methods can only be applied to continuous-valued data. In robotic experiments, however, there are many experiments in which the results are obtained as categories. Nominal variables are defined as variables that are members of an unordered set, such as for example color, or taste. For the following considerations, the interest is in determining whether the two nominal variables are associated or not. Data of two variables can be displayed in a contingency table, which will allow a so-called cross tabulation analysis to be performed. For analyzing the association between X and Y of mobile robots X is assigned as an error in the direction of the robot motion and Y as the error perpendicular to



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
error on tile at X axis	1.5	1.3	1	0.7	0.9	0.8	1.1	1.3	1	1.3	1.4	1	1.1	0.9	1.2	0.8
error on asphalt at X axis	4.85	5.6	3.35	4.25	4.8	3.5	3.4	5.7	4.4	5.2	4	4.5	5	4.2	3.9	5.4

Figure 19. 3D graphical representation of mobile robot error at X axis on Tile and Asphalt.

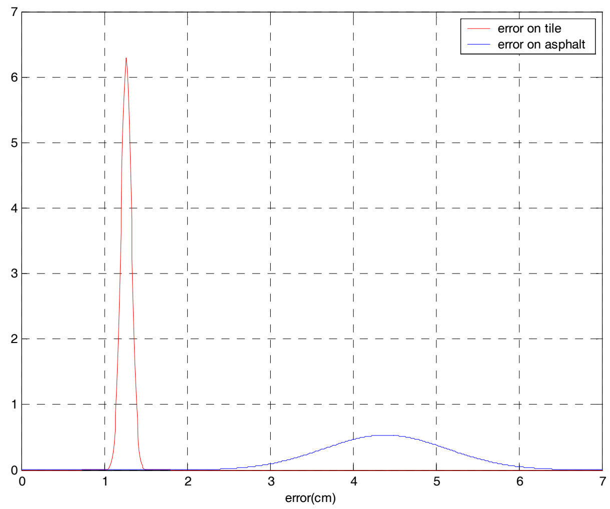
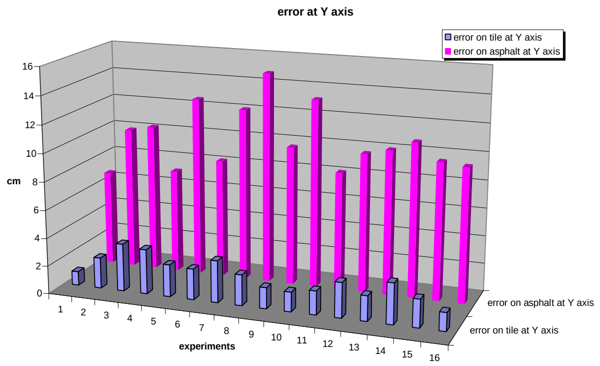


Figure 20. Normal distribution of mobile robot error at X axis on Tile and Asphalt.



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
error on tile at Y axis	1	2.2	3.4	3.2	2.3	2.2	3	2.2	1.5	1.4	1.7	2.5	1.8	2.9	2	1.3
error on asphalt at Y axis	6.8	10.15	10.5	7.4	12.8	8.45	12.3	15	9.9	13.4	8.4	9.9	10.3	11	9.8	9.6

Figure 21. 3D graphical representation of mobile robot error at X axis on Tile and Asphalt.

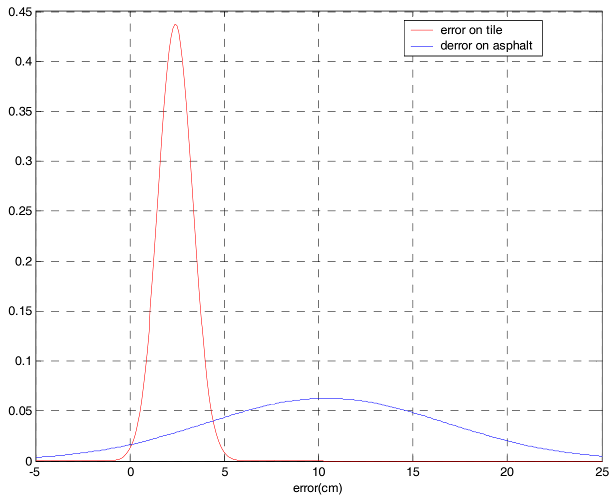


Figure 22. Normal distribution of mobile robot error at Y axis on Tile and Asphalt.

the X direction. A contingency table which states how often each event could happen is created. One test to determine the significance of an association between two variables is the X^2–Test. Equations 5-9 to 5-10 are conducted to accomplish the X^2–Test. Let N_{ij} be the number of events where the variable X has the value i and variable Y has the value j. Let N be the total number of events. Let N_i be the number of events where X has the value I, regardless of Y, N_j the number of events where Y has the value j regardless of the value of X. The null hypothesis in the X^2–Test is that the two variables X and Y have no significant correlation. In order to test this null hypothesis, expected values need to be determined. Under the assumption that the null hypothesis is true, it can therefore yield the table of the expected values n_{ij}:

$$n_{ij} = \frac{N_i N_j}{N} \quad (5\text{-}9)$$

X^2 is defined as:

$$X^2 = \sum_{i,j} \frac{(N_{ij} - n_{ij})^2}{n_{ij}} \qquad (5\text{-}10)$$

The computed value for X^2 in conjunction with the $X^2_{0.05}$ probability function which is available on related tables can be used to determine whether the association between variables I and J is significant or not. The number of degrees of freedom m for a table of size I by J is:

$$m = (I)(J) - I - J + 1 \qquad (5\text{-}11)$$

TABLE 6. Location Error of Sweeper Mobile Robot at Y Axis.

No. of Experiment	Error on Tile (cm)	Error on Asphalt (cm)
1	1	6.8
2	2.2	10.15
3	3.4	10.5
4	3.2	7.4
5	2.3	12.8
6	2.2	8.45
7	3	12.3
8	2.2	15
9	1.5	9.9
10	1.4	13.4
11	1.7	8.4
12	2.5	9.9
13	1.8	10.3
14	2.9	11
15	2	9.8
16	1.3	9.6

By replacing the related parameters in Equation 5-11:

$$m = (3) \times (3) - (3 + 3) + 1 = 4$$

If $X^2 > X^2_{0.05}$ then there is a significant correlation between the variables I and J. The probability for this statement to be wrong is P = % 5. $X^2_{0.05} = 9.5$ is achievedfor its related risk according to the value of m = 4 and P = 0.05. In Table 6 numbers 7, 15 and 35 which are assigned in rows and columns of Table 7 are the restrictions for dividing the variables to nominal variable.

$$0 \le x \le 7 \to A \qquad 7 < x \le 15 \to B \qquad 15 < x \to C$$

$$0 \le y \le 7 \to A \qquad 7 < y \le 15 \to B \qquad 15 < y \to C$$

With respect to Table 6:

$$N_{A,A} = 11 \qquad N_{A,B} = 4 \qquad N_{A,C} = 20$$

$$N_{B,A} = 6 \qquad N_{B,B} = 9 \qquad N_{B,C} = 21$$

$$N_{C,A} = 20 \qquad N_{C,B} = 2 \qquad N_{C,C} = 3$$

By using Equation 5-9, we have:

$$n_{A,A} = \frac{N_{Ai} N_{Aj}}{N} = \frac{(35) \times (37)}{96} = 13.48$$

$$n_{A,B} = \frac{N_{Ai} N_{Bj}}{N} = \frac{(35) \times (15)}{96} = 5.46$$

$$n_{A,C} = \frac{N_{Ai} N_{Cj}}{N} = \frac{(35) \times (44)}{96} = 16.04$$

Now computed values from above relations are placed in pertinent relation for achieving X^2:

$$\chi^2 = \sum_{i,j} \frac{(N_{ij} - n_{ij})^2}{n_{ij}} = \frac{(11 - 13.48)^2}{13.48} +$$

$$\frac{(4 - 5.46)^2}{5.46} + \cdots = 0.456 + 0.390 + \cdots = 27.014$$

TABLE 7. Contingency Table for Nominal Variables of Mobile Robot Error.

Y/X	(A) Low 7	(B) Medium 15	(C) High 35	Number of Events in Related Raw
7 (A) Low	11	6	20	37
15 (B) Medium	4	9	2	15
35 (C) High	20	21	3	44
Number of events in related column	35	36	25	

By considering the desirable condition for hypothesis:

$$X^2 > X^2_{0.05} \Rightarrow 27.014 \rangle 9.5$$

The following conditions indicate the correlation between X and Y.

6. CONCLUSION

The goal of this paper is to develop an instruction for the improvement of mobile robot positioning systems based on odometry positioning techniques. The biggest advantage of this approach is its capability for the improvement of mobile robot error. The entire research can be considered in two discussions. At first, the impacts of different parameters on mobile robot error have been concentrated on. These have been achieved by both practical experiments and theoretical investigations of kinematics and kinetics of mobile robot motion. In the second part statistical process control has been pointed to as a new approach for improvement of mobile robot positioning error. It should be noted that the error of mobile robot has been improved on from 8.35 to 5.67 with the application of designs from the experiments and modeling the process. Experimental tests on Sweeper show the significant position error between paths from asphalt to tile; the accuracy on tile at X and Y axis is 3.42 and 8.19 centimeter respectively are more accurate on asphalt. Finally it should be added that although there were approaches for evaluating mobile robot performance none of them have focused on improvement of error. Statistical process control is proposed as a scientific and reliable approach for evaluation and improvement of mobile robot performance, especially in regard to error.

7. REFERENCES

1. Borenstein, J., Everret, H. R., Feng, L. and Wehe, D., “Mobile Robot Positioning: Sensors and Techniques”, ***Journal of Robotic Systems***, Vol. 14, No. 4, (1997), 231-249.
2. Ojeda, L. and Borenstein, J., “Reduction of Odometry Errors in Over Constrained Mobile Robots”, ***Proceedings of the UGV Technology Conference at the 2000 SPIE AeroSense Symposium***, Orlando, FL, (2003).
3. Nehmzow, U. and Walker, K., “Is the Behavior of a Mobile Robot Chaotic?”, ***AISB Journal***, Vol. 1, No. 4, c SSAISB, http://www.aisb.org.uk, (2003).
4. Korayem, M. H. and Bani Rostam, T., “Design, Modeling and Experimental Analysis of Wheeled Mobile Robots”, ***Mechatronics Symposium***, Sidney, Australia, (2004).
5. Golda, D., Lagnemma, K. and Dobuwsky, S. “Probabilistic and Analyses of High-Speed Rough Terrain Mobile Robots”, ***Proceedings of the 2004 IEEE International Conference on Robotics and Automation***, New Orleans, LA, USA, (2004).
6. "MRL-MID Team Description", Unpublished Report, (2005).

7. Korayem, M. H., "Computational Dynamics of Robots", Publisher Iran University of Science and Technology, Tehran, Iran, (2000).
8. Alexander, J. C. and Maddocks, J. H., "On the Kinematics of Wheeled Mobile Robots", ***Int. J. Robotics Res***., Vol. 8, No. 5, (1989), 15-27.
9. Muir, P. F., "Kinematic Formulation for Wheeled Mobile Robot", ***J. of Robotic Syst***., Vol. 4, No. 2, (1987), 281-340.
10. Sheldon, M., "A First Course in Probability", 4th Edition, Hall, Salt Lake City, (1994).
11. Barrentine, Larry, "An Introduction to Design of Experiments: A Simplified Approach", Publisher: Asq Ar, USA, (1999).
12. Albert, H., Bowker, J. and Lieberman, G., "Engineering Statistics", 2nd-Edition Prentice Hall, Englewood, Cliffs, Stanford, USA, (1972).
13. Nehmzow, U., "Mobile Robots a Practical Introduction", 2nd Edition, Publisher: Springer Verlag, Cambridge, (2003).
14. Korayem, M. H., Bani Rostam, T. and Nakhai, A., "Design, Modeling and Experimental Analysis of Wheeled Mobile Robots", ***International Journal of AMT***, Vol. 28, No. 3-4, (2006), 403-416.