

TRANSIENT ANALYSIS OF M/M/R MACHINING SYSTEM WITH MIXED STANDBYS, SWITCHING FAILURES, BALKING, RENEGING AND ADDITIONAL REMOVABLE REPAIRMEN

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Abstract The objective of this paper is to study the M/M/R machine repair queuing system with mixed standbys. The life-time and repair time of units are assumed to be exponentially distributed. Failed units are repaired on FCFS basis. The standbys have switching failure probability q ($0 \leq q \leq 1$). The repair facility of the system consists of R permanent as well as r additional removable repairmen. Due to impatience, the failed units may balk or renege, with a certain probability, on finding all repairmen busy. Transient probabilities of various states are obtained by solving the set of governing equations via Runge-Kutta method. Expressions for various performance measures are established in terms of transient probabilities of system states. By varying different parameters, the system behavior is examined with the help of numerical illustrations.

Keywords Transient Analysis, Machine Interference, Mixed Standbys, Balking, Reneging, State Dependent Rates, Switching Failures, Runge Kutta Method

چکیده هدف این مقاله مطالعه یک سیستم صف M/M/R برای تعمیر ماشین با حالات رزرو است. فرض می شود که طول عمر و زمان تعمیر ماشین ها از توزیع نمایی پیروی می کنند. ماشین های خراب شده بر اساس قاعده اولین خرابی، اولین تعمیر (FCFS) سرویس می شوند. ماشین های رزرو با احتمال خرابی q ، بطوریکه $0 \leq q \leq 1$ باشد جایگزین می شوند. تسهیلات تعمیر سیستم شامل R تعمیرکار دائمی و r تعمیرکار اضافی موقت است. بدلیل ضرورت انجام کار، وقتی کلیه تعمیرکاران مشغول هستند، ممکن است ماشین های خراب شده، با احتمال معین، بصورت موقت در صف تعمیر قرار داده شوند. احتمال انتقال وضعیت های مختلف با حل یک سری معادلات حاکم از طریق روش رانگ-گوتا بدست می آید. معیارهای عملکرد متعددی برحسب احتمال انتقال مربوط به حالات سیستم ارایه می گردد. با تغییر پارامترهای مختلف، رفتار سیستم با کمک نمایش عددی مورد بررسی قرار می گیرد.

1. INTRODUCTION

Performance modeling of machining systems via queue theoretic approach is often done to deal with various congestion situations in manufacturing/production systems; it can also handle the problem of system design and optimal control at various levels. Any production system cannot work continuously because some interference due to failure of machines as well as unavailability of standbys takes place. In such situations the provision of additional repairmen along with spare

units to replace the operating units is recommended. By the use of spare machines, the system works smoothly without interference loss. There is a wide literature available on machine interference problems with spares as it has been an area of interest for many researchers due to its applications in many organizations working in machining environments.

The machine repair problem with cold standbys was first introduced by Taylor and Jackson [1]. Toft and Boothroyd [2] and Sivazlian and Wang [3] analyzed the M/M/c machine repair problem

with cold and warm standbys, respectively. Machine interference problems with the provision of spares has been studied by Wang [4]; Shawky [5]; Wang and Kuo [6]; Jain and Baghel [7]; Sharma et al. [8]; Ke and Wang [9] and many others.

Many researchers have extensively studied queuing as well as machine repair problems by including the concept of balking and reneging. In cases where all the servers are busy and the queue of failed machines is too long, it is not advisable to put a failed unit for waiting in view of long queue therein. For the time being the failed machines may balk (i.e. not join the queue upon failure) or renege (i.e. leave the queue after waiting for some time); in such situations the failed machines perform some other works in which that particular fault is not an obstruction.

M/M/R machine repair problem with reneging and spares was examined by Jain and Prem Lata [10]. Ke and Wang [11] gave the cost analysis of the M/M/r machine repair problem with balking, reneging and server breakdowns. Cost analysis of finite M/M/R queuing system with balking, reneging and server breakdowns was suggested by Wang and Chang [12]. Wang and Ke [13] analyzed a repairable system with warm standbys, balking and reneging. The M/M/R machine interference model with balking, reneging and spares was considered by Jain et al. [14]. N-Policy for a machine repair system with spares and reneging was proposed by Jain et al. [15].

The balking and reneging behavior of the customers may be checked to some extent, if additional servers are provided apart from permanent servers so as to provide a better grade of service at optimal operating cost conditions. Shawky [16] analyzed single server machine interference model with balking, reneging and an additional server for longer queues. M/M/R machine repair problems with spares and additional servers were analyzed by Jain [17]. Jain et al. [18] examined M/M/C/K/N machine repair problem with balking, reneging, spares and additional repairman. Jain [19] discussed N-Policy for a redundant repairable system with spares and additional repairman. Jain et al. [20] investigated state dependent M/M/C/K/N machining system with mixed spares and removable repairmen.

In the existing literature in most of the cases it

has been considered that on failure of an operating unit, the switching device immediately replaces it by an available spare unit and as such the failure of switching from a standby state to an operating state has not been incorporated. The concept of standby switching failures in the reliability with standby system was considered by Lewis [21]. Recently, Wang et al. [22] analyzed M/M/R machine repair problem with balking, reneging and standby switching failure.

In this paper, we consider an M/M/R machine repair system with mixed standbys, state dependent rates, balking and reneging by incorporating the concept of standby switching failure. There is a provision of additional removable repairmen who turn on one by one according to a threshold policy when all permanent repairmen are busy. The rest of the paper is arranged in the following manner. The model under consideration is described in Section 2. In Section 3, we establish Chapman-Kolmogorov equations governing the model. The computation of probabilities has been discussed in Section 4. Expressions for various system characteristics are given in Section 5. In Section 6, a cost analysis is provided by constructing a cost function in terms of various cost elements. In Section 7, the proposed model is illustrated numerically by taking a real time example from a power plant. The effects of various system parameters are also examined. Finally in Section 8, we summarize our findings and highlight the noble features of the investigation made.

2. MODEL DESCRIPTION

Consider a multi-component repairable machining system consisting of M-operating, Y-cold and S-warm standby units under the care of R permanent and r removable additional repairmen. The lifetimes of operating and warm standby units are exponential distributed with rates λ and $\gamma (< \lambda)$, respectively. The model is based on the following assumptions and notations:

- M operating units are required for the normal functioning of the system, however the system may perform its job in short mode with at least $N (< M)$ operating units.

- When an operating unit fails, a cold-standby unit is immediately substituted for it and the operating unit is sent for repairs to the repair facility. The failed units have to wait in a queue if all the repairmen are busy. When all cold-standby units are utilized, the warm standby-unit if available replaces the failed operating unit.

- When the repair of failed unit is completed, it is as good as a new one and joins the operating group (if there are less than M operating units) otherwise it joins the standby group.

- When a standby unit moves into an operating state, its characteristics are same as that of an operating unit.

- The switching device is subject to failure with probability q during the switching from standby state to operating state.

- β is the probability that a failed unit joins the queue when all permanent repairmen are busy, so that the balking probability is given by $(1-\beta)$ and

$$\beta = 1 \text{ if } n < R; \quad 0 \leq \beta < 1 \text{ if } R \leq n \leq L.$$

- The failed unit may renege according to exponential distribution with parameter α .

- The repair facility consists of R permanent homogeneous repairmen, which repair the failed units according to exponential distribution with rate μ . Apart from permanent repairmen, there is a provision of r additional removable heterogeneous repairmen who render repair of failed units with exponentially distributed rate μ_j ($j = 1, 2, \dots, r$).

- Each repairman can repair only one failed unit at a time and perform repairs according to FCFS discipline.

- The first additional repairman turns on when all standbys are used i.e. at threshold level $Y+S$ and turns off as the number of failed units ceases to $(Y+S)$. If the number of failed units is more than a preassigned threshold value T, the remaining additional repairmen turn on one by one with the additional load of T failed units so that the $(j+1)^{\text{th}}$; $j = 1, 2, \dots, r-1$, repairman will be activated when the number of failed units is more than jT but less than or equal to $(j+1)T$; in case when the number of failed units reduces to threshold level jT , it will be removed.

The following probabilities are defined and associated with different states:

$P_{m,n}(t)$ = Probability that there are m ($m = R, R+1, \dots, R+r$) repairmen and n ($n = 0, 1, \dots, L$) failed units in the system at time t.

The state dependent failure rates for the two cases (i) $R \leq Y$ and (ii) $Y < R$ are:

(i) when $R \leq Y$:

$$\lambda(n) = \begin{cases} M\lambda + S\gamma & ; 0 \leq n < R \\ M\lambda\beta + S\gamma & ; R \leq n < Y \\ M\lambda\beta + (S+Y-n)\gamma & ; Y \leq n < S+Y \\ (M+Y+S-n)\lambda\beta & ; S+Y \leq n < L \end{cases}$$

(ii) when $Y < R$:

$$\lambda(n) = \begin{cases} M\lambda + S\gamma & ; 0 \leq n < Y \\ M\lambda + (S+Y-n)\gamma & ; Y \leq n < R \\ M\lambda\beta + (S+Y-n)\gamma & ; R \leq n < S+Y \\ (M+Y+S-n)\lambda\beta & ; S+Y \leq n < L \end{cases}$$

The state dependent repair rates for both cases are given by:

$\mu(n) =$

$$\mu(n) = \begin{cases} n\mu & ; 1 \leq n \leq R \\ R\mu + (n-R)\alpha & ; R+1 \leq n \leq Y+S \\ R\mu + \mu_1 + (n-R-1)\alpha & ; Y+S < n \leq T \\ R\mu + \sum_{i=1}^{j+1} \mu_i + (n-R+j+1)\alpha & ; jT < n \leq (j+1)T, \\ & j = 1, 2, \dots, r-1 \\ R\mu + \sum_{i=1}^r \mu_i + (n-R+r)\alpha & ; rT < n \leq L \end{cases}$$

where

$$L = M + Y + S - N + 1.$$

3. GOVERNING EQUATIONS

The differential-difference equations governing the

model for two cases when (i) $R \leq Y$ and (ii) $Y < R$ are constructed by using appropriate transition rates as follows:

Case I: $R \leq Y$

$$\frac{d}{dt} P_{R,0}(t) = -[M\lambda + S\gamma]P_{R,0}(t) + \mu P_{R,1}(t) \quad (1)$$

$$\frac{d}{dt} P_{R,1}(t) = -[(M\lambda + S\gamma) + \mu(1)]P_{R,1}(t) + [M\lambda(1-q) + S\gamma]P_{R,0}(t) + 2\mu P_{R,2}(t) \quad (2)$$

$$\begin{aligned} \frac{d}{dt} P_{R,n}(t) = & -[(M\lambda + S\gamma) + n\mu]P_{R,n}(t) + [M\lambda(1-q) + S\gamma]P_{R,n-1}(t) + (n+1)\mu P_{R,n+1}(t) \\ & + \sum_{k=0}^{n-2} M\lambda q^{n-\bar{k}+1}(1-q)P_{R,k}(t), \quad 2 \leq n < R \end{aligned} \quad (3)$$

$$\begin{aligned} \frac{d}{dt} P_{R,R}(t) = & -[(M\lambda\beta + S\gamma) + R\mu]P_{R,R}(t) + [M\lambda(1-q) + S\gamma]P_{R,R-1}(t) + (R\mu + \alpha)P_{R,R+1}(t) \\ & + \sum_{k=0}^{R-2} M\lambda q^{n-\bar{k}+1}(1-q)P_{R,k}(t) \end{aligned} \quad (4)$$

$$\begin{aligned} \frac{d}{dt} P_{R,n}(t) = & -[(M\lambda\beta + S\gamma) + \{R\mu + (n-R)\alpha\}]P_{R,n}(t) + [M\lambda(1-q)\beta + S\gamma]P_{R,n-1}(t) \\ & + [R\mu + (n+1-R)\alpha]P_{R,n+1}(t) + \sum_{k=0}^{R-1} M\lambda q^{n-\bar{k}+1}(1-q)P_{R,k}(t) \\ & + \sum_{k=R}^{n-2} M\lambda\beta q^{n-\bar{k}+1}(1-q)P_{R,k}(t), \quad R+1 \leq n \leq Y \end{aligned} \quad (5)$$

$$\begin{aligned} \frac{d}{dt} P_{R,n}(t) = & -[M\lambda\beta + (S+Y-n)\gamma + \{R\mu + (n-R)\alpha\}]P_{R,n}(t) + [R\mu + (n+1-R)\alpha] \\ & P_{R,n+1}(t) + [M\lambda(1-q)\beta + (S+Y-n)\gamma]P_{R,n-1}(t) + \sum_{k=0}^{R-1} M\lambda q^{n-\bar{k}+1}(1-q)P_{R,k}(t) \\ & + \sum_{k=R}^{n-2} M\lambda\beta q^{n-\bar{k}+1}(1-q)P_{R,k}(t), \quad Y+1 \leq n < Y+S \end{aligned} \quad (6)$$

$$\begin{aligned} \frac{d}{dt} P_{R,Y+S}(t) = & -[M\lambda\beta + \{R\mu + (Y+S-R)\alpha\}]P_{R,Y+S}(t) + [M\lambda(1-q)\beta + \gamma]P_{R,Y+S-1}(t) \\ & + \{R\mu + (Y+S+1-R)\alpha\}P_{R,Y+S+1}(t) + \sum_{k=0}^{R-1} M\lambda q^{n-\bar{k}+1}(1-q)P_{R,k}(t) \\ & + \sum_{k=R}^{Y+S-2} M\lambda\beta q^{Y+S-\bar{k}+1}(1-q)P_{R,k}(t) \end{aligned} \quad (7)$$

$$\begin{aligned} \frac{d}{dt} P_{R+1,Y+S+1}(t) = & -[(M-1)\lambda\beta + R\mu + \mu_1 + (Y+S-R)\alpha]P_{R+1,Y+S+1}(t) \\ & + [M\lambda\beta]P_{R,Y+S}(t) + [R\mu + \mu_1 + (Y+S+1-R)\alpha]P_{R+1,Y+S+2}(t) \\ & + \sum_{k=0}^{R-1} M\lambda q^{n-\bar{k}+1}P_{R,k}(t) + \sum_{k=R}^{Y+S-1} M\lambda\beta q^{Y+S-\bar{k}+1}P_{R,k}(t) \end{aligned} \quad (8)$$

$$\begin{aligned} \frac{d}{dt} P_{R+1,n}(t) = & -[(M+Y+S-n)\lambda\beta + R\mu + \mu_1 + (n-\bar{R}+1)\alpha]P_{R+1,n}(t) + (M+Y+S-n+1) \\ & \lambda\beta P_{R+1,n-1}(t) + \mu(n+1)P_{R+1,n+1}(t), \quad Y+S+2 \leq n < T \end{aligned} \quad (9)$$

$$\begin{aligned} \frac{d}{dt} P_{R+1,T}(t) = & -[(M+Y+S-T)\lambda\beta + R\mu + \mu_1 + (n-\bar{R}+1)\alpha]P_{R+1,T}(t) \\ & + (M+Y+S-T+1)\lambda\beta P_{R+1,T-1}(t) + [R\mu + \mu_1 + (n-\bar{R}+2)\alpha]P_{R+2,T+1}(t), \end{aligned} \quad (10)$$

$$\begin{aligned} \frac{d}{dt} P_{R+j+1,n}(t) = & -[(M+Y+S-n)\lambda\beta + \\ & R\mu + \mu_1 + (n - \overline{R+1})\alpha] P_{R+j+1,n}(t) + \\ & [(M+Y+S-n+1)\lambda\beta] P_{R+j+1,n-1}(t) + R\mu + \\ & \left[\sum_{i=1}^{j+1} \mu_i + (n - \overline{R+j+1})\alpha \right] P_{R+j+1,n+1}(t), \\ & jT < n \leq (j+1)T; j=1,2,\dots,r-1 \end{aligned} \quad (11)$$

$$\begin{aligned} \frac{d}{dt} P_{R+r,n}(t) = & -[(M+Y+S-n)\lambda\beta + \\ & \mu(n)] P_{R+r,n}(t) + \\ & (M+Y+S-n+1)\lambda\beta P_{R+r,n-1}(t) + R\mu + \\ & \sum_{i=1}^r \mu_i + (n - \overline{R+r})\alpha P_{R+r,n+1}(t), \quad rT \leq n < L \end{aligned} \quad (12)$$

$$\begin{aligned} \frac{d}{dt} P_{R+r,L}(t) = \\ \left[R\mu + \sum_{i=1}^r \mu_i + (L - \overline{R+r})\alpha \right] P_{R+r,K}(t) + \\ N\lambda\beta P_{R+r,L-1}(t) \end{aligned} \quad (13)$$

Case II: $R > Y$

The transient state equations for this case are the same as Equations 1-2 and 8-13 and the remaining Equations are given below:

$$\begin{aligned} \frac{d}{dt} P_{R,n}(t) = & -[(M\lambda + S\gamma) + n\mu] P_{R,n}(t) + \\ & [M\lambda(1-q) + S\gamma] P_{R,n-1}(t) + (n+1)\mu P_{R,n+1}(t) \\ & + \sum_{k=0}^{n-2} M\lambda q^{n-\overline{k+1}} (1-q) P_{R,k}(t), \\ & 2 \leq n < Y \end{aligned} \quad (14)$$

$$\begin{aligned} \frac{d}{dt} P_{R,n}(t) = & -[M\lambda + (S+Y-n)\gamma + \\ & n\mu] P_{R,n}(t) + [M\lambda(1-q) + (S+Y-n)\gamma] P_{R,n-1}(t) + \\ & (n+1)\mu P_{R,n+1}(t) + \sum_{k=0}^{n-2} M\lambda q^{n-\overline{k+1}} (1-q) P_{R,k}(t), \\ & Y \leq n < R-1 \end{aligned} \quad (15)$$

$$\begin{aligned} \frac{d}{dt} P_{R,R}(t) = & -[(M\lambda\beta + (S+Y-R)\gamma) + \\ & R\mu] P_{R,R}(t) + [M\lambda(1-q) + (S+Y-R+1)\gamma] \\ & P_{R,R-1}(t) + (R\mu + \alpha) P_{R,R+1}(t) + \\ & \sum_{k=0}^{R-2} M\lambda q^{n-\overline{k+1}} (1-q) P_{R,k}(t) \end{aligned} \quad (16)$$

$$\begin{aligned} \frac{d}{dt} P_{R,n}(t) = & -[(M\lambda\beta + (S+Y-n)\gamma) + \\ & \{R\mu + (n-R)\alpha\}] P_{R,n}(t) + [R\mu + (n+1-R)\alpha] \\ & P_{R,n+1}(t) + [M\lambda(1-q)\beta + (S+Y-n)\gamma] \\ & P_{R,n-1}(t) + \sum_{k=0}^{R-1} M\lambda q^{n-\overline{k+1}} (1-q) P_{R,k}(t) + \\ & \sum_{k=R}^{n-2} M\lambda\beta q^{n-\overline{k+1}} (1-q) P_{R,k}(t), \\ & R+1 \leq n < Y+S \end{aligned} \quad (17)$$

$$\begin{aligned} \frac{d}{dt} P_{R,Y+S}(t) = \\ -[M\lambda\beta + \{R\mu + (Y+S-R)\alpha\}] P_{R,Y+S}(t) + \\ [M\lambda(1-q)\beta + \gamma] P_{R,Y+S-1}(t) + \\ \{R\mu + (Y+S+1-R)\alpha\} P_{R,Y+S+1}(t) + \\ \sum_{k=0}^{R-1} M\lambda q^{n-\overline{k+1}} (1-q) P_{R,k}(t) + \\ \sum_{k=R}^{Y+S-2} M\lambda\beta q^{Y+S-\overline{k+1}} (1-q) P_{R,k}(t) \end{aligned} \quad (18)$$

4. COMPUTATION OF PROBABILITIES

The transient state solution of the model can be obtained by solving the set of transient state differential equations. There are various methods for solving the set of differential equations. We prefer to employ the fourth order Runge-Kutta method. For this purpose we use the software MATLAB in which Runge-Kutta algorithm of fourth order can be implemented by using "ode 45" function.

5. SOME PERFORMANCE INDICES

In this section, we find the expressions for various performance measures in terms of probabilities as follows:

- Expected number of failed units in the system at time t is

$$E\{N(t)\} = \sum_{n=1}^{Y+S} nP_{R,n}(t) + \sum_{n=Y+S+1}^T nP_{R+1,n}(t) + \sum_{j=1}^{r-1} \sum_{n=jT+1}^{(j+1)T} nP_{R+j+1,n}(t) + \sum_{n=rT+1}^L nP_{R+r,n}(t) \quad (19)$$

- Expected number of unused cold and warm standby units at time t are given as

$$E\{UCS(t)\} = \sum_{n=0}^Y (Y-n)P_{R,n}(t) \quad (20)$$

and

$$E\{UWS(t)\} = S \sum_{n=0}^Y P_{R,n}(t) + \sum_{n=Y+1}^{Y+S} (Y+S-n)P_{R,n}(t) \quad (21)$$

- Expected number of operating units at time t is obtained by using

$$E\{O(t)\} = M - \left[\sum_{n=Y+S+1}^T (n - \overline{Y+S})P_{R+1,n}(t) + \sum_{j=1}^{r-1} \sum_{n=jT+1}^{(j+1)T} (n - \overline{Y+S})P_{R+j+1,n}(t) + \sum_{n=rT+1}^L (n - \overline{Y+S})P_{R+r,n}(t) \right] \quad (22)$$

- Expected number of idle permanent repairmen at time t is

$$E\{I(t)\} = \sum_{n=0}^{R-1} (R-n)P_{R,n}(t) \quad (23)$$

- Expected number of busy permanent repairmen at time t is

$$E\{B(t)\} = R - E\{I(t)\} \quad (24)$$

- Expected number of busy removable repairmen at time t is

$$E\{BR(t)\} = \sum_{n=Y+S+1}^T P_{R+1,n}(t) + \sum_{j=1}^{r-1} \sum_{n=jT+1}^{(j+1)T} (j+1)P_{R+j+1,n}(t) + r \sum_{n=rT+1}^L P_{R+r,n}(t) \quad (25)$$

- Average switching failure rate at time t is given by

$$ASF(t) = \sum_{n=1}^{Y+S} M\lambda q P_{R,n-1}(t) \quad (26)$$

- The average rate of failed units which have balked/renege at time t , is obtained using

$$ALR(t) = ABR(t) + ARR(t) \quad (27)$$

Average balking rate at time t ,

$$ABR(t) = \sum_{n=R}^{Y+S} M\lambda(1-\beta)P_{R,n}(t) + \sum_{n=Y+S+1}^T [(M+Y+S-n)\lambda(1-\beta)]P_{R+1,n}(t) + \sum_{j=1}^{r-1} \sum_{n=jT+1}^{(j+1)T} [(M+Y+S-n)\lambda(1-\beta)]P_{R+j+1,n}(t) + \sum_{n=rT+1}^L [(M+Y+S-n)\lambda(1-\beta)]P_{R+r,n}(t) \quad (28)$$

- Average renege rate of failed units at time t ,

$$\begin{aligned}
ARR(t) = & \sum_{n=R+1}^{Y+S} (n-R)\alpha P_{R,n}(t) \\
& + \sum_{n=Y+S+1}^T \left[(n-\overline{R+1})\alpha \right] P_{R+1,n}(t) \\
& + \sum_{j=1}^{r-1} \sum_{n=jT+1}^{(j+1)T} \left[(n-\overline{R+j+1})\alpha \right] P_{R+j+1,n}(t) \\
& + \sum_{n=rT+1}^L \left[(n-\overline{R+r})\alpha \right] P_{R+r,n}(t)
\end{aligned} \tag{29}$$

- Throughput of the system is given by

$$\begin{aligned}
E\{T(t)\} = & \sum_{n=1}^R n\mu P_{R,n}(t) + \sum_{n=R}^{Y+S} [R\mu + (n-R)\alpha] P_{R,n}(t) \\
& + \sum_{n=Y+S+1}^T \left[R\mu + \{\mu_1 + (n-\overline{R+1})\alpha\} \right] P_{R+1,n}(t) \\
& + \sum_{j=1}^{r-1} \sum_{n=jT+1}^{(j+1)T} \left[R\mu + \sum_{i=1}^{j+1} \mu_i + (n-\overline{R+j+1})\alpha \right] P_{R+j+1,n}(t) \\
& + \sum_{n=rT+1}^L \left[R\mu + \sum_{i=1}^r \mu_i + (n-\overline{R+r})\alpha \right] P_{R+r,n}(t)
\end{aligned} \tag{30}$$

6. COST ANALYSIS

In this section, we establish the cost function $E(C)$ in terms of various cost elements, which are defined as follows:

- C_N = Cost per unit time of an operating unit
- C_Y = Cost per unit time for providing a cold spare unit
- C_S = Cost per unit time for providing a warm standby unit
- C_1 = Cost per unit time of per idle permanent repairman
- C_B = Cost per unit time of per permanent repairman when he is busy in providing repair
- C_j = Cost per unit time of j^{th} ($j = 1, 2, \dots, r$) additional repairman when he is busy in providing repair
- C_{LR} = Cost per unit time when one failed unit balks or reneges

C_{SF} = Cost per unit time when one standby unit has switching failure

Now we formulate the expected total cost incurred per unit time as

$$\begin{aligned}
E(C) = & C_N E\{O(t)\} + C_Y E\{UCS(t)\} + C_S E\{UWS(t)\} + \\
& C_1 E\{I(t)\} + C_B E\{B(t)\} + C_{LR} ALR(t) + C_{SF} ASF(t) + \\
& C_1 \sum_{n=Y+S+1}^T P_{R+1,n}(t) + \\
& \sum_{j=1}^{r-1} C_{j+1} \sum_{n=jT+1}^{(j+1)T} P_{R+j+1,n}(t) + \\
& C_r \sum_{n=rT+1}^L P_{R+r,n}(t)
\end{aligned} \tag{31}$$

7. NUMERICAL ILLUSTRATION AND SENSITIVITY ANALYSIS

To illustrate analytical model and validity of computational procedure, we consider the model, which may be applicable in the power plant. It is considered that the power plant requires 70 MW for the normal functioning which is fulfilled by 14 on-line units of 5 MW, however it can perform its job in short mode with at least 4 units of 5 MW. There is a provision of 3 cold standby generators and 2 warm standby generators. The operating online units and the warm standby generators may fail according to exponential distribution with a rate $\lambda = 0.5$ and $\gamma = 0.1$ respectively. When any one of the on-line units fails, it is replaced by an available cold standby generator. The warm standby generator will be used when all three cold standby generators are utilized due to failure of on-line units. It is also realized that there is a possibility of failures during switching of standby generators to an operating on-line state and the switching failure probability is $q = 0.2$. There are $R = 2$ permanent and $r = 2$ additional repairmen available in the repair facility. Let $\beta = 0.8$ and $\alpha = 0.02$ be the joining probability and reneging parameter of failed on-line units, respectively when the permanent repairmen are

busy. The threshold level of failed units at which the second additional repairman turns on is $T = 7$. The permanent repairmen fix the failed component according to exponential distribution with a rate of $\mu = 1$ and the first and second additional repairmen work with a rate $\mu_1 = 0.8$ and $\mu_2 = 0.6$, respectively.

To solve the set of transient state equations for the above model, the coding for programs has been done with the software MATLAB by using “ode 45” function for Runge- Kutta approach to compute the transient probabilities. The probabilities for time $t = 2$ are obtained as:

$$P_{2,0} = 0.0008, P_{2,1} = 0.0046, P_{2,2} = 0.0128, P_{2,3} = 0.0273, P_{2,4} = 0.04930, P_{2,5} = 0.0945, P_{3,6} = 0.1431, P_{3,7} = 0.1740, P_{4,8} = 0.1624, P_{4,9} = 0.1319, P_{4,10} = 0.0923, P_{4,11} = 0.0540, P_{4,12} = 0.0230, P_{4,13} = 0.0160, P_{4,14} = 0.0114, P_{4,15} = 0.0032, P_{4,16} = 0.0007$$

Using the above probabilities the performance measures have been obtained using Equations 19-30 as:

$$E\{N(t)\} = 7.566147, E\{T(t)\} = 3.014633, E\{O(t)\} = 11.2767, E\{UCS(t)\} = 0.0245, E\{UWS(t)\} = 0.1405, ASF(t) = 0.1329, ABR(t) = 1.1223, ARR(t) = 0.2315, E(C) = 2888.02.$$

Now we examine the effects of different parameters on various performance measures and the expected total cost. The graphical presentation has been done in Figures 1-6, by setting default parameters as $M = 14, Y = 3, S = 2, N = 4, R = 2, r = 2, T = 7, \gamma = 0.1, \beta = 0.8, \mu = 1, \mu_1 = 0.8, \mu_2 = 0.6, \alpha = 0.02, q = 0.2$. For Figures 1, 3, 5 and Figures 2, 4, 6 we set $\lambda = 0.5$ and $\lambda = 0.3$, respectively and the profiles have been examined regarding the expected number of failed units in the system and throughput for different values of $\lambda, \gamma, \mu, \mu_1, \mu_2, \alpha, \beta, q$. It is noted that the expected number of failed units $E\{N(t)\}$ and throughput of the system $E\{T(t)\}$ initially increase sharply as time t increases and become almost constant after some time in all cases.

From Figures 1(a) and (b) the effects of failure rate of operating units (λ) and standby units (γ) have been explored respectively, on the expected number of failed units in the system. It is observed that $E\{N(t)\}$ increases for the increasing values

of λ and γ as expected; however the effect of λ is more prominent. Figures 2(a)-(b) show the

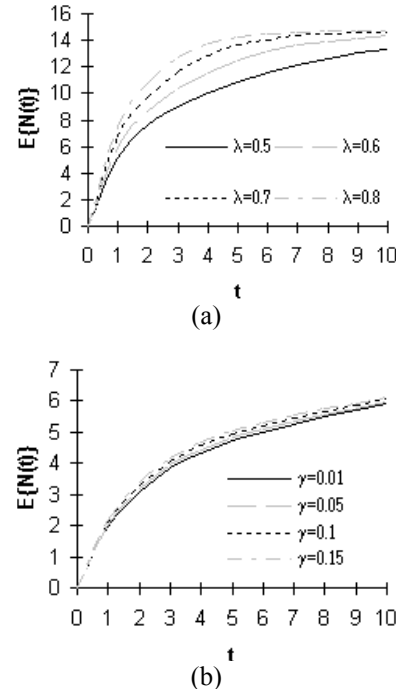


Figure 1. Expected number of failed units in the system by varying time t and (a) λ , (b) γ .

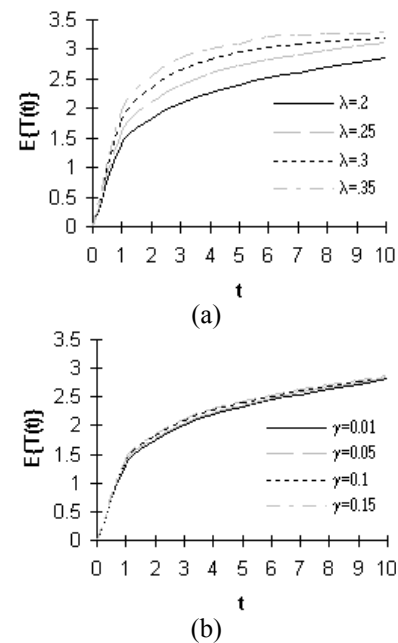


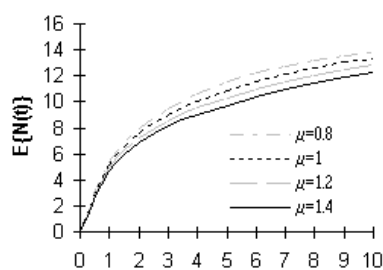
Figure 2. Throughput of the system by varying time t and (a) λ , (b) γ .

increasing trend of $E\{T(t)\}$ for different values of λ and γ ; but the effect of γ is not much significant. Figures 3(a)-(c) and 4(a)-(c) display the effect of repair rates μ , μ_1 , μ_2 on $E\{N(t)\}$ and $E\{T(t)\}$, respectively. As expected, $E\{N(t)\}$ decreases but $E\{T(t)\}$ increases as repair rate increases, however the effect of repair rate of permanent repairmen is more significant in comparison to the repair rate of additional repairmen.

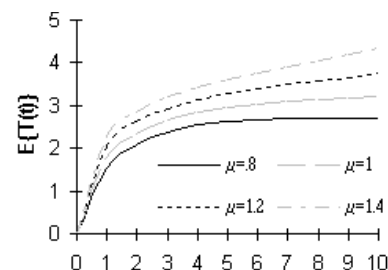
The effects of the reneging parameter on $E\{N(t)\}$ and $E\{T(t)\}$ are depicted in Figures 5(a) and 6(a), respectively. It is observed that $E\{N(t)\}$ decreases but $E\{T(t)\}$ increases for increasing values of α . To exhibit the effect of β on $E\{N(t)\}$ and $E\{T(t)\}$, we

set $q = 0.1$ in Figures 5(b) and 6(b) and notice that both $E\{N(t)\}$ and $E\{T(t)\}$ increase with the increment in the joining probability β .

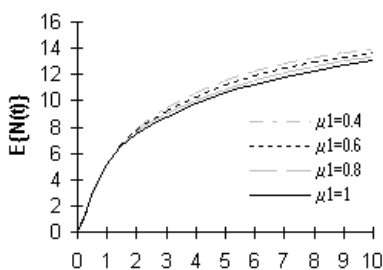
In Tables 1-3, we display the expected number of operating units $E\{O(t)\}$, expected number of unused cold standbys $E\{UCS(t)\}$ and warm standby units $E\{UWS(t)\}$, expected number of idle permanent repairmen $E\{I(t)\}$, expected number of busy permanent repairmen $E\{B(t)\}$, expected number of busy removable repairmen $E\{BR(t)\}$, average switching failure rate $ASF(t)$, average balking rate $ABR(t)$, average reneging rate $ARR(t)$ and total expected cost $E(C)$ by varying different input parameters and time t . The



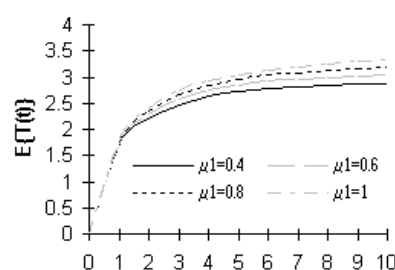
(a)



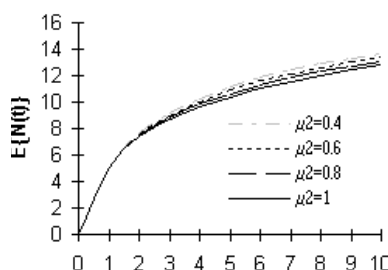
(a)



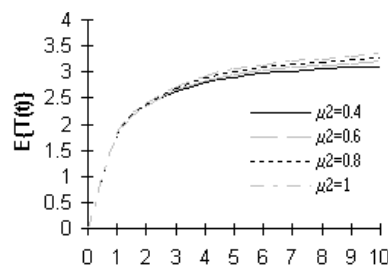
(b)



(b)



(c)



(c)

Figure 3. Expected number of failed units in the system by varying time t and (a) μ , (b) μ_1 , (c) μ_2 .

Figure 4. Throughput of the system by varying time t and (a) μ , (b) μ_1 , (c) μ_2 .

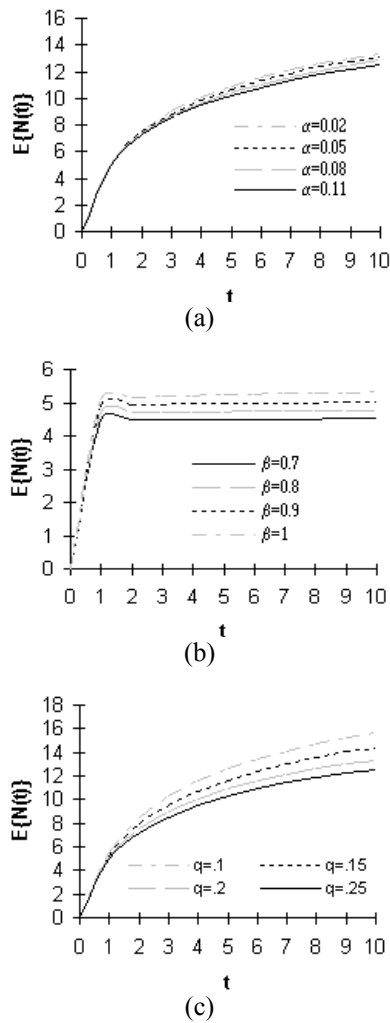


Figure 5. Expected number of failed units in the system by varying time t and (a) α , (b) β (c) q .

default parameters are chosen as $\lambda = 0.3$, $\gamma = 0.05$, $\beta = 0.8$, $\mu = 0.8$, $\mu_1 = 0.6$, $\mu_2 = 0.4$, $\alpha = 0.02$, $q = 0.1$, $C_N = \text{Rs. } 200/\text{day}$, $C_Y = \text{Rs. } 150/\text{day}$, $C_S = \text{Rs. } 100/\text{day}$, $C_1 = \text{Rs. } 80/\text{day}$, $C_B = \text{Rs. } 125/\text{day}$, $C_1 = \text{Rs. } 100/\text{day}$, $C_2 = \text{Rs. } 100/\text{day}$, $C_{LR} = \text{Rs. } 200/\text{day}$, $C_{SF} = \text{Rs. } 100/\text{day}$. It is noted that as time increases $E\{B(t)\}$, $E\{BR(t)\}$, $ABR(t)$ and $ARR(t)$ increase whereas $E\{O(t)\}$, $E\{UCS(t)\}$, $E\{UWS(t)\}$, $ASF(t)$ and $E(C)$ decrease. The results for different values of failure rates λ and γ are summarized in Table 1. It is observed that the number of busy repairmen increases as the failure rate increases. Also increment in λ and γ results in an increase in average balking and renegeing rates. Table 2 displays the results for various

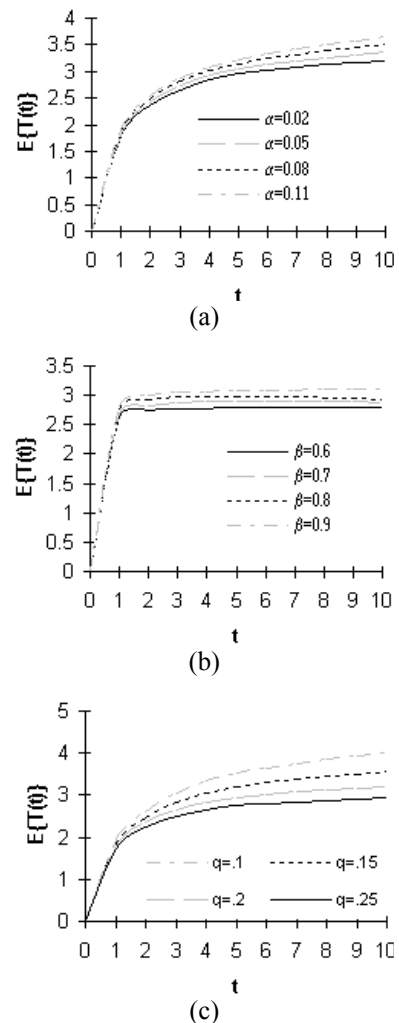


Figure 6. Throughput of the system by varying time t and (a) μ , (b) μ_1 , (c) μ_2 .

performance measures by varying repair rates μ , μ_1 and μ_2 . It is noted that by increasing the repair rates, the number of busy repairmen and average balking and renegeing rates can be reduced. Tables 3 and 4 summarize results for various performance measures and total expected cost $E(C)$ by varying (α, β) and q , respectively.

Overall, based on numerical experiment we conclude that

- The expected number of failed units $E\{N(t)\}$, and the throughput of the system $E\{T(t)\}$ both initially increase sharply as time t increases, but after some time attain asymptotically constant values as expected in physical situations.

TABLE 1. Some Performance Measures for Different Values of t and (λ, γ) .

(λ, γ)	t	$E\{O(t)\}$	$E\{UCS(t)\}$	$E\{UWS(t)\}$	$E\{B(t)\}$	$E\{BR(t)\}$	ASF(t)	ABR(t)	ARR(t)	E(C)
(0.1,0.02)	0	14.00	3.00	2.00	0.00	0.00	0.1400	0.0000	0.0000	3624.00
	1	14.00	1.88	1.85	0.96	0.00	0.1310	0.0743	0.0002	3496.95
	2	13.98	1.43	1.76	1.28	0.02	0.1279	0.1276	0.0010	3444.25
	3	13.95	1.24	1.71	1.41	0.04	0.1264	0.1571	0.0023	3418.28
	4	13.92	1.15	1.69	1.46	0.06	0.1263	0.1763	0.0035	3405.70
	5	13.90	1.11	1.69	1.49	0.08	0.1275	0.1907	0.0046	3401.16
(0.2,0.04)	0	14.00	3.00	2.00	0.00	0.00	0.2800	0.0000	0.0000	3638.00
	1	13.95	1.10	1.69	1.51	0.04	0.2508	0.3443	0.0023	3449.33
	2	13.72	0.60	1.35	1.77	0.20	0.2132	0.4886	0.0119	3345.86
	3	13.39	0.42	1.10	1.85	0.39	0.1805	0.5509	0.0244	3259.08
	4	13.08	0.33	0.95	1.88	0.57	0.1586	0.5876	0.0396	3189.45
	5	12.80	0.28	0.85	1.90	0.71	0.1447	0.6142	0.0616	3136.25
(0.3,0.05)	0	14.00	3.00	2.00	0.00	0.00	0.4200	0.0000	0.0000	3652.00
	1	13.79	0.61	1.39	1.77	0.15	0.3291	0.7079	0.0090	3418.64
	2	12.98	0.22	0.78	1.93	0.62	0.1989	0.8463	0.0391	3198.38
	3	12.08	0.11	0.46	1.97	1.04	0.1237	0.8594	0.1017	3004.47
	4	11.31	0.07	0.31	1.98	1.34	0.0848	0.8667	0.2578	2867.87
	5	10.66	0.05	0.23	1.99	1.54	0.0638	0.8683	0.5431	2787.73

TABLE 2. Some Performance Measures for Different Values of t and (μ, μ_1, μ_2) .

(μ, μ_1, μ_2)	t	$E\{O(t)\}$	$E\{UCS(t)\}$	$E\{UWS(t)\}$	$E\{B(t)\}$	$E\{BR(t)\}$	ASF(t)	ABR(t)	ARR(t)	E(C)
(0.8,0.6,0.4)	0	14.00	3.00	2.00	0.00	0.00	0.4200	0.0000	0.0000	3652.00
	1	13.79	0.61	1.39	1.77	0.15	0.3291	0.7079	0.0090	3418.64
	2	12.98	0.22	0.78	1.93	0.62	0.1989	0.8463	0.0391	3198.38
	3	12.08	0.11	0.46	1.97	1.04	0.1237	0.8594	0.1017	3004.47
	4	11.31	0.07	0.31	1.98	1.34	0.0848	0.8667	0.2578	2867.87
	5	10.66	0.05	0.23	1.99	1.54	0.0638	0.8683	0.5431	2787.73
(1,0.8,0.6)	0	14.00	3.00	2.00	0.00	0.00	0.4200	0.0000	0.0000	3652.00
	1	13.83	0.70	1.46	1.73	0.12	0.3400	0.6791	0.0076	3435.96
	2	13.22	0.32	0.94	1.89	0.49	0.2366	0.8413	0.0305	3268.15
	3	12.59	0.20	0.68	1.94	0.81	0.1750	0.8484	0.0678	3131.51
	4	12.08	0.14	0.54	1.96	1.04	0.1419	0.8515	0.1436	3037.62
	5	11.67	0.12	0.46	1.96	1.21	0.1237	0.8591	0.2707	2979.51
(1.2,1,0.8)	0	14.00	3.00	2.00	0.00	0.00	0.4200	0.0000	0.0000	3652.00
	1	13.86	0.78	1.51	1.68	0.10	0.3493	0.6493	0.0064	3451.04
	2	13.41	0.42	1.10	1.85	0.38	0.2713	0.8058	0.0238	3326.62
	3	12.98	0.31	0.90	1.89	0.62	0.2275	0.8239	0.0469	3236.80
	4	12.64	0.26	0.80	1.91	0.79	0.2064	0.8357	0.0848	3180.64
	5	12.38	0.23	0.76	1.92	0.92	0.1976	0.8361	0.1422	3148.54

TABLE 3. Some Performance Measures for Different Values of t and (α, β) .

(α, β)	t	$E\{O(t)\}$	$E\{UCS(t)\}$	$E\{UWS(t)\}$	$E\{B(t)\}$	$E\{BR(t)\}$	ASF(t)	ABR(t)	ARR(t)	E(C)
(0.01,0.7)	0	14.00	3.00	2.00	0.00	0.00	0.4200	0.0000	0.0000	3652.00
	1	13.83	0.67	1.60	1.76	0.12	0.3762	1.1639	0.0037	3549.53
	2	13.14	0.31	1.11	1.91	0.55	0.2795	1.5020	0.0173	3406.28
	3	12.29	0.20	0.80	1.94	0.99	0.2093	1.6257	0.0464	3243.00
	4	11.51	0.14	0.63	1.96	1.34	0.1668	1.6865	0.1248	3105.73
	5	10.83	0.11	0.52	1.97	1.61	0.1407	1.7223	0.2837	3004.99
(0.02,0.8)	0	14.00	3.00	2.00	0.00	0.00	0.4200	0.0000	0.0000	3652.00
	1	13.79	0.61	1.39	1.77	0.15	0.3291	0.7079	0.0090	3418.64
	2	12.98	0.22	0.78	1.93	0.62	0.1989	0.8463	0.0391	3198.38
	3	12.08	0.11	0.46	1.97	1.04	0.1237	0.8594	0.1017	3004.47
	4	11.31	0.07	0.31	1.98	1.34	0.0848	0.8667	0.2578	2867.87
	5	10.66	0.05	0.23	1.99	1.54	0.0638	0.8683	0.5431	2787.73
(0.03,0.9)	0	14.00	3.00	2.00	0.00	0.00	0.4200	0.0000	0.0000	3652.00
	1	13.75	0.56	1.22	1.78	0.18	0.2898	0.3251	0.0159	3308.31
	2	12.83	0.16	0.55	1.95	0.68	0.1438	0.3653	0.0658	3044.51
	3	11.89	0.07	0.28	1.98	1.08	0.0750	0.3547	0.1765	2848.85
	4	11.11	0.03	0.16	1.99	1.33	0.0450	0.3381	0.4415	2735.01
	5	10.47	0.02	0.11	1.99	1.47	0.0309	0.3221	0.8827	2689.53

TABLE 4. Some Performance Measures for Different Values of t and q .

q	t	$E\{O(t)\}$	$E\{UCS(t)\}$	$E\{UWS(t)\}$	$E\{B(t)\}$	$E\{BR(t)\}$	ASF(t)	ABR(t)	ARR(t)	E(C)
0.05	0	14.00	3.00	2.00	0.00	0.00	0.2100	0.0000	0.0000	3631.00
	1	13.82	0.68	1.55	1.75	0.13	0.1829	0.7550	0.0078	3442.40
	2	13.01	0.27	0.92	1.92	0.61	0.1174	0.9229	0.0380	3230.95
	3	12.05	0.14	0.57	1.96	1.07	0.0759	0.9504	0.0997	3027.94
	4	11.19	0.09	0.39	1.98	1.41	0.0532	0.9446	0.2549	2876.13
	5	10.46	0.06	0.29	1.98	1.65	0.0404	0.9294	0.5464	2781.38
0.1	0	14.00	3.00	2.00	0.00	0.00	0.4200	0.0000	0.0000	3652.00
	1	13.79	0.61	1.39	1.77	0.15	0.3291	0.7079	0.0090	3418.64
	2	12.98	0.22	0.78	1.93	0.62	0.1989	0.8463	0.0391	3198.38
	3	12.08	0.11	0.46	1.97	1.04	0.1237	0.8601	0.1017	3004.47
	4	11.31	0.07	0.31	1.98	1.34	0.0848	0.8480	0.2578	2867.87
	5	10.66	0.05	0.23	1.99	1.54	0.0638	0.8299	0.5431	2787.73
0.2	0	14.00	3.00	2.00	0.00	0.00	0.8400	0.0000	0.0000	3694.00
	1	13.72	0.50	1.12	1.81	0.20	0.5311	0.6328	0.0118	3370.51
	2	12.92	0.16	0.55	1.95	0.65	0.2840	0.7245	0.0420	3139.25
	3	12.13	0.07	0.30	1.98	1.00	0.1648	0.7203	0.1079	2966.03
	4	11.50	0.04	0.20	1.99	1.22	0.1099	0.7015	0.2682	2857.49
	5	10.98	0.03	0.15	1.99	1.36	0.0825	0.6815	0.5448	2801.76

- The expected number of failed units in the system $E\{N(t)\}$ increases with the increasing values of λ , γ , β but decreases with μ , μ_1 , μ_2 , α and q ; the patterns of graphs match with realistic situations as failure rates and balking probability (repair rate and reneging parameter) may cause increment (decrement) in queue length.
- The throughput of the system $E\{T(t)\}$ increases as the parameters λ , γ , μ , μ_1 , μ_2 , β , α increase but decreases with the increasing values of switching failure probability q .
- Average balking and reneging rates increase with the failure rates of operating and standby units, respectively; but these can be reduced by increasing the repair rates. This pattern is in agreement with the real time situations.

8. CONCLUSION

In this paper, M/M/R machining system with mixed spares, balking and reneging was presented. The provision of standby switching failure makes the model more feasible in depicting real time machining system with spares. When all the permanent servers are busy and the queue of failed machines is long, the provision of mixed standbys and additional removable repairmen according to a threshold policy to maintain the desired availability of the real time production/manufacturing system may be helpful for a system designer to ensure the high grade of service (GOS) at optimum cost.

Runge-Kutta method was employed to establish transient state solution of the model and provided various performance measures as well as the cost function in a transient state. Numerical results are provided to highlight the effect of various system parameters on the performance measures. By taking numerical illustration, it has been shown that the combination of additional servers and spare-part support is of great importance in many realistic situations of machining environment.

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