# ECONOMIC PRODUCTION QUANTITY MODELS WITH SHORTAGE, PRICE AND STOCK-DEPENDENT DEMAND FOR DETERIORATING ITEMS

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**Abstract** This investigation presents an economic production quantity (EPQ) model for deteriorating items with stock-dependent demand and shortages. It is assumed that a constant fraction of the on-hand inventory deteriorates and demand rate depends upon the amount of the stock level. Expression for various optimal indices as well as cost analysis are provided. By taking numerical illustration, sensitivity analysis has been carried out. For cost optimization, Newton's method is employed.

**Keywords** Economic Production Quantity, Deterioration, Stock-Dependent Demand, Cost Function, Newton's Method

**چکیده** در این مقاله، مدل مقدار تولید اقتصادی برای کالاهای فاسد شدنی با کمبود و تقاضای وابسته به موجودی مورد مطالعه قرار گرفته است. چنین فرض شده که کسری ثابت از موجودی از دست رفته، فاسد شده است و نرخ تقاضا وابسته به سطح موجودی است که برای حالات بهینه مختلف و همچنین برای هزینه های متفاوت تجزیه و تحلیل شده است. آنالیز حساسیت توسط مثال عددی بیان شده و برای بهینه سازی هزینه، از روش نیوتن استفاده شده است.

## 1. INTRODUCTION

In recent years, most researches in the area of inventory control have been oriented towards the development of more realistic and practical models for decision makers. Recently, deteriorating of items in inventory systems has become an interesting topic due to its practical importance. In the competitive market situation, the customers are influenced by the marketing policies such as the attractive display of items in the showroom or in big malls. The display of items in large number has a motivational effect on the buyers and attracts the people to buy more, so the demand is influenced by stock status.

It is worthwhile to have a look at the recent studies by researchers on an EPQ model for a single-item inventory having a stock-dependent demand rate. In the traditional inventory model, as described by Silver and Peterson [12], sales at a retail level may be proportional to the amount of inventory displayed. Gupta and Vrat [6] have introduced the stock dependent phenomena in modeling inventory systems assuming the consumption rate to be a function of the order quantity. Datta and Pal [2] considered the demand rate as a linear function of on-hand inventory in developing the inventory models for deteriorating items.

The manufacturing flexibility which is capable of adjusting the production rate with the variability in the market demand is known as volume flexibility [11]. Urban [14] considered the inventory system in which the demand rate of the product is a function of the on-hand inventory. Giri et al. [5] extended the model of Urban [14] to the case of items deteriorating overtime. Padmanabhan and Vrat [7] introduced an EOQ model for perishable items under stock dependent selling rates. Ray and Chaudhari [8] discussed an EOQ model with stock-dependent demand, shortage, inflation and time discounting of the different costs and prices being associated with the system.

Ray et al. [9] studied the inventory problem with a stock-dependent demand rate and two levels of storage, rented warehouses (RW) and owned warehouses (OW). Giri and Chaudhuri [4] investigated an inventory model with deteriorating items and discussed both the cases of non-linear time-dependent and stock-dependent holding costs. Dye [3] developed a deteriorating inventory model with stock dependent demand including the conditions of allowable shortage and permissible delay in payments. Hart and Brady [17] developed an optimal control model for cost-effective management of pollution, including two state variables, pollution stock and ecosystem quality. Papachristos and Skouri [18] considered a model where the demand rate is a convex decreasing function of the selling price and the backlogging rate is a time-dependent function, which ensures that the rate of backlogged demand increases as the waiting time to replenishment point decreases. Mondal et al. [19] developed an inventory model for ameliorating items. These items include the fast growing animals like ducks, pigs, broilers, etc. in poultry farm, high breed fish in pond, etc. When these items stay at farm or pond or in the sales counter or distribution centre, the stock either increases due to growth or decreases due to death. Sana and Chaudhari [10] considered a volume flexible manufacturing system for a deteriorating item with an inventory-level-dependent demand rate.

Subbaiah et al. [13] considered the demand rate dependent on stock status assuming the life time of the perishable items to be random and follows the three-parameter Weibull distribution, which includes the constant, increasing and decreasing rates of deterioration for different values of the parameters. Teng and Chang [16] established an economic production quantity (EPQ) model for deteriorating items when the demand rate depending not only the on-display stock level but also on the selling price per unit. Maiti and Mathi [15] proposed an appropriate solution to the contradiction faced during the inventory of displayed damageable items where both demand and damageability are stock-dependent. Chen and Chen [1] developed a tactical-level decision model that solves the production-scheduling problem taking into account the dynamic nature of customer demand, which is partially controllable through pricing schemes.

In the present paper, an EPQ model with shortage has been developed by incorporating the deterioration effect and stock-dependent demand rate. In Section 2, the fundamental assumptions for the proposed EPQ model are outlined along with notations used for mathematical formulation. In Section 3, the mathematical model is developed by constructing the differential equations, which are analytically solved. Then the total cost function is constructed and the necessary conditions for an optimal solution are outlined in Section 4. In Section 5, some special cases of the model are deduced by choosing appropriate values of parameters. Numerical illustrations and sensitivity analysis are provided in the subsequent Section 6. Finally conclusions are drawn in Section 7.

## 2. ASSUMPTIONS AND NOTATIONS

An Economic Production Quantity (EPQ) model was developed for a single-item inventory having a stock-dependent demand rate. The mathematical model of the economic production quantity has been developed on the basis of the following assumptions.

A single item is considered over a prescribed period of T units of time. $D(I(t), p)$ is the demand rate, which is a function of the stock on-display $I(t)$, and constant selling price (p) within the production cycle. Here it is assumed that $D(I(t), p)$ is equal to $\alpha(p)+\beta I(t)$, where $\alpha(p)$ is a non-negative function of p and $\beta$ is a non-negative constant. Shortages are allowed and backlogged. The maximum allowable amount of displayed stock is P and the initial and ending inventory levels are zero. A constant fraction of on-hand inventory deteriorates per unit of time and deterioration of the units is considered only after those receiving them in the inventory.

The following notations are used for mathematical formulation of the problem:

| | |
|---|---|
| I(t) | Inventory level at any time t |
| $\theta$ | The constant deterioration rate |
| R | Maximum inventory level |
| Q | Unfilled order backlogged |
| C | Setup cost per cycle |
| $C_d$ | The cost of a deteriorated item |
| $C_i$ | Inventory carrying cost per unit |
| $C_s$ | Shortage cost per unit |
| $t_1$ | The production run time |
| T | The duration of a production cycle, where $T = t_1 + t_2 + t_3 + t_4$ |
| K | The constant production rate |
| TC | Total average cost |

## 3. MATHEMATICAL MODEL AND ANALYSIS

At the beginning the stock is assumed to be zero so that the economic production quantity level starts at a time t = 0, and reaches P maximum level after $t_1$ time units have elapsed. The production is then stopped and the stock level declines continuously and the inventory level becomes zero at time $t = t_2$. At this time, the shortage starts developing and reaches to level Q at time $t = t_3$. The fresh production starts to clear the backlog by the time $t = t_4$. The aim in the present investigation is to find the optimal values of $t_1$, $t_2$, $t_3$, $t_4$, P and Q that minimize total cost (TC) over the time horizon [0,T].

The differential equations describing the stock status during the period $0 \leq t \leq T$ can be constructed as follows:

$$\frac{dI(t)}{dt} = K - D(I(t), p) - \theta I(t) \tag{1}$$

$$\frac{dI(t)}{dt} = -D(I(t), p) - \theta I(t) \tag{2}$$

$$\frac{dI(t)}{dt} = -D(I(t), p) \tag{3}$$

$$\frac{dI(t)}{dt} = K - (I(t), p) \tag{4}$$

The boundary conditions are

$I(t) = 0$ at $t = 0$, $t_1+t_2$ and T; $I(t_1) = R$ and $-I(t_3) = Q$

The solutions of Equations 1-4 with the above boundary conditions are

$$I(t) = \frac{K - \alpha(p)}{\theta + \beta}\left(1 - e^{-(\theta+\beta)t}\right) \tag{5}$$

$$I(t) = \frac{\alpha(p)}{\theta + \beta}\left(e^{(\theta+\beta)(t_1 + t_2 - t)} - 1\right) \tag{6}$$

$$I(t) = \frac{\alpha(p)}{\beta}\left(e^{\beta(t_1 + t_2 + t_3 - t)} - 1\right) \tag{7}$$

and

$$I(t) = \frac{K - \alpha(p)}{\beta}\left(1 - e^{\beta(T - t)}\right) \tag{8}$$

From Equations 5 and 6 using $I(t_1) = R$, we

$$R = \frac{K - \alpha(p)}{\theta + \beta}\left(1 - e^{-(\theta+\beta)t_1}\right) = \frac{\alpha(p)}{\theta + \beta}\left(e^{(\theta+\beta)t_2} - 1\right) \tag{9}$$

and

$$Q = \frac{\alpha(p)}{\beta}\left(e^{\beta t_3} - 1\right) = \frac{K - \alpha(p)}{\beta}\left(1 - e^{\beta t_4}\right) \tag{10}$$

Thus, $t_1$ and $t_2$ are related by the equation

$$t_2 = \frac{1}{\theta + \beta}\log\left[\frac{K + (\alpha(p) - K)e^{-(\theta+\beta)t_1}}{\alpha(p)}\right] \tag{11}$$

Again $t_3$ and $t_4$ are related by the equation

$$t_3 = \frac{1}{\beta}\log\left[\frac{K - (K - \alpha(p))e^{\beta t_4}}{\alpha(p)}\right] \tag{12}$$

## 4. COST ANALYSIS

The deterioration cost for the period (0,T) is

$$C_d\left\{\int_0^{t_1}\theta I(t)\,dt+\int_0^{t_2}\theta I(t)\,dt\right\}=$$

$$C_d\theta\left\{\frac{K-\alpha(p)}{\theta+\beta}\left[t_1+\frac{e^{-(\theta+\beta)t_1}}{\theta+\beta}-\frac{1}{\theta+\beta}\right]+\right.$$

$$\left.\frac{\alpha(p)}{\theta+\beta}\left[e^{(\theta+\beta)(t_1+t_2)}\left(-\frac{e^{-(\theta+\beta)t_2}}{\theta+\beta}+\frac{1}{\theta+\beta}\right)-t_2\right]\right\} \tag{13}$$

The inventory carrying cost over the period (0,T) is

$$C_i\left\{\int_0^{t_1}I(t)\,dt+\int_0^{t_2}I(t)\,dt\right\}=$$

$$C_i\left\{\frac{K-\alpha(p)}{\theta+\beta}\left[t_1+\frac{e^{-(\theta+\beta)t_1}}{\theta+\beta}-\frac{1}{\theta+\beta}\right]+\right.$$

$$\left.\frac{\alpha(p)}{\theta+\beta}\left[e^{(\theta+\beta)(t_1+t_2)}\left(-\frac{e^{-(\theta+\beta)t_2}}{\theta+\beta}+\frac{1}{\theta+\beta}\right)-t_2\right]\right\} \tag{14}$$

The shortage cost can be obtained as

$$C_s\left\{-\int_0^{t_3}I(t)\,dt+\int_0^{t_4}I(t)\,dt\right\}=$$

$$C_s\left[\frac{\alpha(p)e^{\beta(t_1+t_2)}}{\beta^2}-\frac{\alpha(p)t_3}{\beta}+\frac{(K-\alpha(p))t_4}{\beta}-\right.$$

$$\left.\frac{Ke^{\beta(t_1+t_2+t_3)}}{\beta^2}+\frac{(K-\alpha(p))e^{\beta T}}{\beta^2}\right] \tag{15}$$

Now the total average cost of the inventory system is

TC = Setup cost + deterioration cost + inventory carrying cost + shortage cost

$$=\frac{C}{T}+\left(\frac{C_d\theta+C_i}{T}\right)\left\{\frac{K-\alpha(p)}{\theta+\beta}\left[t_1+\frac{e^{-(\theta+\beta)t_1}-1}{(\theta+\beta)}\right]+\right.$$

$$\left.\frac{\alpha(p)}{(\theta+\beta)}\left[e^{(\theta+\beta)(t_1+t_2)}\left(\frac{e^{-(\theta+\beta)t_2}+1}{(\theta+\beta)}\right)-t_2\right]\right\}+\frac{C_s}{T}$$

$$\left[\frac{\alpha(p)e^{\beta(t_1+t_2)}-Ke^{\beta(t_1+t_2+t_3)}+(K-\alpha(p))e^{\beta T}}{\beta^2}\right.$$

$$\left.+\frac{(K-\alpha(p))t_4-\alpha(p)t_3}{\beta}\right] \tag{16}$$

Using Maclaurin series for approximation, Equation 16 becomes

$$TC=\frac{C}{T}+\left(\frac{C_d+C_i}{T}\right)\left\{\frac{\alpha(p)}{\theta+\beta}\right.$$

$$\left.\left[1+2(\theta+\beta)(t_1+t_2)\left(\frac{1}{\theta+\beta}-t_2\right)\right]\right\} \tag{17}$$

$$+\frac{2C_s}{\beta T}\left(Kt_4-\alpha(p)(t_3+t_4)\right)$$

The problem is now to obtain the optimal values for $t_1$, $t_2$, $t_3$, $t_4$, Q, and R such that TC in 17 is maximized. For this purpose, equate the first derivatives of TC with respect to $t_1$ and $t_2$, equal to zero, thus:

$$C(1+F'(t_1))+\frac{\alpha(p)}{\theta+\beta}\left\{(C_d\theta+C_i)(1+F'(t_1))\right.$$

$$\left[1+2(\theta+\beta)(t_1+F(t_1))\left(\frac{1}{\theta+\beta}-F(t_1)\right)\right]$$

$$-(C_d\theta+C_i)T\left[2(\theta+\beta)\right.$$

$$\left.\left.\left[(1+F'(t_1))\left(\frac{1}{\theta+\beta}-F(t_1)\right)-(t_1+F(t_1))F'(t_1)\right]\right]\right\}$$

$$+2C_s(1+F'(t_1))\left[Kt_4-\alpha(p)(F(t_4)+t_4)\right]=0 \tag{18}$$

and

$$C\left(1+F'(t_4)\right)+\frac{\alpha(p)}{\theta+\beta}\left(C_d\theta+C_i\right)\left(1+F'(t_4)\right)$$
$$\left[1+2(\theta+\beta)\left(t_1+F(t_1)\right)\left(\frac{1}{\theta+\beta}-F(t_1)\right)\right]$$
$$+\frac{2C_s}{\beta}\left(1+F'(t_4)\right)\left[Kt_4-\alpha(p)\left(F(t_4)+t_4\right)\right]-$$
$$\frac{2C_sT}{\beta}\left[K-\alpha(p)\left(F(t_4)+1\right)\right]=0 \quad (19)$$

where

$$F(t_1)=\frac{1}{\theta+\beta}\log\left[\frac{\alpha(p)-(\alpha(p)-K)(\theta+\beta)t_1}{\alpha(p)}\right] \quad (20)$$

$$F(t_4)=\frac{1}{\beta}\log\left[\frac{\alpha(p)-(K-\alpha(p))\beta t_4}{\alpha(p)}\right] \quad (21)$$

$$F'(t_1)=\left[\frac{K-\alpha(p)}{\alpha(p)-(\alpha(p)-K)(\theta+\beta)t_1}\right] \quad (22)$$

$$F'(t_4)=\left[\frac{(\alpha(p)-K)}{\alpha(p)-(K-\alpha(p))\beta t_4}\right] \quad (23)$$

To solve the two simultaneous non-linear Equations 18 and 19 Newton methods is used and we obtain the optimal values of $t_1$ and $t_4$. Then the values of $t_1$ and $t_4$ are put in Equations 9-12 and 16 to obtain the optimal values of R, Q, $t_2$, $t_3$ and average total cost. The algorithmic procedure of Newton's method has been outlined in the Appendix.

## 5. SPECIAL CASES

Here some special cases are considered for economic production quantity model by setting appropriate parameter values.

### Case I

In this case, the deterioration rate is not considered, i.e. $\theta = 0$, so that the total cost of the model becomes

$$TC=\frac{C}{T}+\left(\frac{C_i}{T}\right)\left\{\frac{K-\alpha(p)}{\beta}\left[t_1+\frac{e^{-\beta t_1}-1}{\beta}\right]+\right.$$
$$\left.\frac{\alpha(p)}{\beta}\left[e^{\beta(t_1+t_2)}\left(\frac{e^{-\beta t_2}+1}{\beta}\right)-t_2\right]\right\}+\frac{C_s}{T}$$
$$\left[\frac{\alpha(p)e^{\beta(t_1+t_2)}-Ke^{\beta(t_1+t_2+t_3)}+(K-\alpha(p))e^{\beta T}}{\beta^2}\right.$$
$$\left.+\frac{(K-\alpha(p))t_4-\alpha(p)t_3}{\beta}\right] \quad (24)$$

where

$$t_2=\frac{1}{\beta}\log\left[\frac{K+(\alpha(p)-K)e^{-\beta t_1}}{\alpha(p)}\right],$$

$$t_3=\frac{1}{\beta}\log\left[\frac{K-(K-\alpha(p))e^{\beta t_4}}{\alpha(p)}\right]$$

### Case II

Consider the EPQ model without shortage, i.e. $C_s = 0$, so that the total cost-function reduces to

$$TC=\frac{C}{T}+\left(\frac{C_d\theta+C_i}{T}\right)\left\{\frac{K-\alpha(p)}{\theta+\beta}\left[t_1+\frac{e^{-(\theta+\beta)t_1}-1}{(\theta+\beta)}\right]+\right.$$
$$\left.\frac{\alpha(p)}{(\theta+\beta)}\left[e^{(\theta+\beta)(t_1+t_2)}\left(\frac{e^{-(\theta+\beta)t_2}+1}{(\theta+\beta)}\right)-t_2\right]\right\} \quad (25)$$

where

$$t_2=\frac{1}{\theta+\beta}\log\left[\frac{K+(\alpha(p)-K)e^{-(\theta+\beta)t_1}}{\alpha(p)}\right],$$

$$t_3=\frac{1}{\beta}\log\left[\frac{K-(K-\alpha(p))e^{\beta t_4}}{\alpha(p)}\right]$$

## 6. NUMERICAL ILLUSTRATION

In order to explore the effect of system parameters on optimal cost, the numerical results are computed and summarized in Tables 1-6. Table 1(a-c) displays the optimal cost for different cost elements $C_i$, $C_d$, $C_s$ and varying values of θ. It is noted from Table 1(a), that the total cost (TC) increases as the inventory carrying cost ($C_i$) increases, but decreases with the increment in α(p) and deterioration rate (θ). Table 1(b) shows the effect of cost of deteriorating item ($C_d$), β and deterioration rate (θ); the total cost (TC) increases as the value of β increases and the cost of deteriorating item. The effect of shortage cost ($C_s$), α(p) and θ on the total cost is shown in Table 1(c); the decreasing trend of total cost is noted with the increase in α(p) and θ but reverse effect of $C_s$.

From Table 2, it is observed that as the value of 'θ' increases, the values of $t_1$ and $t_2$ decrease but the values of $t_3$, $t_4$ increase. Maximum inventory level (R) and total average cost (TC) decrease as the deterioration rate (θ) increases. As the value of 'K' increases, the values of $t_i$ (i = 1, 2, 3, 4) decrease but the maximum inventory level (R) and total cost (TC) increase.

It is noted from Table 3, that as the value of θ increases, the value of $t_i$ (i = 1, 2, 3, 4), R and TC decrease. The values of $t_1$ and $t_4$ increase as the α(p) increases but $t_2$, $t_3$, R and TC decrease with increments in α(p). In Table 4, it can be seen that $t_i$ (i = 1, 2, 3, 4) and R decrease as the value of β increases, but TC increases.

Table 5 shows the effect of α(p) and β on $t_i$ (i = 1, 2, 3, 4), R and TC. The values of $t_i$ and R decrease as the β increases, but value of TC increases as β increases. As α(p) increases, the values of $t_1$ and $t_4$ increase but other values decrease. Table 6 displays the effect of K and β on $t_i$, R and TC. The values of $t_1$, $t_2$, $t_3$ and R decrease as β increases, but $t_4$ and TC increase. On increasing the value of K, both $t_1$ and $t_4$ decrease but other values increase.

Overall it can be concluded that as with the increase the production, maximum inventory level and total cost increase. Maximum inventory level (R) and total average cost (TC) decrease as the deterioration rate (θ) increases. So, the building inventory is profitable and the production run continues until the inventory reaches the maximum allowable level.

## 7. CONCLUSION

This study presents an economic production quantity model for deteriorating items in which the demand has been considered as the function of the selling price and the stock on display. In particular, shortages are allowed and backlogged. This assumption is more realistic in the market.

Furthermore, the results of the sensitivity analysis are also consistent with the economic incentives. In future research on this problem, it would be of interest to incorporate the effect of more realistic demand rates in this model. On the other hand, the possible extension of this work may relax the assumption of constant deterioration rate.

## 8. APPENDIX

To solve Equations 18 and 19, the Newton method was used. Denoting L.H.S. of Equations 18 and 19 by $f_1(t_1,t_4)$ and $f_2(t_1,t_4)$, the outlines of the algorithm to determine $t_1$ and $t_4$ are as follows:

**Algorithm**

**Step 1.** Evaluate the function

$$F(t_k) = \begin{bmatrix} f_1\left(t_1, t_4\right) \\ f_2\left(t_1, t_4\right) \end{bmatrix}$$

with the initial condition $t_k = 0$, where k = 1,4.

**Step 2.** Evaluate the Jacobian

$$J(t_k) = \begin{bmatrix} \dfrac{\partial f_1\left(t_1, t_4\right)}{\partial t_1} & \dfrac{\partial f_1\left(t_1, t_4\right)}{\partial t_4} \\ \dfrac{\partial f_2\left(t_1, t_4\right)}{\partial t_1} & \dfrac{\partial f_2\left(t_1, t_4\right)}{\partial t_4} \end{bmatrix}$$

**TABLE 1. Optimal Values of TC for Different Values of (a) $C_i$, α(P) and θ, (b) $C_d$, β and θ and (c) $C_s$, α(p) and θ.**

(a)

| $C_i$ | α(p) | TC | | | |
|---|---|---|---|---|---|
| | | θ = 0.1 | θ = 0.2 | θ =0.3 | θ = 0.4 |
| 0.3 | 0.1 | 37.92 | 29.31 | 25.84 | 24.30 |
| | 0.2 | 19.92 | 17.74 | 17.08 | 17.05 |
| | 0.3 | 15.35 | 14.69 | 14.76 | 15.14 |
| 0.5 | 0.1 | 57.74 | 42.20 | 35.66 | 32.46 |
| | 0.2 | 29.13 | 24.55 | 22.73 | 22.03 |
| | 0.3 | 21.70 | 19.74 | 19.15 | 19.15 |
| 0.7 | 0.1 | 77.56 | 55.10 | 45.48 | 40.61 |
| | 0.2 | 38.34 | 31.36 | 28.37 | 27.01 |
| | 0.3 | 28.05 | 24.79 | 23.54 | 23.16 |

**(b)**

| $C_d$ | β | TC | | | |
|---|---|---|---|---|---|
| | | θ = 0.1 | θ = 0.2 | θ = 0.3 | θ = 0.4 |
| 0.5 | 0.1 | 8.92 | 9.31 | 9.87 | 10.48 |
| | 0.2 | 11.04 | 11.80 | 12.61 | 13.47 |
| | 0.3 | 14.32 | 15.16 | 16.11 | 17.12 |
| 1 | 0.1 | 9.55 | 10.39 | 11.38 | 12.40 |
| | 0.2 | 11.80 | 13.19 | 14.62 | 16.08 |
| | 0.3 | 15.39 | 17.13 | 18.92 | 20.78 |
| 1.5 | 0.1 | 10.18 | 11.47 | 12.88 | 14.35 |
| | 0.2 | 12.55 | 14.58 | 16.62 | 18.70 |
| | 0.3 | 16.46 | 19.10 | 21.73 | 24.43 |

**(c)**

| $C_s$ | α(p) | TC | | | |
|---|---|---|---|---|---|
| | | θ = 0.1 | θ = 0.2 | θ = 0.3 | θ = 0.4 |
| 0.6 | 0.1 | 37.92 | 29.31 | 25.84 | 24.30 |
| | 0.2 | 19.92 | 17.74 | 17.08 | 17.05 |
| | 0.3 | 15.35 | 14.69 | 14.76 | 15.14 |
| 0.9 | 0.1 | 54.28 | 41.12 | 35.68 | 33.15 |
| | 0.2 | 26.83 | 23.28 | 22.02 | 21.70 |
| | 0.3 | 19.63 | 18.33 | 18.13 | 18.41 |
| 1.2 | 0.1 | 70.64 | 52.93 | 45.52 | 42.00 |
| | 0.2 | 33.74 | 28.82 | 26.96 | 26.36 |
| | 0.3 | 23.91 | 21.97 | 21.51 | 21.68 |

**TABLE 2. Optimal Solution for Different Values of K and θ.**

| K | θ | $t_1$ | $t_2$ | $t_3$ | $t_4$ | R | TC |
|---|---|---|---|---|---|---|---|
| 10 | 0.1 | 0.034 | 2.67 | 2.93 | 0.03 | 16 | 50118 |
| | 0.5 | 0.027 | 1.96 | 2.92 | 0.04 | 12 | 32510 |
| | 0.9 | 0.022 | 1.54 | 2.91 | 0.05 | 9 | 25878 |
| 20 | 0.1 | 0.012 | 3.33 | 3.69 | 0.004 | 35 | 261634 |
| | 0.5 | 0.009 | 2.44 | 3.65 | 0.009 | 25 | 142633 |
| | 0.9 | 0.007 | 1.92 | 3.61 | 0.012 | 20 | 102690 |
| 30 | 0.1 | 0.006 | 3.70 | 4.08 | 0.0003 | 53 | 703904 |
| | 0.5 | 0.004 | 2.71 | 4.05 | 0.0036 | 39 | 347123 |
| | 0.9 | 0.003 | 2.14 | 4.02 | 0.0051 | 30 | 235829 |

**TABLE 3. Optimal Solution for Different Values of α(p) and θ.**

| α(p) | θ | $t_1$ | $t_2$ | $t_3$ | $t_4$ | R | TC |
|---|---|---|---|---|---|---|---|
| 10 | 0.1 | 0.0024 | 2.67 | 2.9444 | 0.0232 | 164 | 20328 |
| | 0.5 | 0.0011 | 1.96 | 2.9365 | 0.0396 | 120 | 13698 |
| | 0.9 | 0.0006 | 1.54 | 2.9258 | 0.0346 | 95 | 11300 |
| 20 | 0.1 | 0.0026 | 1.99 | 2.1972 | 0.1592 | 145 | 8443 |
| | 0.5 | 0.0023 | 1.46 | 2.1687 | 0.1864 | 106 | 6788 |
| | 0.9 | 0.0021 | 1.15 | 2.1321 | 0.1804 | 84 | 6196 |
| 30 | 0.1 | 0.0194 | 1.57 | 1.7346 | 0.5382 | 127 | 5422 |
| | 0.5 | 0.0151 | 1.15 | 1.7311 | 0.5531 | 94 | 4837 |
| | 0.9 | 0.0126 | 0.91 | 1.7298 | 0.5446 | 74 | 4687 |

**TABLE 4. Optimal Solution for Different Values of β and θ.**

| β | θ | $t_1$ | $t_2$ | $t_3$ | $t_4$ | R | TC |
|---|---|---|---|---|---|---|---|
| 1 | 0.1 | 0.0008 | 4.812 | 5.2933 | 0.0006 | 180 | 14344700 |
| | 0.5 | 0.0006 | 3.5288 | 5.2534 | 0.0003 | 132 | 5268242 |
| | 0.9 | 0.0004 | 2.7859 | 5.1254 | 0.0002 | 104 | 3017854 |
| 2 | 0.1 | 0.0004 | 2.5206 | 2.6466 | 0.0012 | 94 | 17247560 |
| | 0.5 | 0.0003 | 2.1173 | 2.4789 | 0.0005 | 79 | 9147928 |
| | 0.9 | 0.0003 | 1.8252 | 2.2584 | 0.0001 | 68 | 5846210 |
| 3 | 0.1 | 0.0003 | 1.7075 | 1.7644 | 0.0020 | 64 | 18417550 |
| | 0.5 | 0.0002 | 1.5123 | 1.5454 | 0.0004 | 57 | 11628880 |
| | 0.9 | 0.0002 | 1.3572 | 1.3658 | 0.0001 | 51 | 8125149 |

**TABLE 5. Optimal Solution for Different Values of α(p) and β.**

| α(p) | β | $t_1$ | $t_2$ | $t_3$ | $t_4$ | R | TC |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 0.0065 | 3.8833 | 4.0775 | 0.0002 | 55 | 801962 |
| | 2 | 0.0036 | 1.989 | 2.0387 | 0.0008 | 28 | 860958 |
| | 3 | 0.0025 | 1.3368 | 1.3591 | 0.0019 | 19 | 882317 |
| 2 | 1 | 0.0096 | 3.2069 | 3.3672 | 0.0074 | 53 | 289222 |
| | 2 | 0.0056 | 1.6425 | 1.6836 | 0.0056 | 27 | 305369 |
| | 3 | 0.004 | 1.104 | 1.1224 | 0.0039 | 18 | 311158 |
| 3 | 1 | 0.0114 | 2.8042 | 2.9444 | 0.0235 | 51 | 163098 |
| | 2 | 0.0069 | 1.4363 | 1.4722 | 0.0165 | 26 | 170485 |
| | 3 | 0.005 | 0.9653 | 0.9814 | 0.0159 | 18 | 173120 |

**TABLE 6. Optimal Solution for Different Values of K and β.**

| K | β | $t_1$ | $t_2$ | $t_3$ | $t_4$ | R | TC |
|---|---|---|---|---|---|---|---|
| 10 | 1 | 0.0362 | 2.8042 | 2.9444 | 0.0331 | 17 | 54369 |
| | 2 | 0.0223 | 1.4363 | 1.4722 | 0.0352 | 9 | 56834 |
| | 3 | 0.0164 | 0.9653 | 0.9814 | 0.0482 | 6 | 57715 |
| 20 | 1 | 0.0126 | 3.4891 | 3.6635 | 0.0031 | 36 | 292787 |
| | 2 | 0.0072 | 1.7871 | 1.8317 | 0.0041 | 19 | 311296 |
| | 3 | 0.005 | 1.2011 | 1.2211 | 0.0063 | 12 | 317959 |
| 30 | 1 | 0.0065 | 3.8833 | 4.0775 | 0.0003 | 55 | 801961 |
| | 2 | 0.0036 | 1.989 | 2.0387 | 0.0008 | 28 | 860957 |
| | 3 | 0.00255 | 1.3368 | 1.3591 | 0.0019 | 19 | 882316 |

**Step 3.** Solve the linear system

$$J(t_k)\Delta t = F(t_k)$$

where

$$\Delta t = \begin{bmatrix} t_1^{(1)} \\ t_4^{(1)} \end{bmatrix}$$

**Step 4.** Compute the next point

$$t_{k+1} = t_k + \Delta t$$

Now repeat the process till we get the perfect result.

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