

SPATIALLY VARIED FLOW PROFILES IN A V-SHAPED SIDE-CHANNEL

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Abstract Spatially varied flow in open channels occurs in a large variety of hydraulic structures as well as road/bridge surface drainage channels. The equation of motion for spatially variable flow in an open channel, being produced by the lateral or a vertical inflow, has been treated previously. However, these are not applicable to the phenomenon of runoff from a plane surface, which is an example of spatially varied flow. This paper reviews the state of the subject and presents the experimental data applying a practical equation of motion mathematically derived. Several series of experiments are carried out in a v-shaped bottom channel. These experiments are used to model specific rainfall intensity and discharge. For this particular channel shape, longitudinal water surface profiles are plotted and are compared with the profiles given by the equation of motion treated for this channel shape. Analysis of the results agrees well with the experimental data and the proposed equation for supercritical flows. The results, however, show that the profiles slope does not normally increase as the flow discharge increases at the end of the channel. This result confirms that the flow resistance in spatially varied flow as well as the critical depth position along the channel and cross sectional shape should be taken into account.

Keywords: spatially varied flow, water surface profile, hydraulic structures, surface drainage side-channel, V-shaped channel.

چکیده جریان گسسته در کانالهای روباز، در سازه های هیدرولیکی و همچنین در کانال های زهکش کنار جاده ها یا پلها اتفاق می افتد. معادله حرکت برای جریان گسسته در کانال روباز، در حالت جریان با ورودی جانبی و یا جریان مستقیم در کانال از قبل بدست آمده است. با این وجود، آن معادلات برای پدیده جریان ناشی از بارش روی سطح چندان کاربردی نبوده است، اگرچه آن نیز به عنوان نمونه ای از جریان گسسته می باشد. در این مقاله، موضوع به نحو خوبی بیان شده و داده های آزمایشگاهی به منظور بکارگیری معادله حرکت کاربردی مورد تحلیل قرار گرفته است. سری آزمایشات متعددی در کانالی با کف V- شکل انجام یافته است. این آزمایشات به منظور مدل نمودن دبی جریان ناشی از بارش ویژه ای تدوین یافته است. برای کانالی با شکل مقطع خاص قید شده، پروفیل های سطح آب طولی حاصل از آزمایشات با پروفیل های حاصل از معادله جریان گسسته مقایسه شده اند. آنالیز نتایج نشان می دهد که در شرایط جریان فوق بحرانی، داده های آزمایشگاهی با معادله ریاضی جریان تطابق خوبی دارد. در عین حال، نتایج نشان می دهد که شیب عمومی امتداد پروفیل های سطح آب، متناسب با افزایش دبی جریان در انتهای کانال، افزایش پیدا نمی کند. این نتیجه مؤید آن است که مقاومت جریان در شرایط جریان گسسته و همچنین در وضعیت عمق بحرانی جریان در طول کانال و در مقطع آن بایستی به دقت مورد بررسی قرار بگیرد.

1. INTRODUCTION

Free surface flow in a channel with its discharge varying along the flow is known as “*spatially varied flow*”. Examples are flows at bridges,

highways and urban stormwater drainage channels, overland flows, flow in roof gutters, and regulator overflow from sewers. Simplified approximate forms of 1D equations for spatially varied flow have been used to solve all these problems.

Routing flood with lateral flows and side channel spillways or weirs are some other examples. The phenomenon, which occurs in a variety of circumstances, provides the proper use of the energy and momentum approaches, although it can lead to problems of some difficulty. According to Chow (1959), applying some certain assumptions the simplified 1D spatially varied flow equations can then be solved by stepwise integration for steady flow and by the method of characteristics or finite-difference schemes for unsteady flows.

In the past, a considerable attention has been paid on the prediction of free surface profiles in open channels with spatially varied increasing discharge such as Ackers (1957), Smith (1967), Yen & Wenzel (1970), Fox & Goodwill (1971), Yen (1971), Yoon & Wenzel (1971), and Hager (1985). However only limited experimental observations have been presented such as those of Hinds (1926), Farney & Markus (1962), Bremen & Hager (1989), Katz et al (1995) and Beecham et al (2005). As pointed out by Bremen & Hager (1989), the cross-sectional profile usually accounted for traditional rectangular or trapezoidal channels. The effect of the bottom slope on the free surface profile has not yet been systematically analysed. However, longitudinal surface profile for shallow channels, i.e., stormwater flows and resistance due to the spatially varied flow need to be investigated for different channel shapes. Rigorous analysis of spatially varied flow must consider the effects of changes in momentum and hydrostatic pressure on the water surface profile. This has been done by some investigators Woo & Brater (1962), and Yoon & Wenzel (1971), while others have simplified the analysis by ignoring flow momentum and pressure, considering only the effects of friction and gravity e.g. Emmett (1970).

In this paper, a particular emphasise is focused on the free surface profiles in a complicate V-shaped bottom channel shape (Figure 1) of rectangular section having a V-shaped bottom. Discussions on the effect of bottom channel slope by means of the

sub and supercritical flows are also addressed herein.

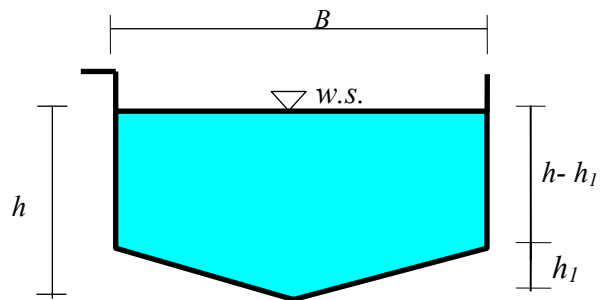


Figure 1. V-shaped bottom drainage channel cross-section.

2. DYNAMIC EQUATION OF SPATIALLY VARIED FLOW

It has been acknowledged that the one-dimensional analysis of flow has made an acceptable solution for the majority of the problems related to the design of such structures. According to Favre (1933) and French (1985), an equation for the free surface profile for a side channel flow is given by:

$$\frac{dh}{dx} = \frac{S_0 - S_f - \left(2 - \frac{V \cos \phi}{U}\right) \frac{QQ_x}{gA^2} + \frac{Q^2}{gA^3} \frac{\partial A}{\partial x}}{1 - \frac{Q^2}{gA^3} \frac{\partial A}{\partial h}} \quad (1)$$

where $h(x)$ is the average depth of flow; $S_0 = S_0(x)$ is the channel bottom slope; $S_f (= \tau_b / \rho g R)$ is the frictional slope; τ_b is the mean energy boundary shear stress; R is the hydraulic radius; $A = A(h, x)$ is the flow cross-sectional area; $Q = Q(x)$ is the channel flow discharge at any station along

chainage; $Q_x=dQ/dx$ is the lateral inflow discharge per unit length of the channel; U is the mean velocity ($=Q/A$); and $V\cos\phi$ is the streamwise velocity component of the lateral inflow made a ϕ angle. Regarding to the friction head, h_f , between two sections $h_f=S_f dx$, where S_f is given by:

$$S_f = \left(\frac{nQ}{AR^{2/3}} \right)^2 \quad (2)$$

A more general equation for the free surface profile has been developed by Yen & Wenzel (1970). However according to Bremen & Hager (1989) since it contains some unspecified parameters it is not readily applicable. Equation (1) has been used by Bremen & Hager (1989) to predict surface profile of a rectangular side channel spillway. In the same manner using equation (1) for a V-shaped bottom channel (see Figures 1 and 2 for notations), one obtain:

$$\frac{dh}{dx} = [S_0 - S_f - \left\{ 2 - \frac{S_0 \sqrt{2g(Z_0 + Z_s + h_2 + h - h_1 + S_0 x)}}{U} \right\} \frac{QQ_x}{gA^2} + \frac{Q^2 B_x}{gB^3 (h - 0.5h_1)^2}] / (1 - Fr^2) \quad (3)$$

in which B_x is the width variation ($=\theta$ which is the width divergence, where $B=b+\theta x$ and b is the channel width at $x=0$), see Figure (2) for other notations, and Fr is the Froude number defined as:

$$Fr^2 = \frac{Q^2}{gA^3} \frac{\partial A}{\partial h} = \frac{U^2}{gh_D} \quad (4)$$

where $h_D (=A/B)$ is known as the hydraulic depth and B is the top width. Equation (3) is referred as the extended equation for the free surface profile since the term representing the lateral momentum contribution $V\cos\phi$ is normally not included (Chow, 1959). Using one boundary condition e.g.

$h(x)=h_0$, it may be solved by a standard numerical integration technique. When the flow is in critical condition ($Fr=1$), a control point is established, and the integration of equation (3) must be initiated at this point. However, according to Chow (1959) and Hager (1985), the distinction between singular ($[dh/dx]=[0/0]$) and critical ($[dh/dx] \rightarrow \infty$) points may be made the direction of computation which is in the upstream direction for the case of subcritical flows ($Fr<1$) and in the downstream direction for the case of supercritical flows ($Fr>1$).

3. EXPERIMENTAL EQUIPMENT AND PROCEDURE

The experiments were conducted in a 9m long, 1.10m wide and 0.30m depth, tilting flume which was specially designed and built to undertake further hydraulic tests on the V-shaped channels, simulating flow in a roadway with a nominal 5% crossfall. The longitudinal slope of the flume and roadway could be varied from zero to a maximum positive gradient of 2.2%. The flume was centrally pivoted so that the slope could be set manually through two 1-tone Neer Drive jacks, operating together. The slope was controlled by a scale read to ± 0.2 mm, thus enabling slopes to be set to an accuracy of between 0.1% and 2% from 0.016 to 0.001. Two pumps with two separate delivery systems were used and two electro-magnetic (ABB Kent-Taylor MagMaster) flow metering equipment were installed in series with pipelines. The discharges reading accuracy in both pipelines were 0.5%. This system allowed the water to be introduced not only into the channel longitudinally, but also laterally through the roadway side of channel, thus simulating runoff from a roadway surface. The longitudinal main flow was supplied through a 150mm pipeline and a pump with a maximum capacity of 70 l s^{-1} . Experimental V-shaped channel was made on one side of the flume (Figure 2a & b).

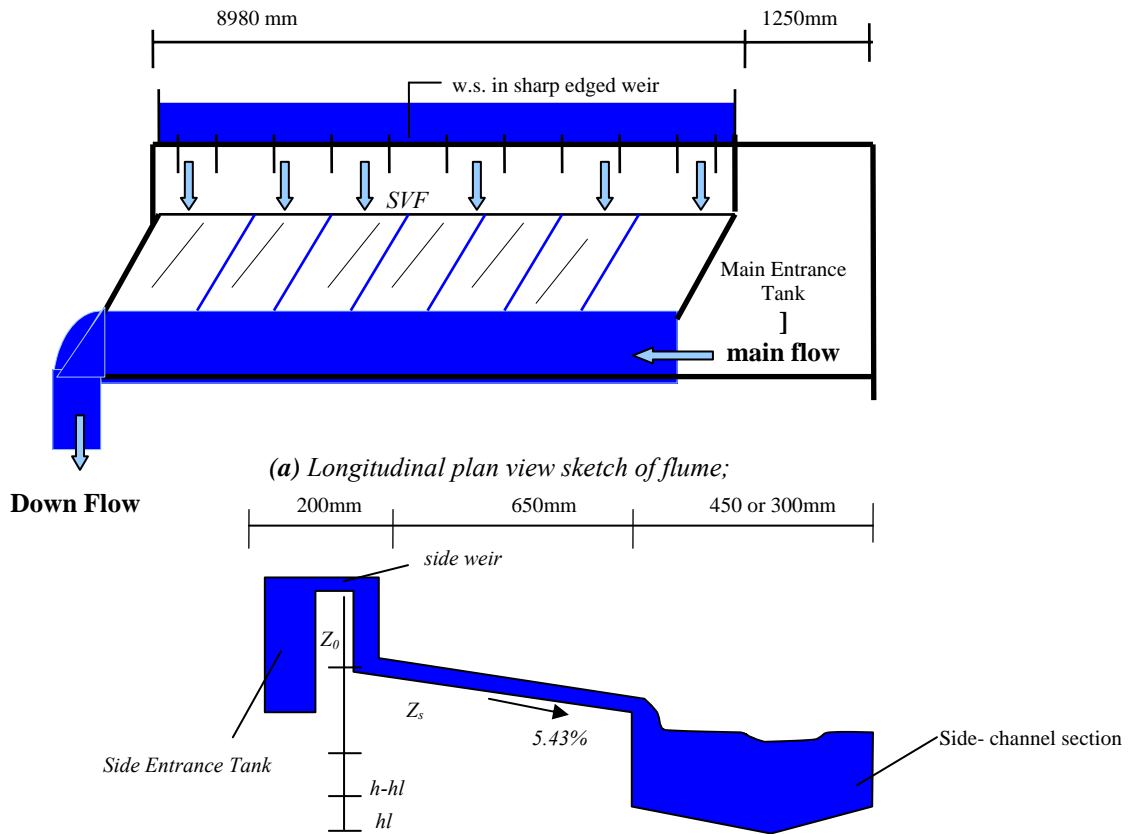


Figure 2a & b. Definition sketch for the experimental rig.

The lateral inflow was supplied through a 100mm pipeline, through two connected distributor head tanks, and then via ten 50mm diameter flexible tubes of equal length into a 9m long \times 100mm wide \times 350mm height distributor box made in 5 panels of 1.80m connected together. Total length of a 9m long sharp edged side weir on one side of the flume was designed for spatially flow. The maximum capacity of the pump delivering the lateral inflow was 40 l s^{-1} . Figures (2a & b) show the dimensions and different views of the flume used for the present research work. Uniform flow is required in order to establish the

stag-discharge relationships, tailgate setting and consequently water surface profiles for the case of adding spatially varied flow onto the main flow especially in subcritical flow conditions. Measurements were undertaken for both subcritical and supercritical flows conditions. In subcritical flows, for each slope and main flow discharge, uniform flow was obtained by firstly undertaking a series of preliminary experiments and plotting depth, slope and tailgate setting for several M1 and M2 water surface profiles. It was therefore possible to estimate the precise normal depth and tailgate setting numbers for a certain main flow discharge.

When spatially varied flow was added onto the flow the tailgate was established for the main flow and water surface profile measurements were undertaken. In the case of supercritical flows, the S2 profile at the upstream end of the channel was excluded and depth readings were only taken in that part of the channel in which uniform flow occurred. Figure (3) shows three S2 water surface profiles in the experimental channel. It may be seen from this Figure that for the higher flows and steeper channel, large perturbations are occurred in water surface.

4. PRESENTATION OF THE EXPERIMENTAL RESULTS AND DISCUSSION

Using elementary notions of momentum, a criterion is derived for the considered side-channel water surface profiles. The criterion is examined by referring to some experiments carried out by the author. In this examination it is noted incidentally that the law of resistance for the spatially variable discharge is at variance with laws ordinarily ascribed to the constant discharge in open

channels. The results of some experiments in both subcritical and supercritical regimes are reported here and are compared with the predictions from dynamic equation. Figures (4&5) are shown drawn some selected water surface profiles for different flow discharges (main-flow and spatially variable flow) at various channel bed slopes, considering with and without momentum, respectively. A comparison of these two Figures realizes that at the same main and spatially flow discharges, when we consider momentum (Figure 5), water surface profiles are getting higher. It can be seen that for both cases, the steepest the channel the lower the water surface profile. It was also observed that for the case of subcritical flow regime water surface profiles give rise in comparison with the uniform flow, by contrast in supercritical flow regime this is not the case. This result illustrates that the longitudinal overall friction factor will considerably vary. In earlier studies by Fox & Goodwill (1971) and others quoted by Henderson (1966), it is reported that the coefficient of friction in spatially varied flow is generally much higher than the corresponding value in steady uniform flow. This has been confirmed by the present experimental results as well.

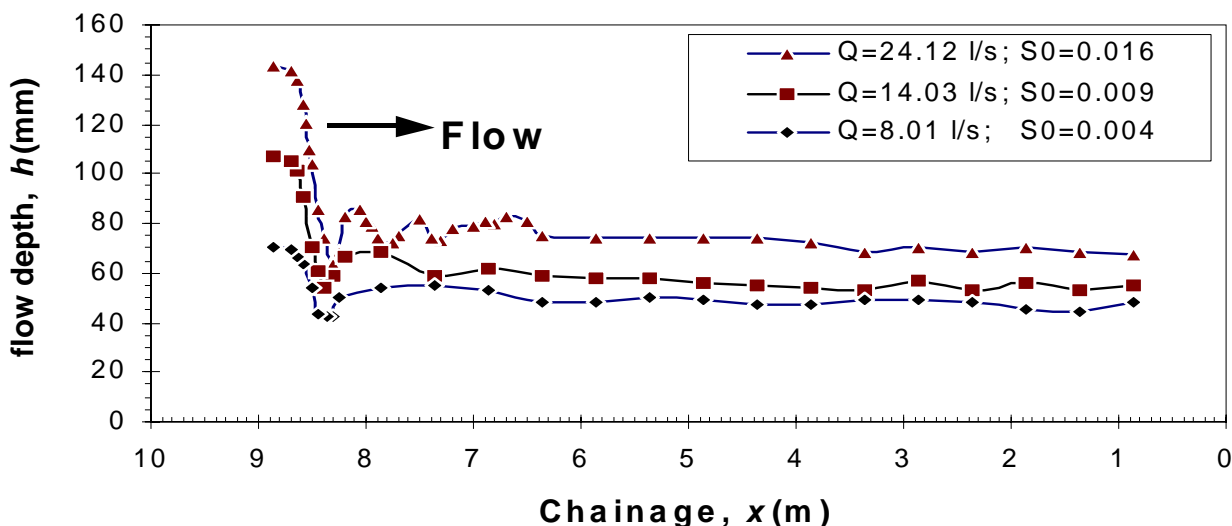


Figure 3. Selected typical S2 profiles in supercritical flow conditions.

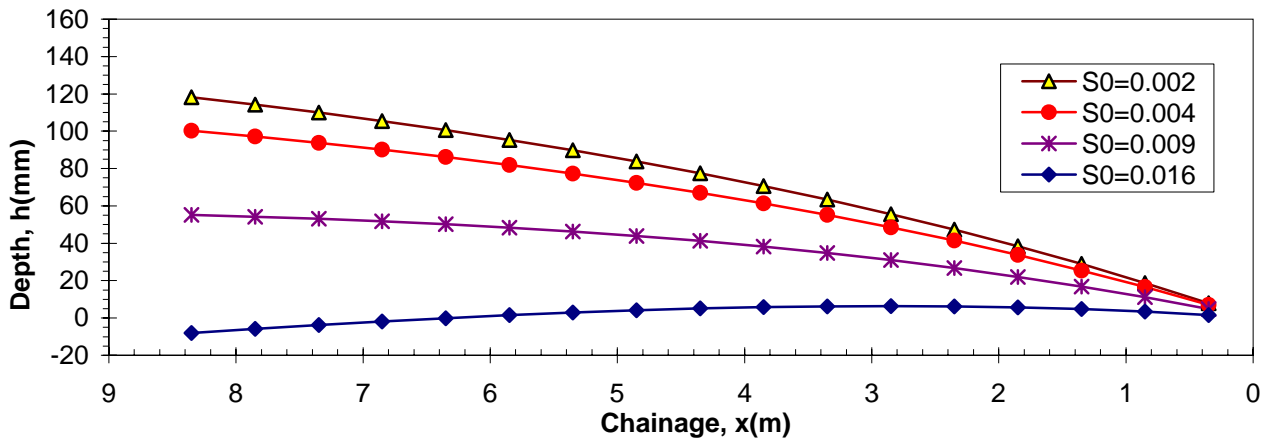


Figure 4. Water surface profiles for $Q=5$ (l/s) & $q=2.5$ (l/s) without momentum.

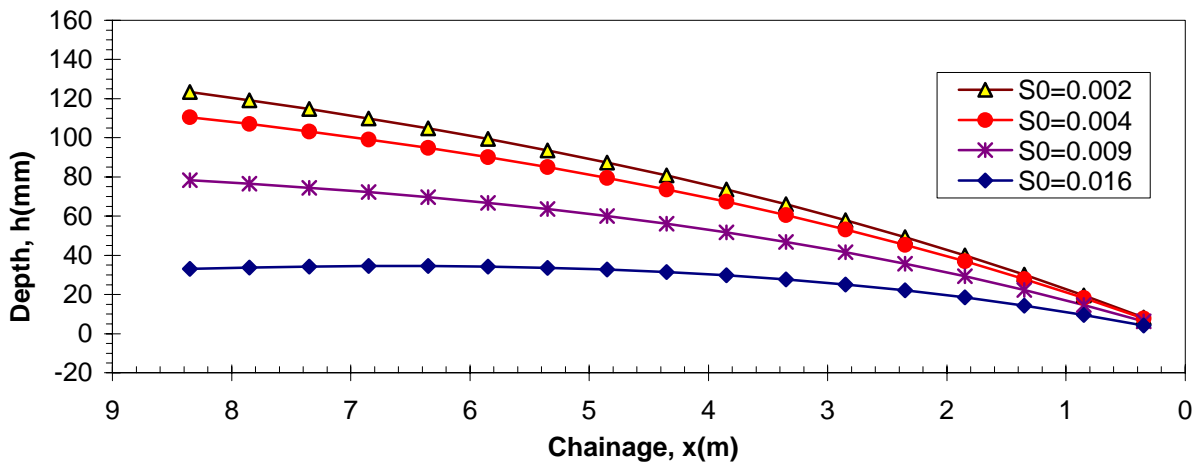


Figure 5. Water surface profiles for $Q=5$ (l/s) & $q=2.5$ (l/s) with momentum.

An attempt was made to compare the dynamic equation (eq. 3) with the experimental results. The selected results are presented in Figures (6-9). As

can be seen from Figures (6&7) for the case of subcritical flows, equation (3) underestimates the water surface profiles.

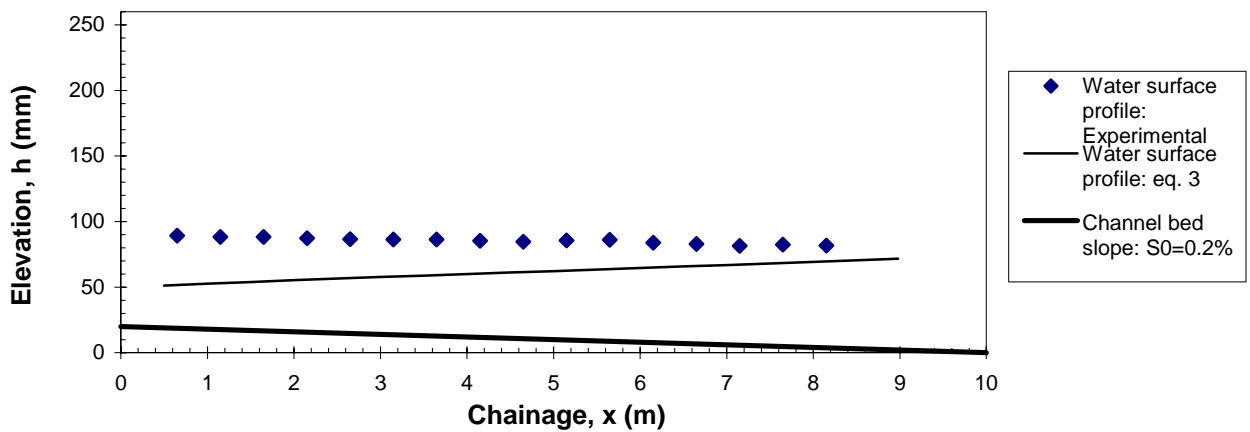


Figure 6. Water surface profiles (a comparison of experimental results & eq. 3) for $Q=2.52$ (l/s) & $q=5$ (l/s) at $S_0=0.2\%$ (subcritical flow).

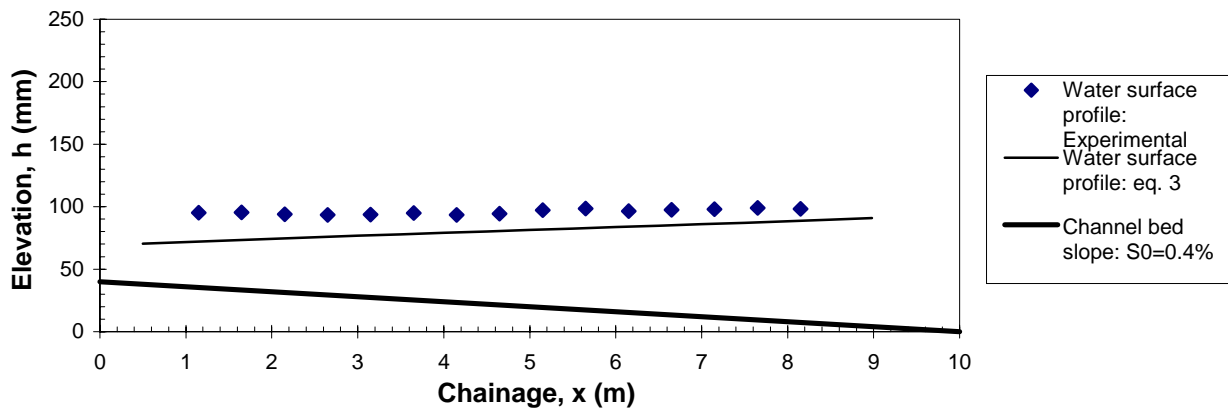


Figure 7. Water surface profiles (a comparison of experimental results & eq. 3) for $Q=2.52$ (l/s) & $q=5$ (l/s) at $S_0=0.4\%$ (subcritical flow).

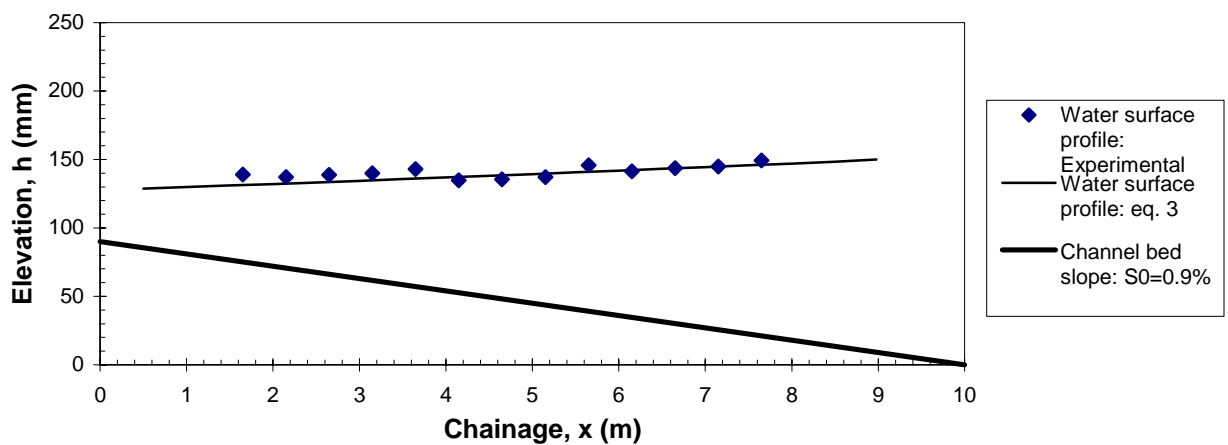


Figure 8. Water surface profiles (a comparison of experimental results & eq. 3) for $Q=5$ (l/s) & $q=5$ (l/s) at $S_0=0.9\%$ (supercritical flow).

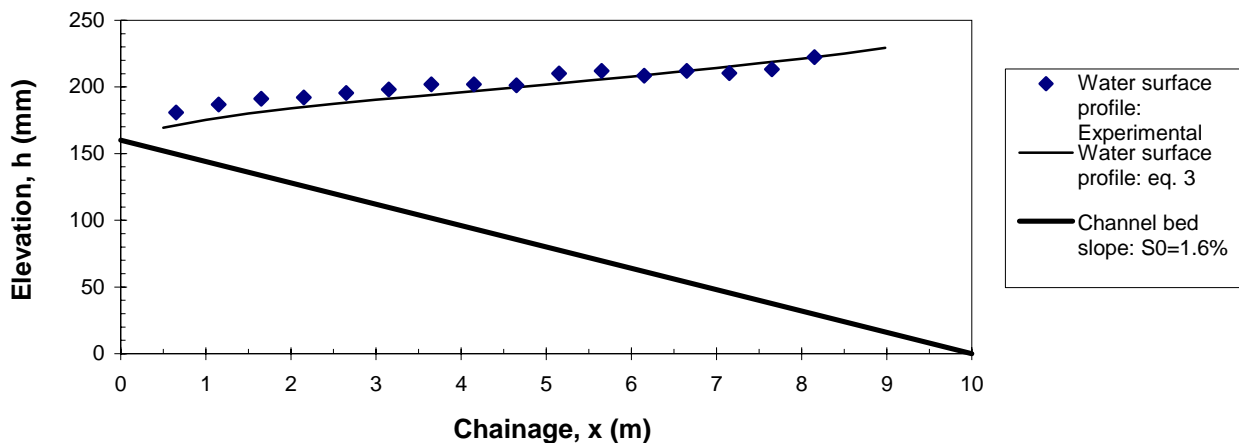


Figure 9. Water surface profiles (a comparison of experimental results & eq. 3) for $Q=0$ & $q=12.34$ (l/s) at $S_0=1.6\%$ (super critical flow).

However for the case of super critical flow regime, equation (3) well predicts the water surface profiles both with and without spatially varied flow condition as can be observed in Figures (8&9). The reason is that in case of subcritical flow momentum transfer dose not affect too much on flow structure, however for super critical flow this is not the case.

5. CONCLUDING REMARKS

Using the experimental results and together with the proposed dynamic equation for a V-shaped bottom channel the main conclusions are derived as follows.

In order to investigate the characteristics of the spatially varied flow in a V-shaped bottom channel, a series of experiments has been carried out. A differential equation based on the conservation of momentum has been derived for this channel with nonuniform discharge. Various flow discharges at different channel bed slopes are examined for both main surface drainage side channel and increasing spatially varied flow in the

conditions of subcritical and supercritical flows.

The results shows that at the same main and spatially varied flow discharges, when we ignore the effect of momentum (Figure 6), water surface profiles are getting lower which is not the case for considering momentum effects.

A comparison of the experimental results and the dynamic equation shows that for the case of steep slope channels (super critical flow regime) this equation well predicts water surface profiles, but it gives underestimate results for the mild slope channel (see Figures 6-9).

It is well recognized that this study is not complete in the sense that the effects of surface roughness were not considered. However analysis of the results shows that hydraulic friction is appreciably increased in spatially variable flows. This part of the research is still in process.

6. ACKNOWLEDGEMENT

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7. NOMENCLATURE

A : transverse cross-sectional area; geometry factor;
 b : channel bed width;
 B : top width of the channel;
 f : Darcy-Weisbach friction factor;
 Fr : Froude number;
 g : acceleration due to gravity or a constant;
 h : flow depth;
 h_1 : water depth of the triangular bottom section of the channel V-shaped bottom;
 h_f : friction head losses;
 h_D : hydraulic depth ($=A/B$);
 M_1 : water surface profile of a mild slope channel;
 M_2 : water surface profile of a mild slope channel;
 n : Manninig's roughness coefficient;
 Q : flow discharge;
 R : hydraulic radius ($=A/P$);
 s : side channel slope (1:s, vertical: horizontal);
 S_2 : water surface profile of a steep slope channel;
 S_0 : channel bed slope;
 S_e : longitudinal energy slope;
 S_f : friction slope;
 U : mean velocity in the longitudinal direction, x ;
 V : velocity component in transverse direction, y ;
 x : streamwise co-ordinate; gradient of velocity profile; chainage;
 y : transverse co-ordinate; local vertical depth at $x=x$;
 z : vertical co-ordinate and a constant;
 Z_0 : flume channel dimension (see Figure 2a&b);
 Z_s : flume channel dimension (see Figure 2a&b);
 θ : acute angle between bottom parts of channel and horizontal line;
 ϕ : acute angle between bottom parts of channel and vertical line;
 ρ : fluid density;
 τ_b : bed shear stress;

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