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## RESEARCH NOTE

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# GMDH TYPE NEURAL NETWORKS AND THEIR APPLICATION TO THE IDENTIFICATION OF THE INVERSE KINEMATIC EQUATIONS OF ROBOTIC MANIPULATORS

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**Abstract** In this paper, it is shown that group method of data handling (GMDH)-type neural networks and their application can be effectively used to acquire the inverse kinematic equations of a Puma760 robot manipulator based on the numerical data of its motion. The aim of such modeling is to show the accuracy of GMDH-type neural networks. For evaluating the accuracy of the obtained equations, a new trajectory is employed to demonstrate whether the models are still valid or not. Finally, the best results are used to define the inverse kinematic equations.

**Keywords** GMDH, Neural Networks, Inverse Kinematics, Puma760.

**چکیده** در این مقاله به چگونگی استفاده از شبکه های عصبی از نوع GMDH برای شناسایی مشخصات سینماتیکی رباتها اشاره شده است. پس از مدلسازی و ایجاد ساختار شبکه های عصبی از نوع GMDH، که با استفاده از داده های عددی بدست آمده از حرکت ربات ارائه می شود، شناسایی دقیق به انجام رسیده و توانایی پیش بینی برای حرکتها دیگر ربات مورد ارزیابی قرار گرفته است. با توجه به تأیید اعتبار شناسایی انجام شده، از بهترین نتایج برای ارائه معادلات سینماتیک معکوس یک ربات PUMA 760 استفاده شده است.

## 1. INTRODUCTION

The inverse kinematic equations of robotic manipulators are used to identify the angular positions of the links, when the Cartesian coordinate of end-effector is known (Craig 1989). Since identification and modeling of such equations using input-output data is one of the time-consuming and complicated problems in the control of robotic manipulators, several system identification techniques are applied to model the inverse kinematics of robotic manipulators.

Neural networks, which is one of the main components of soft-computing and has shown great ability in solving complex non-linear system identification and control problems, could be effectively used for this purpose.

Among several methodologies that have been expanded, group method of data handling (GMDH) algorithm is a self-organizing approach by which gradually complicated models are generated based on the evaluation of their performances on a set of multi-input single-output data pairs. In this way, GMDH was used to

overcome the difficulty of knowing a priori knowledge of mathematical model of the system. The main idea of GMDH is to build an analytical function in a feedforward network based on a quadratic node transfer function whose coefficients are obtained using regression technique.

## 2. KINEMATICS OF A PUMA760 ROBOT

PUMA760 is a spatial 6 DOF robotic manipulator with all revolute joints (Fu et al. 1990). A schematic diagram of this robot is shown in Figure 1.

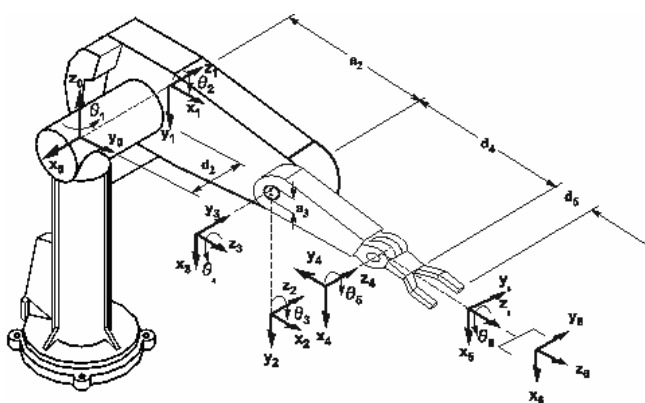


Figure 1. A Schematic Diagram of a Puma760 Robot

In order to obtain the kinematic equations, the Denavit-Hartenberg parameters of this robot are shown in Table 1. Using such parameters, appropriate transfer matrices of links and also the kinematic equations can be found.

Table 1. The Denavit-Hartenberg Parameters

Joint i	$\theta_i$	$\alpha_i$	$a_i$	$D_i$
1	90	-90	0	0
2	0	0	700	300
3	90	90	-33	0
4	0	-90	0	915.1
5	0	90	0	0
6	0	0	0	170

The considered trajectory of a Puma760 is shown in Figure 2.

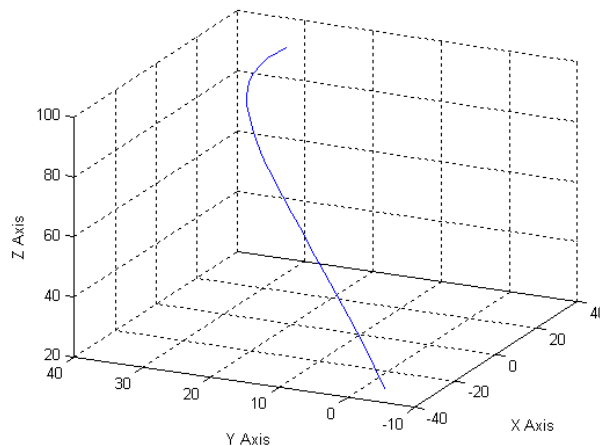


Figure 2. The Considered Trajectory

## 3. USING GMDH TYPE NEURAL NETWORKS IN MODELING

The classical GMDH algorithm can be represented as a set of neurons in which different pairs of them in each layer are connected through a quadratic polynomial and thus produce new neurons in the next layer. Such representation can be used to model the mapping inputs to outputs. The formal definition of the identification problem is to find a function  $\hat{f}$  so that can be approximately used instead of actual one,  $f$ , in order to predict output  $\hat{y}$  for a given input vector  $X=(x_1, x_2, x_3, \dots, x_n)$  as close as possible to its actual output  $y$ . Therefore, given  $M$  samples of multi-input single-output data pairs define the following equations:

$$y_i = f(x_{i1}, x_{i2}, x_{i3}, \dots, x_{in}), \quad i = 1, 2, \dots, M. \quad (1)$$

It is now possible to train a GMDH-type neural network to predict the output values  $\hat{y}_i$ , for any given input vector  $X=(x_{i1}, x_{i2}, x_{i3}, \dots, x_{in})$ . It means,

$$\hat{y}_i = \hat{f}(x_{i1}, x_{i2}, x_{i3}, \dots, x_{in}) \quad i = 1, 2, \dots, M. \quad (2)$$

The problem is now to determine a GMDH-type neural network so that the square of difference between the predicted output and the actual one to be minimized.

$$\sum_{k=1}^M \left[ (\hat{f}(x_{i1}, x_{i2}, x_{i3}, \dots, x_{in}) - y_i)^2 \right] \rightarrow \text{Min} . \quad (3)$$

General connection between inputs and output variables can be expressed by a complicated polynomial of the form

$$\hat{y} = a_0 + \sum_{i=1}^m a_i x_i + \sum_{i=1}^m \sum_{j=1}^m a_{ij} x_i x_j + \sum_{i=1}^m \sum_{j=1}^m \sum_{k=1}^m a_{ijk} x_i x_j x_k + \dots \quad (4)$$

which is known as the Ivakhnenko polynomial (Farlow 1984). However, for most applications, the quadratic form of only two variables is used in the form of

$$\hat{y} = G(x_i, x_j) = a_0 + a_1 x_i + a_2 x_j + a_3 x_i^2 + a_4 x_j^2 + a_5 x_i x_j \quad (5)$$

to predict the output  $y$ . The coefficients  $a_i$  in equation (5) are calculated using regression techniques (Ivakhnenko 1981; Iba et al. 1986) so that the difference between actual output,  $y$ , and the calculated one,  $\hat{y}$ , for each pairs of  $(x_i, x_j)$  as input variables to be minimized. Indeed, it can be seen that a tree of polynomials is constructed using the quadratic form given in equation (5), whose coefficients are obtained in a least-squares sense. In this way, the coefficients of each quadratic function  $G_i$  are obtained to optimally fit the output in the whole set of input-output data pair,

$$r^2 = \frac{\sum_{i=1}^M (y_i - G_i())}{\sum_{i=1}^M y_i^2} \rightarrow \text{Min} \quad (6)$$

In basic form of the GMDH algorithm, all the possibilities of two independent variables out of total  $n$  input variables are taken in order to

construct the regression polynomial in the form of equation (5) that best fits the dependent samples  $(y_i, i = 1, 2, \dots, M)$  in a least-squares sense.

Consequently,  $\binom{n}{2} = n(n-1)/2$  neurons will be built up in the second layer of the feedforward network from the samples  $\{(y_i, x_{ip}, x_{iq}), (i=1, 2, \dots, M)\}$  for different  $p, q \in \{1, 2, \dots, M\}$  (Farlow 1984). In other words, it is now possible to construct  $M$  data triples  $\{(y_i, x_{ip}, x_{iq}), (i=1, 2, \dots, M)\}$  from samples using such  $p, q \in \{1, 2, \dots, M\}$  in the following form:

$$\begin{bmatrix} x_{1p} & x_{1q} & : & y_1 \\ x_{2p} & x_{2q} & : & y_2 \\ \dots & \dots & \dots & \dots \\ x_{Mp} & x_{Mq} & : & y_M \end{bmatrix}$$

Using the quadratic sub-expression in the form of equation (5) for each row of  $M$  data triples, the following matrix equation can be readily obtained as

$$Aa = Y \quad (7)$$

where  $a$  is the vector of unknown coefficients of the quadratic polynomial in equation (5):

$$a = \{a_0, a_1, a_2, a_3, a_4, a_5\} \quad (8)$$

and

$$Y = \{y_1, y_2, y_3, \dots, y_M\}^T \quad (9)$$

is the vector of output values from samples. It can also be readily seen that

$$A = \begin{bmatrix} 1 & x_{1p} & x_{1q} & x_{1p}x_{1q} & x_{1p}^2 & x_{1q}^2 \\ 1 & x_{2p} & x_{2q} & x_{2p}x_{2q} & x_{2p}^2 & x_{2q}^2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & x_{Mp} & x_{Mq} & x_{Mp}x_{Mq} & x_{Mp}^2 & x_{Mq}^2 \end{bmatrix} \quad (10)$$

The least-squares technique from multiple-regression analysis leads to the solution of the normal equations in the form of

$$a = (A^T A)^{-1} A^T Y \quad (11)$$

which determines the vector of the best coefficients of the quadratic equation (5) for the whole set of M data triples. However, such solution directly from normal equations is rather susceptible to round off error and, more importantly, to the singularity of these equations.

#### 4. STRUCTURAL IDENTIFICATION OF GMDH TYPE NEURAL NETWORKS

Three different approaches for structural identification of GMDH-type networks are presented as follows (Nariman-Zadeh et al. 2002):

**Method I:** *Increasing Selection Pressure Approach (ISP)*

In this approach, only one parameter, called selection pressure, is sequentially increased in different layers in order to determine the number of neurons in each layer and also the number of layers in network. The main steps of this approach are described as follows:

**Method II:** *Pre-specified Structural Design Approach (PSD)*

In this approach, the number of layers in the network and also the number of neurons in each layer is pre-specified.

**Method III:** *Error Driven Structural Approach (EDS)*

In this approach, the numbers of layers as well as the number of neurons in each layer are determined according to a threshold error for equation (6). In addition, unlike two previous approaches, some of input variables or generated neurons in different layers can be included in subsequent layers. It is, therefore, evident that the structure of such network may be more complicated than those generated in previous methods.

#### 5. GMDH TYPE NEURAL NETWORKS MODELING OF THE INVERSE KINEMATIC EQUATIONS OF A PUMA760 ROBOT

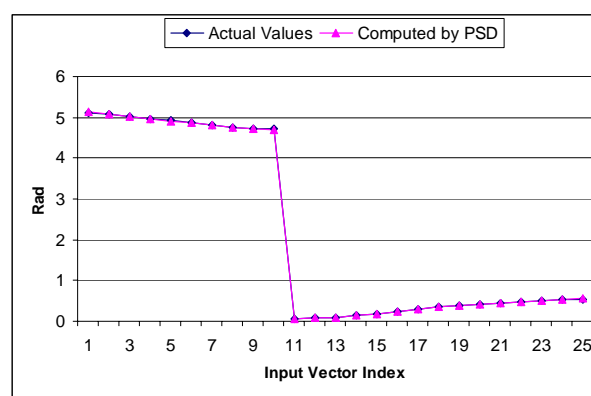
Three methods discussed previously are used to design GMDH-type network systems for a set of actual input–output data in a series of Puma760 trajectory obtained from Roboworks ([www.newtonium.com](http://www.newtonium.com)).

Selected parameters of interest in this multi-input six-output system, which affect the main and local coordinates of gripper, are  $X, Y, Z, T, A, O$ . The  $X, Y$  and  $Z$  are defined as the hand coordinates of the gripper and also  $T, A$  and  $O$  are defined as the Tool, Altitude and Orientation angles of the gripper respectively (Fu et al. 1987).

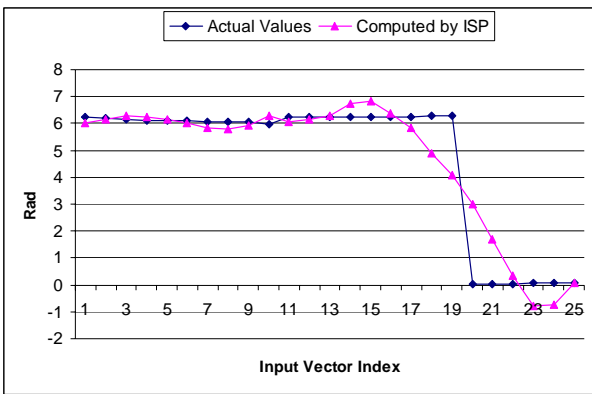
The complete data set consists of a total number of 25 input–output actual data considering different value of inputs and six output parameters.

In order to model these six-input six-output set of data, each of the three methods previously mentioned was used separately in conjunction with singular value decomposition (SVD) approach for the coefficient of the quadratic polynomials. The actual data obtained from the Roboworks constitute six sets of 25 six-input single-output data used independently by these three methods.

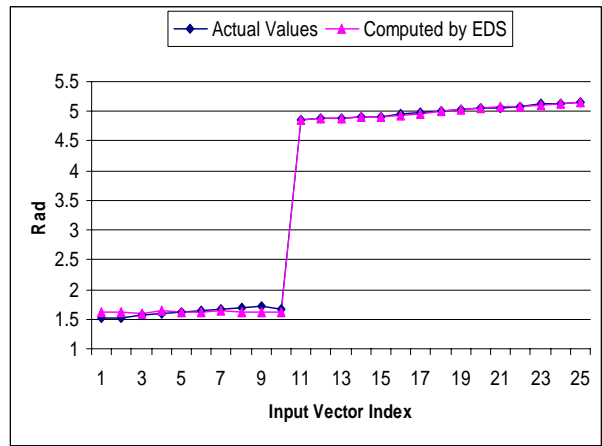
Figures 3-8 show the modeling behaviour of identified networks. Accordingly, figures 9-14 show the structure of identified networks.



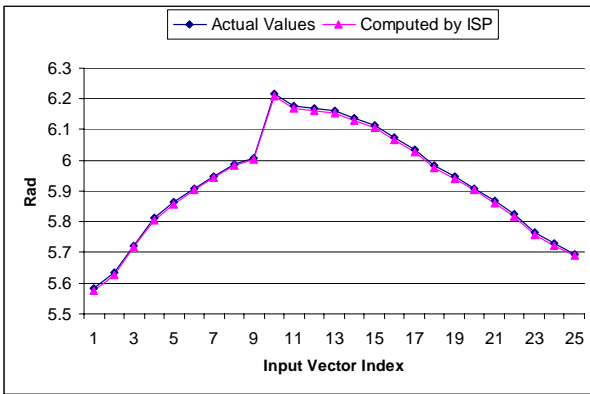
**Figure 3.** Variation of joint 1 angle with input data samples: comparison of actual values with computed values by method II (PSD) – 2 layers



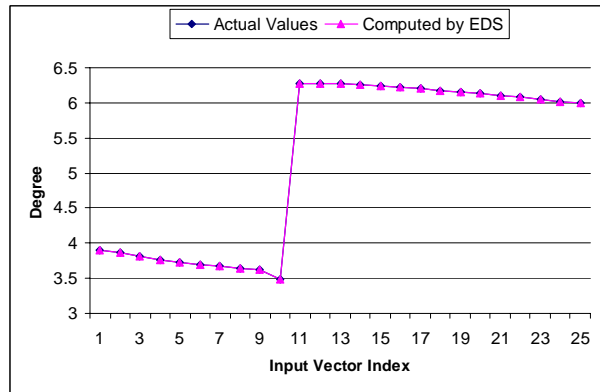
**Figure 4.** Variation of joint 2 angle with input data samples: comparison of actual values with computed values by method I (ISP) – 2 layers



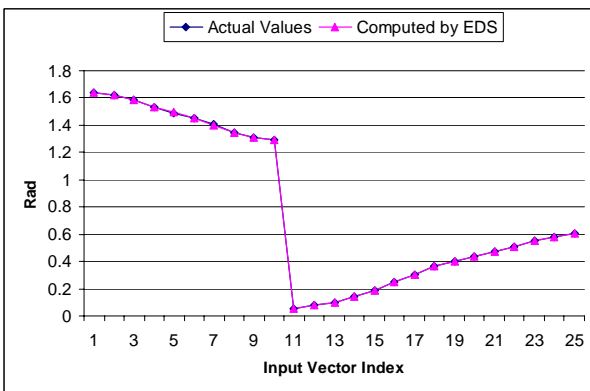
**Figure 7.** Variation of joint 5 angle with input data samples: comparison of experimental values with computed values by method III (EDS) – 4 layers



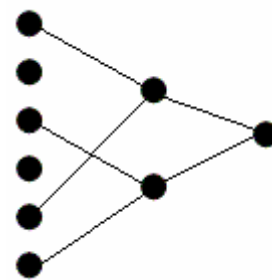
**Figure 5.** Variation of joint 3 angle with input data samples: comparison of experimental values with computed values by method I (ISP) – 2 layers



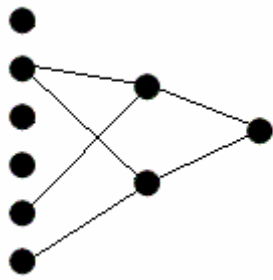
**Figure 8.** Variation of joint 6 angle with input data samples: comparison of experimental values with computed values by method III (EDS) – 4 layers



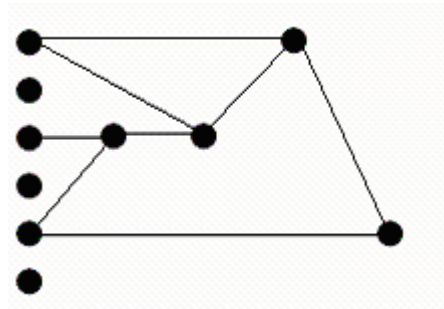
**Figure 6.** Variation of joint 4 angle with input data samples: comparison of experimental values with computed values by method III (EDS) – 4 layers



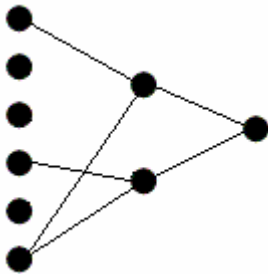
**Figure 9.** GMDH-type network obtained by method II (PSD) for Joint 1



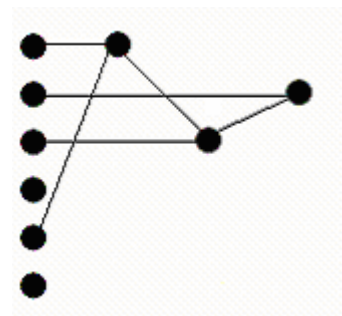
**Figure 10.** GMDH-type network obtained by method I (ISP) for Joint 2



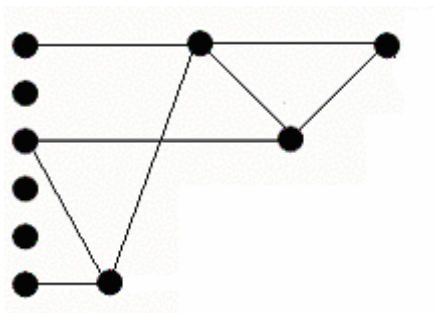
**Figure 13.** GMDH-type network obtained by method III (EDS) for Joint 5



**Figure 11.** GMDH-type network obtained by method I (ISP) for Joint 3



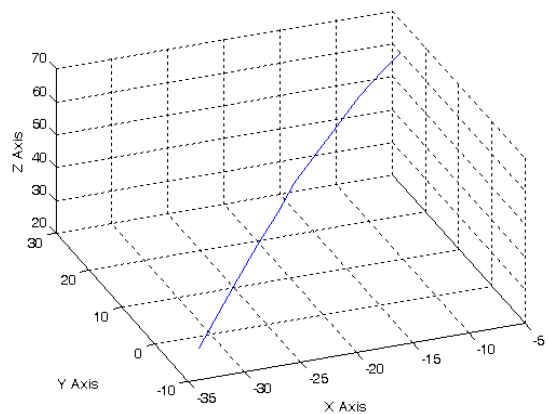
**Figure 14.** GMDH-type network obtained by method III (EDS) for Joint 6



**Figure 12.** GMDH-type network obtained by method III (EDS) for Joint 4

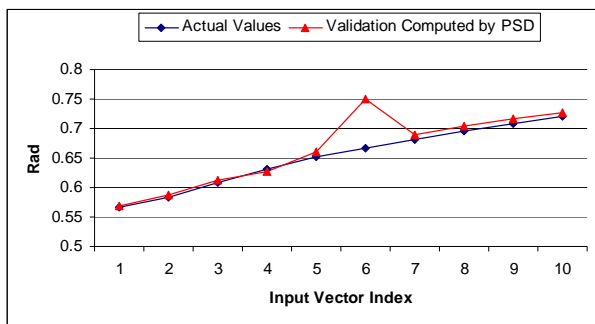
In order to demonstrate the accuracy of such modeling, a new trajectory is employed to demonstrate whether the models are still valid or not.

The new trajectory is shown in Figure 15.



**Figure 15.** The new trajectory of a Puma760 Robot

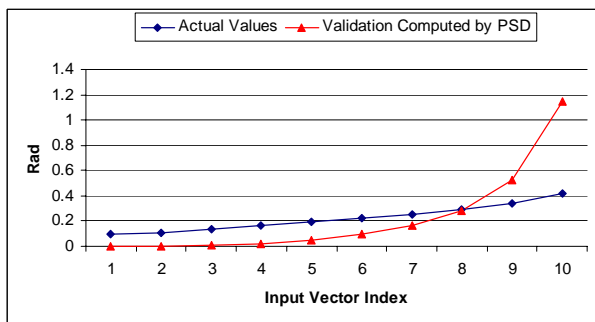
In this way, Figures 16-21 show the validation of the obtained GMDH-type networks by contrast of three different approaches. The best obtained network for each joint is used to define the inverse kinematic equations which are indicated in Equations 12-17.



**Figure 16.** Validation of Joint 2 inverse kinematic equations obtained by PSD – 2 Layers

By comparison of graphs in three different approaches, it is evident that the PSD method reveals the best result. Therefore, the inverse kinematic equations of joint 1 are concluded as follows:

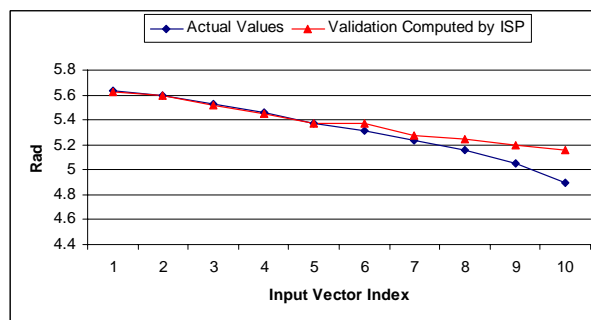
$$\begin{aligned}
 Y &= 0.0003 + 0.0834H_1 + 0.9332H_2 + 16.5379H_1H_2 - 8.1296H_1^2 - 8.4116H_2^2 \\
 ,H_1 &= 0.4989 - 0.0124X - 0.0286A + 0.0003XA + 0.5582A^2 \\
 ,H_2 &= 4.9610 + 0.0097Z + 32.3641O - 0.2847ZO + 0.0001Z^2 + 0.9346O^2
 \end{aligned}
 \tag{12}$$



**Figure 17.** Validation of Joint 2 inverse kinematic equations obtained by ISP and PSD – 2 Layers

By comparison of graphs in three different approaches, it is evident that the both ISP and PSD methods reveal the best result. Therefore, the inverse kinematic equations of joint 2 are shown as follows:

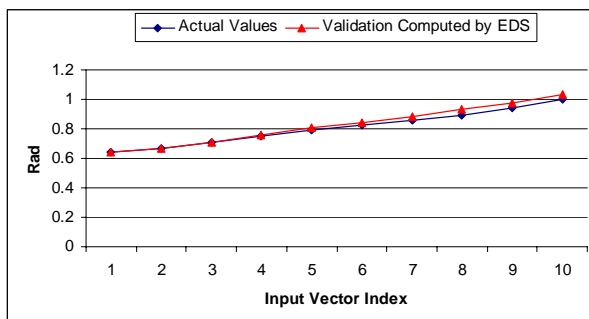
$$\begin{aligned}
 Y &= -0.1817 + 4.9810H_1 - 3.5545H_2 - 0.6043H_1H_2 - 0.0738H_1^2 + 0.6124H_2^2 \\
 ,H_1 &= 68.1428 - 3.0333Y - 0.4215Z - 0.8855YZ + 0.0203Y^2 + 0.0024A^2 \\
 ,H_2 &= 15.7548 - 0.4215Y + 57.4692O - 1.7071YO + 0.0046Y^2 + 2.3713O^2
 \end{aligned}
 \tag{13}$$



**Figure 18.** Validation of Joint 3 inverse kinematic equations obtained by ISP – 2 Layers

By comparison of graphs in three different approaches, it is evident that the ISP method reveals the best result. Therefore, the inverse kinematic equations of joint 3 are shown as follows:

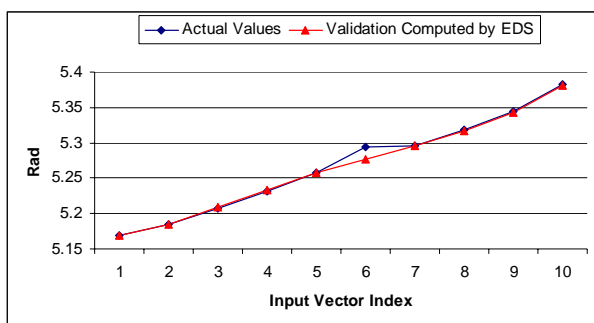
$$\begin{aligned}
 Y &= 1.9209 + 16.2267H_1 - 15.8713H_2 + 1.1267H_1H_2 - 1.8225H_1^2 + 0.7496H_2^2 \\
 ,H_1 &= 5.0068 + 0.0027X - 1.0276O + 0.0330XO + 0.0001X^2 + 0.5252O^2 \\
 ,H_2 &= 5.1970 - 0.0018T - 1.2276O - 0.0015TO + 0.0001T^2 - 0.3842O^2
 \end{aligned}
 \tag{14}$$



**Figure 19.** Validation of Joint 4 inverse kinematic equations obtained by EDS – 4 Layers

By comparison of graphs in three different approaches, it is evident that the EDS method reveals the best result. Therefore, the inverse kinematic equations of joint 4 are shown as follows:

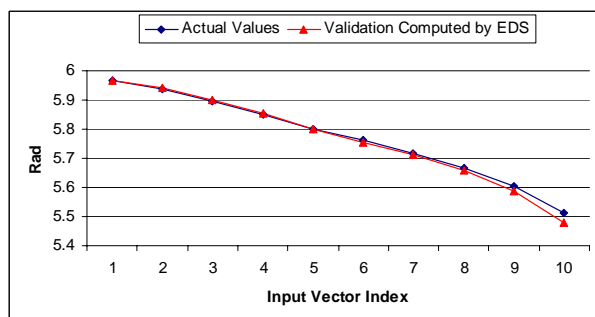
$$\begin{aligned}
 Y &= 0.0004 - 0.0762H_2 + 1.0718H_3 - 275.0498H_2H_3 + 137.6804H_2^2 + 137.3714H_3^2 \\
 ,H_3 &= -0.0528 - 0.0006Z + 1.0180H_2 + 0.0003ZH_2 + 0.0097H_2^2 \\
 ,H_2 &= 0.5430 - 0.0166X - 0.2149H_1 + 0.0108XH_1 + 0.0001X^2 + 0.3953H_1^2 \\
 ,H_1 &= 1.5304 + 0.0076Z + 7.5824O - 0.0582ZO + 0.3991O^2
 \end{aligned}
 \tag{15}$$



**Figure 20.** Validation of Joint 5 inverse kinematic equations obtained by EDS – 4 Layers

By comparison of graphs in three different approaches, it is evident that the EDS method reveals the best result. Therefore, the inverse kinematic equations of joint 5 are shown as follows:

$$\begin{aligned}
 Y &= 0.0697 - 0.0164H_3 + 0.9721A + 0.0096AH_3 - 0.0031H_3^2 + 0.0028A^2 \\
 ,H_3 &= -0.3607 - 0.0109Z + 1.7327H_2 + 0.0012ZH_2 - 0.1296H_2^2 \\
 ,H_2 &= -1.2548 - 0.0056X + 2.4H_1 - 0.0002XH_1 - 0.2260H_1^2 \\
 ,H_1 &= 5.8117 + 0.0048Z + 0.0004O - 0.0001Z^2 - 0.4272O^2
 \end{aligned}
 \tag{16}$$



**Figure 21.** Validation of Joint 6 inverse kinematic equations obtained by EDS – 4 Layers

$$\begin{aligned}
 Y &= 0.9300 + 0.0044H_3 + 0.5690Y - 0.0010YH_3 + 0.0490Y^2 \\
 ,H_3 &= -0.8661 + 0.0231Y + 1.1749H_2 - 0.0025YH_2 - 0.0001Y^2 - 0.0082H_2^2 \\
 ,H_2 &= 1.9655 + 0.0113Z + 0.0782H_1 - 0.0027ZH_1 + 0.1063H_1^2 \\
 ,H_1 &= 6.0544 + 0.0108X + 0.0052A - 0.0001AX - 0.0001X^2 - 0.2331A^2
 \end{aligned}
 \tag{17}$$

It can be seen clearly that the performance of method III in the GMDH-type neural network modeling of the inverse kinematic equations in most cases is superior to those of both methods I and II.

## 6. CONCLUSION

The effectiveness of GMDH-type of neural networks which could model the complex systems without having specific knowledge of the systems, is shown in this paper.

The results presented in this paper clarified that GMDH-type networks can precisely model the inverse kinematic equations of a Puma760 robot manipulator.

In addition, it is clear that this approach can be used for identifying the inverse kinematic equations of the n DOF robotic manipulators. Moreover, it has been shown that SVD can effectively improve the accuracy of such GMDH-type networks which can be constructed by each of the three methods.

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