RESEARCH NOTE

MATHEMATICAL PROGRAMMING APPROCH TO ALLOCATE LOCAL OR NATIONAL RESOURCES FOR BRIDGE MAINTENANCE REHABILITATION AND REPLACEMENT PLANNING

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Abstract Today's, the transportation facilities such as terminals, street, bridge, etc, represent the major investment in highway network. Every year tremendous resources should be invested to maintain these facilities. Among them, the Bridge Management System (B.M.S.) has been necessitated by large imbalance between extensive bridge repair and maintenance needs and limited available budget. So the main purpose of this research study is to develop an optimization methodology to allocate the limited resource among the most bridge maintenance demanded projects. The reduction of user cost or increased the user benefit can be considered as the main objective function of the developed Dynamic Programming Optimization Model. The results of the implementing of optimization model shows a tremendous cost savings, and as a result, more demanded bridge requiring the maintenance can be selected.

Key Words Optimization, Resource Allocation, Bridge Management

چکیده پلها به عنوان مهمترین عناصر راه، وظیفه سنگین اتصال شبکه های درون و برون شهری را به عهده دارند. عملکرد موثر و کارای آنها سبب روانی بیشتر ترافیک، جلوگیری از راه بندان ها و آلودگی هوا، ایمنی و مدیریت مناسب تر حمل و نقل می‌شود. از طرفی به عنوان یکی از مهمترین تسهیلات شهری استراتژیک، بخش عظیمی از بودجه سرمایه‌ای و جاری کشور را بخود اختصاص می‌دهند. مدیریت پل (Bridge Management) امروزه توجه زیادی را به مسائل نگهداری و تعمیرات این سرمایه عظیم کشوری معطوف داشته است. با توجه به منابع محدود مالی، وجود یک برنامه ریزی منظم که بتواند از منابع محدود به بهترین نحو در نگهداری و تعمیر پل ها استفاده کند، ضروری است. تحقیقات انجام شده گسترش مدل برنامه ریزی پویا (دینامیکی) را با توجه به منابع محدود مالی در حالتهای مختلف انجام می‌دهد. کاربرد مدل گسترش داده شده به عنوان یک زیر سیستم بهینه سازی در برنامه های مدیریت پل ها توصیه گردیده است. نتایج حاصله از کاربرد مدل نشان می‌دهد که مدیریت حمل و نقل در کشور با داشتن یک زیر سیتم اطلاعاتی موثر براحتی نمی‌تواند به عنوان یک وسیله موثر و قابل اعتماد از ستادهای مدل در تخصیص بهینه منابع در نگهداری، بازسازی و نوسازی پل ها استفاده نماید.

1. INTRODUCTION

Today's one of the most important issues in transportation management is how to maintain and upgrade the bridge system in the region. The State of Indiana has a large number of bridges that need immediate attention. About 21% of them are functionally obsolete, and about 10% of them were rated as structurally deficient [1]. In 1983 about 58% of North Carolina's bridges were classified as deficient by the sufficiency rating [2]. This is a fact that this tremendous investment is becoming depreciated before their normal lives coming.

Faced with budget constraints and the extensive bridge repair and replacement needs, it is necessary for decision makers to have an efficient tool for selecting and allocating the most effective and efficient bridge alternative maintenance in his

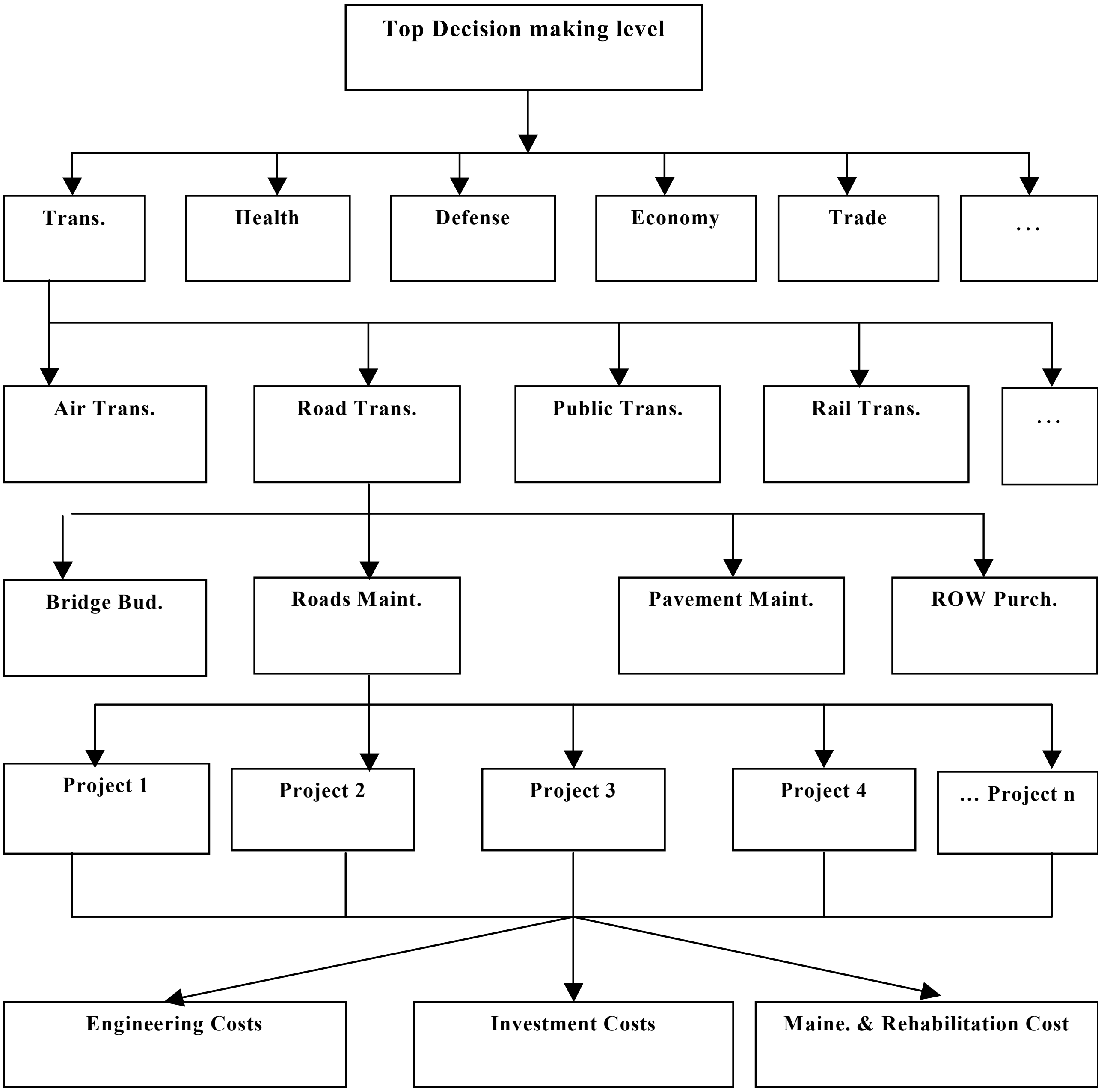

Figure 1. Various level of decision making sectors for resource allocation (top-down or button-up planning).

organization. Unfortunately, the main problem faced by most transportation organization agencies is that the cost of meeting this need is more than the available budgeted funds, and many sound and effective alternative may be omitted or ignored because of the insufficient and limited resources. Also lack of the powerful and effective managerial decision making tools will contribute to this problem and make the decision making process to be very complicated.

The main purpose of this research study is to develop a mathematical decision tool to assist the manager to have an efficient tool for selecting bridge projects among many alternatives so that to maximize the total user benefits. At the network level of decision-making, a comprehensive system would be such a tool for managing a bridge system with thousands of different bridges. So the major objective of a bridge management system is to assist bridge managers in making consistent and cost-effective decision that is related to maintenance, rehabilitation and replacement of bridges on a system wide basis.

Decision-making in transportation project management especially in bridge management system is usually performed at two different levels, i.e., at individual project level and at the network level. In both cases, the user benefits and the governmental costs are the main bases that would be considered for optimization.

2. STUDY BACKGROUND

Harness and Santa [3] developed a priority setting procedure. The procedure selects bridge projects by successively sub setting feasible projects according to a set of criteria. Shirole and Hill [4] developed a system approach for bridge rehabilitation and replacement decision-making. In their investigations, they use the adequacy for future use and the present value of total cost as criteria for decision-making. Johnston and Zia [5] developed a level-of-service system for bridge project evaluation. Load capacity, clear deck width, and vertical roadway under clearance and over clearance were chosen as major factors to measure bridge deficiencies. The State Of Nebraska has also developed a priority-ranking model (6). This model includes four bridge attributes

(a): Single vehicle load capacity;
(b): Clear bridge deck width;
(c): Vertical roadway Over/ Under clearance;
(d): Estimated remaining life.

Lu and Litton [8] developed a strategic planning scheme for a pavement rehabilitation and maintenance management system. A resource allocation problem was formulated in a zero-one integer programming model that maximize the overall effectiveness of all proposed maintenance and rehabilitation activities subject to the some factors such as available supplies, equipment, work force, budget, and minimum distress rating and pavement rating constraints [9,10].

Reviewing the various approaches and methodologies in transportation facility planning, it is clear that most of the models and procedures suffer for a lack of suitable optimization models in which to be able to work at individual project level as well as at system-wide or network level. Actually, presently in the area of management, only techniques of ranking and life cycle analysis bridge management currently being applied. The ranking techniques used in bridge management are deterministic and usually do not yield optimal solutions [11,12].

So the main objective of this study is developing an optimization procedure to allocate the limited resources among different alternative and integrate all activities related to the management of bridge in a comprehensive system or individual project level. The developed methodology enables the decision-maker to apply the proposed optimization model in any level of hierarchy of governmental decision-making. Figure 1 represents the various decision-making levels in which a decision can be made by governmental authority for bridge resource allocation.

3. MULTI-PERIOD OPTIMIZATION PROCEDURE

Despite the fact that the problem of allocating the budget to bridge projects often are not a single period problem, but mostly they should be considered as a multi-period optimization problem. Unfortunately none of the available methods can perform such function. The bridge problems can be approached in two dimensions, namely, the time act dimension and the network dimension.

4. CONVENTIONAL OPTIMIZATION APPROACH

Most of the developed optimization procedures consider the problem of bridge projects as a single objective problem having an objective function as maximizing the user benefits, saved user costs or minimizing the governmental cost and expenditures, lost user or system benefits, and etc. in which are independent to the time and period of resource allocation. One type of this formulation is shown below:

$$\text{MINIMIZE: } Z= \sum_{i}\sum_{j}\sum_{k} (LB)_{ijk} * X_{ijk} \qquad (1)$$

Where:

Z: the total of lost benefit in the system,
N: the total number of bridges,
T: total period of time,
i: the number of alternative improvement, i = 1,2,m,
J: the number of bridge, j = 1,2,n
X_{ijk}: a zero - one variable, it is equal one if alternatives improvement m (k, I) is selected, otherwise is equal zero.

If we consider the total available budget in each period of k for each bridge j is equal to $(TB)_{jk}$ and equation 1the amount of governmental expenditure for building or allocating for improvement i, for bridge j in the period of k is equal to $(C)_{ijk}$, so we can arrange the following constraints:

$$\sum_{j}\sum_{i} (C)_{ijk} * X_{ijk} \leq (TB)_{jk} \qquad (2)$$

Where:

$(C)_{ijk}$: The governmental expenditure to implement the alternative improvement i for bridge j in the period of k;
$(X)_{ijk}$: It is a zero-one variable, it is equal to one, if alternative improvement of m (k,I) for bridge j in the period of k is selected, otherwise is equal to the zero.

Different procedure and methodologies have been developed to evaluate and accessing the bridge qualitative as well as quantitative impacts of alternatives. In this article also a special methodology has been developed to compute and evaluate the selected alternative as an individual or system level, and it is called the "Alternative Economic Assessment Analysis", (AEAA).

5. ALTERNATIVE ECONOMIC ASSESSMENT ANALYSIS (AEAA)

Engineering economy studies directed to public projects almost delineated the benefits to be derived from the project as a first step. This is in contrast to the consideration of profitability as a first step in evaluating an activity of private enterprise. When a public project is considered, the question to be decided is: Will it result in the greatest possible enhancement of the general welfare in terms of economic, social, cultural, or other public satisfactions as judge by the people in the governmental unit concerned? The second step in evaluating a public project involves an analysis of cost to the governmental agency. One common method for determining the cost of constructing and operating a public activity is to call for competitive bids from private organizations. The lowest bid received, after allowances for the bidder's ability to discharge the terms of his contract, is then a measure of cost.

6. USER SAVING COST CALCULATION

The AEAA module has been developed in this study to identify and evaluate any maintenance, improvement program or any other related activities that makes the bridge functional and work at the defined level of service. The concept of governmental economical analysis has been used to analyze the alternative projects. As in the public projects, the benefits to the user or user saving cost have been considered in the AEAA module. The user saving cost can be classified as:

(a) The cost of traffic delay related to the bridge such as bridge construction time, etc.
(b) The cost of travel time,
(c) The cost of environmental pollution resulted from lack of facility,
(d) Fuel and service cost,
(e) Depreciation cost,
(f) Toll road cost,
(g) Any other factors depends on the decision maker's view

All the data and the necessary information should be computed as a " Before-After Analysis". This means that the analyst should provide all the data related to the facility before and after its implementation. In the other word, the quantified difference between these two situations, determine the user benefit for the proposed facility. In the

planning stage, it is very common to predict or for caste the information related to the " After" analysis since we have no real information about the impact of the proposed bridge alternative, so we can estimate it with the help as current information's.
User saving cost = user cost before bridge – user cost after bridge improvement

The user saving cost can be computed for the entire life cycle of the bridge. In AEEA module, for the sake of simplicity, we consider the user benefits as a function of four factors, i.e., time, fuel, depreciation, and accidents, so:

User Saving Cost = F (User Travel Time, Fuel, Depre. Acc. Cost)

The following notations has been used:

∇D: User saving travel time,
∇F: User saving fuel cost,
∇S: User saving Depreciation cost,
∇A: User saving accident cost.
TUj: Total user saving when implementing project j.

$$TU_j = \nabla D_j + \nabla F_j + \nabla S_j + \nabla A_j \quad (3)$$

It is note-worthy that all the above factors should be in a quantitative manner and if some factor is in a qualitative shape, they should be converted to the quantitative one. In this nag ands, so many models and procedures can be found in reference [14].

$$\nabla D_j = (T1j-T2j) * (ADT)_j * C_j * OU_j * 365 \quad (4)$$

$$\nabla F_j = (L1j-L2j) * (ADT)_j * W_j * 365 \quad (5)$$

$$\nabla S_o = (S1j-S2j) * (ADT)_j * 365 \quad (6)$$

$$\nabla A_j = \{(ACC1 - ACC2)_j\} * Ac_j \quad (7)$$

Where:

$T1_j$: Travel time spend before improvement j,
$T2_j$: Travel time spend after improvement j
ADT_j: Average Daily Traffic on bridge j,
C_j: The cost of spending one-hour time,
OU_j: Occupancy rate on bridge j,
$L1_j$: Fuel consumption before improvement j (liter per equivalent car),
$L2_j$: Fuel consumption after improvement j,
W_j: The price of a unit of fuel,
$S1_j$: The cost of depreciation before improvement j,
$S2_j$: The cost of depreciation after improvement j,
$(ACC1)_j$: Equivalent accident frequencies before improvement j,
$(ACC2)_j$: Equivalent accident frequencies after improvement j,
AC_j: Average unit accident cost.

So if the alternative bridge improvement j has been selected, the total annual user benefit can be computed as follow:

$$TU_j = (T1_j-T2_j) * (ADT)_j * 365 * C_j * OU_j + (L1_j-L2_j) * (ADT)_j * 365 * W_j + (S1_j-S2_j) * (ADT)_j * 365 + (ACC1-ACC2)_j * AC_j \quad (8)$$

Simplified the model as:

$$TU_j = [(ADT)_j * 365]\{[(T1_j-T2_j) * C_j * OU_j] + [(L1_j + L2_j) * W_j] + [(S1_j-S2_j)] + [(ACC-ACC1)_j * AC_j]\} \quad (9)$$

7. GOVERNMENTAL COST ANALYSIS

After evaluating the user benefit, then, we should compute the governmental cost for implementing the bridge improvement alternatives. It is very usual to consider three parts of cost elements, i.e., investment cost, maintenance cost, and salvage value especially when we are considering the bridge rehabilitation or replacement program. The time value of money has also been considered in our calculations. As the user benefit is calculated annually, so the governmental cost should also be converted to the annual bases. The following notation has been used:

TC_j: The total present worth value of bridge j,
IC_j: The cost of investment for bridge j,
OM_j: The annual operating maintenance for bridge j,

S_j:	The salvage or residual value of bridge j at the end its life cycle.
(p/a, i%, n):	The present worth factor of annual operating maintenance with i% interest rate for n years
(p/f, i%, n):	The present worth factor of future salvage value considering with i% interest rate for n years
ATC_j	Annual total cost of bridge j

So the total present value of a bridge during its entire life can be calculated as:

$$TC_j = IC_j + OM_j\ (p/a, i\%, n) - S_j\ (p/f, i\%, n) \quad (10)$$

If we convert the above equation as an annual uniform equivalent, we can multiply it by capital recovery factor as:

$$ATC_j = TC_j\ (a/p, i\%, n) \quad (11)$$

In which ATC_j is the annual equivalent cost of alternative j that can be paid by the government or transportation agency, n and i also are the bridge life and the minimum rate of return in which the transportation agency is willing to invest.

As it was mentioned, the information related to the computation of TU_j and ATC_j can be calculated by AEAA module and are used in the optimization model as an input. There are some other procedures related to the calculation of user benefit or user saving cost in which some other factors were considered in the model. Factors such as capacity, traffic distribution, bridge inefficiency rating, vertical alignments and physical conditions have been considered in those studies [7-9].

8. DYNAMIC PROGRAMMING OPTIMIZATION PROCEDURE DEVELOPMENT FOR MULTI-PERIODS RESOURCE ALLOCATION

Dynamic programming (DP) is a recursive optimization procedure popularized by Richard Bellman [13]. DP. Breaks down an optimization problem in N decision variables into a series of N independent single variable optimization. It is based upon what is known as the principle of optimality.

The optimal set of decisions in sequence decision process has the property that whatever the initial budget level, decision point, and decisions are up to that point, the remaining decision constitutes an optimal sequence of decisions for the remaining problem. An important advantage of DP is that it determines absolute (global) maximal or rather than relative (local) optima. It can easily handle integrality and non-negativity of decision variables. Furthermore, the principle of optimality assures that DP results in not only the optimal solution of a problem, but also the optimal solutions of sub-problem. This is a fact that has been considerably viewed in this research to allocate the optimal resources during the planning period. For example, for a 5-years period planning, DP gives the optimal project selections for the entire 5-year development program as well as the optimal project selections for any period less than 5 years. These optimal solutions of the sub-periods are often of interest to bridge programmers. In virtually all other optimization techniques, certain kinds of constraints can cause significant problems. In the other words, the imposition of integrality on the variables of a problem will destroy the utility of these computational methods, however, in DP the requirement that some or all of the variables be integers greatly simplifies the computation process. Similar considerations apply to such restriction as non-negativity of decision variables [14].

The DP divides the federal and state (general or local budgets) of each year into several possible spending portions, and chooses the optimal spending policy, which maximizes the system user saving costs, or minimizes the total system lost benefits.

DP Formulation For the sake of simplicity, define the following notation consistent with conventional DP terminology:

Fi(di):	The user saving resulted from decision d_i in the location i
Xd_i:	The cost of making decision d_i in the location I,

$D_i = \{ a_i, b_i, c_i, \ldots\}$. The set of alternatives, A_i, b_i, c_i,… are the alternatives improvement in location i. So the objective function can be written as:

TABLE 1. The Output of AEEA Module, which Can Be Considered as Data Input to DP.

TYPE OF BRIDGE	ALTERNATIVE IMPROVEMENT	USER SAVING COST $ (1000)	GOVRNMENTAL COST $ (1000)
RIGIDE FRAME	a1: PAVED SHOULDER b1: UP-GRADE SAFETY c1:BRIDGE TRAFFIC MANAGEMENT d1:PAVEMENT REPLACEMENT	30,000. 50,000. 45,000. 50,000.	2000. 4000. 3500. 650.
CONTINUOS WITH SUSPENDED SPAN	a2: GENERAL MAINTENANCE b2: WATERWAY IMPROVEMENT	20,000. 30,000.	4000. 2000.
STEEL BRIDGE	a3: MEDIAN BARRIER IMPROVEMENT b3: TRUSSES IMPROVEMENT	5,000. 15,000.	1000. 3000.

TABLE 2. Result of Budget Allocation in Stage One.

B_1	a_1	b_1	c_1	d_1	N_1	D_1*	$V_1(B_1)$
0.	-	-	-	-	0.	N1	0.
1000.	-	-	-	-	0.	N1	0.
2000.	30,000.	-	-	-	0.	A1	30,000.
3000.	30,000.	-	-	-	0.	A1	30,000.
4000.	30,000.	50,000.	45,000.	-	0.	B1	50,000.
5000.	30,000.	50,000.	45,000.	-	0.	B1	50,000.

Maximise : $Z = \sum Fi(d_i)$ (12)

Subject to:

$\sum Xd_i < TB$ (13)

TB is the total available budget.

In the formulation of DP the total budget can be divided into n increments, and is called "state", also every location of bridge is viewed as a "stage", so if we consider the Vi (Bi) as a maximum of user benefit as a function of allocated budget B_i in the stage of i, the recursive equation can be written as:

$$V_i (B_i) = MAX.. \{F_i(d_i) + V_{i-1} (B_{i-1} = B_i - X_{di})\} \quad (14)$$

The principle of optimality is best explained through use of a small case study. Three locations with three types of bridge have been considered. Each location or bridge type has the number of

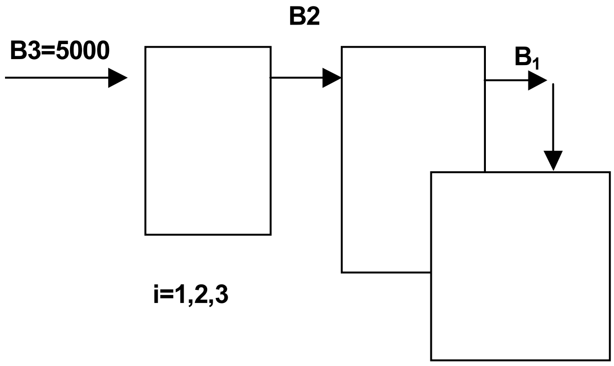

Figure 2. The Concept of Recursive Approach Process in DP, as in the Example problem.

alternative improvements, which should be investigated. For each bridge, the AEEA module calculates the user saving costs and the governmental expenditure for all related alternatives. For the following year, the government just allocates about $5000,000, for bridge maintenance, repair, and other related activities in which makes the indicated bridge to effectively functioning. The following information in Table 1 has been provided for input use in the DP model.

A full description for stage 1 to stage 3 can be found in reference [15] and as shown in Figure 2, the recursive approach can be written as follows:

Stage 1

$$V_1(B_1) = MAX. \{F_1(d_1)\} \quad (15)$$

$$D_1 = \{a_1,b_1,c_1,d_1\} \quad (16)$$

The state of the system is the available budget levels, so:

$$D_1 = \{a_1,b_1,c_1,d_1\} \quad (17)$$

Also the budget of 5000 can be divided into the five increments of size $ 1000 as shown in Table 2. In all the stages, there in one alternative named "do-nothing" in which it is shown by N_i, this alternative means that keep the present situation and take no alternative improvement for the indicated bridge. Tables 3 and 4 show the results of budget allocations in stage two and three respectively.

Stage2

$$V_2 (B_2) = MAX. \{F_2(d_2) + V_1 (B_2\text{-}Xd_2)] \quad (18)$$

$$d_2 = \{a_2, b_2\} \quad (19)$$

Stage 3

$$V_3 (B_3 = 5000) = MAX. \{F3 (d_3) + V2 (5000 – Xd_3)\} \quad (20)$$

So, with regards to the DP calculation, a different bridge type can be obtained as optimum resource allocation for the three types shown in Table 5.

9. MODEL IMPLEMENTATION

The AEEA module utilizes the optimization model with the necessary data and information's. The row data input also can be extracted from the available database systems or from the Area Bridge Inventory and Inspection System for AEEA module. Figure 3 demonstrates how the proposed optimization model for a multi-period planning can allocate the limited resources among different bridge improvement alternatives in a multi-period planning situation.

10. CONCLUSION

The development of bridge management system is a recent initiative compared to the current age of our nation's bridge inventory. Bridges improvement projects as a most strategic maintenance activity has shown a tremendous user cost savings, environmental effects, traffic smoothness, social and economical development and many other impacts, which makes the bridge management program as an important issues in today's limited resources.

So the main objective of this research study was to provide an effective and efficient managerial decision tools to assist him to allocate optimally

TABLE 3. The Result of Budget Allocation in Stage Two.

B_2	A_2	b_2	N_2	D_1*	$V_1(B_1)$
0.	-	-	0.	N2	0.
1000.	-	-	0.	N2	0.
2000.	-	30,000	30,000	b2 or N2	30,000.
3000.	-	30,000.	30,000.	b2 or N2	30,000.
4000.	20,000.	60,000.	50,000.	b2	60,000.
5000.	20,000.	60,000.	50,000.	b2	60,000.

TABLE 4. The Result of Budget Allocation in Stage Three.

B_3	a_3	b_3	N_3	D_3*	$V_3(B_3)$
5000.	65,000.	45,000.	60,000.	A3	65,000.

TABLE 5. The Final Result of DP Optimization Approach.

Type of bridge	Allocated budget	Selected Alternative	Cumulative User Benefits
Rigid Frame	2,000,000.	a_1	30,000,000
Continues With Suspended Span	2,000,000.	B_2	60,000,000.
Steel Bridge	1,000,000.	a_3	65,000,000.

the limited resources among the most cost effective bridge improvement alternatives.

In any bridge improvement program, so many decision variables are involved. It is a fact that, the bridge problems have two dimensions; the time and network dimensions. This complexity makes these problems very difficult and a very time consuming problem in the area of transportation facility assessment. For these reasons the concept of D.P. has been widely considered in this research study.

The concept of Dynamic Programming approach has been used and developed to allocate the limited resources among different bridge improvement projects, especially when there is a multi period situation has been involved.

The development of the proposed optimization approach provide a number of significant contributions towards enhancing the bridge maintenance decision process, namely; (a) the model provide a framework for incorporating key safety and serviceability into the different alternative bridge improvement, (b) the AEAA module provides a user a functional framework to define, analyze, and assess the bridge improvement alternative in a very easy and applicability manner, (c) the optimization procedure provides the decision maker to allocate in the different level of authority to allocate optimally the

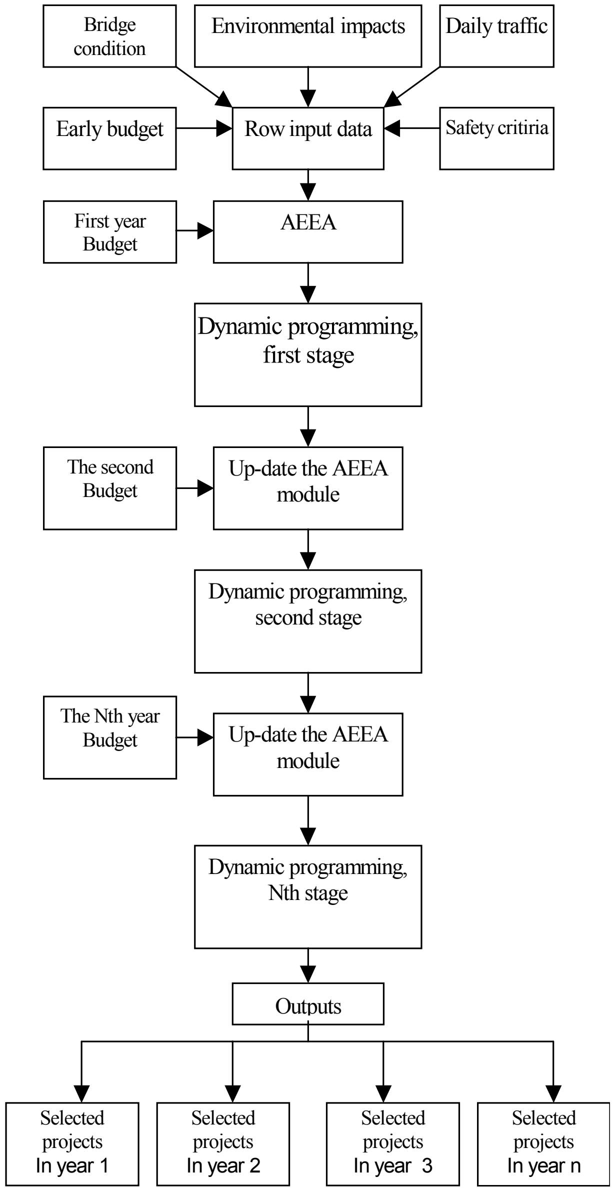

Figure 3. the model implementation procedure.

scare funds among different effective and efficient alternatives.

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